Equilibrium Structural Evolution

The value of complementary perspectives

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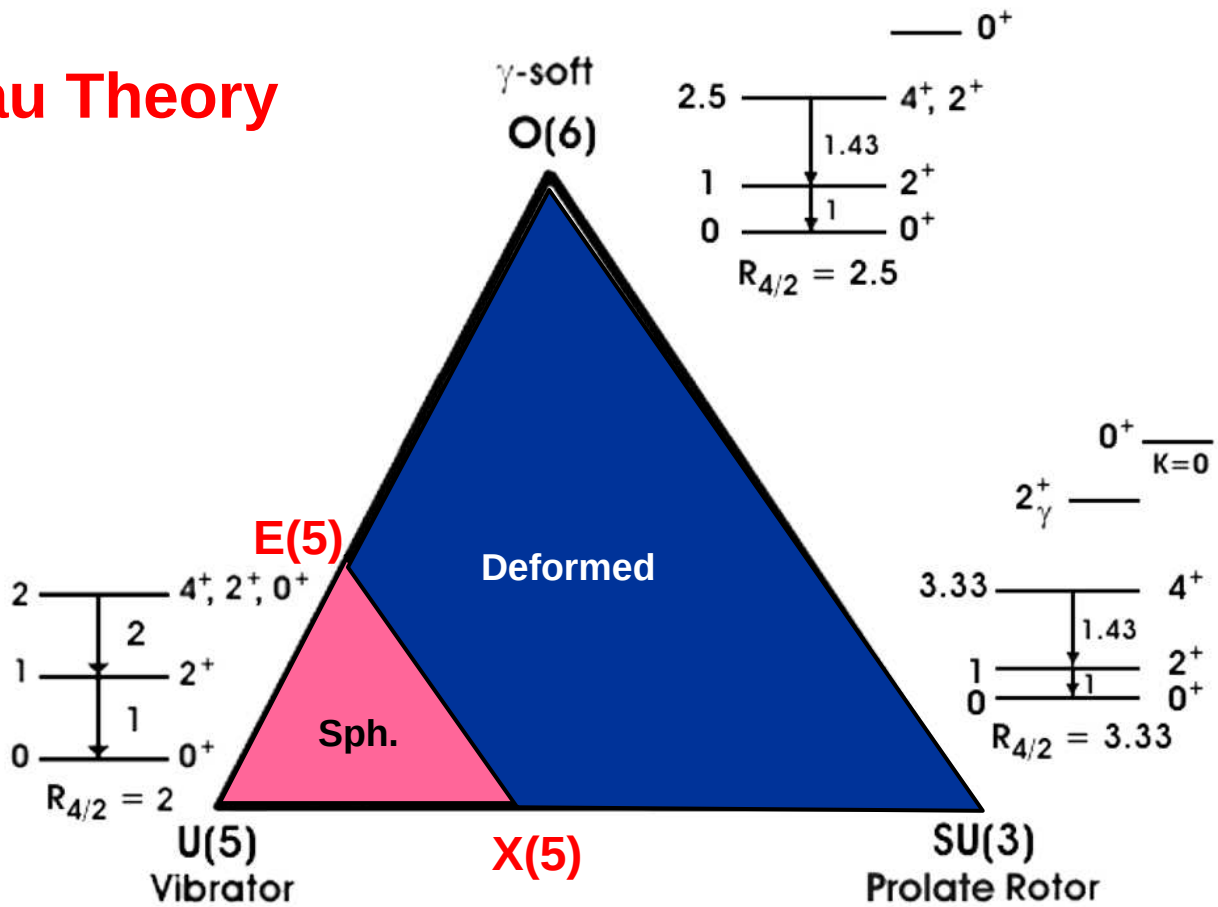
Microscopic and Macroscopic Approaches

Micro	Macro
Single part. D.o.F.	Collective D.o.F.
Large basis wave shell model functions	Simple configurations in collective basis
Particle quantum numbers (configurations, seniority)	Many-body quantum numbers (e.g., coll. modes, representation labels, rot. ang. mom., K)

Need to look at both perspectives: what (symmetries, geometry) and why (shell structure, interactions – pairing, pn)

Classifying Structure -- The Symmetry Triangle

Landau Theory



Most nuclei do not exhibit the idealized symmetries but rather lie in transitional regions. Mapping the triangle.

