Heavy-Ion Fusion Using Density-Constrained TDHF

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Microscopic Methods for PES (CHF-ATDHF)

- Advantages

  - Fully microscopic, self-consistent description of nuclear PES
  - Use same microscopic interaction used in g.s. calculation
  - Gives global information on collective potential
  - Quantization via ATDHF

- Shortcomings

  - Artificial introduction of constraining operators
  - Collective motion not necessarily confined in constrained phase space
  - Static adiabatic approximation
  - Most energetically favorable state requires sudden rearrangement
  - No reason why dynamical system should move along the valley of PES
  - CHF calculations seldom produce the correct saddle-point

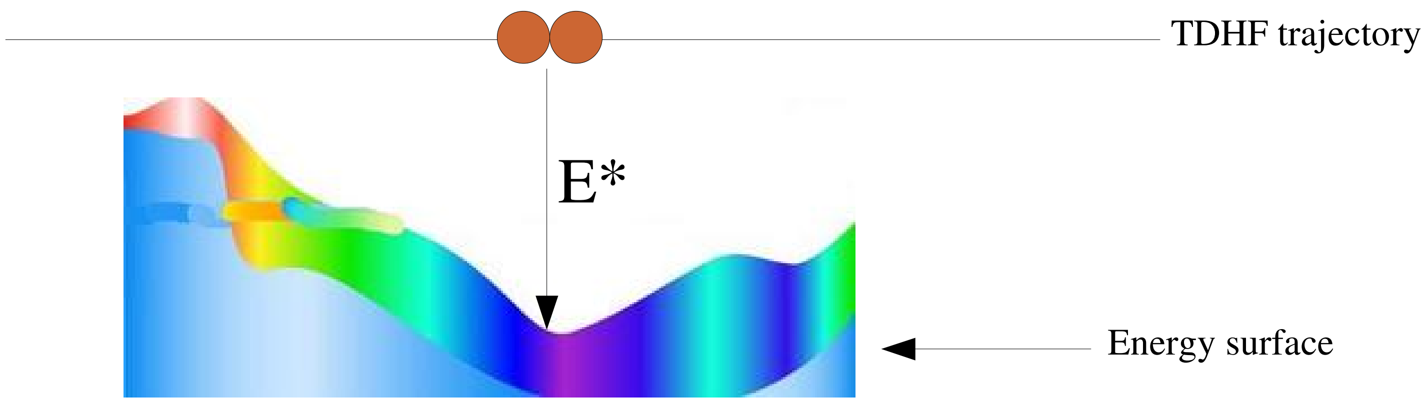
Möller, Sierk, Iwamoto, PRL 92, 072501 (2004)

Phenomenological Fusion Barriers

- All information taken from static properties of nuclei
  - These may be correct prior to nuclear overlap but different after
- Use frozen densities when nuclei overlap
  - Typically Fermi densities fitted to experimental data
- Phenomenological heavy-ion interaction potentials
  - Wood-Saxon, Proximity, double-folding, W-A
  - Several free parameters
- Use few excited states (2-3)
  - Coupling potentials for excitations derived in simple models (rigid rotor, harmonic vibrator)
  - B(EL) values taken from experimental data
- Rotating frame approximation
  - To reduce the number of channels
- Allow for neutron transfer
  - Based on Q-values

Density-Constrained TDHF

- Project <u>unhindered</u> TDHF evolution onto the dynamical PES
  - system selects its evolutionary path by itself
  - constrains all collective degrees of freedom



- Extract internal excitation energy
  - hold the instantaneous TDHF density frozen
  - minimize the energy

Implementation

- Generalize the ordinary method of constraints

  - for a single constraint → $\hat{H} \rightarrow \hat{H} + \lambda \hat{Q}$

  - for a set of constraints → $\hat{H} \rightarrow \hat{H} + \sum_i \lambda_i \hat{Q}_i$

  - for density constraint → $\hat{H} \rightarrow \hat{H} + \int d^3r\, \lambda_q(r)\, \hat{\rho}_q(r)$

NEW!

- Works as accurately as a single constraint

  - numerical method for steering the solution to TDHF density is given in:

1. Cusson et al. , Z. Phys. A320, 475 (1985)
2. Umar et al. , Phys. Rev. C32, 172 (1985)

Ion-Ion Potential

*Umar, Oberacker, Phys. Rev. C **74**, 021601(R) (2006)*

- Total energy in terms of the excitation energy is:

$$E_{TDHF} = T_R + V + E^* \longrightarrow V = E_{TDHF} - T_R - E^* = E_{DC}$$

- E_{DC} contains the binding energies of the two nuclei

$$V(R) \rightarrow E_{DC}(R) - E_{A_1} - E_{A_2} \longrightarrow$$ Subtract binding energies

- Asymptotically correct (no normalization needed):

$$E_{DC}(R_{max}) = E_{A_1} + E_{A_2} + \frac{Z_1 Z_2 e^2}{R_{max}} \longrightarrow V(R_{max}) = \frac{Z_1 Z_2 e^2}{R_{max}}$$

- Contains all of the dynamics present in TDHF; neck formation, particle exchange, deformation to all orders ...

A New Generation TDHF Code

- Unrestricted 3-D Cartesian geometry
  - No fixed reaction plane
  - No reflection symmetry (+z/-z)
- Basis-Spline discretization for high accuracy
- Coded in Fortran-95 and $OMP

- Use of modern Skyrme forces with **all** the terms (time even/odd)
- No time-reversal symmetry assumed

- Much improved fusion cross sections for light systems

  *Umar, Oberacker, Phys. Rev. C **73**, 054607 (2006)*
- It is possible to calculate cross sections for deformed nuclei

  *Umar, Oberacker, Phys. Rev. C **74**, 124606 (2006)*
- Calculate EOS for finite nuclei

  *Umar, Oberacker, Phys. Rev. C **76**, 024316 (2007)*

Additional Terms in Energy Density

New Terms

t_0, t_3 →

$$\frac{1}{4} t_0 x_0 \vec{s}^{\,2} - \frac{1}{4} t_0 (\vec{s}_n^{\,2} + \vec{s}_p^{\,2}) + \frac{1}{24} \rho^{\alpha} t_3 x_3 \vec{s}^{\,2} - \frac{1}{24} t_3 \rho^{\alpha} (\vec{s}_n^{\,2} + \vec{s}_p^{\,2})$$

t_1, t_2 →

$$\frac{1}{32}(t_2 + 3t_1) \sum_q \vec{s}_q \cdot \nabla^2 \vec{s}_q - \frac{1}{32}(t_2 x_2 - 3 t_1 x_1) \vec{s} \cdot \nabla^2 \vec{s} +$$

$$\frac{1}{8}(t_1 x_1 + t_2 x_2)(\vec{s} \cdot \vec{T} - \overleftrightarrow{J}^{2}) + \frac{1}{8}(t_2 - t_1) \sum_q (\vec{s}_q \cdot \vec{T}_q - \overleftrightarrow{J}_q^{2})$$

t_4 →

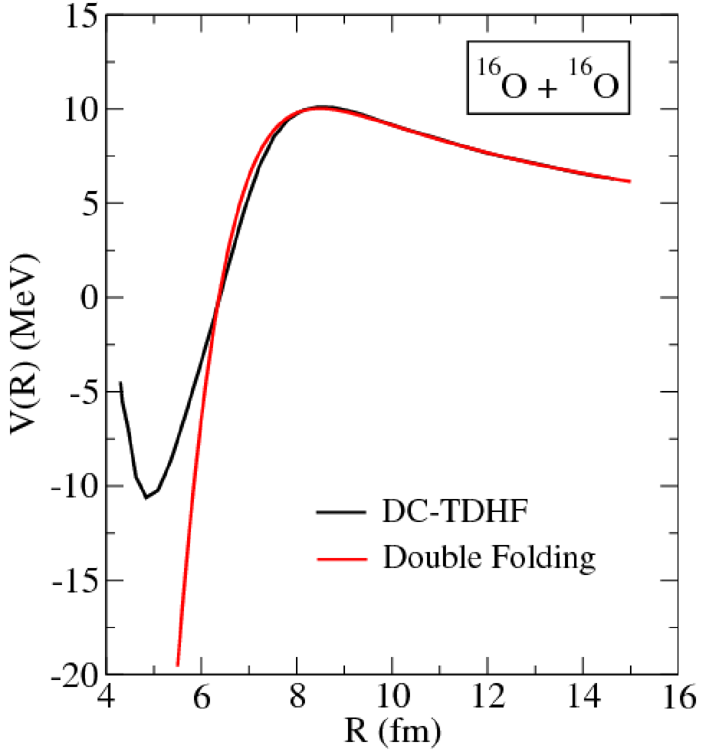
$$-\frac{t_4}{2} \sum_{qq'} (1 + \delta_{qq'}) [\vec{s}_q \cdot \nabla \times \vec{j}_{q'} + \rho_q \overleftrightarrow{\nabla} \cdot \overleftrightarrow{J}\,]$$

