Barrier Distributions for Sub- and Near-Barrier Fusion

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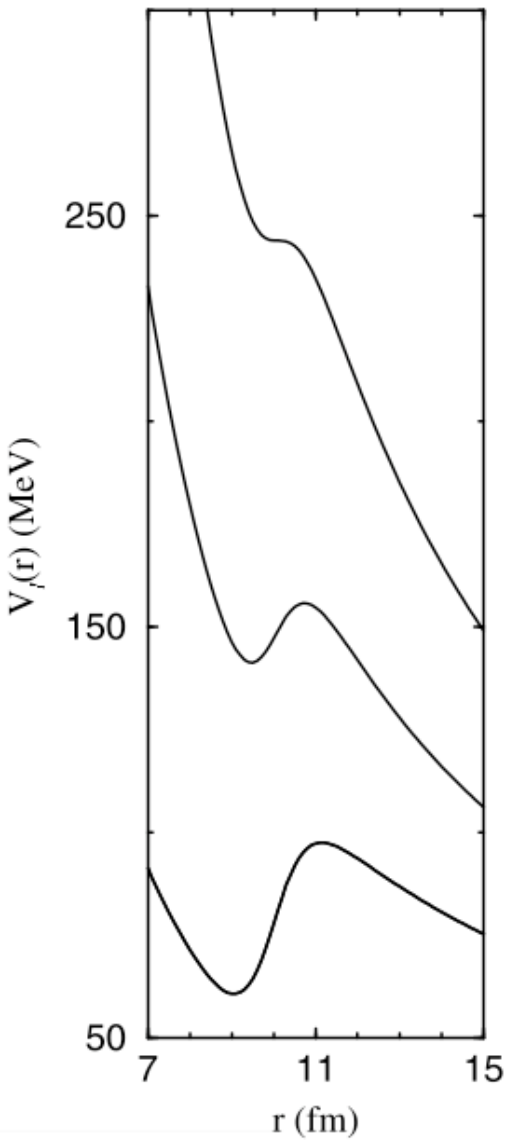
Subbarrier Fusion - Experimental Observables



$$\sigma(E) = \sum_{\ell=0}^{\infty} \sigma_\ell(E),$$

$$\langle \ell(E) \rangle = \frac{\sum_{\ell=0}^{\infty} \ell \sigma_\ell(E)}{\sum_{\ell=0}^{\infty} \sigma_\ell(E)}.$$

$$\sigma_\ell(E) = \frac{\pi \hbar^2}{2\mu E}(2\ell+1)T_\ell(E),$$

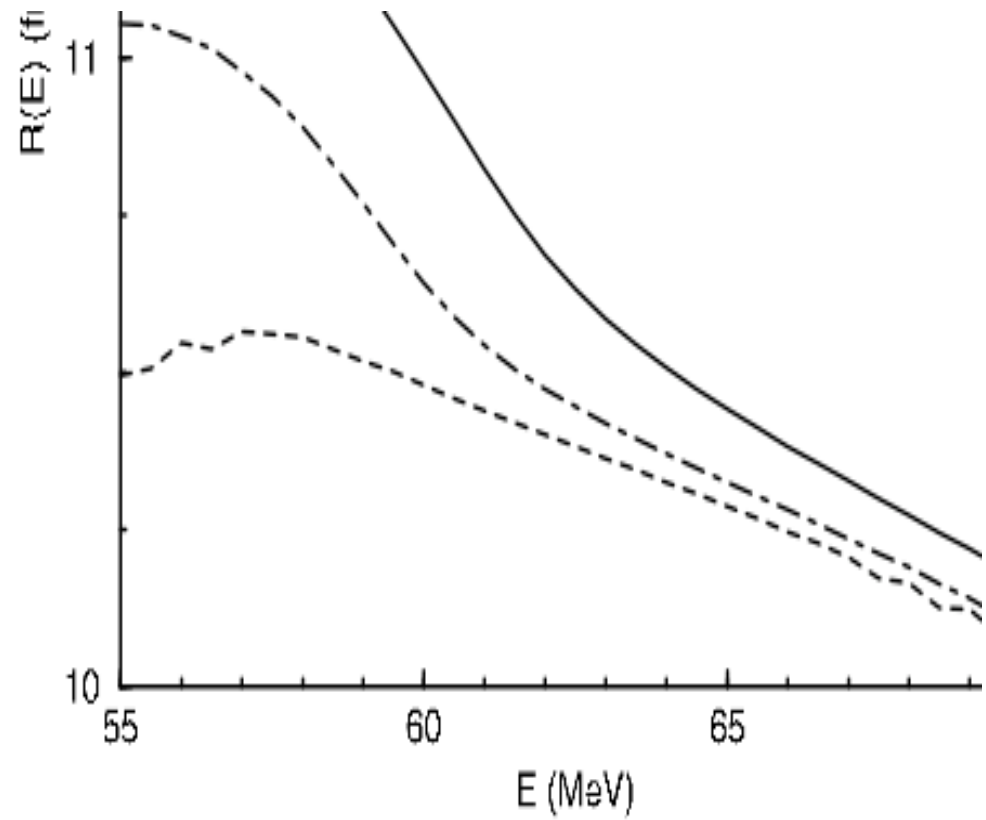
One-dimensional Model

$$T_\ell(E) = \left[1 + \exp\sqrt{\frac{2\mu}{\hbar^2}}\int_{r_{1\ell}}^{r_{2\ell}} dr\left[V_0(r) + \frac{\hbar^2\ell(\ell+1)}{2\mu r^2} - E\right]^{1/2}\right]^{-1}.$$