Critical Point Symmetries

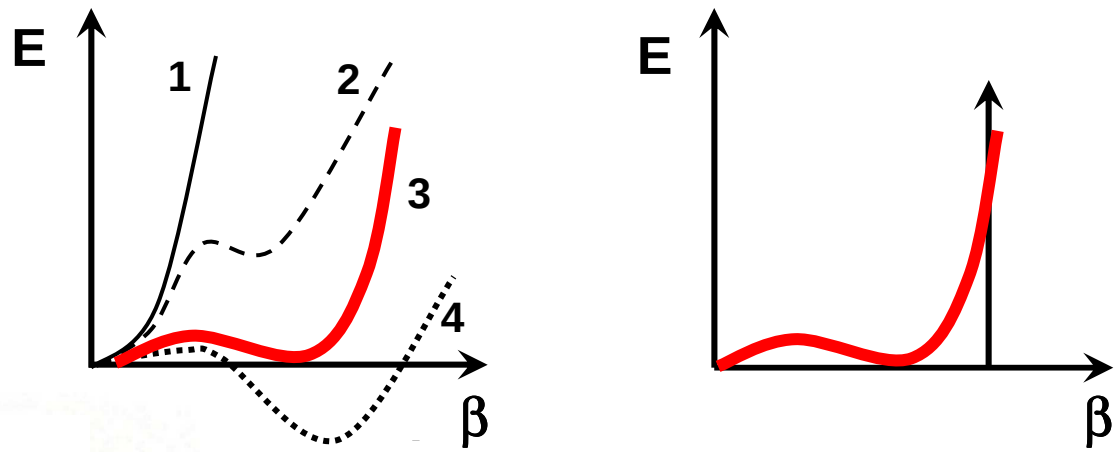
Iachello

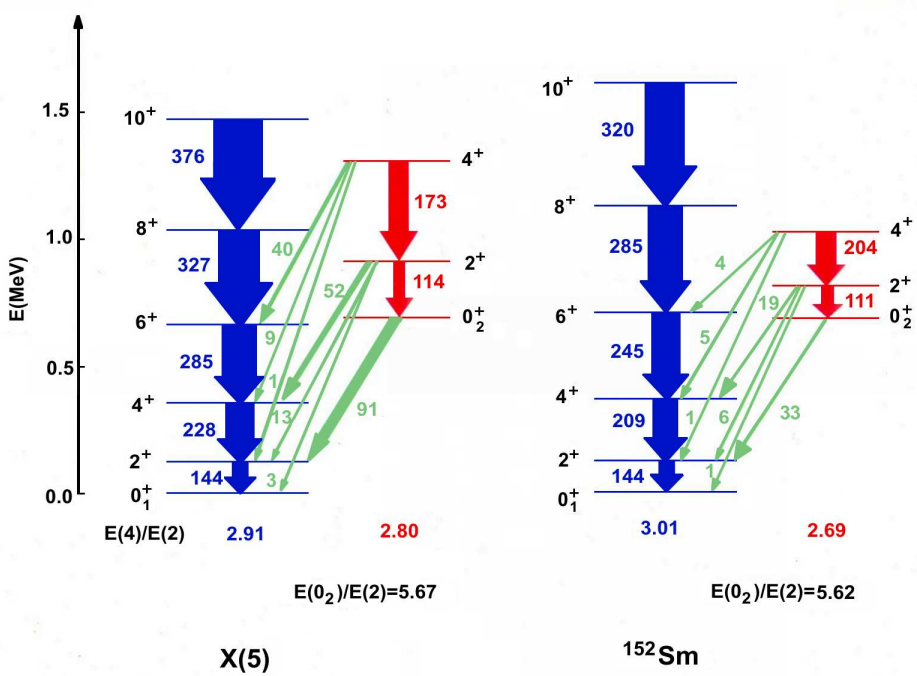
X(5)