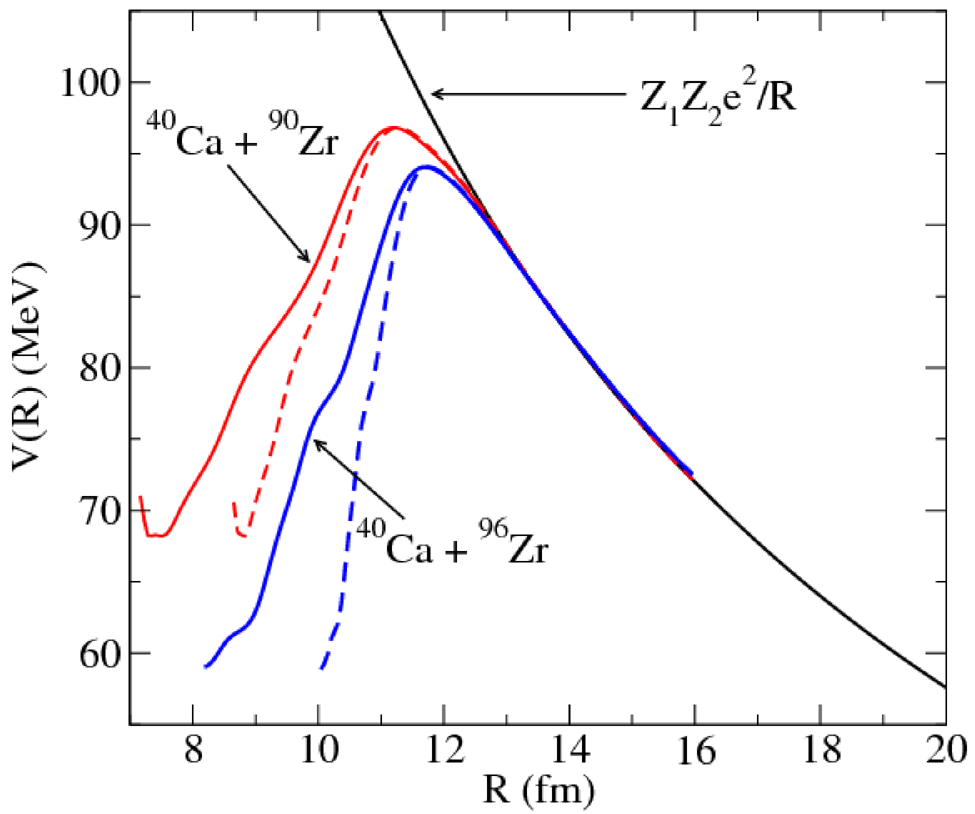
Old Term

$$\rho \tau - \vec{j}^{\,2}$$

Comparison to Empirical Fusion Potentials

- DC-TDHF potential contains no parameters and normalization

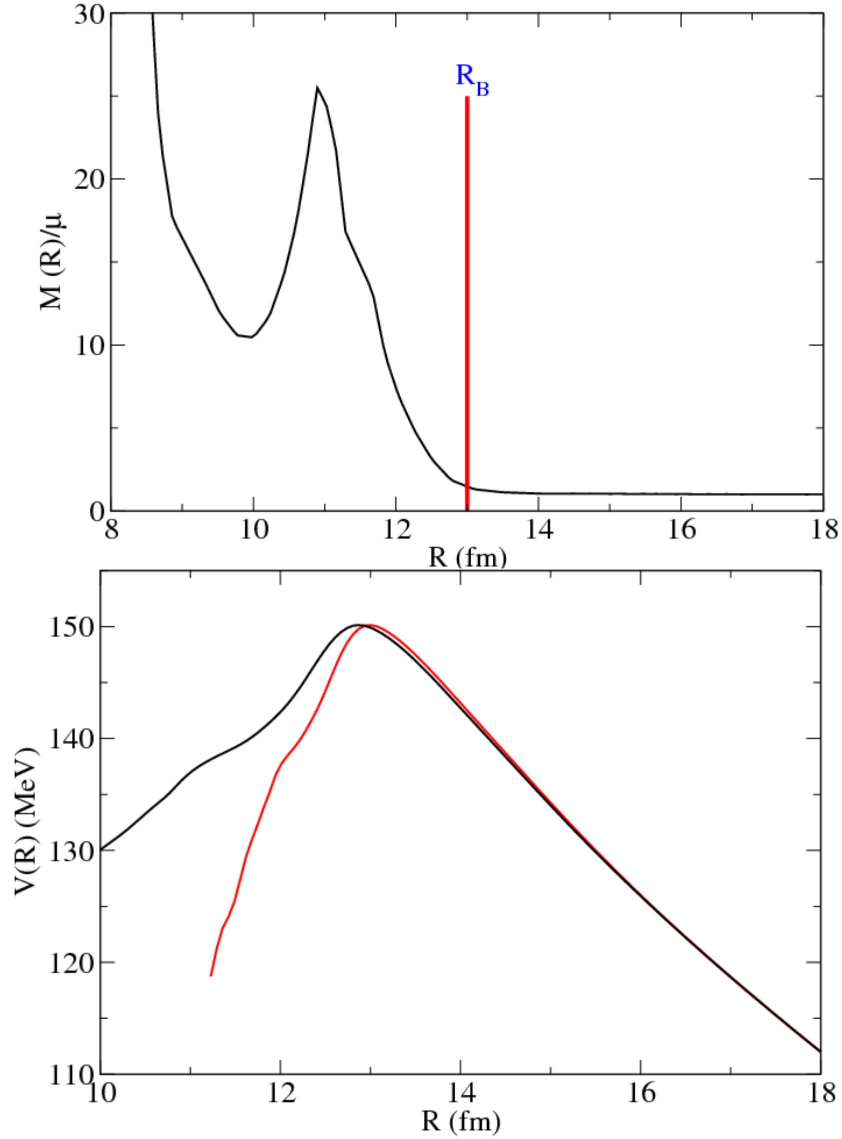




Double folding:
M3Y effective NN interaction
densities from electron scattering

- All comparisons show incredible agreement!

Dynamical Effective Mass



$$E_{c.m.} = \frac{1}{2} M(R) \dot{R}^2 + V(R)$$

TDHF DC-TDHF

- Typical CHF type peak

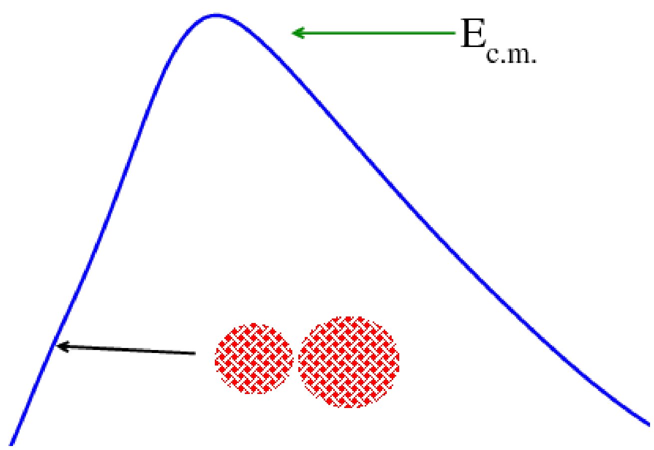
  - Because we are over the barrier!

$$M(R) = \frac{2(E_{c.m.} - V(R))}{\dot{R}^2}$$

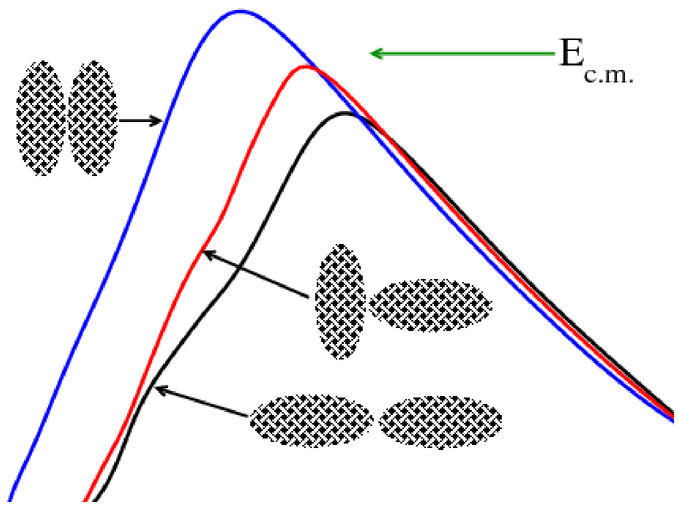
- Transform effect to V(R)