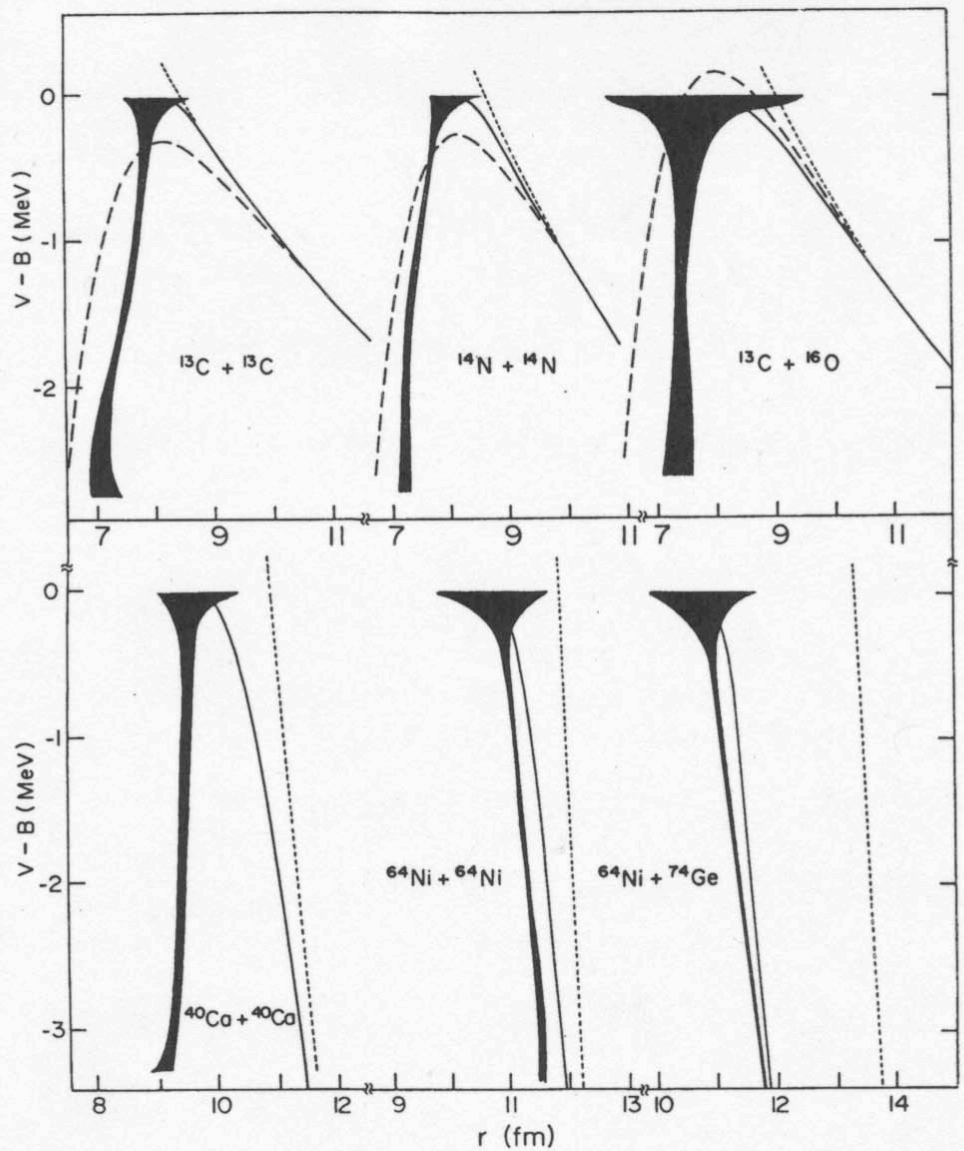
$$T_\ell \simeq T_0\left[E - \frac{\ell(\ell+1)\hbar^2}{2\mu R^2(E)}\right]$$

$$E\sigma(E) = \pi R^2(E)\int_0^E dE' T_0(E')$$

$$\frac{dT_0(E)}{dE} \sim \frac{1}{\pi R^2(E)}\frac{d^2}{dE^2}(E\sigma(E)) + \mathcal{O}(\frac{dR}{dE}).$$