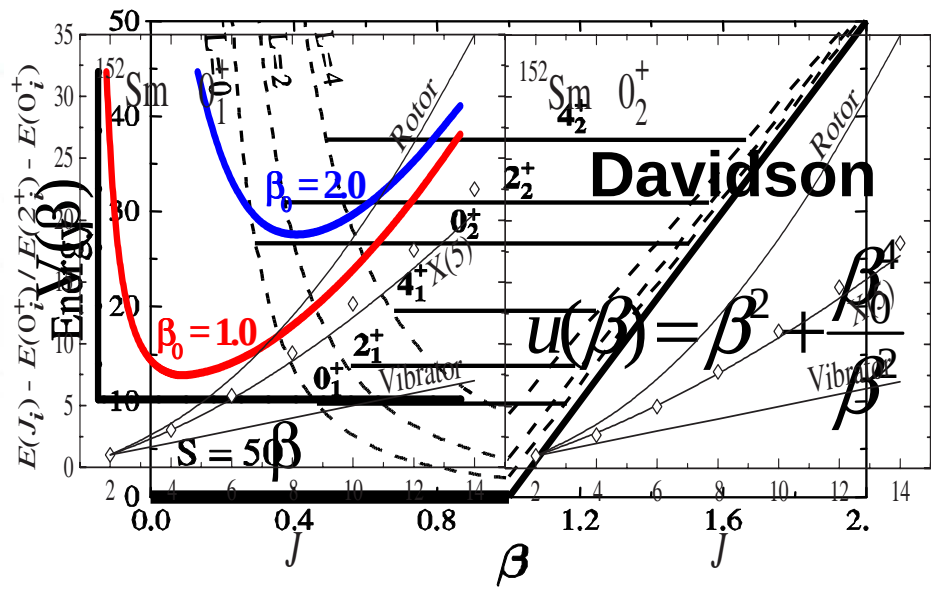
First Order Phase Transition – Phase Coexistence