$$d\bar{R} = \left(\frac{M(R)}{\mu}\right)^{\frac{1}{2}} dR$$

Initial Alignment for Deformed Nuclei

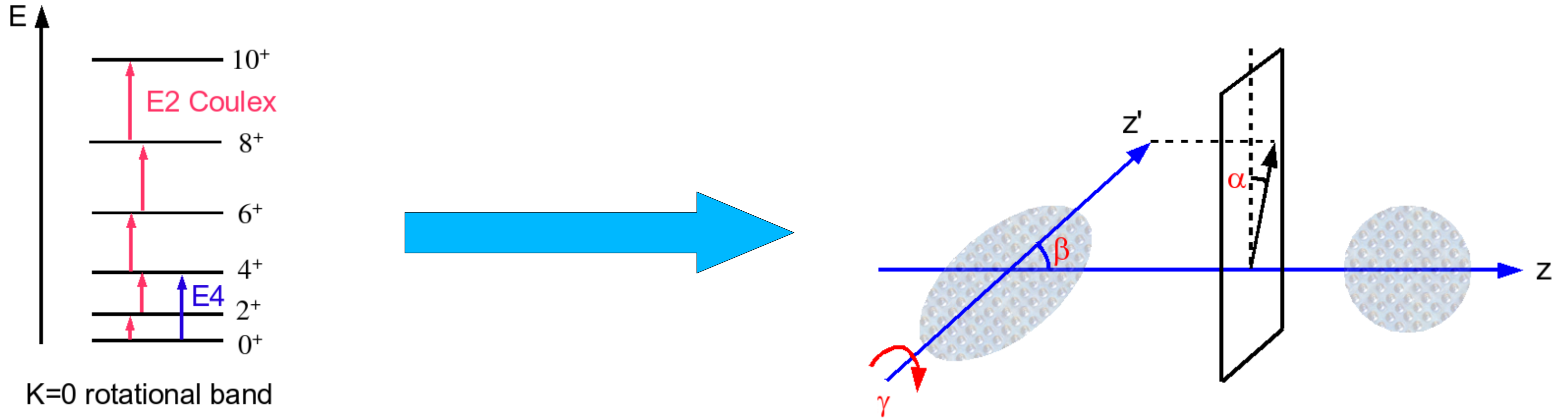


Spherical nuclei → single barrier



Deformed nuclei → barrier distribution

- Dynamical alignment due to Coulomb excitation



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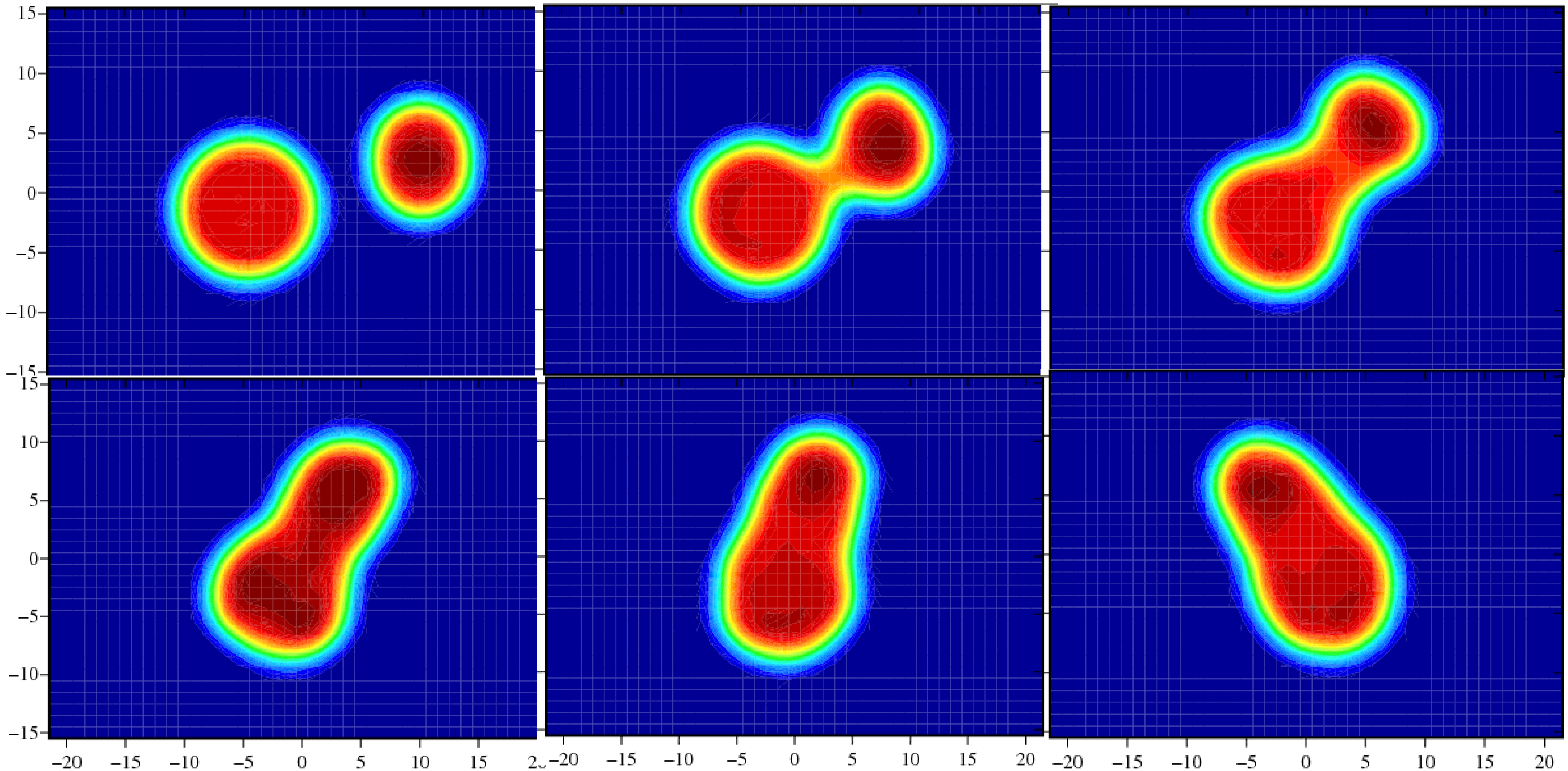
$^{64}Ni + ^{132}Sn$ Fusion

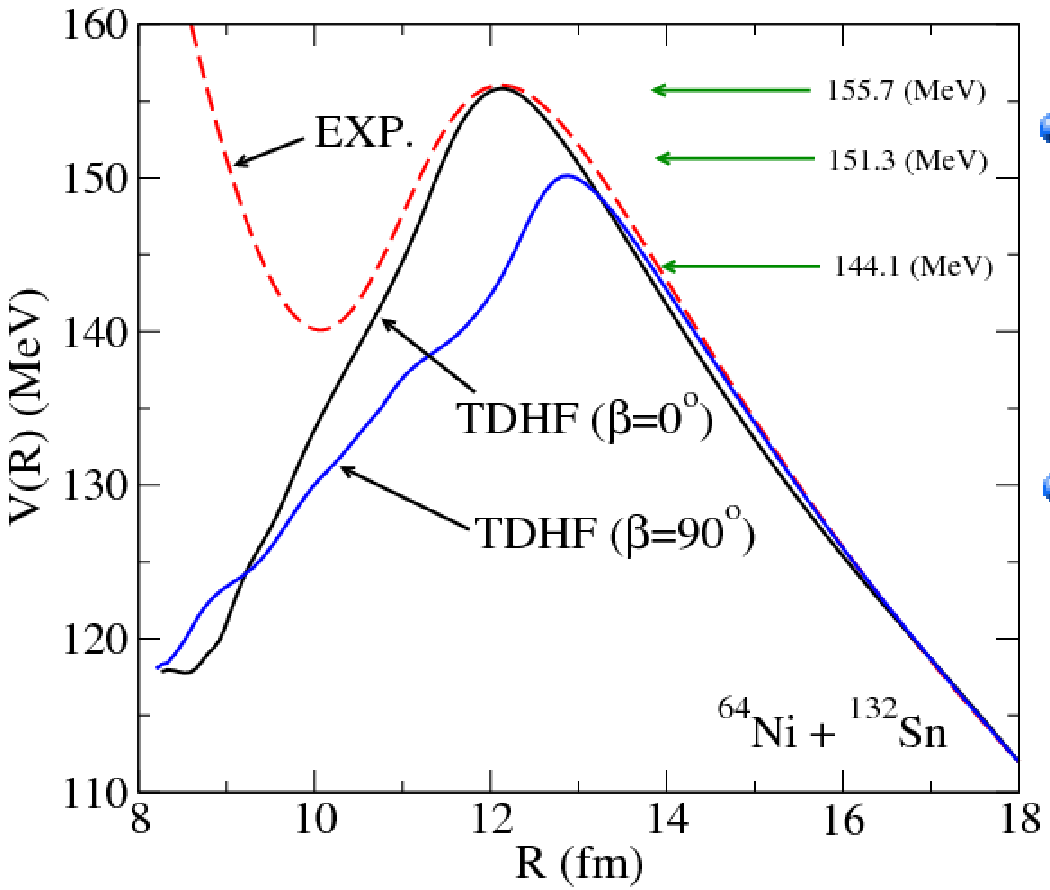
- 3D HF (SLy4) gives oblate deformation for ^{64}Ni: $Q_{zz}(n) = -0.85$ b, $Q_{zz}(p) = -0.59$ b