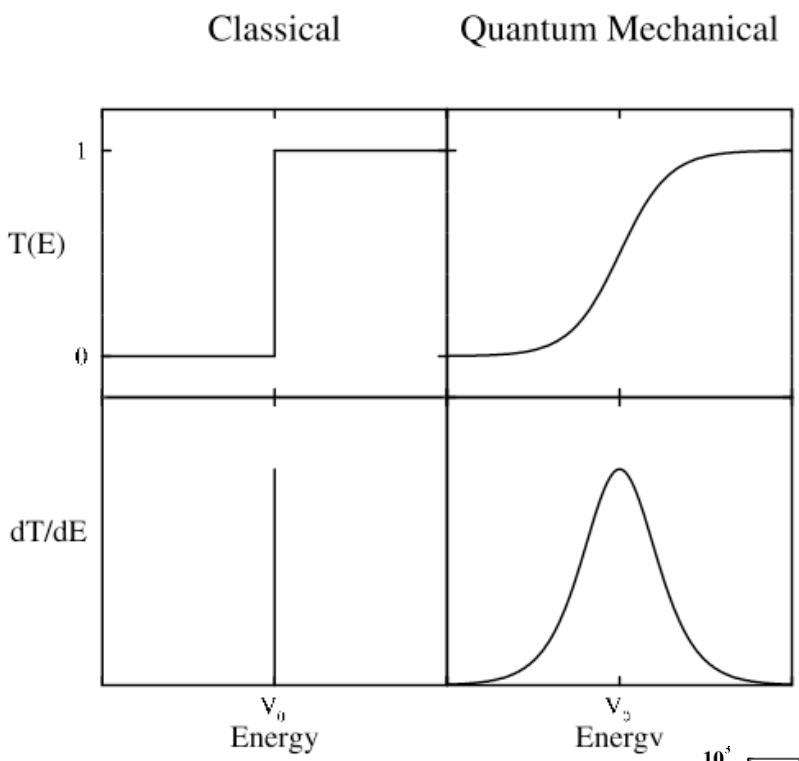


One-dimensional Inversion

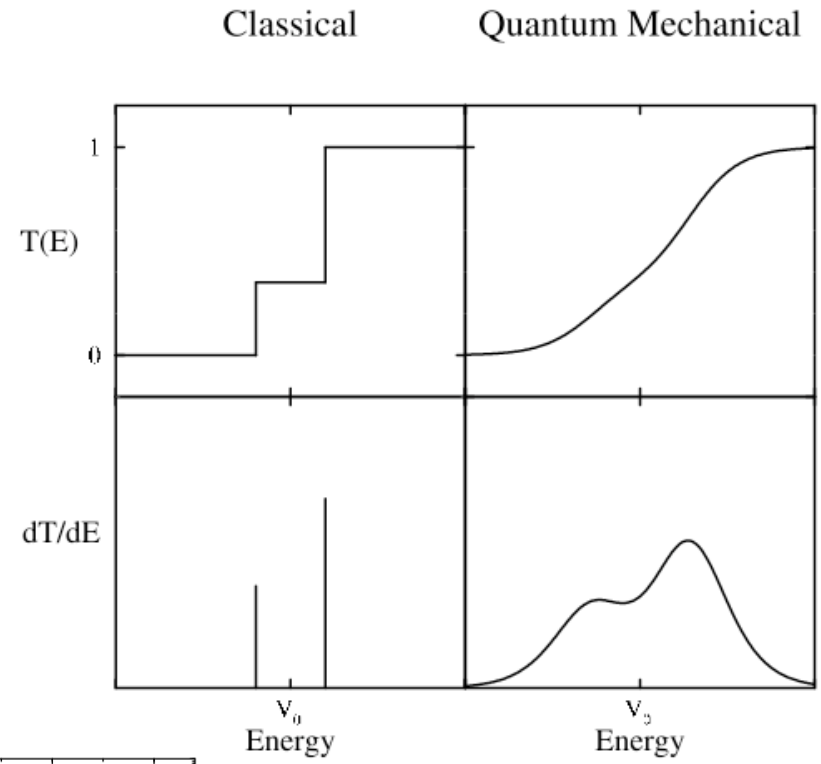


Balantekin, Koonin, Negele, 1983

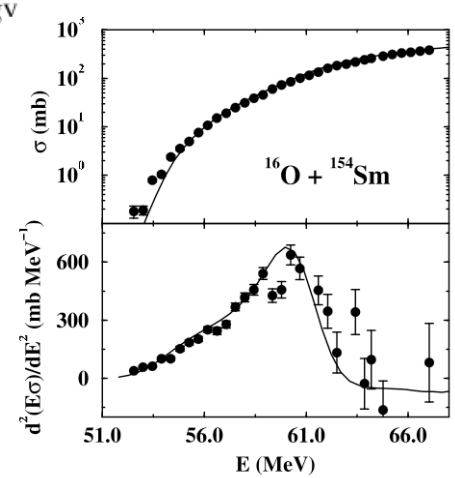
Classical versus Quantum Tunneling



One-dimensional



Multi-dimensional



Methods of calculating fusion cross sections

Potential barrier ⇒ parabola	CCDEF ✗ CCFUS ✗
Parabola + $T_L(E)$ ⇒ $T_L = T_0\,[E - L(L+1)/2\mu R^2]$	Wong's formula ✗
Full Coupled-Channels Calculations	CCFULL ✓

