Density Matrix Renormalization Group Approach for many body open quantum systems

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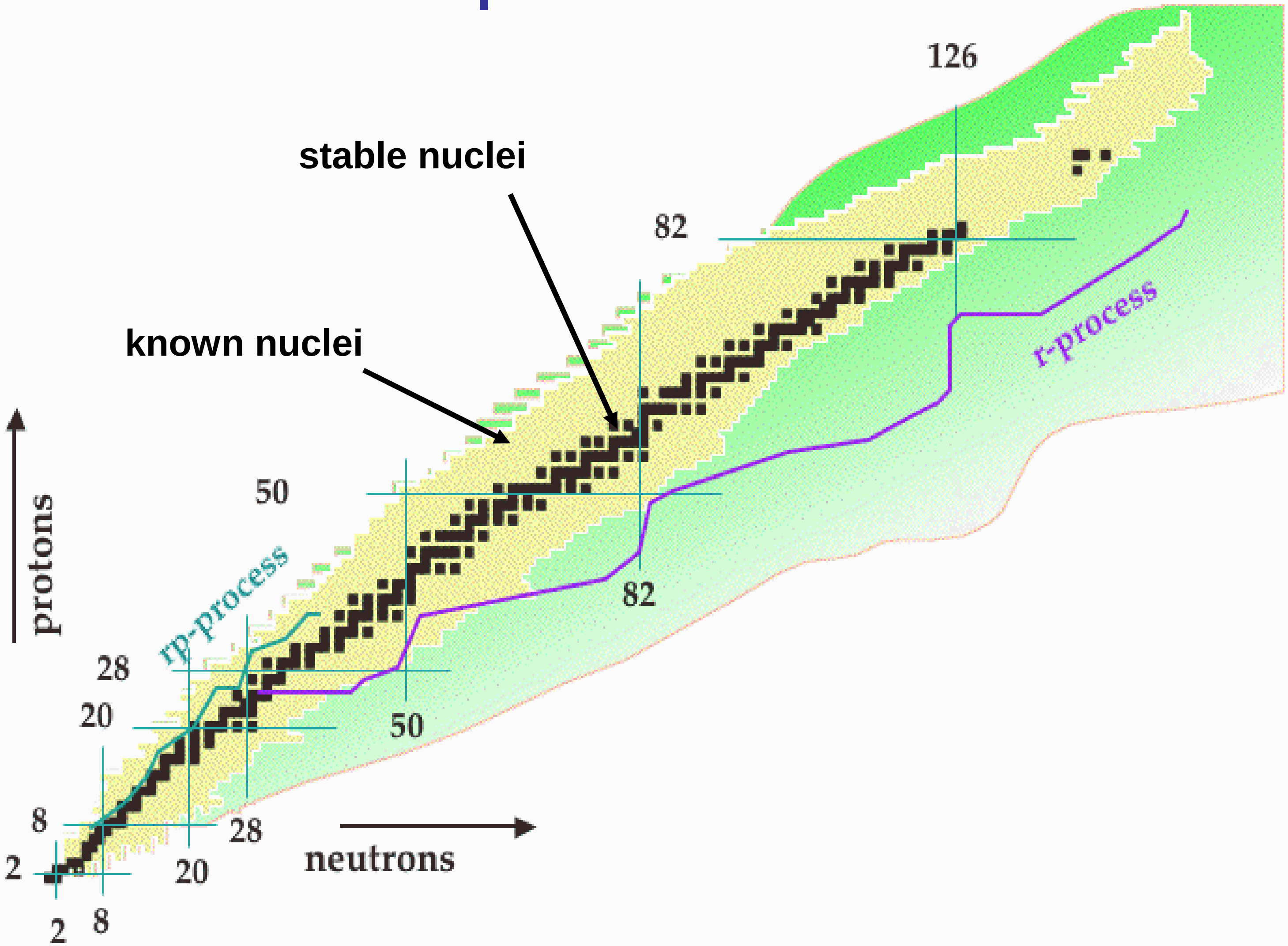
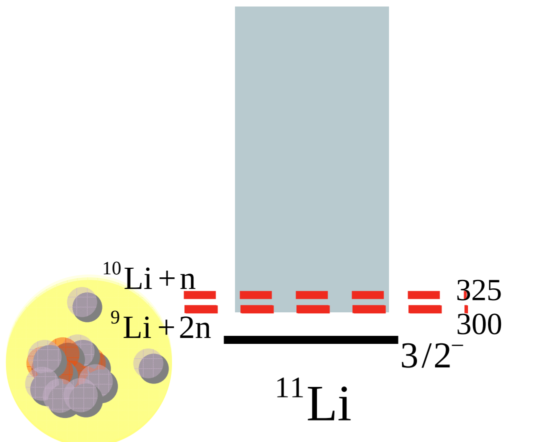
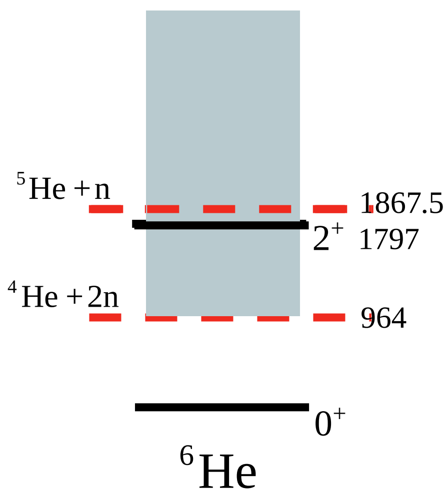
Nuclear Landscape