- $^{64}Ni + ^{132}Sn$, $E_{cm} = 176$ MeV, SLy5, b = 3 fm

^{64}Ni + ^{132}Sn Limiting Barriers

Umar, Oberacker, *Phys. Rev. C **74**, 061601(R) (2006)*
Umar, Oberacker, *Phys. Rev. C **76**, 014614 (2007)*

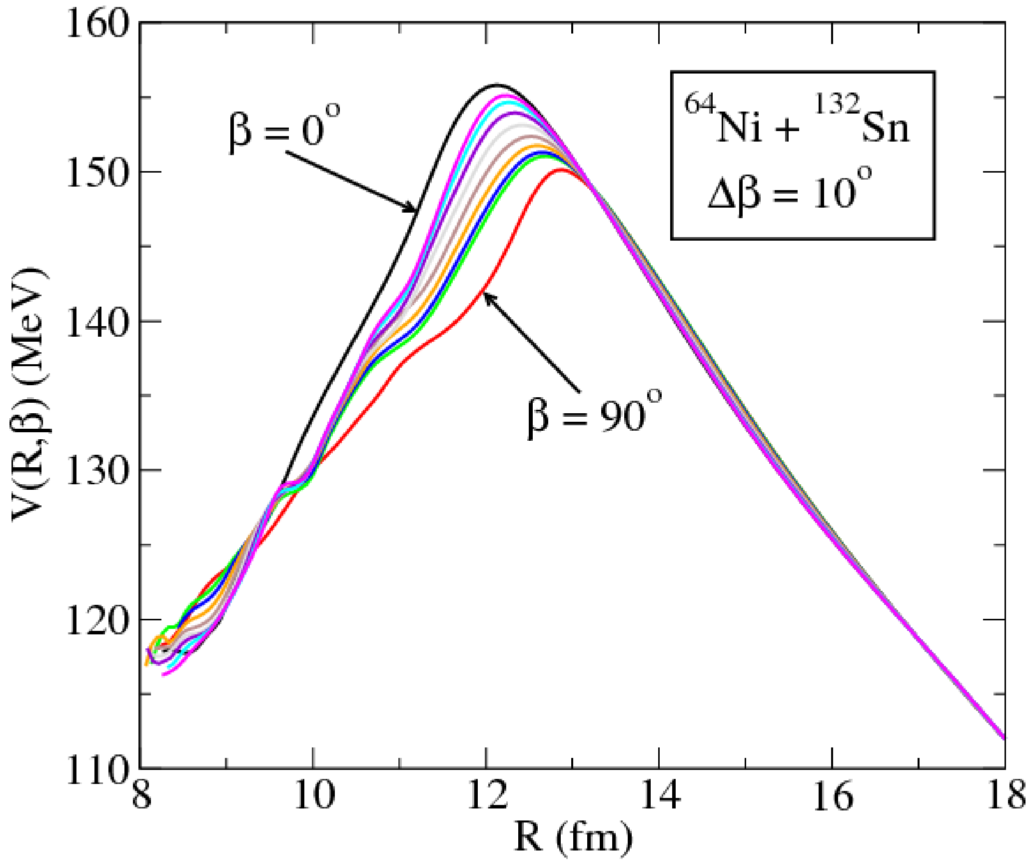


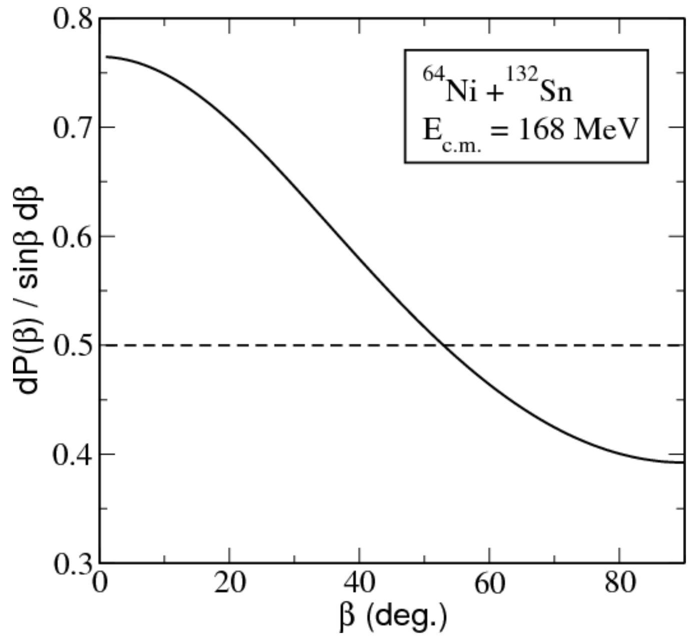
Liang et al., PRL 91, 152701-1 (2003)

- Barrier for $\beta=0^{o}$ agrees with empirical
  - No parameter/normalization in TDHF
  - V_B = 155.81 MeV
  - R_B = 12.12 fm
- Barrier for $\beta=90^{o}$ lower
  - V_B = 150.13 MeV
  - R_B = 12.87 fm

$^{64}Ni + ^{132}Sn$ Complete Set of Barriers

Umar and Oberacker, Phys. Rev. C76, 014614 (2007)





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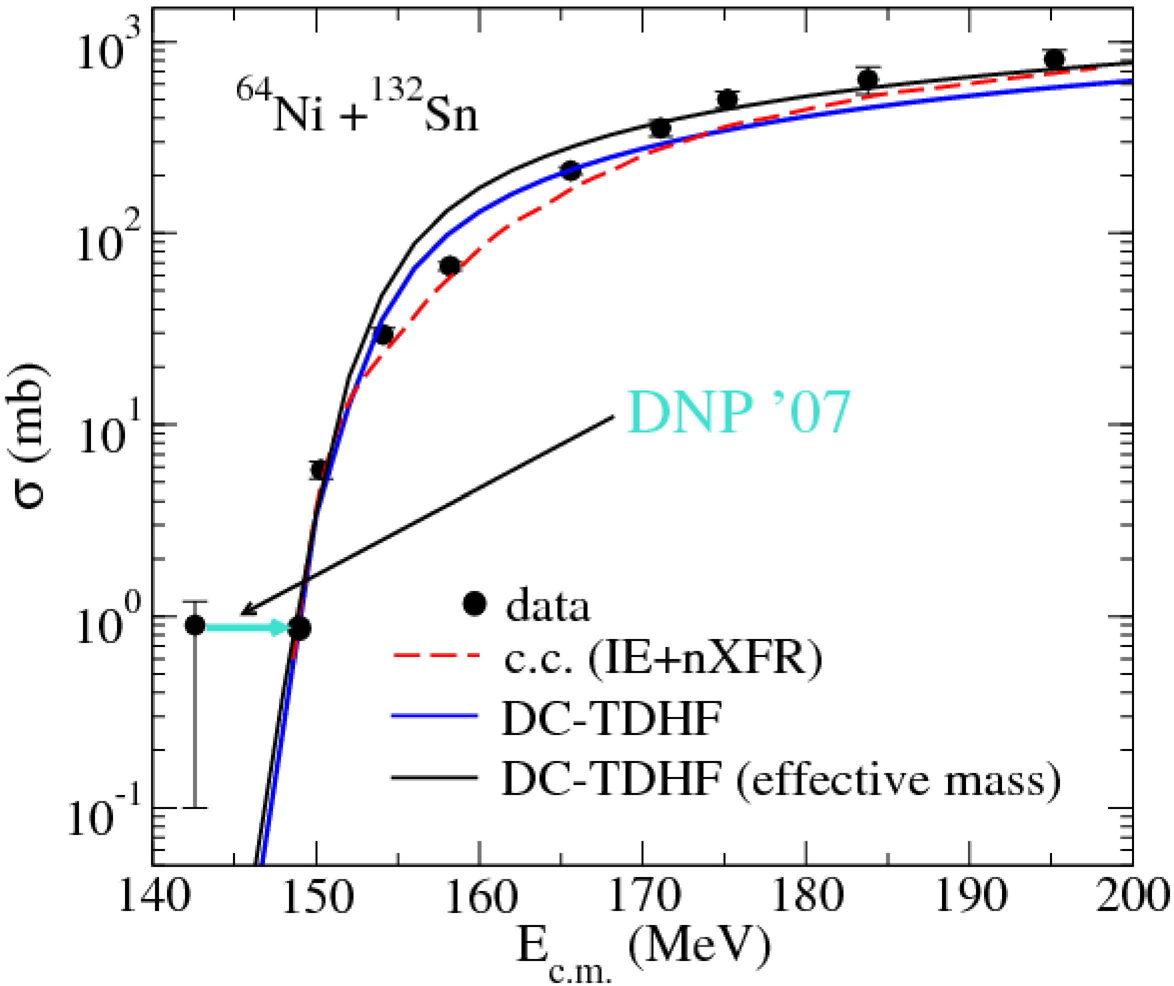
$^{64}Ni + ^{132}Sn$ Fusion Cross-Section

