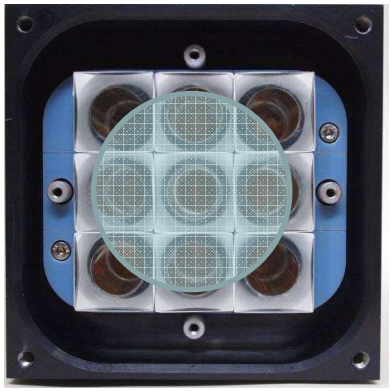
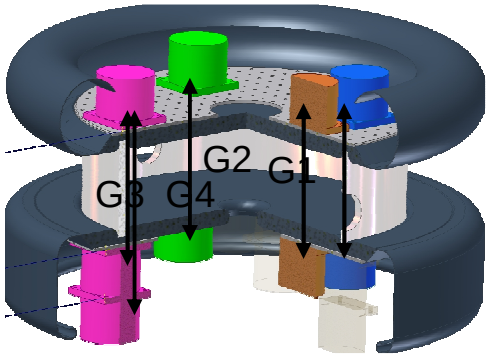
The Hg comagnetometer and the geometric phase induced false nEDM

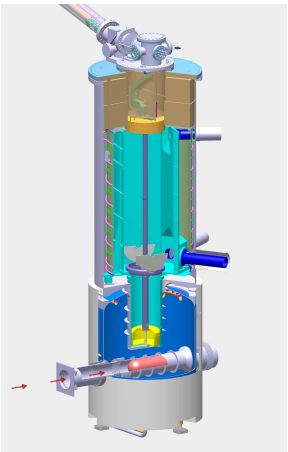
Dominique Rebreyend

nEDM workshop

Oak Ridge, 11-13/10/2012



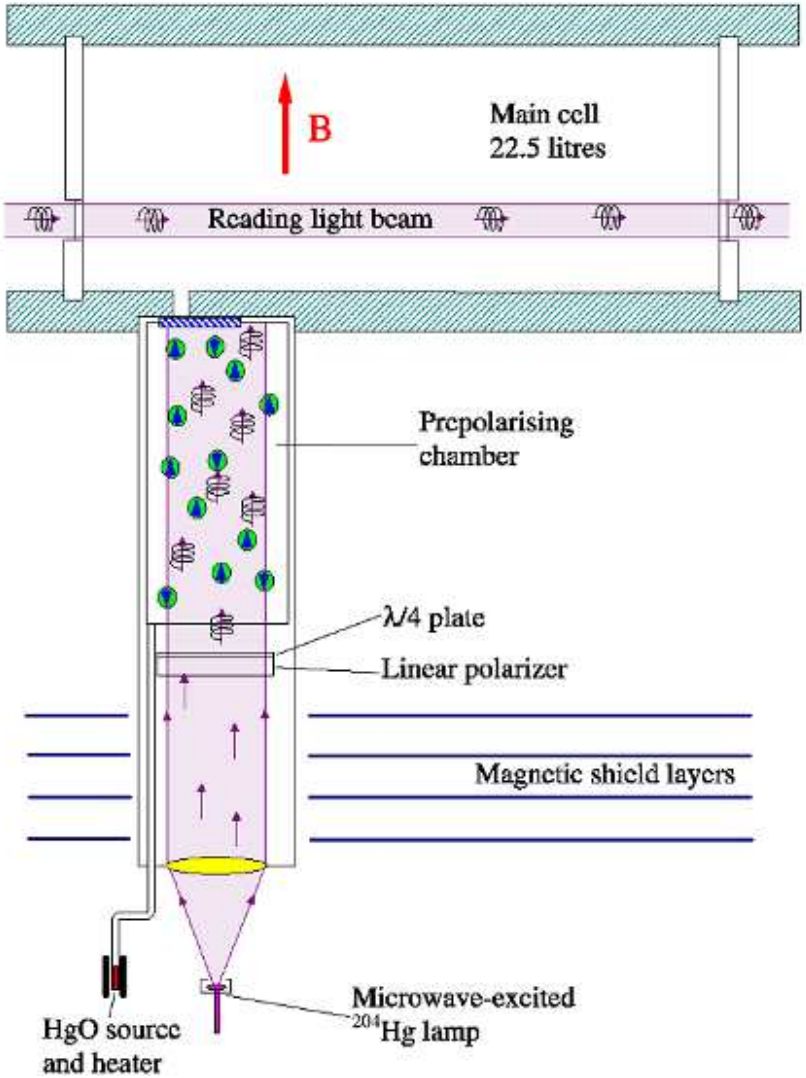


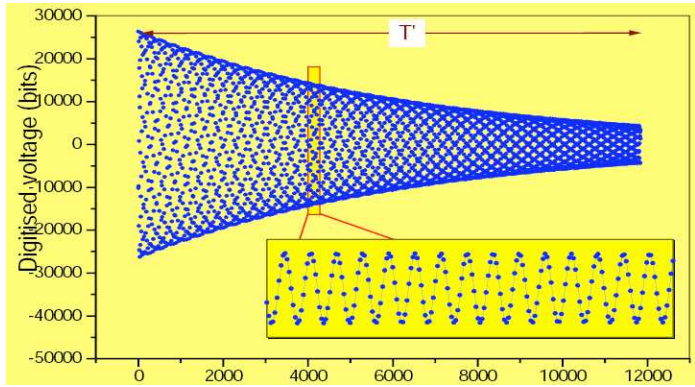


Outline

- The Hg co-magnetometer
- Geometric phase issue
- Measurement of Hg GPE
- An improved theory to calculate GPE