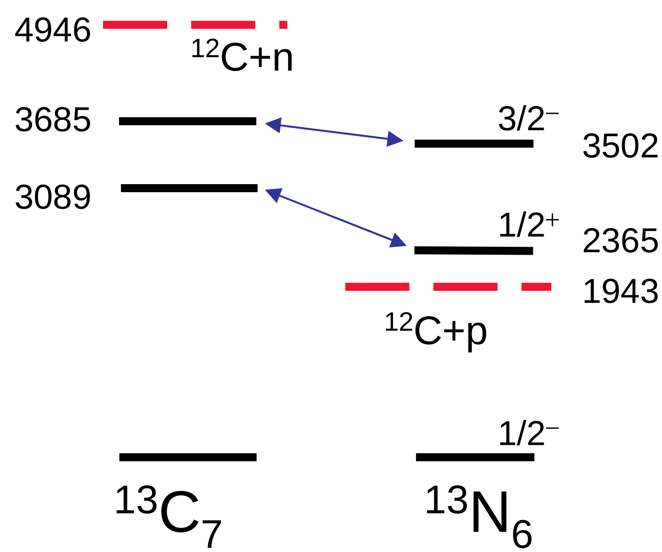


Halo structures

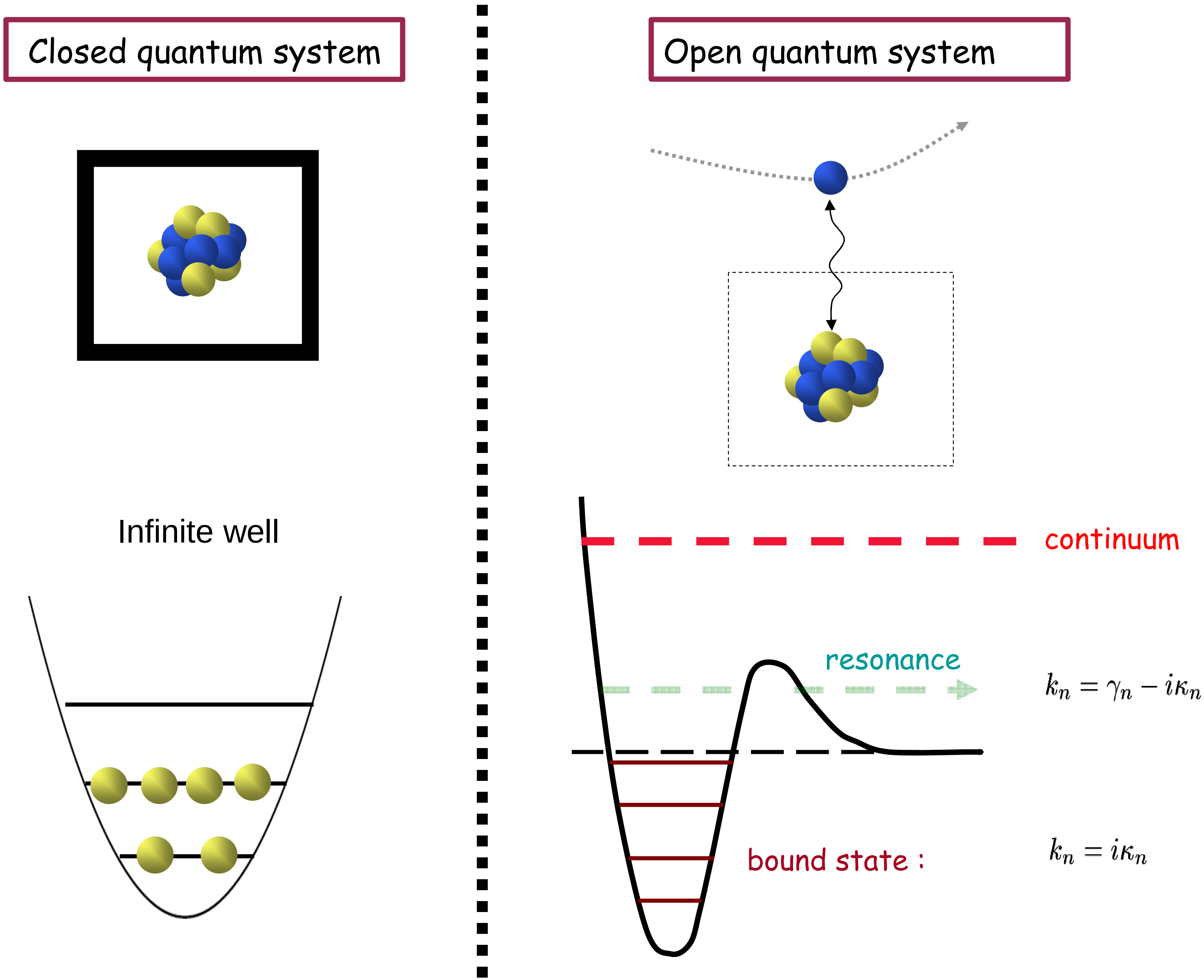




Thomas-Ehrmann effect

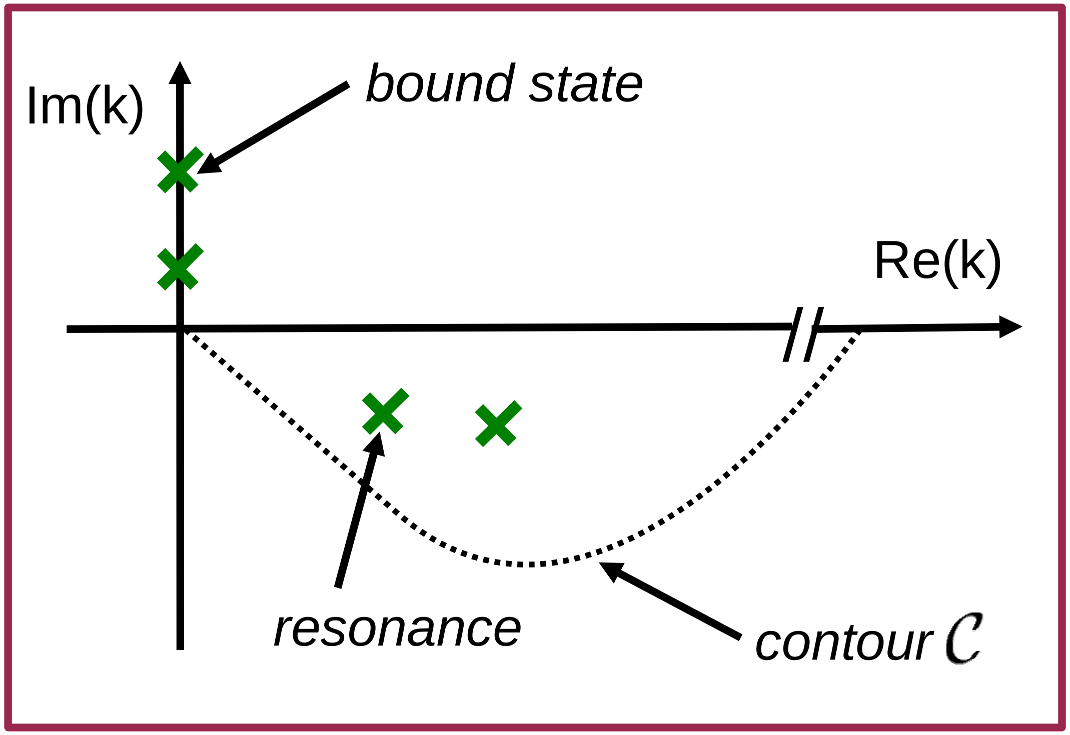


Spectra and matter distribution modified by the proximity of scattering continuum


Closed quantum system
Open quantum system
Infinite well
continuum
resonance
$k_n = \gamma_n - i\kappa_n$
bound state :
$k_n = i\kappa_n$

Gamow Shell Model

N. Michel et al., PRL. 89, (2002) 042502
R. Id Betan et al., PRL. 89, (2002) 042501
G. Hagen et al, PR C71, (2005) 044314
J.R et al., PRL. 97, (2006) 110603



$$\boxed{\sum_{\text{pole}} |u_n\rangle\langle\tilde{u}_n| + \int_{\mathcal{C}} dk |u_k\rangle\langle\tilde{u}_k| = 1}$$

(Berggren completeness relation)

$u(r) \sim H^{+}_{l,\eta}(kr)$ (bound, resonant state)

$u(r) \sim C^{+}H^{+}_{l,\eta}(kr) + C^{-}H^{-}_{l,\eta}(kr)$ (complex-continuum state)

- intrinsic hamiltonian :

$$\boxed{H = \sum_{i=1}^{n}\left[\frac{\mathbf{p_i}^2}{2\mu} + U_i\right] + \sum_{j>i=1}^{n}\left[V_{ij} + \frac{1}{A_c}\mathbf{p_i}\mathbf{p_j}\right]}$$

- discretization of contour $\mathcal{C}$:

$$\boxed{\sum_{\mathbf{pole}} |u_n\rangle\langle\tilde{u}_n| + \sum_i |u_i\rangle\langle\tilde{u}_i| \simeq 1}$$

- complex symmetric hamiltonian

- need a discretization precise enough to have completeness !

Density Matrix Renormalization Group (DMRG)