Umar and Oberacker, Phys. Rev. C76, 014614 (2007)

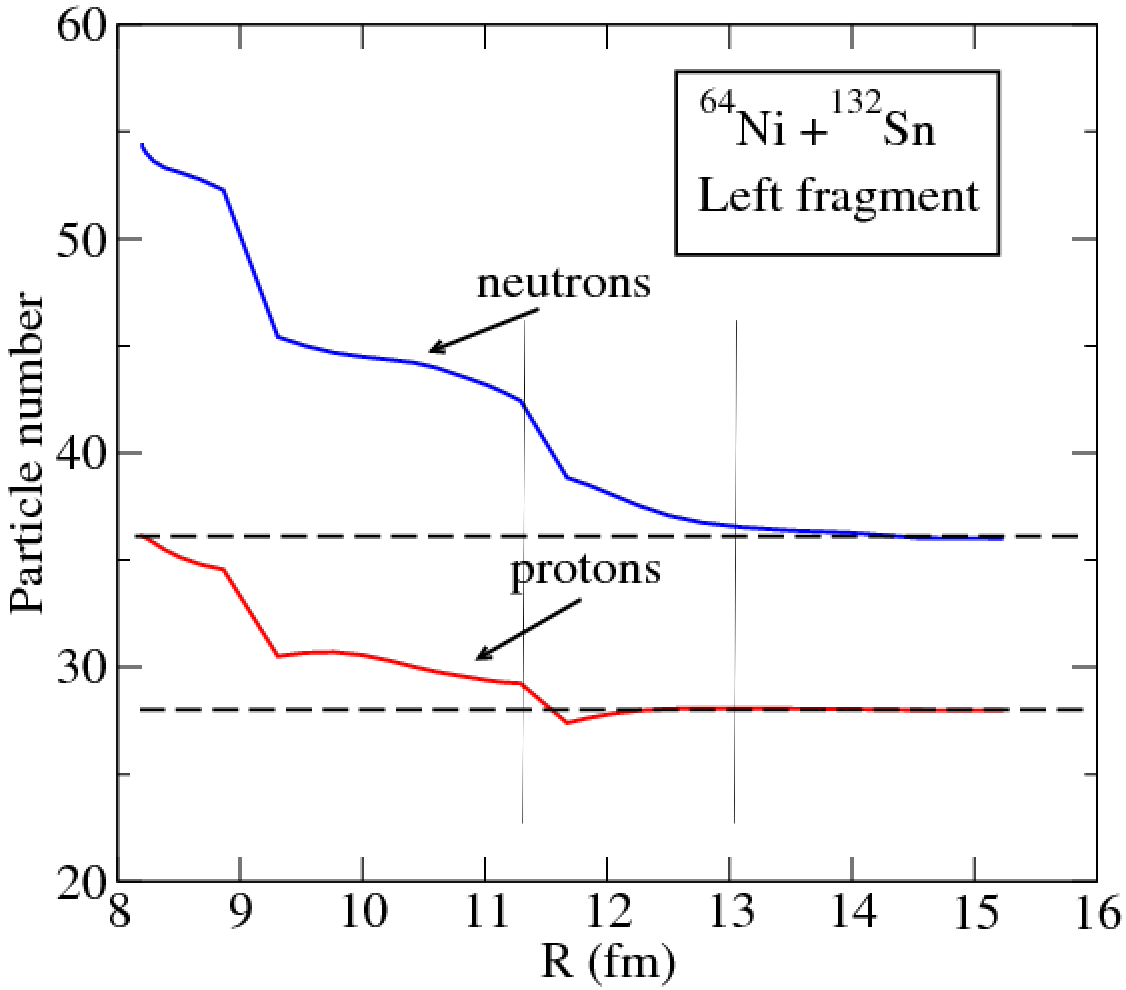


- Use IWBC
- Average over orientations

$$\sigma_f(E_{c.m.}) = \int_0^1 d\cos(\beta)\, P(\beta)\, \sigma(E_{c.m.}, \beta)$$

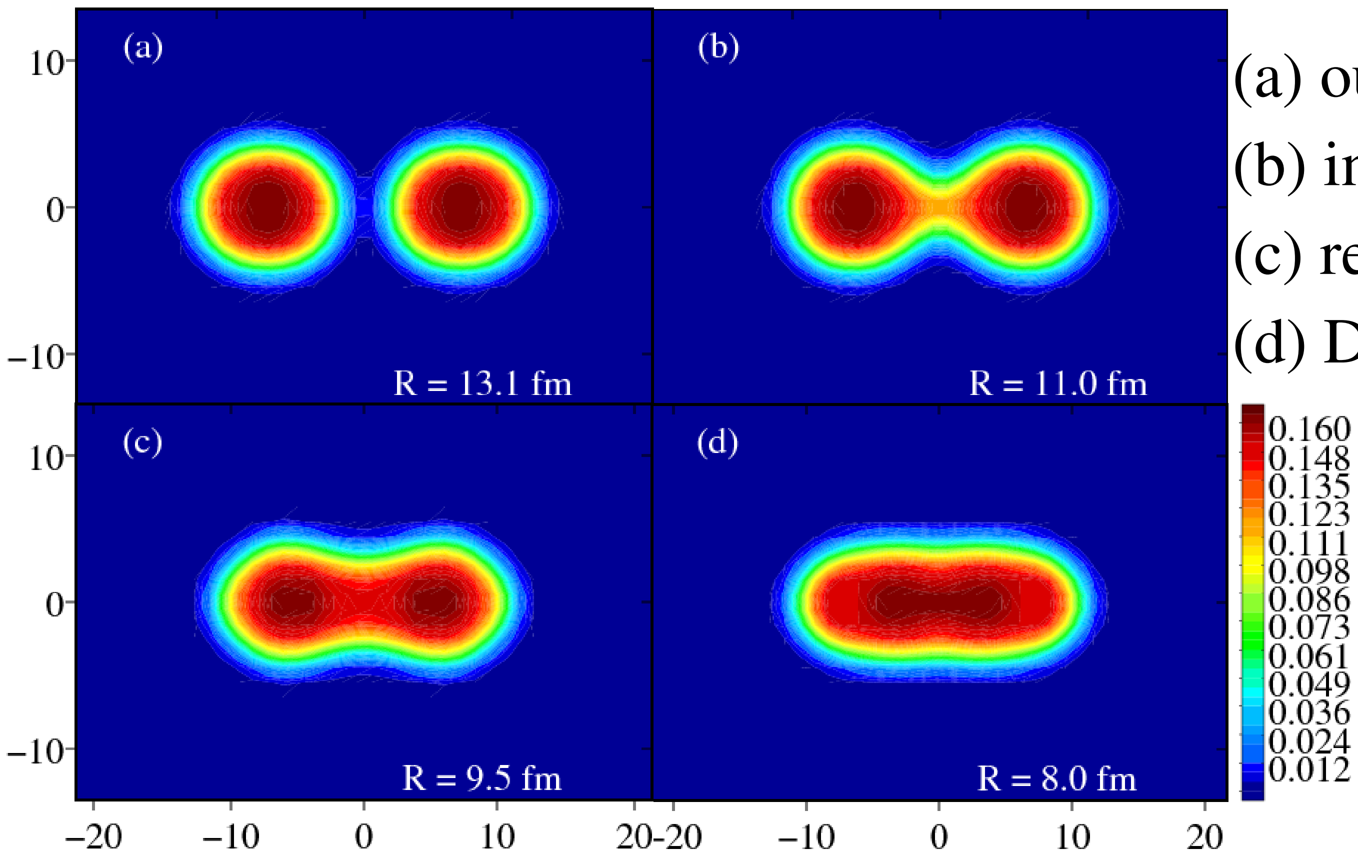
<u>Exp. Data</u>
J.F. Liang *et al.*,
PRL 91, 152701 (2003)
PRC 75, 054607 (2007)

$^{64}Ni + {}^{132}Sn$ Particle Transfer



^{64}Ni + ^{64}Ni Fusion (β_1=90^o, β_2=90^o)

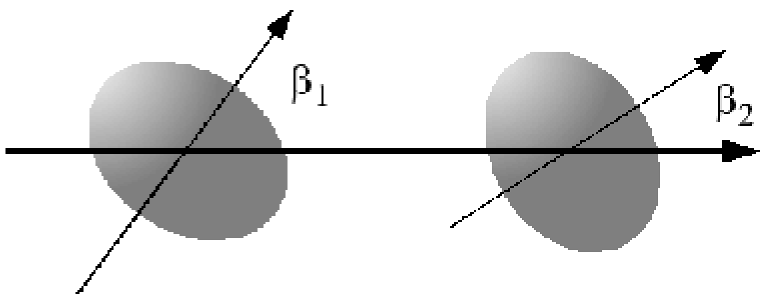
- Interesting neutron rich identical system