The Hg co-magnetometer: an ideal tool to compensate for B drift



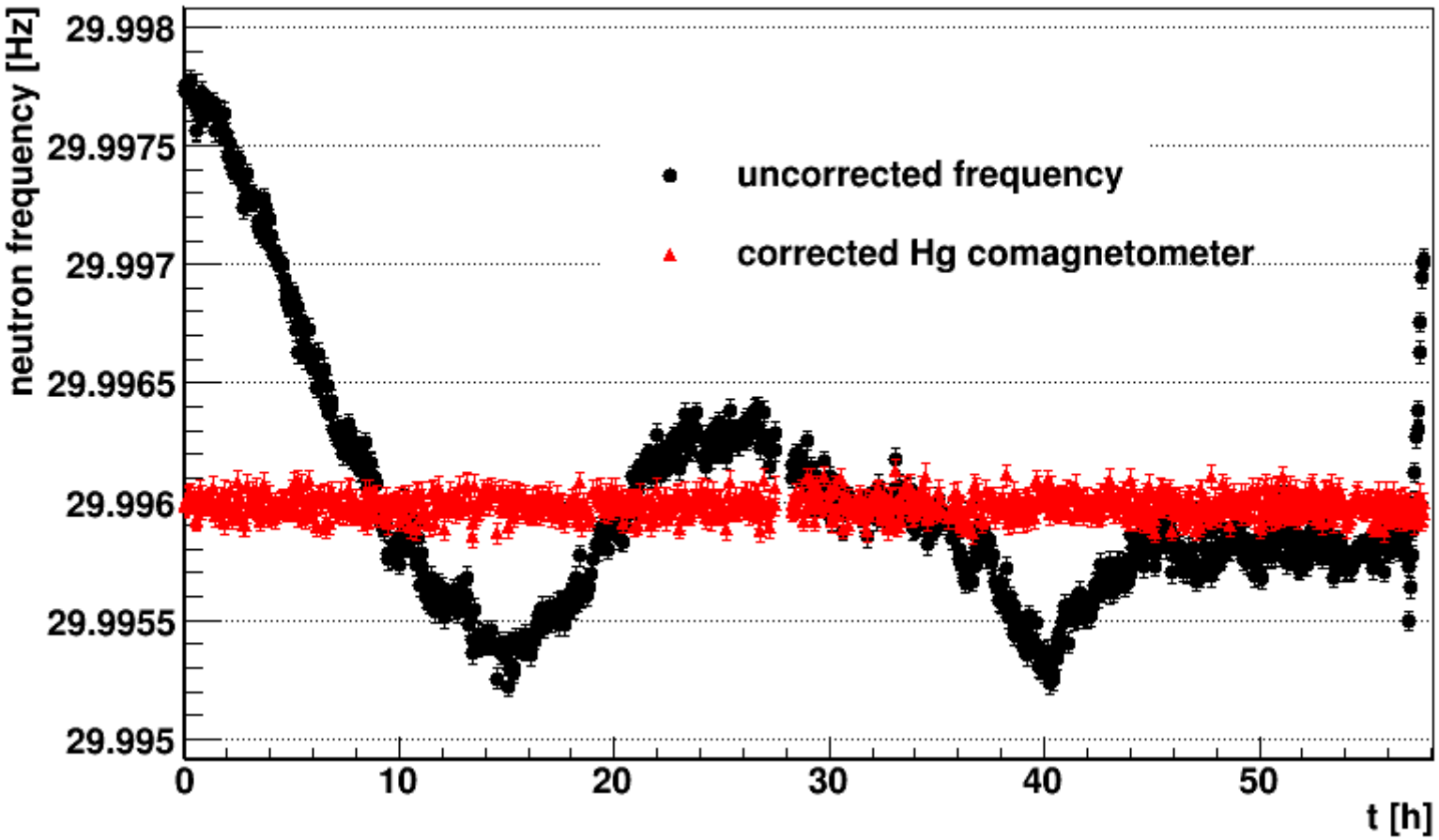


The Hg co-magnetometer measures the same *volume* and *time*-averaged magnetic field as seen by the neutron.

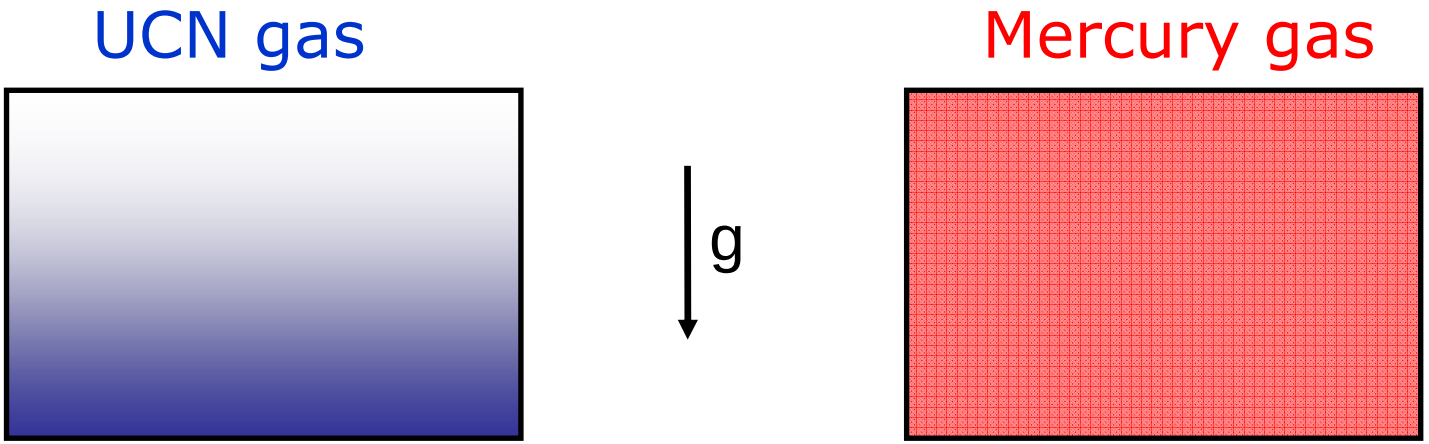
$$R = \frac{f_{\mathrm{n}}}{f_{\mathrm{Hg}}} = \frac{\gamma_{\mathrm{n}}}{\gamma_{\mathrm{Hg}}}$$

is independent of magnetic field fluctuations.