$$V(r) = V_B + \partial V/\partial r|_B$$

0

$$+ (1/2!)\, \partial^2 V/\partial r^2|_B\, (r-r_0)^2$$

CCDEF
CCFUS

$$+ (1/3!)\, \partial^3 V/\partial r^3|_B\, (r-r_0)^3$$

CCFULL

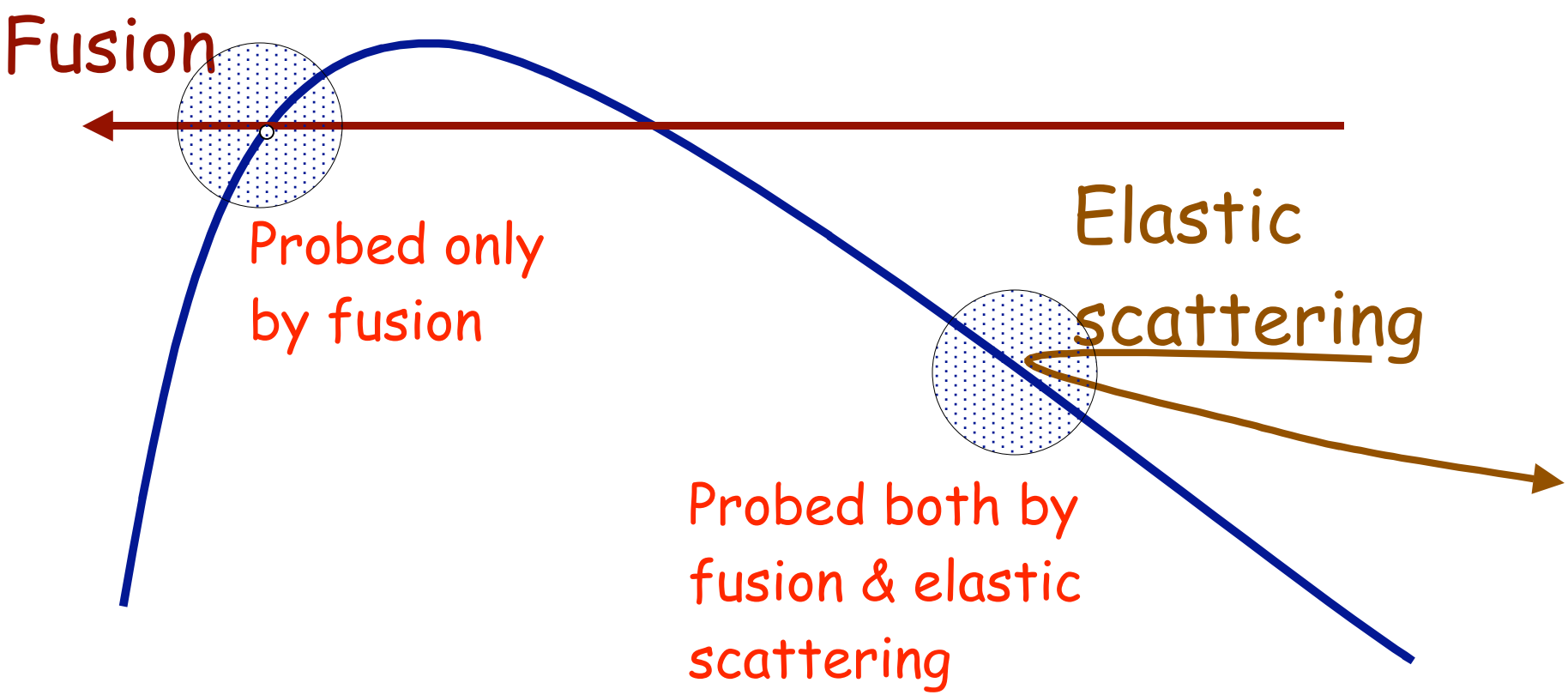
$$+ \ldots$$

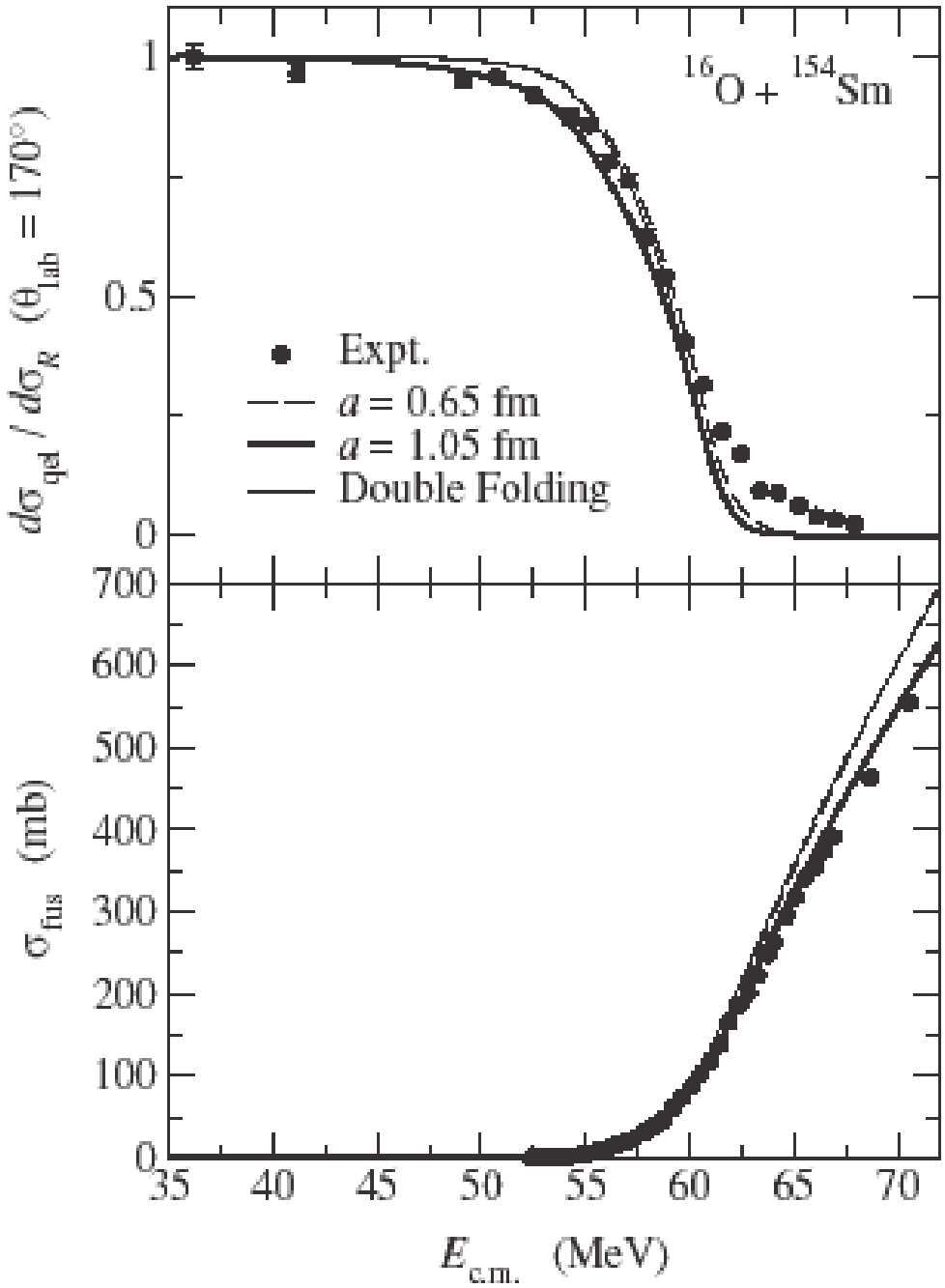
Similar nonlinear
effects for couplings

Halo Nuclei

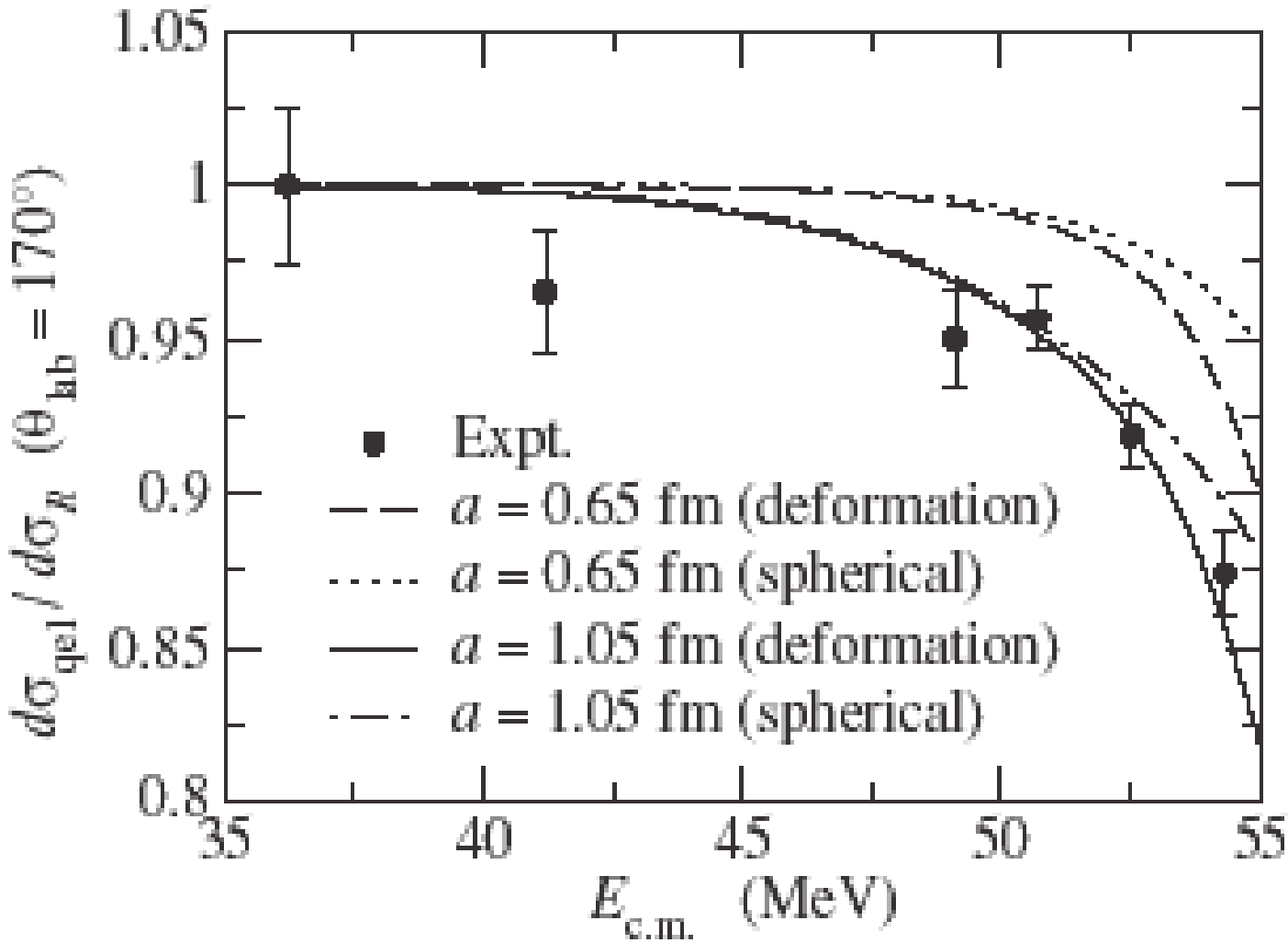
- Understanding the effects of barrier penetration and transfer processes
- Weakly-bound systems with a positive Q-value may lead to a reduction of σ_{fusion}
- Separating complete and incomplete fusion (theorists & experimentalists do it differently)