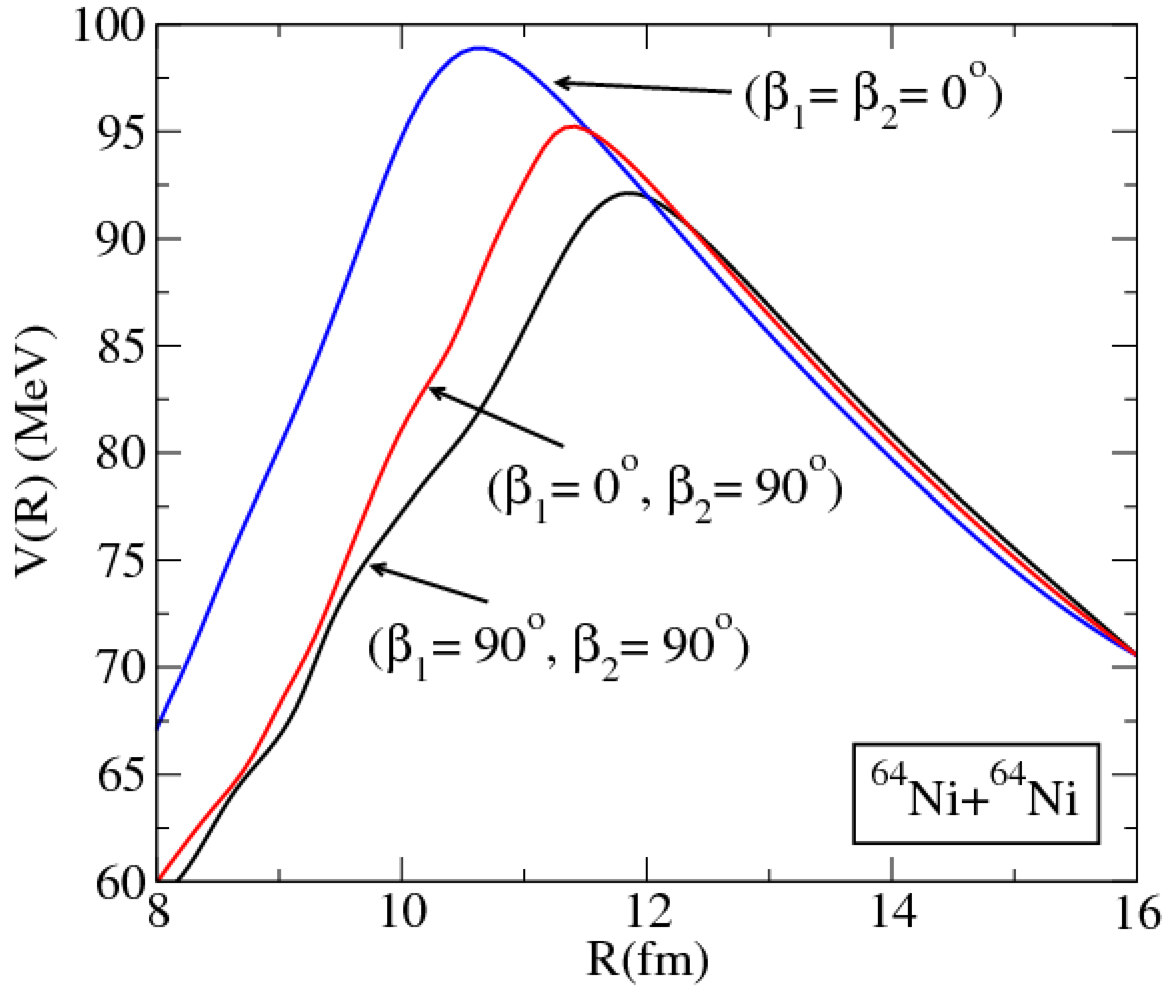
(a) outer turning point
(b) inner turning point
(c) reorientation of the core
(d) Density at the minimum

Turning points for E_{cm} = 86 MeV

^{64}Ni + ^{64}Ni Limiting Barriers

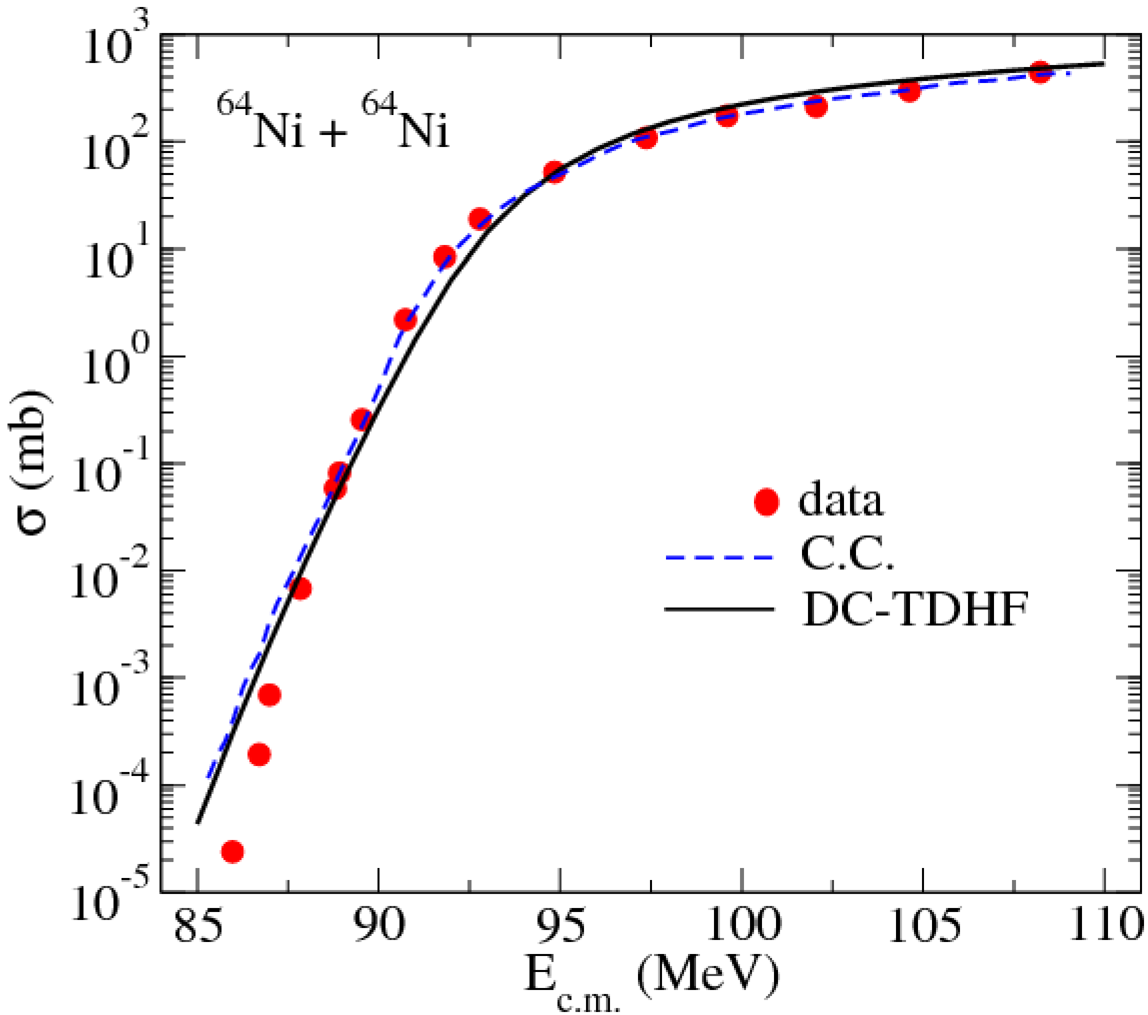


- Variation with Euler angle α_i is negligible!



arXiv:0709.3972v1 [nucl-th]