Correction of B-field drift with the Hg-comagnetometer



Ideal…but not perfect: the gravitational effect

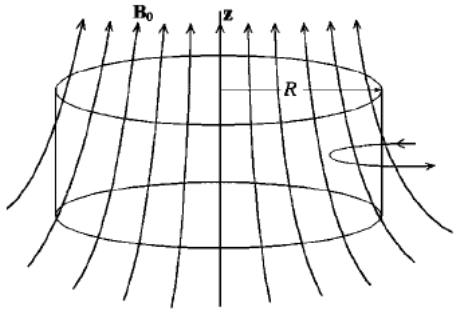


Center of gravity height difference: $\Delta h \approx 2.5\,\mathrm{mm}$

$$R = \frac{f_{\mathrm{n}}}{f_{\mathrm{Hg}}} = \frac{\gamma_{\mathrm{n}}}{\gamma_{\mathrm{Hg}}}\left(1 + \frac{\Delta h}{B_0}\frac{\partial B}{\partial z}\right)$$

The geometric phase frequency shifts

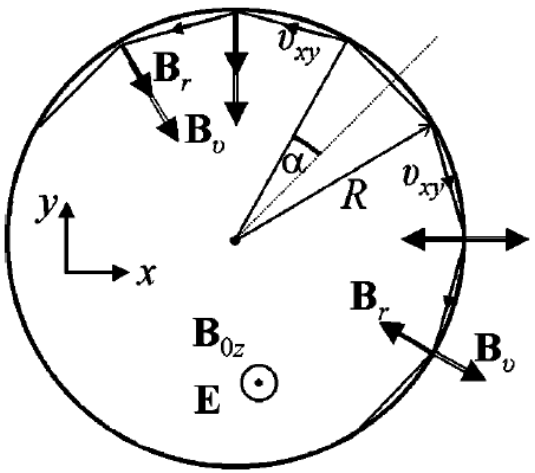
Transverse magnetic field



+

Motional (transverse) field

$$B = \frac{1}{c^2}\,\mathbf{v}\times\mathbf{E}$$



Ramsey-Bloch-Siegert shift due to transverse rotating field:

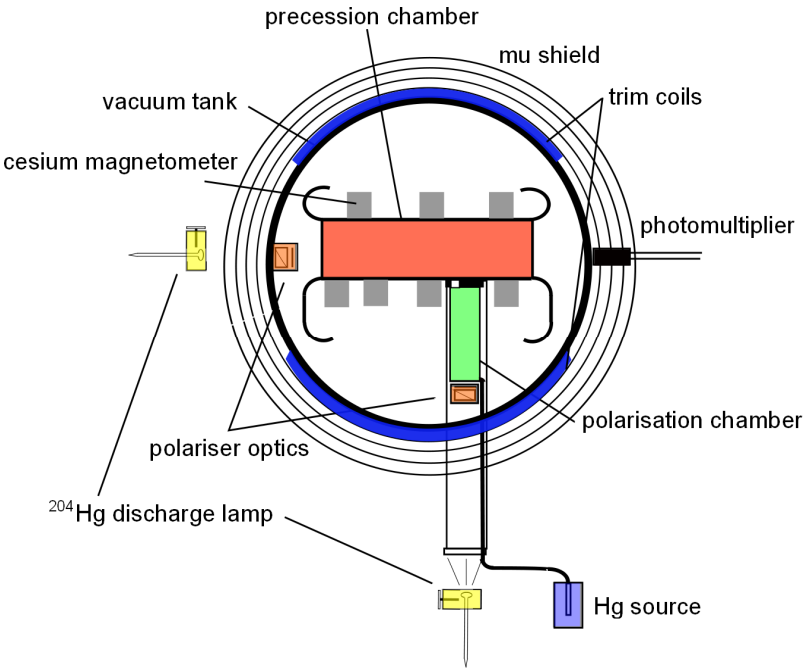
$$\delta\omega = \delta\omega_{B^2} + \delta\omega_{E^2} + \delta\omega_{BE}$$

False n/Hg EDM

$$d_{\mathrm{Hg}}^{\mathrm{False}} = -\frac{\hbar\,\gamma_{\mathrm{Hg}}^2}{32\,c^2}\,D^2\,\frac{\partial B_z}{\partial z} \quad \Rightarrow \quad d_{\mathrm{n}}^{\mathrm{False}} = \frac{\gamma_{\mathrm{n}}}{\gamma_{\mathrm{Hg}}}\,d_{\mathrm{Hg}}^{\mathrm{False}}$$

Expression valid for uniform gradient with cylindrical symmetry.
(Pendlebury et al, PRA **70** 032102 (2004))

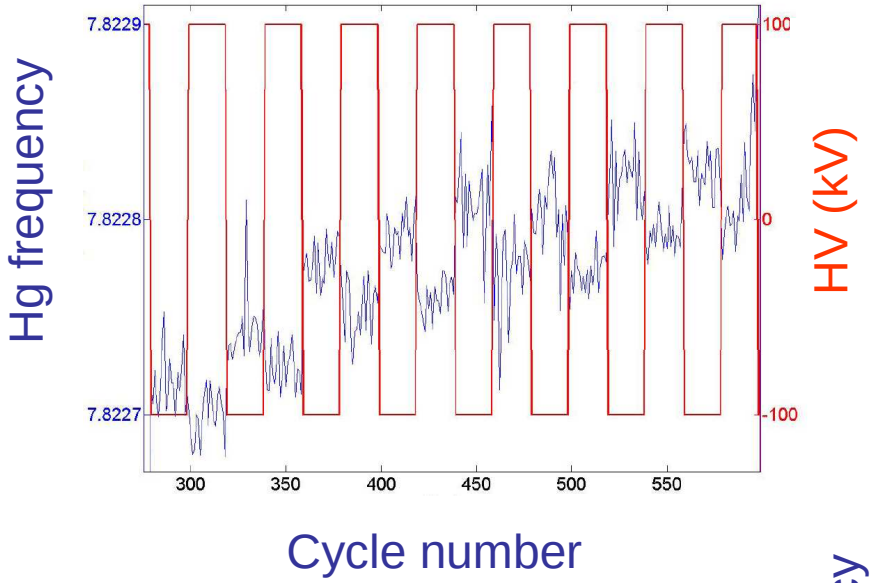
Direct measurement of GPE: experimental setup