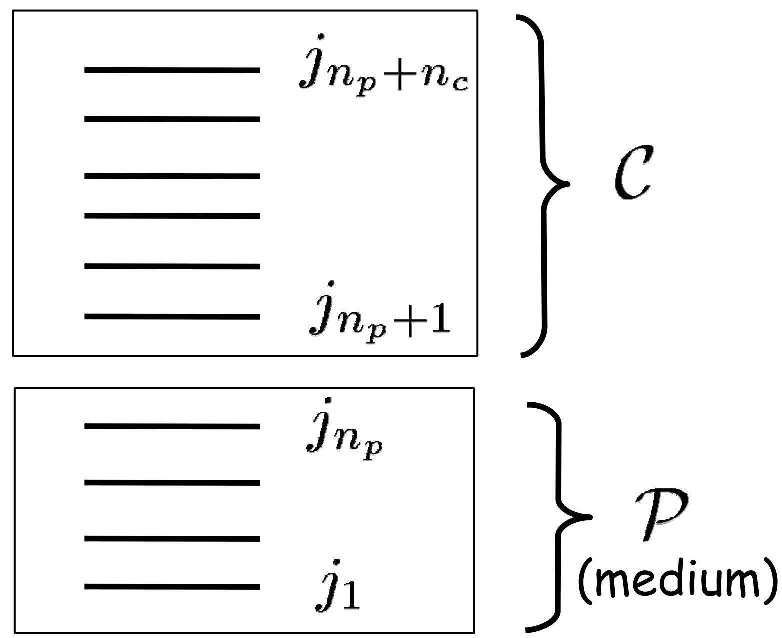
S.R. White, PRL 69 (1992) 2863
O. Legeza et al., PRB 67 (2003) 125114
U. Schollwock, RMP 77 (2005) 259
S. Pittel et al.,PRC73 (2006) 014301

application to lattice models, spin chain, quantum dots, atomic nuclei.......

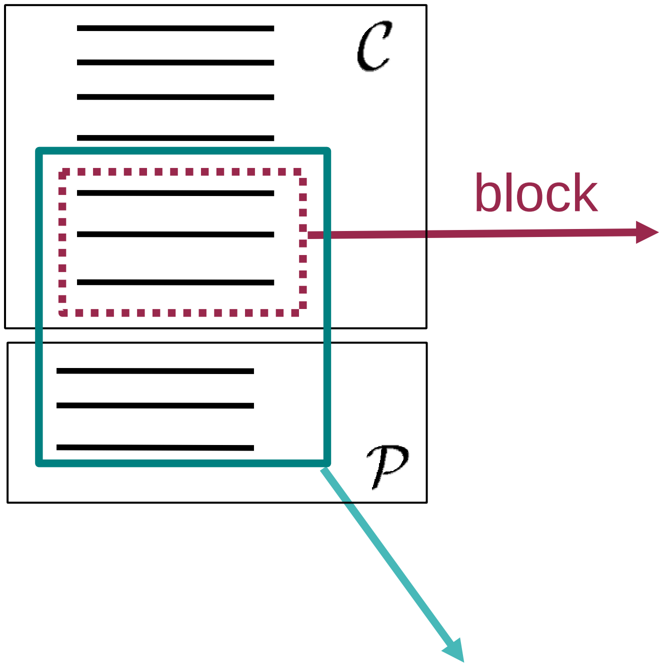
GSM+DMRG

$$|\Psi\rangle^J = \sum_{p,c} \Psi_{pc} \left[|p\rangle^{J_p} |c\rangle^{J_c}\right]^J$$

DMRG $\longrightarrow$ truncation in $\mathcal{C}$



- construction of a block in $\mathcal{C}$:



$$\boxed{\begin{array}{l} \{|c\rangle\} \equiv |(j_{n_p+1})^{L}_{n_p+1}..(j_i)^{L_i}_{n_i}..\,;K..K_i..\,\rangle^{J_p} \\ a^\dagger, \quad [a^\dagger a^\dagger]^\Gamma \;, \;.\;.\;. \end{array}}$$