^{64}Ni + ^{64}Ni Fusion Cross-Section



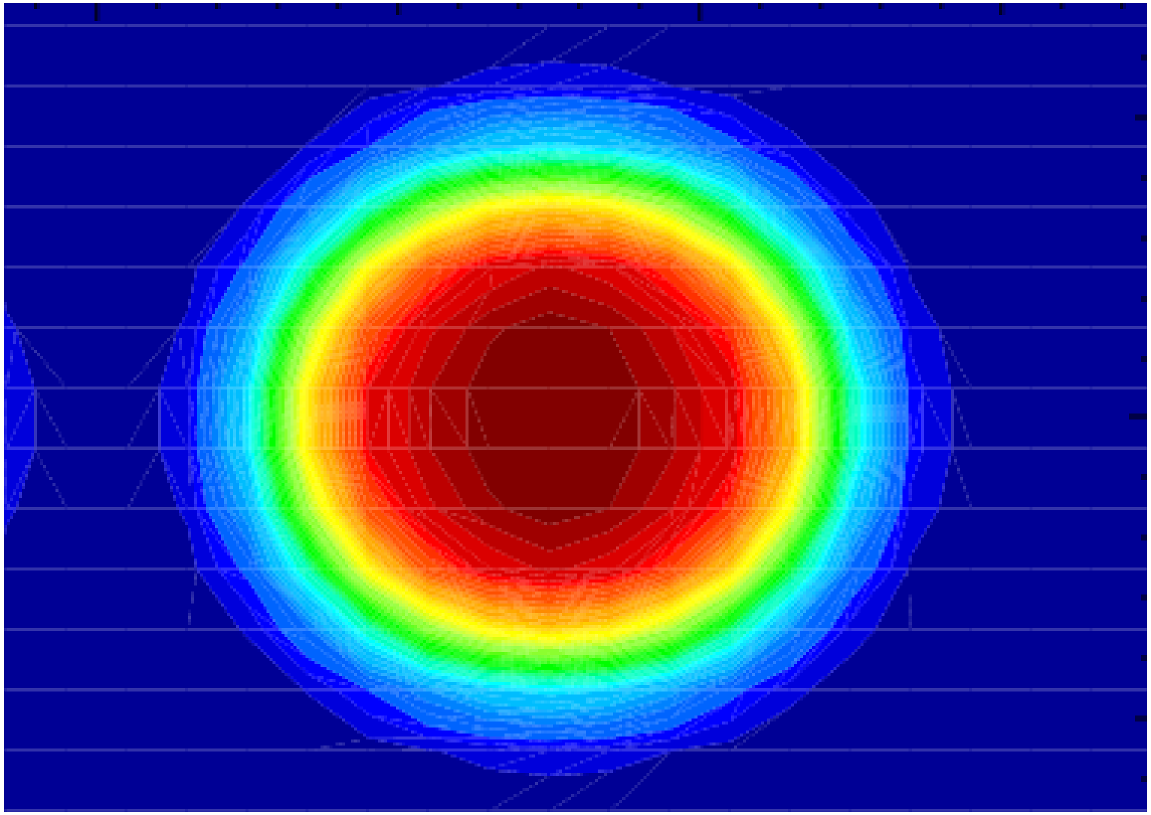
- Problem at low energies

- Compression potential
  Mişicu, Esbensen, PRL 96, 112701 (2006)
- Modify inner turning point
  Ichikawa, Hagino, Iwamoto,
  PRC 75, 064612 (2007)

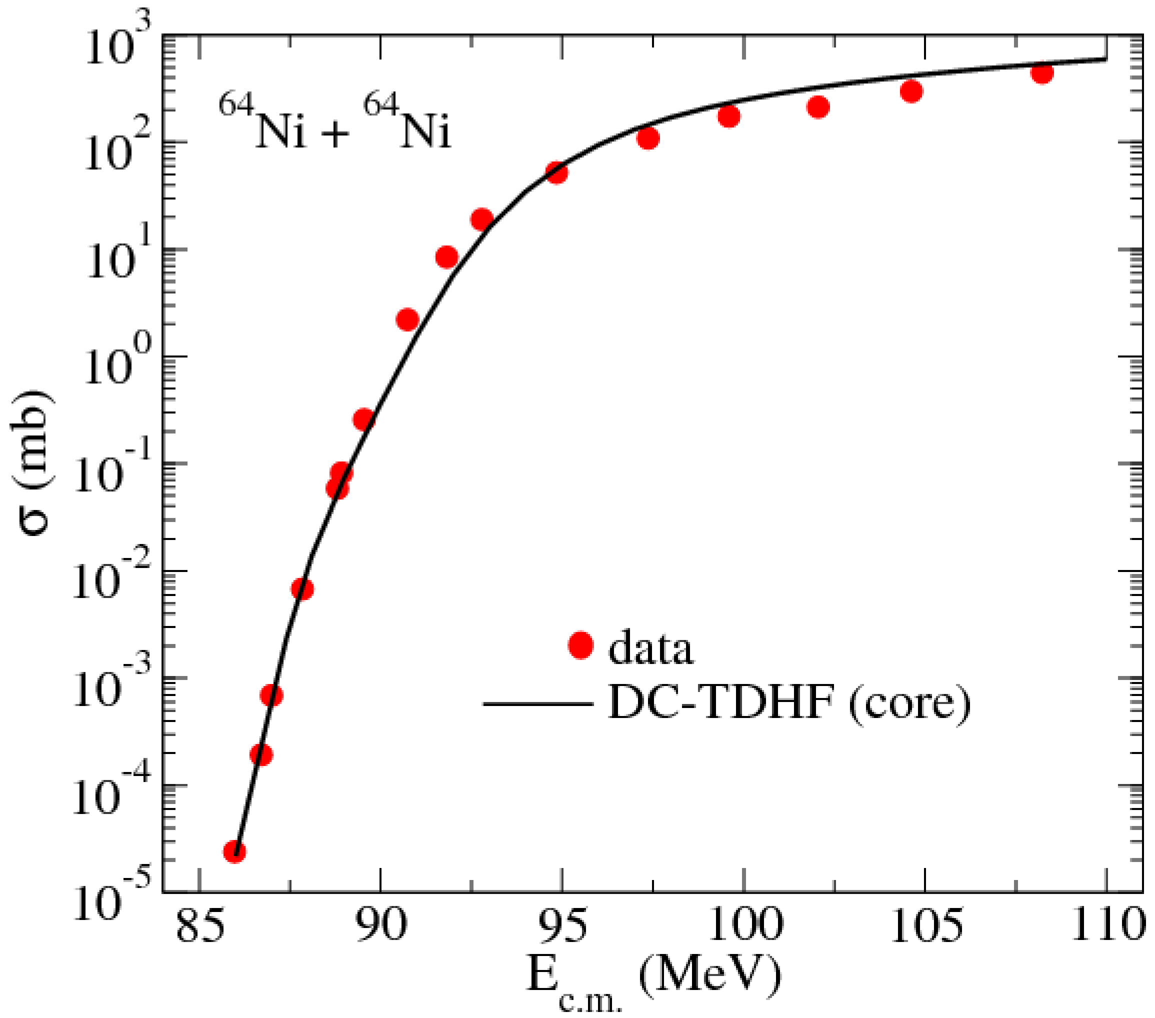
<u>Exp. Data</u>
C.L. Jiang *et al.*,
PRL 93, 012701 (2004)

Conjecture - Skin versus Core Orientation

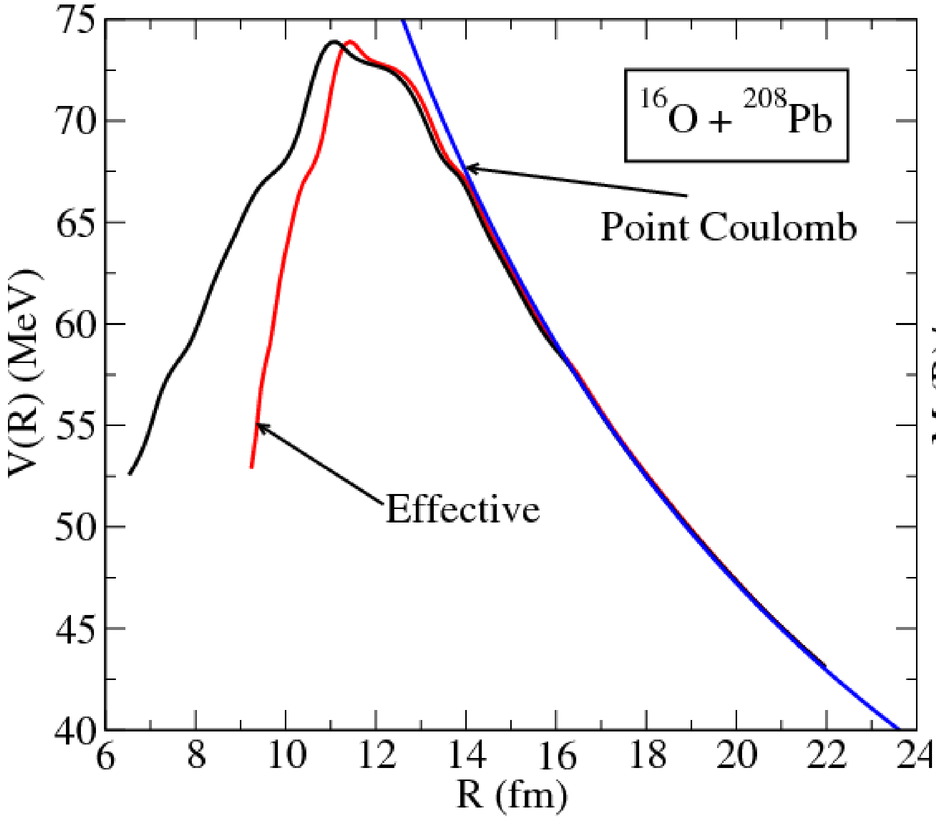
- Neutron rich systems have extended outer skins
- Orientation of core in ^{64}Ni perpendicular to outer surface
- Top of the barrier primarily determined by outer surface
- Lower energies imply larger overlaps for inner turning point
- Ambiguity in which orientation to choose in angle averaging