- Vertical gradients obtained via:
  1. Cs magnetometers:
     $G = (\langle CsM_{up} \rangle - \langle CsM_{down} \rangle)/23\,cm$
  2. Field maps:
     All trim coils (30) mapped during dedicated campaign with fluxgates
- Gradients varied using 2 saddle trim coils
- E = 100 kV/12 cm
- Change HV polarity every 20 cycles

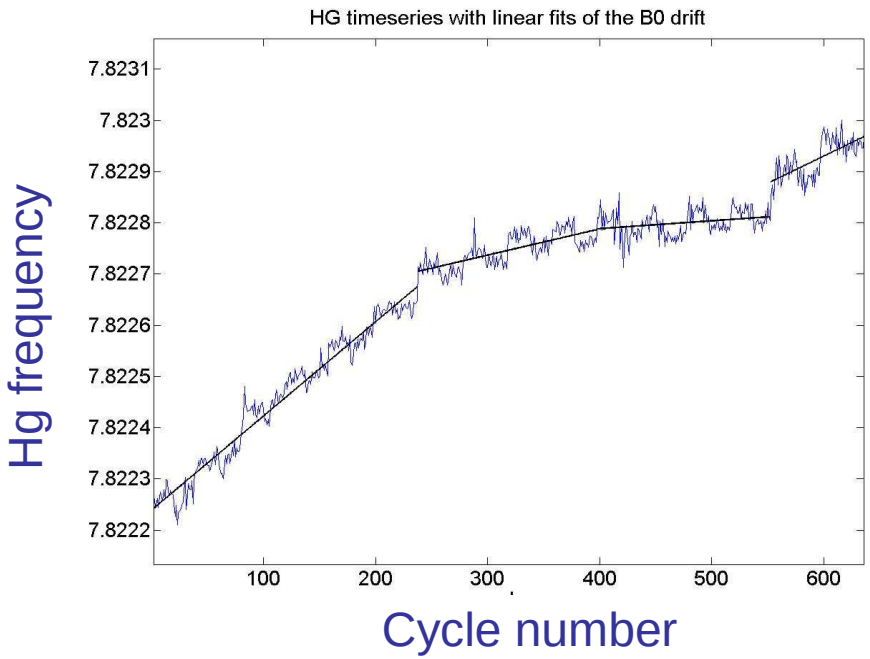
Marlon Horras's PhD thesis (July 2012)

Frequency shift extraction



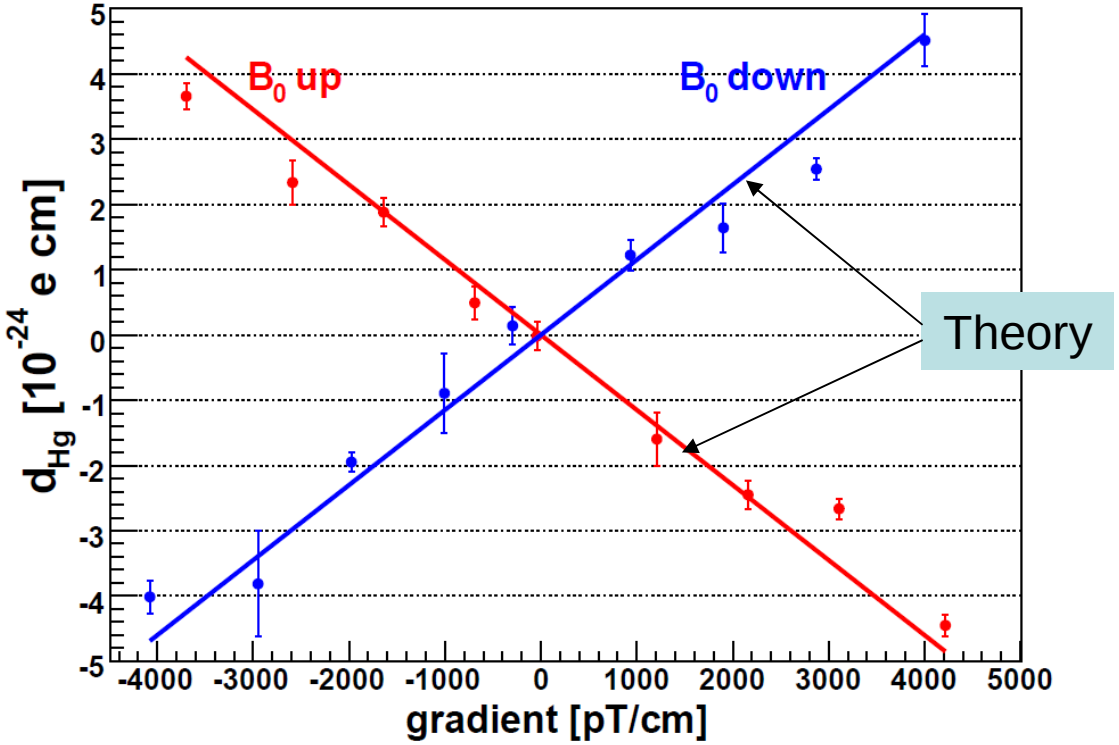
Frequency shifts associated to HV polarity changes clearly visible.

Long term magnetic field drift corrected by linear fit over selected periods.