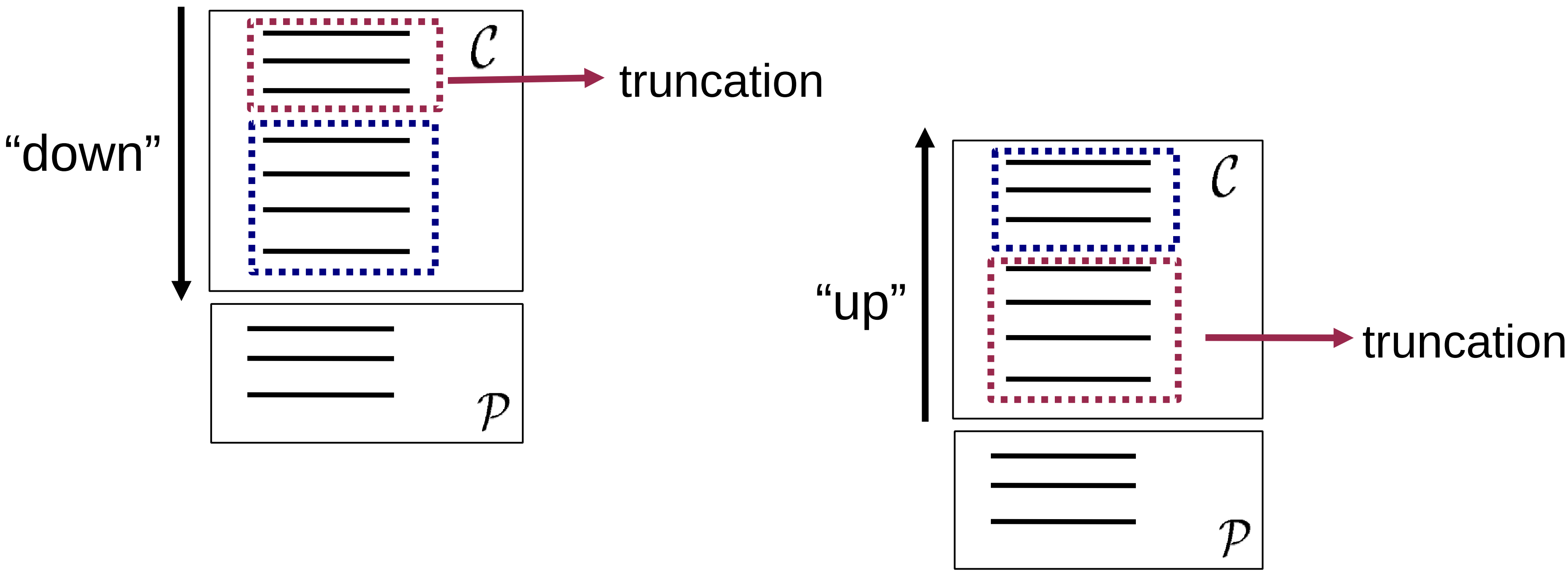
- diagonalization in the superblock : $\boxed{|\Psi\rangle^J = \sum_{p,c} \Psi_{pc} \left(|p\rangle^{J_p}|c\rangle^{J_c}\right)^J}$

- truncation with the density matrix :