Diffuseness for both fusion and quasi-elastic scattering = 1.5 to 2 × diffuseness for elastic scattering





Hagino, Takehi, Balantekin, Takigawa, 2005


1.05
1
0.95
0.9
0.85
0.8
35
40
45
50
55
$d\sigma_{qel} / d\sigma_R$ (θ_{lab} = 170°)
$E_{c.m.}$ (MeV)
Expt.
a = 0.65 fm (deformation)
a = 0.65 fm (spherical)
a = 1.05 fm (deformation)
a = 1.05 fm (spherical)

Fusion of unstable nuclei

- Coupling to a resonance state

  Balantekin and Takigawa

- Molecular Bond Formation (^{9}Li + ^{11}Li)

  Balantekin and Bertulani

Fusion of asymmetric systems (low $V_{nuclear}/V_{coulomb}$ in the tail of the potential) is well understood. Can we use this knowledge to probe neutron rich nuclei?

Multi-Channel WKB Approximation

Hagino and Balantekin, 2004

Coupled-channel equations

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}u_{nn_0}(x)+\sum_m\left[V_{nm}(x)+(\epsilon_m-E)\delta_{n,m}\right]u_{mn_0}(x)=0.$$

Boundary Conditions

$$u_{nn_0}(x)\longrightarrow T_{nn_0}e^{-ik_nx}\quad(x\longrightarrow-\infty),$$

$$\longrightarrow\delta_{nn_0}e^{-ik_nx}+R_{nn_0}e^{ik_nx}\quad(x\longrightarrow\infty).$$

Local transmission matrix

$$\boldsymbol{\tau}(x) \equiv \frac{1}{2}[i\mathbf{q}(x)^{-1}\mathbf{u}'(x) + \mathbf{u}(x)],$$

$$\mathbf{q}(x) = \{2m[E - \mathbf{W}(x)]/\hbar^2\}^{1/2}$$

$$\tau_{nm}(x) \to T_{nm}e^{-ik_n x} \quad (x \to -\infty),$$

$$\to \delta_{n,m}e^{-ik_n x} \quad (x \to \infty).$$

$$\boldsymbol{\tau}'(x) = -\frac{1}{2}\mathbf{q}^{-1}(x)\mathbf{q}'(x)[1 + \boldsymbol{\rho}(x)]\boldsymbol{\tau}(x) - i\mathbf{q}(x)\boldsymbol{\tau}(x),$$

$$\boldsymbol{\rho}(x) \equiv [i\mathbf{q}^{-1}(x)\mathbf{u}'(x) - \mathbf{u}(x)] \cdot [i\mathbf{q}^{-1}(x)\mathbf{u}'(x) + \mathbf{u}(x)]^{-1}$$

Exact equation

$$\boldsymbol{\tau}'(x) = -\frac{1}{2}\mathbf{q}^{-1}(x)\mathbf{q}'(x)[1+\boldsymbol{\rho}(x)]\boldsymbol{\tau}(x) - i\mathbf{q}(x)\boldsymbol{\tau}(x),$$

Approximate as

$$\boldsymbol{\tau}'(x) = -\frac{1}{2}\mathbf{q}^{-1}(x)\mathbf{q}'(x)\boldsymbol{\tau}(x) - i\mathbf{q}(x)\boldsymbol{\tau}(x).$$

Discrete iterative solution

$$\boldsymbol{\tau}(-\infty) = \frac{1}{\sqrt{\mathbf{q}(-\infty)}}\left(\prod_i e^{i\mathbf{q}(x_i)\Delta x}\right)\sqrt{\mathbf{q}(\infty)}.$$

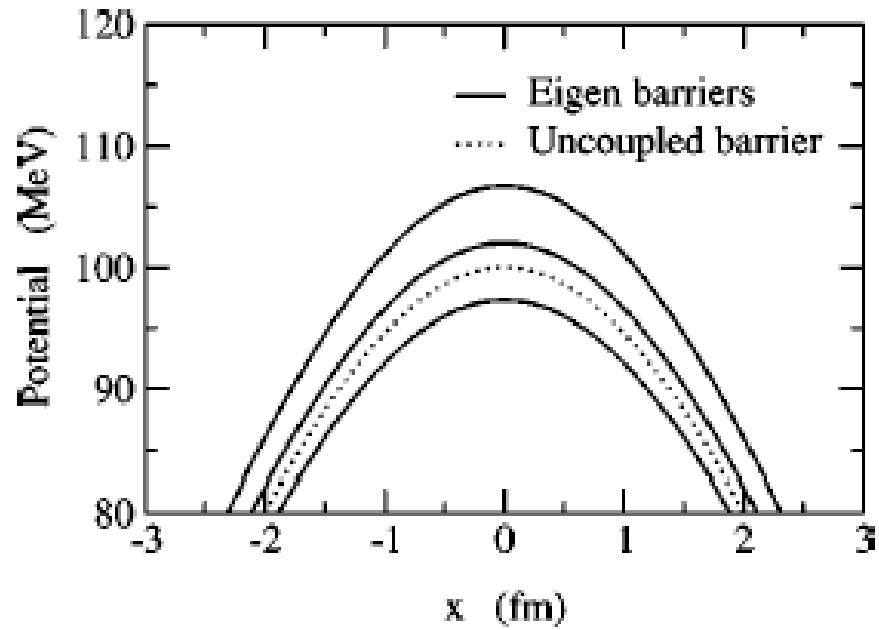
$$P = \sum_n \left|\left\langle n \left| \prod_i e^{i\mathbf{q}(x_i)\Delta x} \right| n_0 \right\rangle\right|^2.$$