Frequency shift extracted from residuals

Measurement of GPE:
1) gradients extracted from **Cs-OPM**



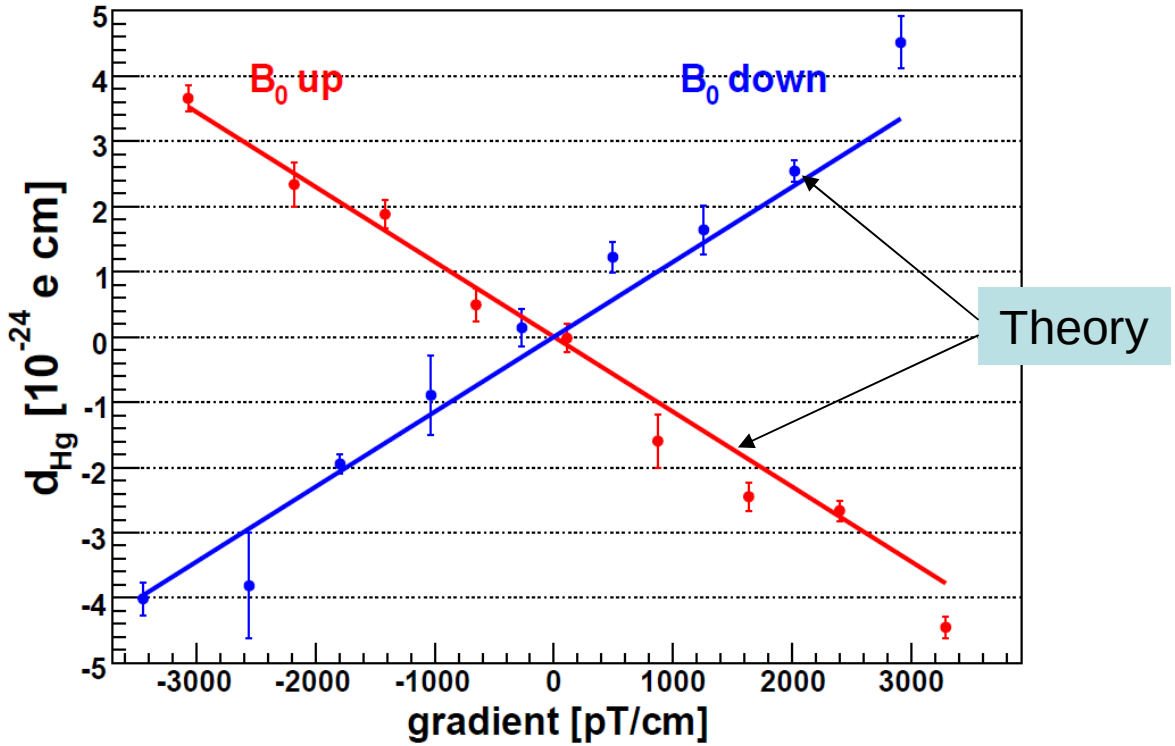
Theory : $\frac{\hbar\gamma_{\mathrm{Hg}}^2}{32c^2}D^2 = 1.15\times10^{-27} e\,\mathrm{cm}/(\mathrm{pT}/\mathrm{cm})$

Fitted slope: $(0.98\pm0.03)\times10^{-27} e\,\mathrm{cm}/(\mathrm{pT}/\mathrm{cm})$

$\chi^2/N_{\mathrm{dof}} = 38/17$

15 % agreement !

Measurement of GPE:
2) gradients extracted from **field maps**



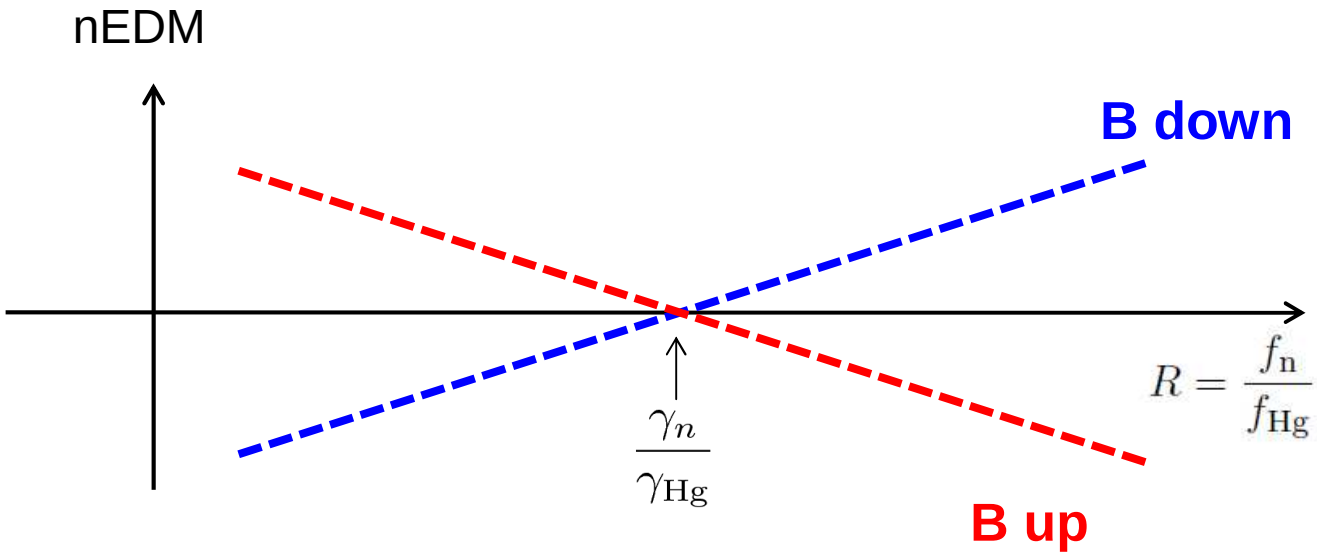
Theory : $\frac{\hbar\gamma_{\mathrm{Hg}}^2}{32c^2}D^2 = 1.15 \times 10^{-27} e\,\mathrm{cm}/(\mathrm{pT}/\mathrm{cm})$

Fitted slope: $(1.24 \pm 0.03) \times 10^{-27} e\,\mathrm{cm}/(\mathrm{pT}/\mathrm{cm})$

$\chi^2/N_{dof} = 26/17$

7 % agreement !