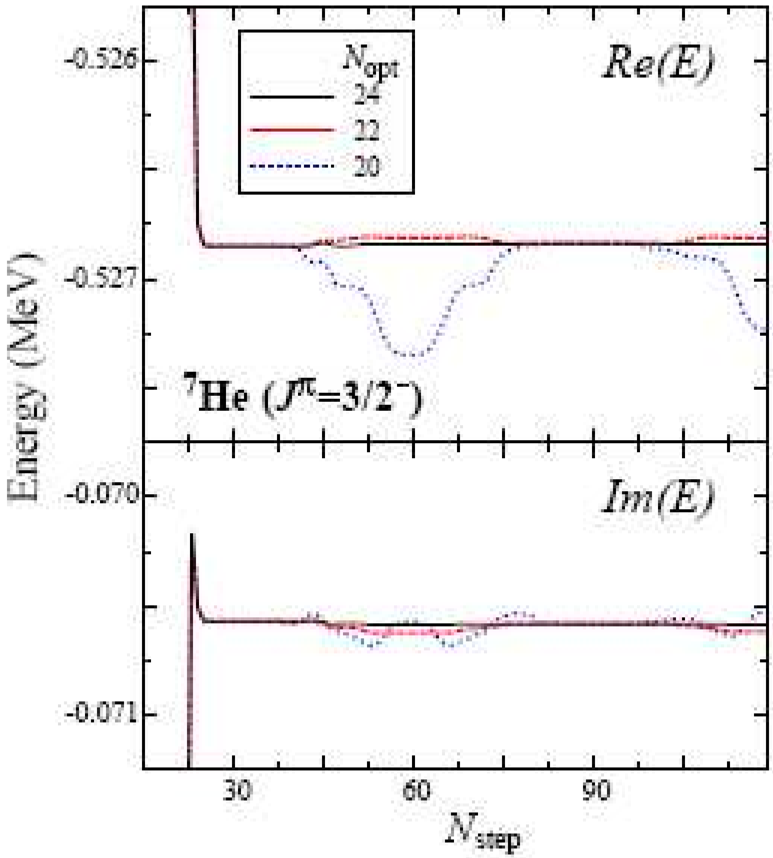
$$\boxed{\rho^{J_c}_{c,c'} = \sum_p \Psi_{pc}\Psi_{pc'}}$$

eigenstates with “largest” eigenvalues are kept

sweeping phase



sweeping until convergence is reached ....



$^7\mathrm{He}$, $J^\pi = 3/2^-$

- ✗ alpha core + 3 neutrons
- ✗ Woods-Saxon + Surface Gaussian Interaction :

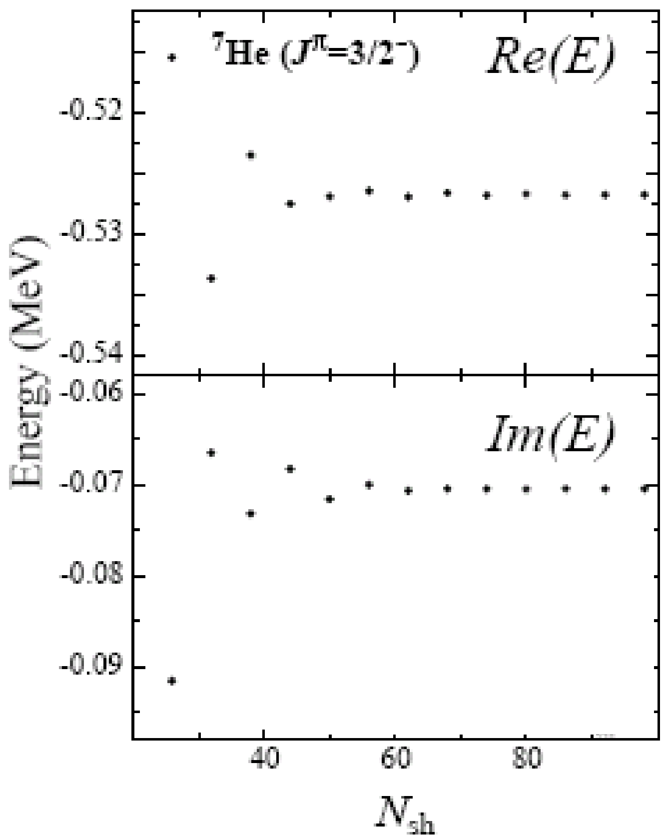
$$V_{i,j}^{J,T} = V_0(J,T)\exp\left[-\left(\frac{\mathbf{r_1}-\mathbf{r_2}}{\mu}\right)^2\right]\delta(|\mathbf{r_1}|+|\mathbf{r_2}|-2R_0)$$

- ✗ pole space : $0p_{3/2}$,$0p_{1/2}$
- ✗ continuum space : p3/2 ,p1/2 (30 shells each)

Shell Model dimension=83948
largest matrix in DMRG=1143

(J.R *et al.*, Phys.Rev.Lett. 97 (2006) 110603)