Example: A three-channel problem

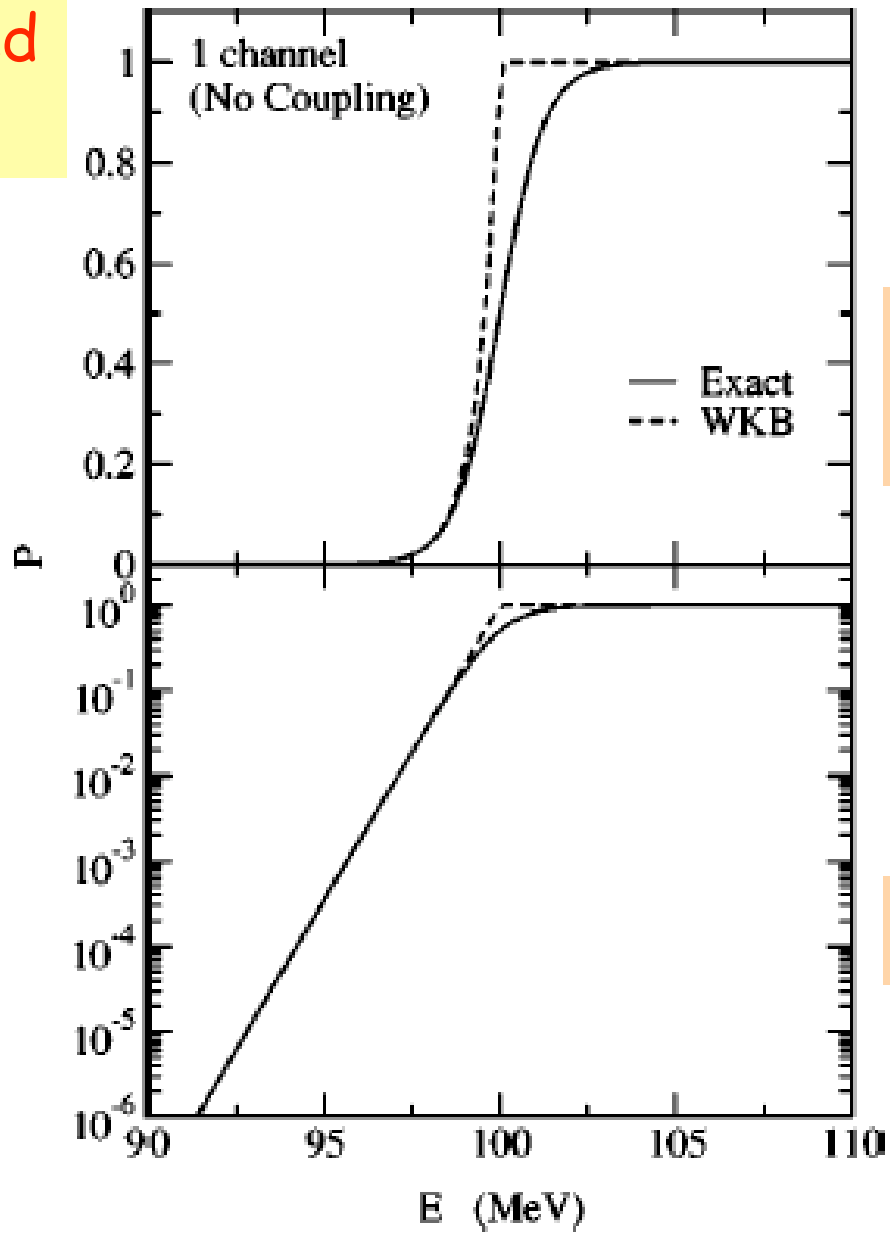
$$\mathbf{W}(x) = \begin{pmatrix} V(x) & F(x) & 0 \\ F(x) & V(x)+\epsilon & F(x) \\ 0 & F(x) & V(x)+2\epsilon \end{pmatrix},$$