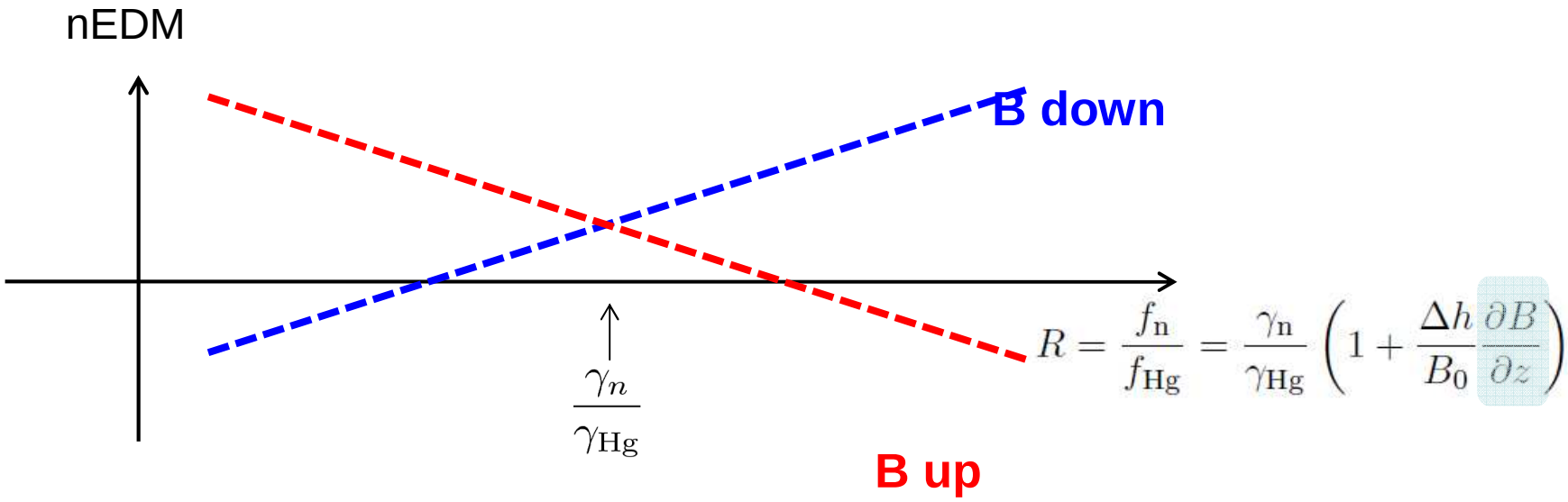
RAL-Sussex strategy to eliminate GPE: the crossing-point analysis



$$R = \frac{f_{\mathrm{n}}}{f_{\mathrm{Hg}}} = \frac{\gamma_{\mathrm{n}}}{\gamma_{\mathrm{Hg}}}\left(1 + \frac{\Delta h}{B_0}\frac{\partial B}{\partial z}\right)$$

$$d_{\mathrm{n}}^{\mathrm{False}} = \frac{\hbar \gamma_{\mathrm{n}} \gamma_{\mathrm{Hg}}}{32 c^2} D^2 \frac{\partial B_z}{\partial z}$$

RAL-Sussex strategy to eliminate GPE: the crossing-point analysis



$$d_{\mathrm{n}}^{\mathrm{False}} = \frac{\hbar \gamma_{\mathrm{n}} \gamma_{\mathrm{Hg}}}{32c^2} D^2 \frac{\partial B_z}{\partial z} (1 + E)$$

Enhancement factor

Strategy inappropriate for general case, in particular for *local dipoles* → Need for an improved theory

Redfield theory of relaxation (statistical theory)

Lamoreaux&Golub, PRA **71** 032104 (2005)

$$\delta\omega = \frac{1}{2}\int_0^\infty d\tau \cos(\omega_0\tau)\ \langle \omega_x(0)\omega_y(\tau) - \omega_y(0)\omega_x(\tau)\rangle$$
$$+\frac{1}{2}\int_0^\infty d\tau \sin(\omega_0\tau)\ \langle \omega_x(0)\omega_x(\tau) + \omega_y(0)\omega_y(\tau)\rangle.$$

ensemble average

$$\omega_x = \gamma B_x - \frac{\gamma}{c^2} v_y E$$

$$\omega_y = \gamma B_y - \frac{\gamma}{c^2} v_x E$$

$$\delta\omega = \delta\omega_{B^2} + \delta\omega_{E^2} + \delta\omega_{BE}$$

In the non adiabatic regime

$$d^{Hg}_{False} = -\frac{\hbar\gamma^2_{Hg}}{2\,c^2}\ \langle x B_x + y B_y\rangle$$

volume average

Pignol&Roccia, Phys. Rev. A 85, 042105 (2012)

Case 1: large scale inhomogeneities

General expression for a magnetic field with first order inhomogeneities (5 parameters):

$$
\begin{aligned}
B_x &= G_x\,x + Q_y\,z + Q_z\,y \\
B_y &= G_y\,y + Q_x\,z + Q_z\,x \\
B_z &= B_0 - (G_x + G_y)z + Q_x\,y + Q_y\,x
\end{aligned}
$$

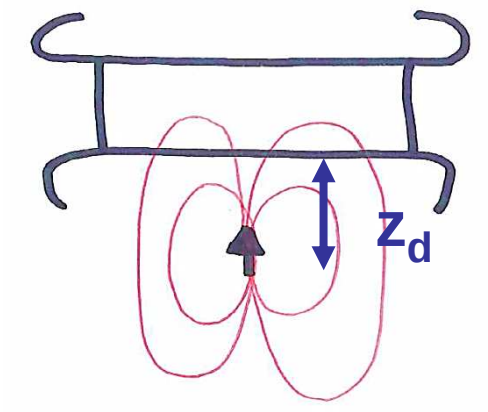
$$
d_{\mathrm{Hg}}^{\mathrm{False}} = -\frac{\hbar\,\gamma_{\mathrm{Hg}}^2}{32\,c^2}\,D^2\,(G_x + G_y)
$$

(Trap with cylindrical symmetry)

$$
= \frac{\hbar\,\gamma_{\mathrm{Hg}}^2}{32\,c^2}\,D^2\,\left\langle \frac{\partial B_z}{\partial z} \right\rangle
$$

Same expression as for uniform gradient with cylindrical symmetry.