$^7\mathrm{He}$, $J^\pi = 3/2^-$



(N_{opt} = 22)

Convergence of the real (top) and imaginary part (bottom) of the g.s. energy as a function of the total number of shells N_{sh} .

total dimension ($\propto N_{sh}^3$) :

6149 ⟷ 332171

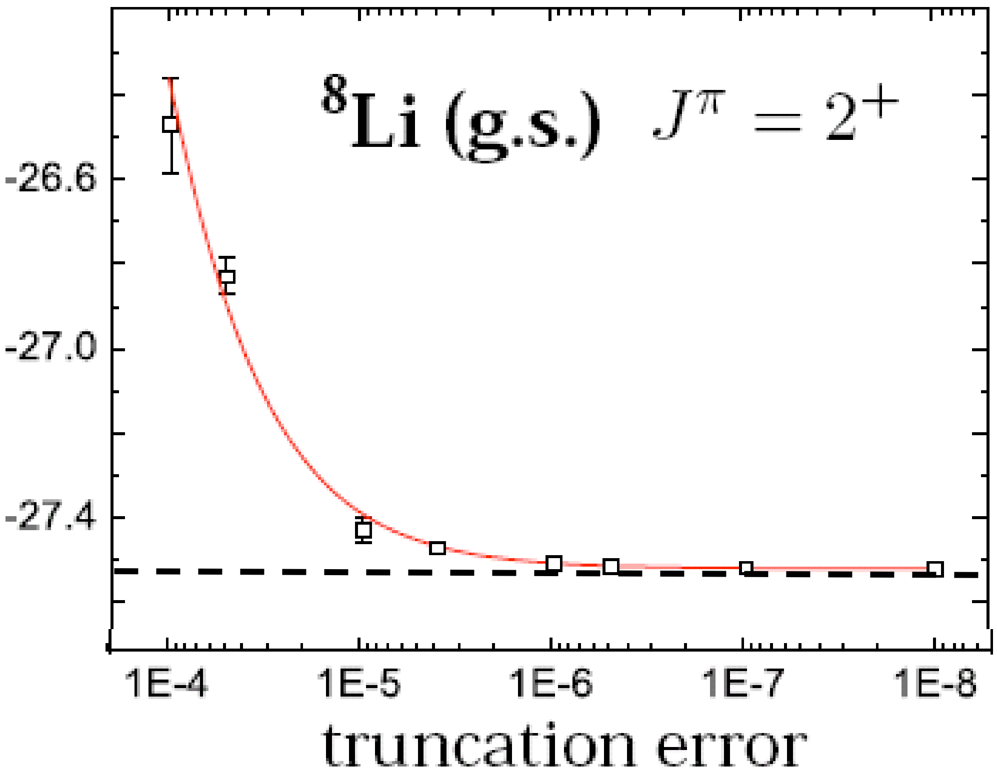
DMRG dimension:

941 ⟷ 1001

Very good scaling with number of shells !

'Dynamic' truncation

$$\boxed{\left|1 - \mathcal{R}e\left(\sum_{i=1}^{N_\rho} w_i\right)\right| < e}$$



✗ pole space : $0p_{3/2}$, $0p_{1/2}$ (p/n)

✗ continuum space :

$\left\{ \begin{array}{l} p_{3/2}, p_{1/2} \text{ complex continuum} \\ s_{1/2}, d_{5/2} \text{ real continuum} \end{array} \right.$

✗ 52 shells in total

Shell Model dimension:190616

e=1E-6:

6 % of total dimension

precision ~13 keV

e=1E-8:

16 % of total dimension
precision ~0.24 keV

Conclusion and Outlook

- Gamow Shell Model+DMRG in J-scheme

-> very good scaling with number of shells !

- The code is now parallelized

-> study of heavier systems : ^{11}Li …

- implementation of realistic interactions.