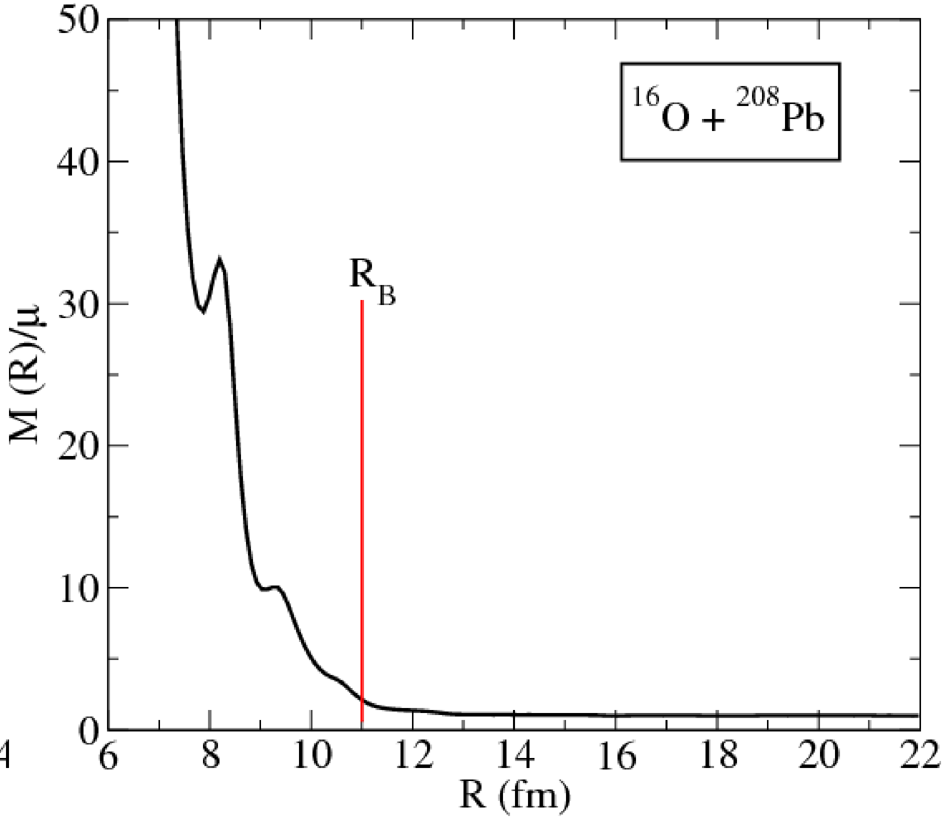


For Low Energies use Core Orientation

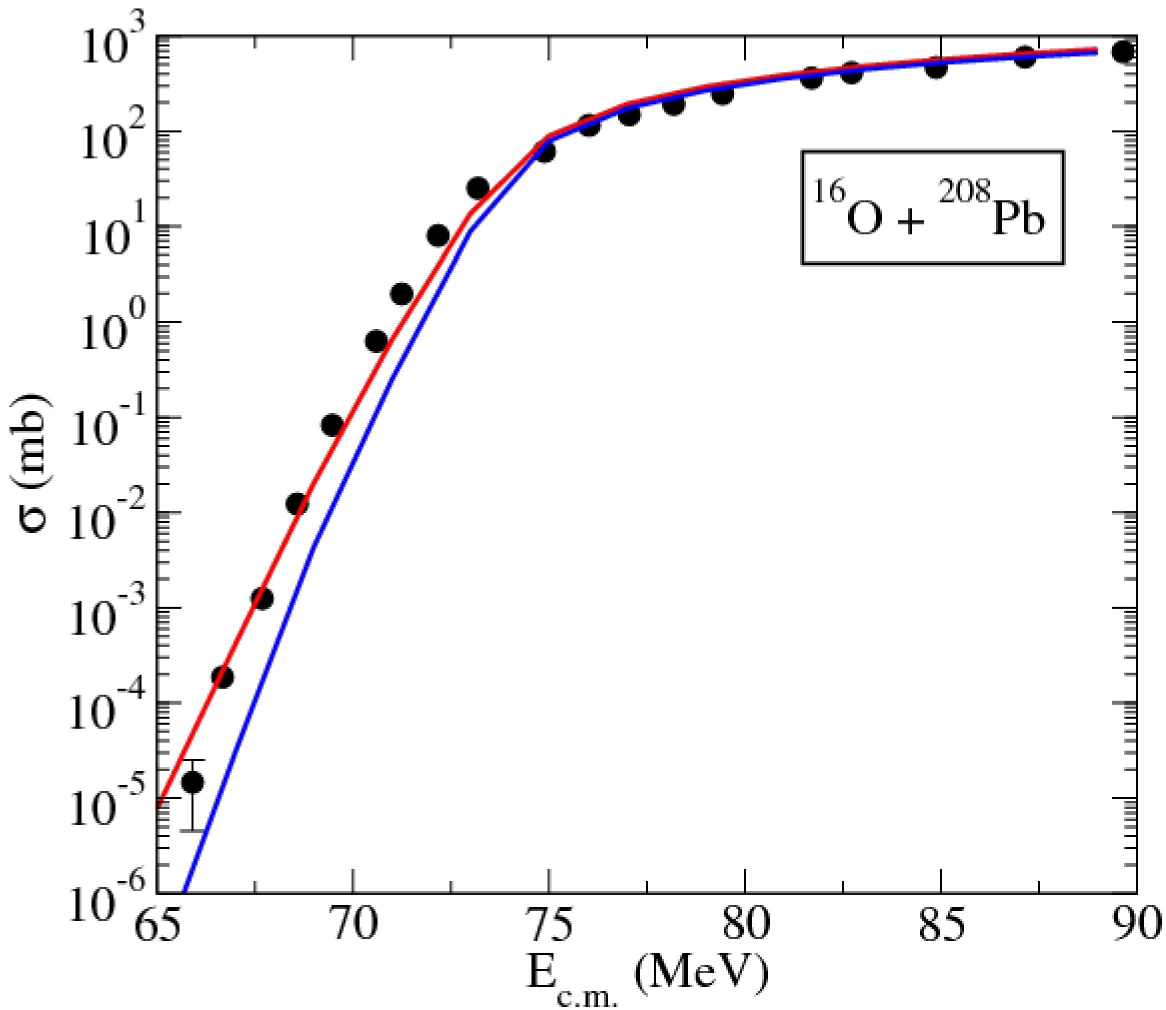


^{16}O + ^{208}Pb Fusion Barrier - Preliminary





$^{16}O + ^{208}Pb$ Fusion Cross - Section



M. Dasgupta et al., PRL 99, 192701 (2007)

Summary and Conclusions

- Density-constrained TDHF provides a microscopic method for extracting ion-ion potentials that incorporates the mean-field dynamics
- The method is not limited to TDHF, can be used in any dynamical approach (TDHFB, time-dependent DFT, and beyond)
- Use is limited by the extent of how well the nucleus is described by Hartree-Fock and the effective interaction.

TDHF and C.M. Corrections - I

- Initial state has no cross-channel coupling

- Each nucleus is boosted by multiplying with a phase factor

$$\Phi_j \rightarrow \exp(i\boldsymbol{k}_j \cdot \boldsymbol{R}_j)\Phi_j \quad \text{and} \quad \boldsymbol{R}_j = \frac{1}{A_j}\sum_{i=1}^{A_j} r_i$$

- They are placed in the center-of-mass of the entire system

$$A_1\boldsymbol{R_1} + A_2\boldsymbol{R_2} = 0$$

- Total initial momentum has expectation zero and relative momentum is $2\hbar\boldsymbol{k}$

- Physics of the collision is determined by <u>relative momenta</u>

TDHF and C.M. Corrections - II

- Initial states are interpreted as wave packets whose centers of momentum are shifted by **k**, and travel without spreading
- We are not free to change the intrinsic wave packet contained in the Slater determinant
- Various estimates for the uncertainties/spread is available
- Calculations most accurate for early stages of time-evolution
- DC-TDHF is not sensitive to spread in beam-energy
- We have already shown agreement with point Coulomb and double-folding potential for O+O

TDHF and C.M. Corrections - III

- Run TDHF with simple c.m. Corrections included in static and dynamic
- Perform DC-TDHF calculation – result is mostly a constant shift