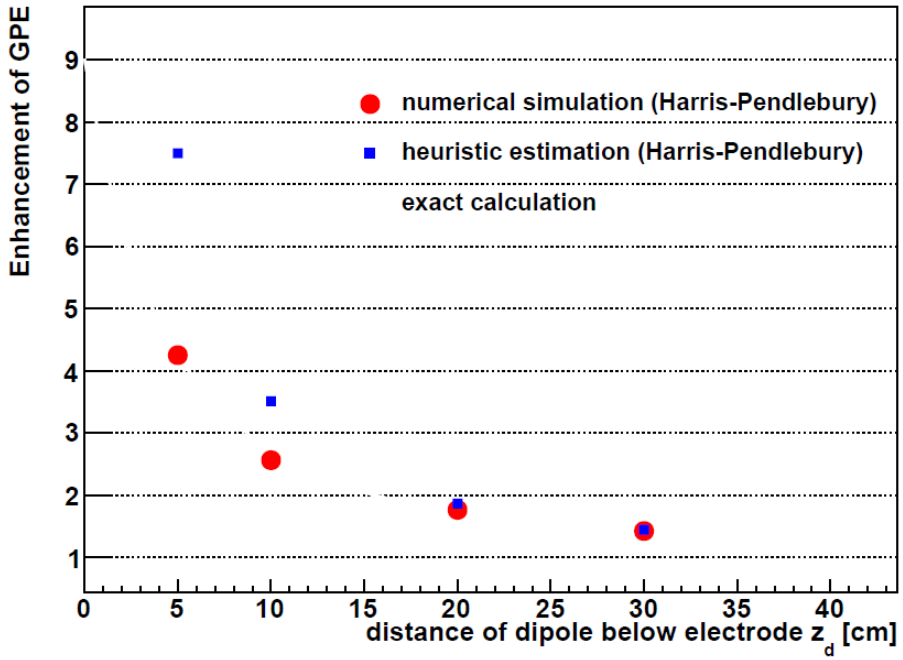
Case 2: a local dipole on axis



$$1 + E = -\frac{4}{R^2}\frac{\langle xB_x + yB_y \rangle}{\left\langle \frac{\partial B_z}{\partial z} \right\rangle}$$

Enhancement factor calculable analytically when dipole on axis

Numerical simulation / Analytical result



Harris&Pendlebury, PRA 73 (2006)

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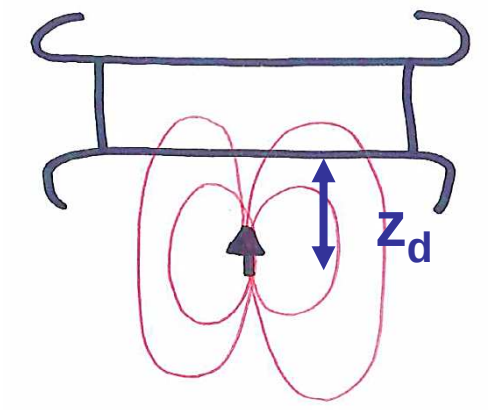

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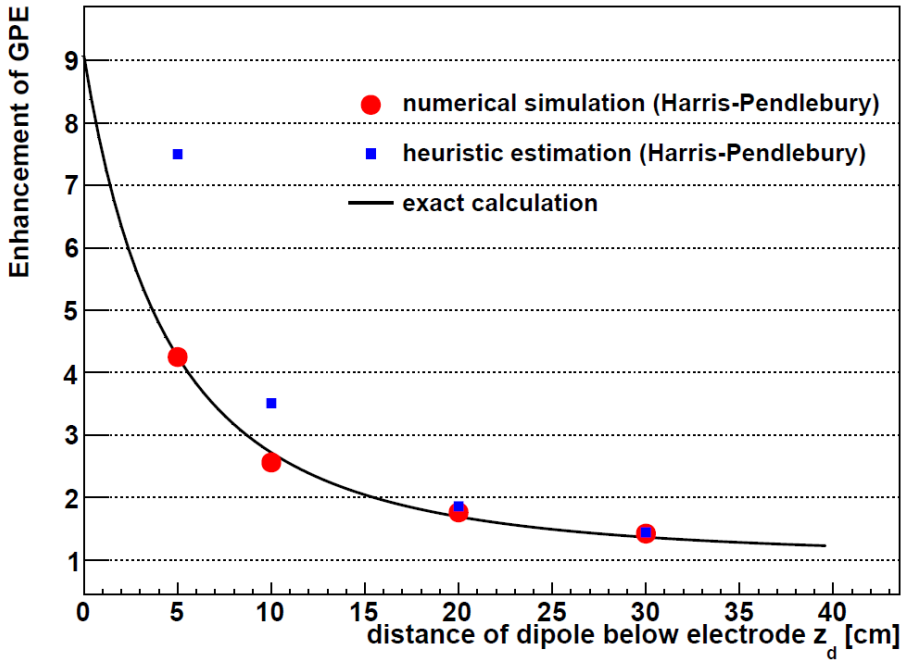
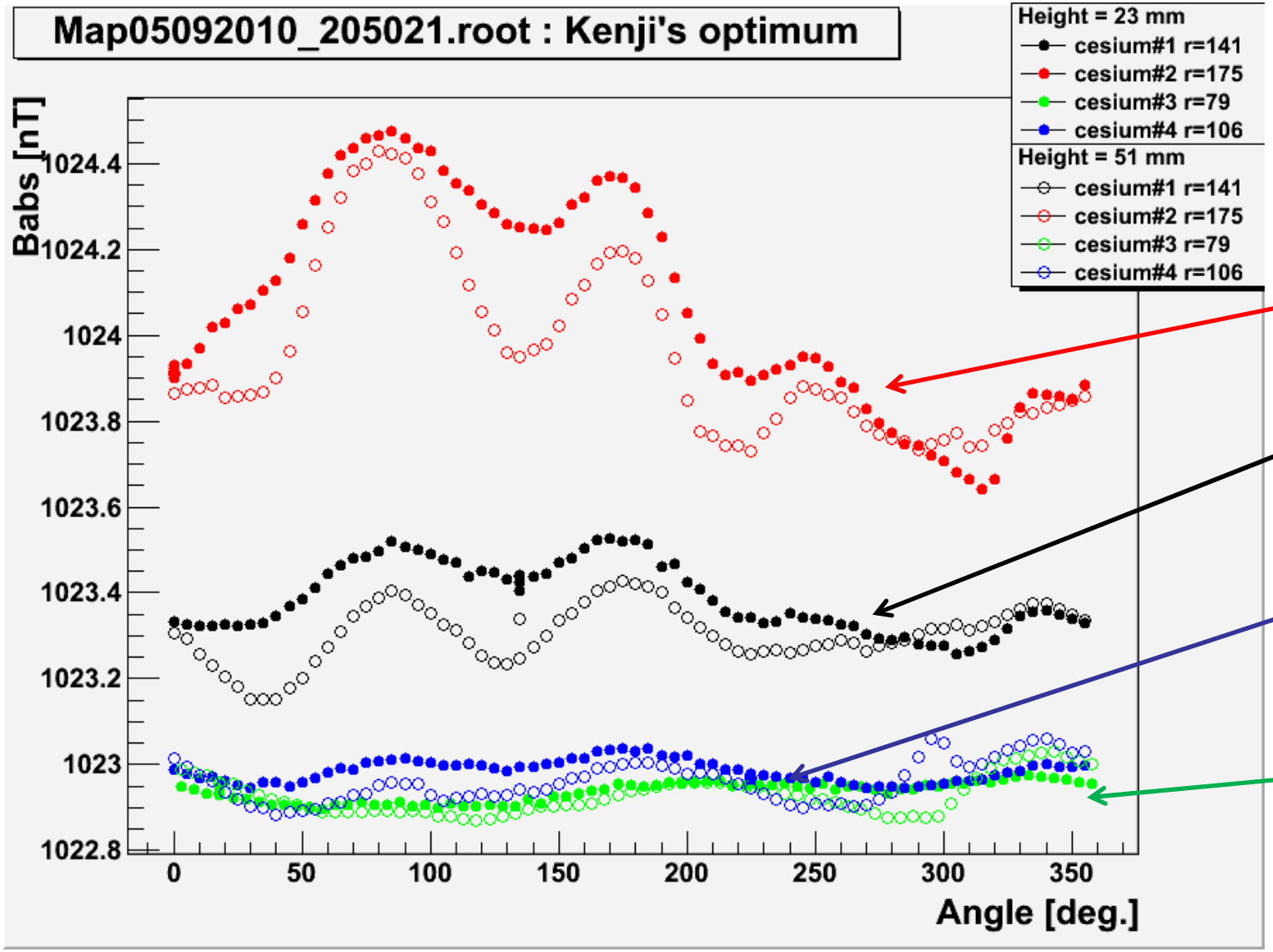