$$V(x) = V_0 e^{-x^2/2s^2},$$

$$F(x) = F_0 e^{-x^2/2s_f^2}.$$



Uncoupled barrier

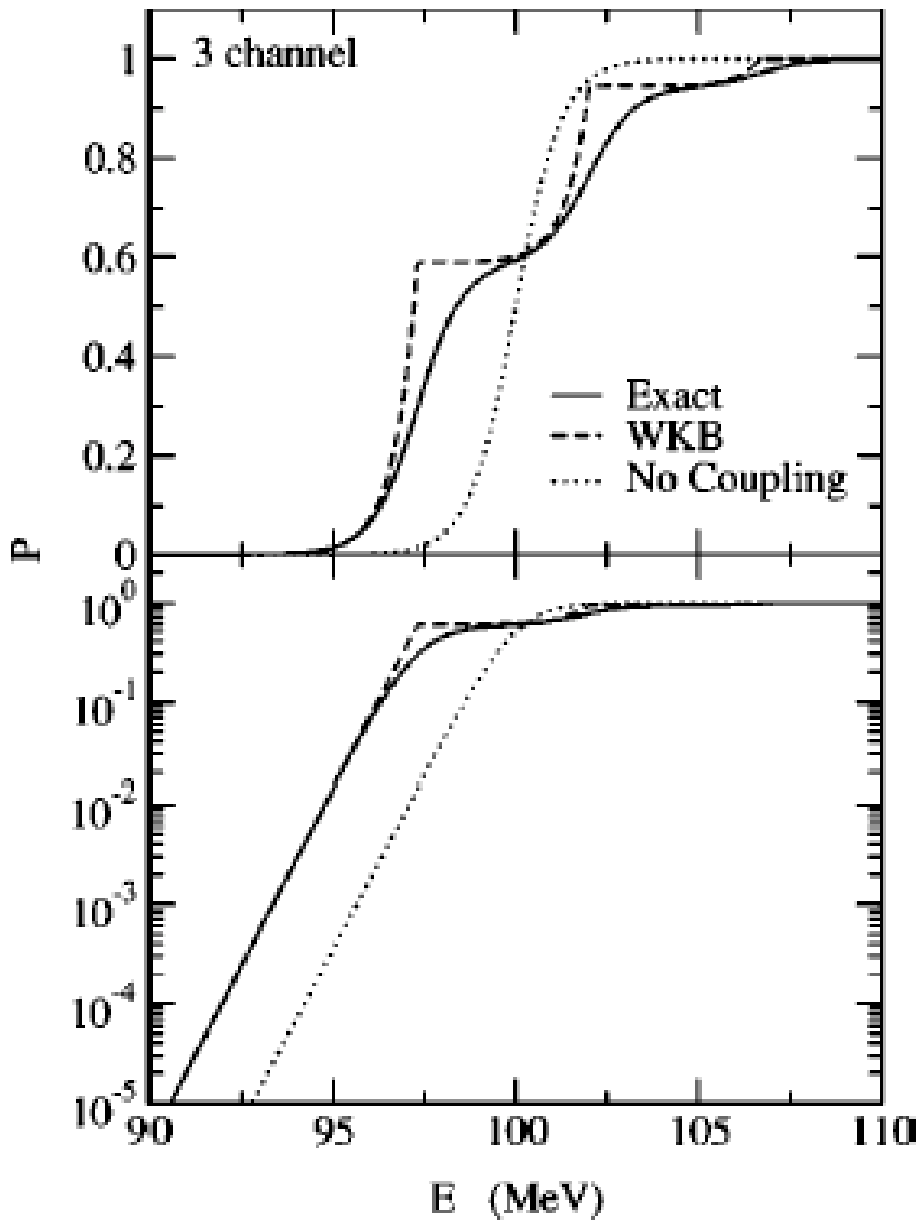


Linear scale

Log scale

Hagino, Balantekin 2004

Coupled barrier

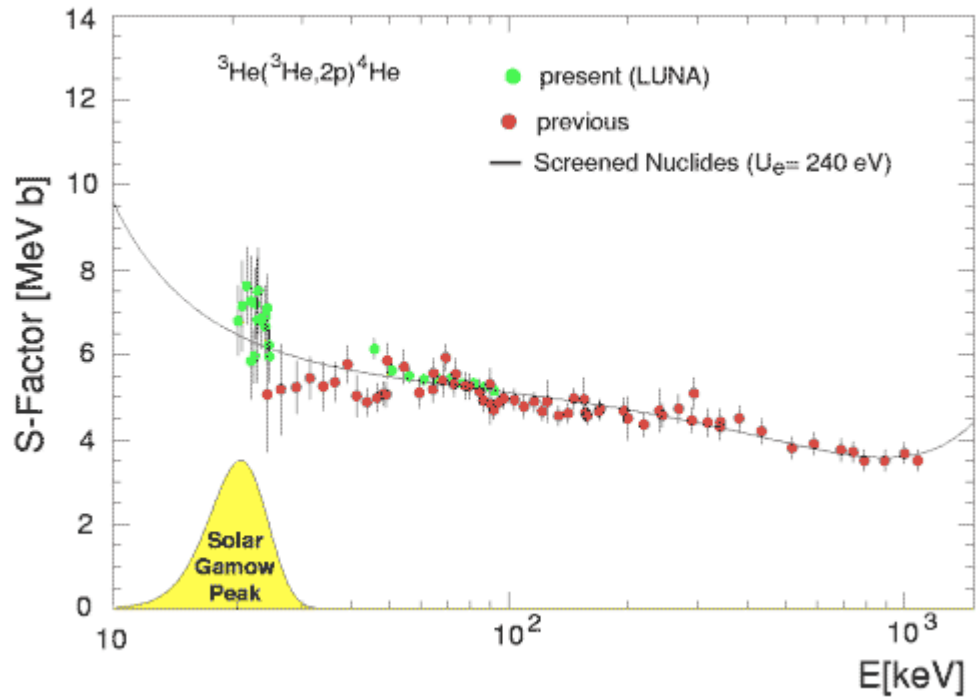


Fusion in astrophysical settings

Additional issues:

- Stellar Screening
- Laboratory Screening

Adelberger, et al. 1998



Laboratory Electron Screening

- Adiabatic approximation: Constant energy shift to the Coulomb potential.

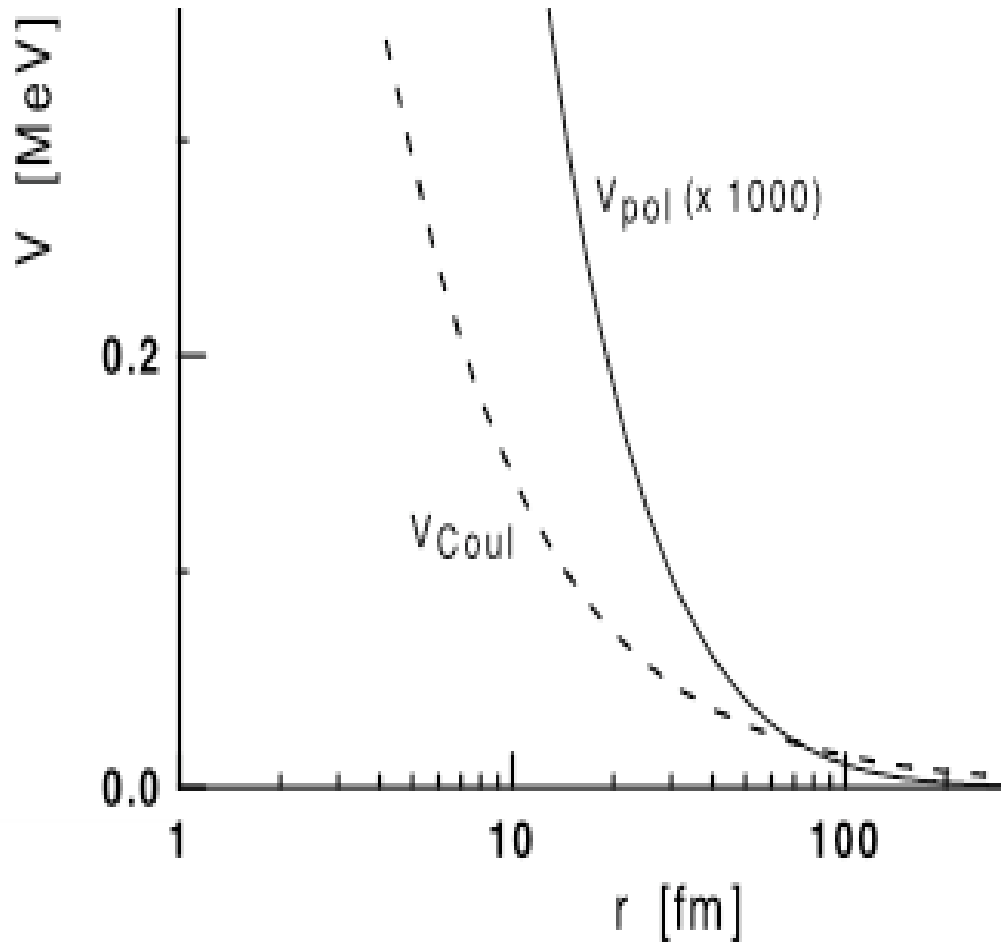
Langanke et al.

- Data requires even larger shifts.
- Puzzle: Adiabatic approximation typically overpredicts. (→ excluded physics?)