Harris&Pendlebury, PRA 73 (2006)

Pignol&Roccia, Phys. Rev. A 85, 042105 (2012)

Challenge: measure local dipoles close to the electrode surfaces ?

In situ: mapping with Cs magnetometers



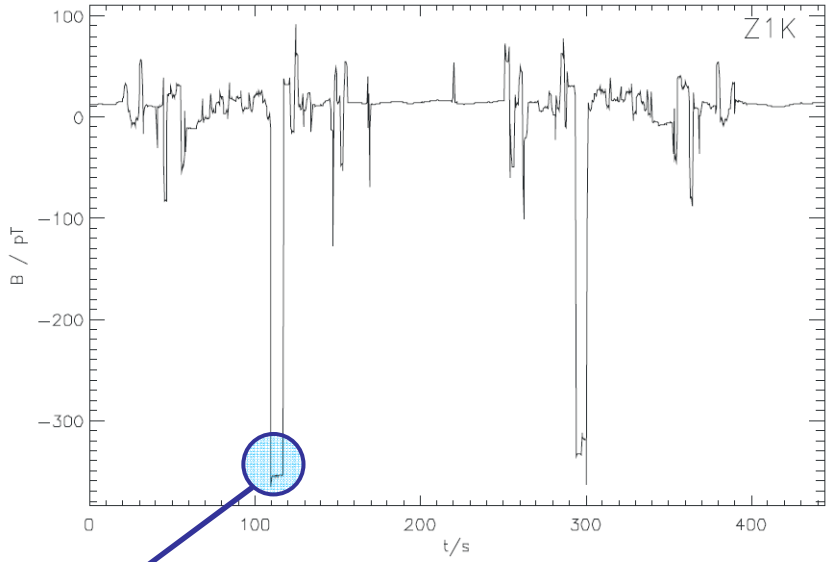
Max variation 300 pT

Max variation 200 pT

Max variation 100 pT

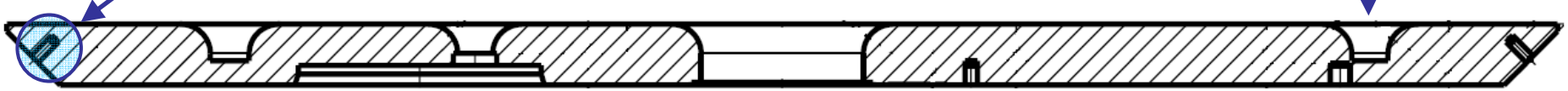
Max variation 50 pT

Prior/post measurement control: magnetic scan at PTB Berlin



Scan of the bottom electrode using shielded room BMSR-2 at PTB (Nov. 2011)

Groove insulating ring



Approximate dipole position

$$x = 31 \text{ cm}, \;\; z = -0.6 \text{ cm}$$

Approximate dipole strength

$$p = \frac{\mu_0}{4\pi} m = 26.5 \text{ nT cm}^3$$

We would then quote a systematic effect

$$\Delta d_n = 0.4 \times 10^{-27} \; e \text{ cm}$$

Summary

- Direct measurement of GPE using 2 methods to measure vertical gradients.
- New theory to calculate GPE valid for arbitrary shape of the magnetic field (including local dipoles).

Goal for GPE systematics: 0.4×10^{-27} *e* cm