Laboratory screening

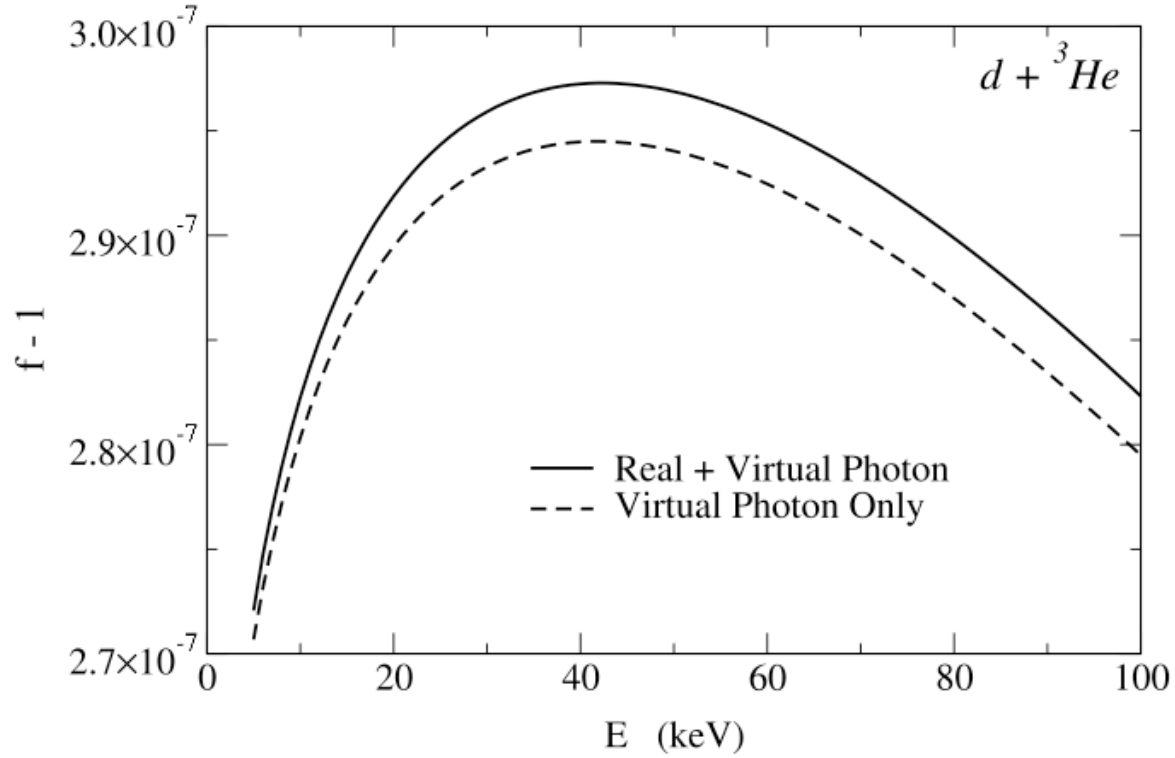
- Other Effects (vacuum polarization,relativity, Bremsstrahlung and atomic polarization) are very small.

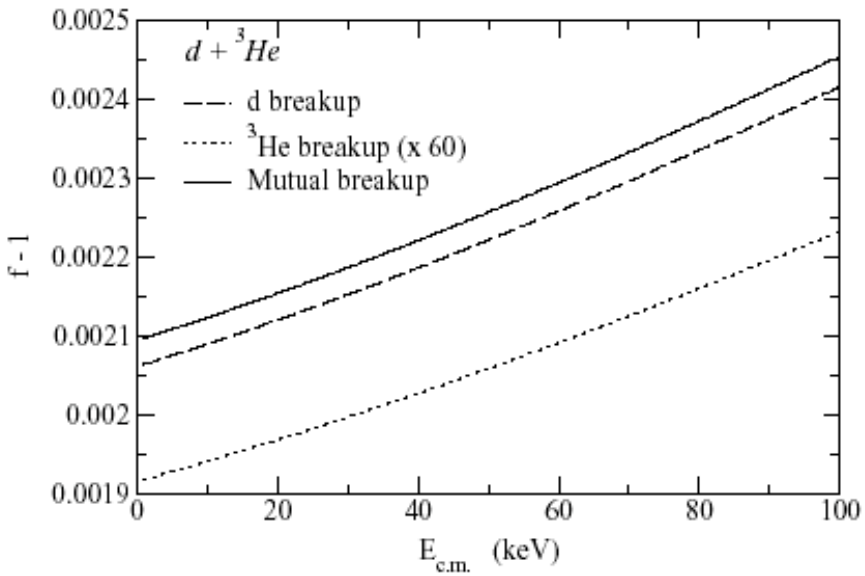
Balantekin, Bertulani, Hussein, 1997

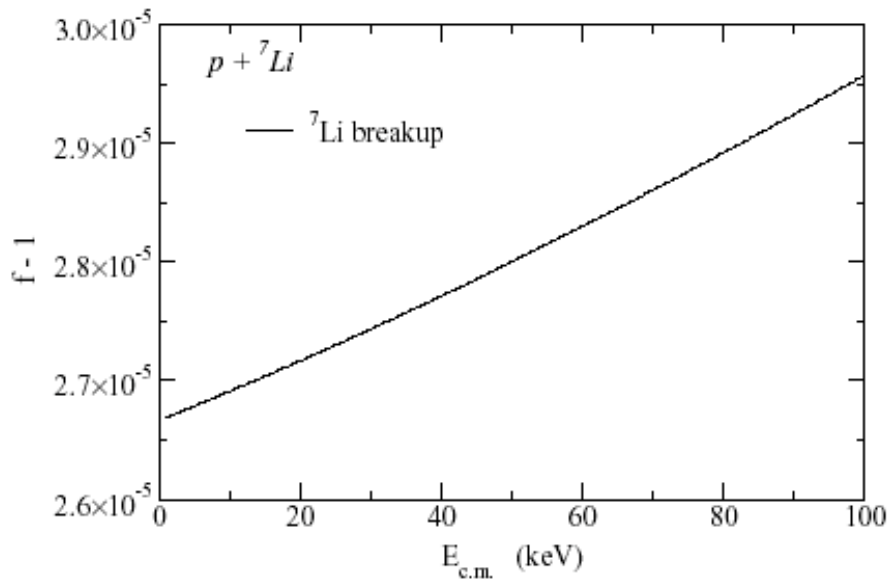


Electromagnetic Effects in Tunneling

- Suggestion: Virtual photon emission during tunneling may increase probability Flambaum and Zelevinsky, 1999
- Radiation field can be eliminated using the path integral formalism. Enhancement is extremely small.⇒ Discrepancy in screening is still a puzzle Hagino and Balantekin, 2002.







Enhancement of the probability due to deuteron breakup is also too small

Hagino, Hussein, Balantekin, 2003

Concluding questions

- For asymmetric systems the barrier is well-outside the region where nuclei touch. Multidimensional barrier penetration is conceptually well-defined. Do we really understand the fusion of such nuclei? What is the large diffuseness telling us?

- What happens when nuclei fuse at energies well-below the barrier?

- Do we understand how we should theoretically formulate the fusion of unstable nuclei? What can we learn by studying fusion of nuclei off the line of stability?