Energy surface changes with valence nucleon number

Bessel equation





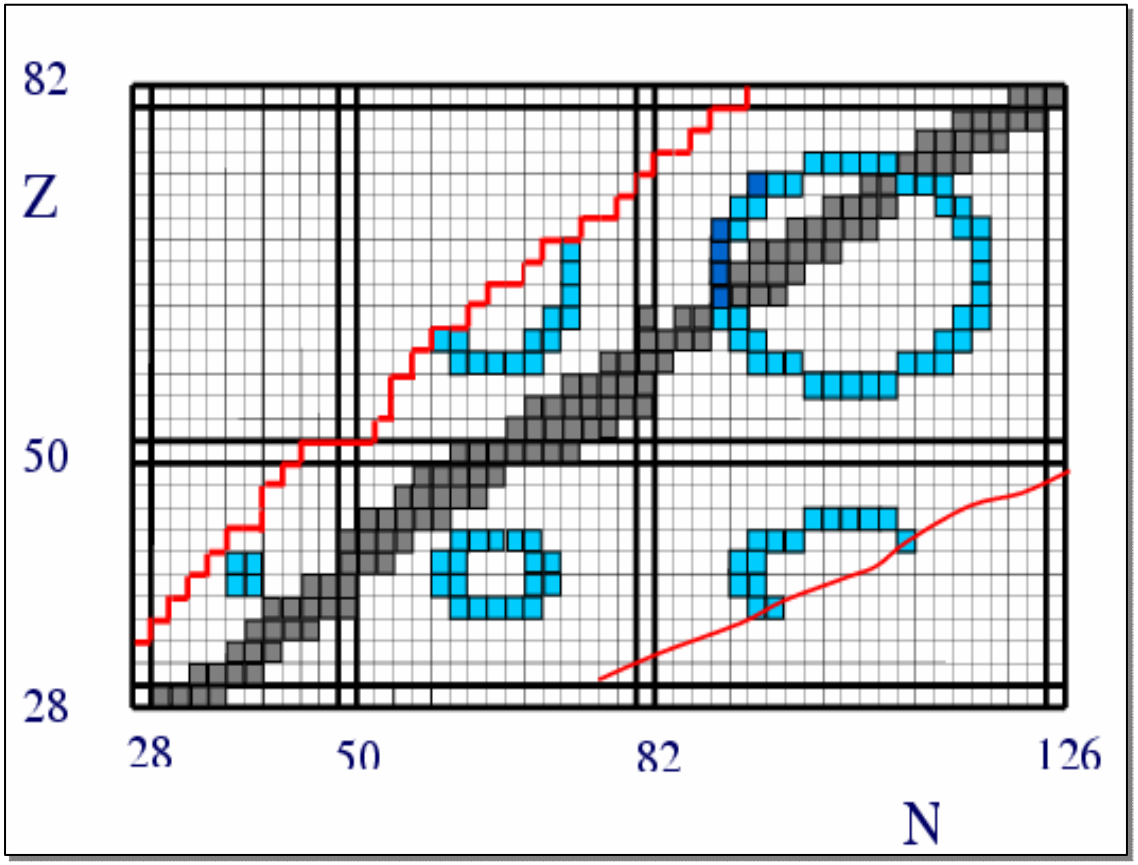


Contours define locus of possible X(5) nuclei and enclose regions of deformation

p-n / pairing

$$P = \frac{N_p N_n}{N_p + N_n} \sim \frac{p - n}{pairing}$$

P ~ 5



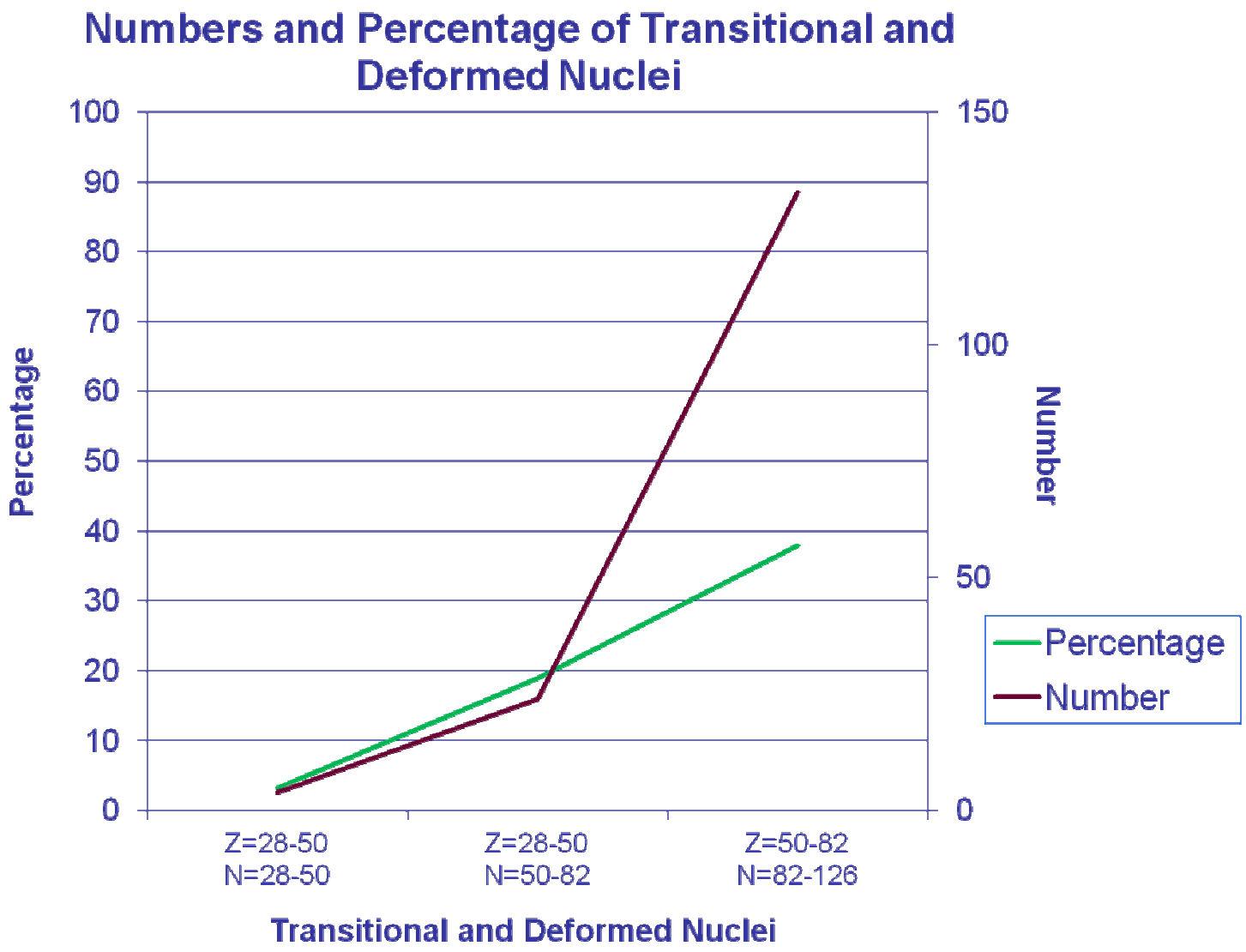
Global mass-dependence of structure:

Why the structure of heavy nuclei is qualitatively different from light nuclei

- **Competition of residual interactions: pairing and p-n compete differently in heavy nuclei than in light nuclei.**

WHY?

- **Higher particle angular momenta, greater shell model degeneracies, UPO**
- **Larger shell size allows the development of larger and more varied regions of deformed nuclei and new forms of collectivity**

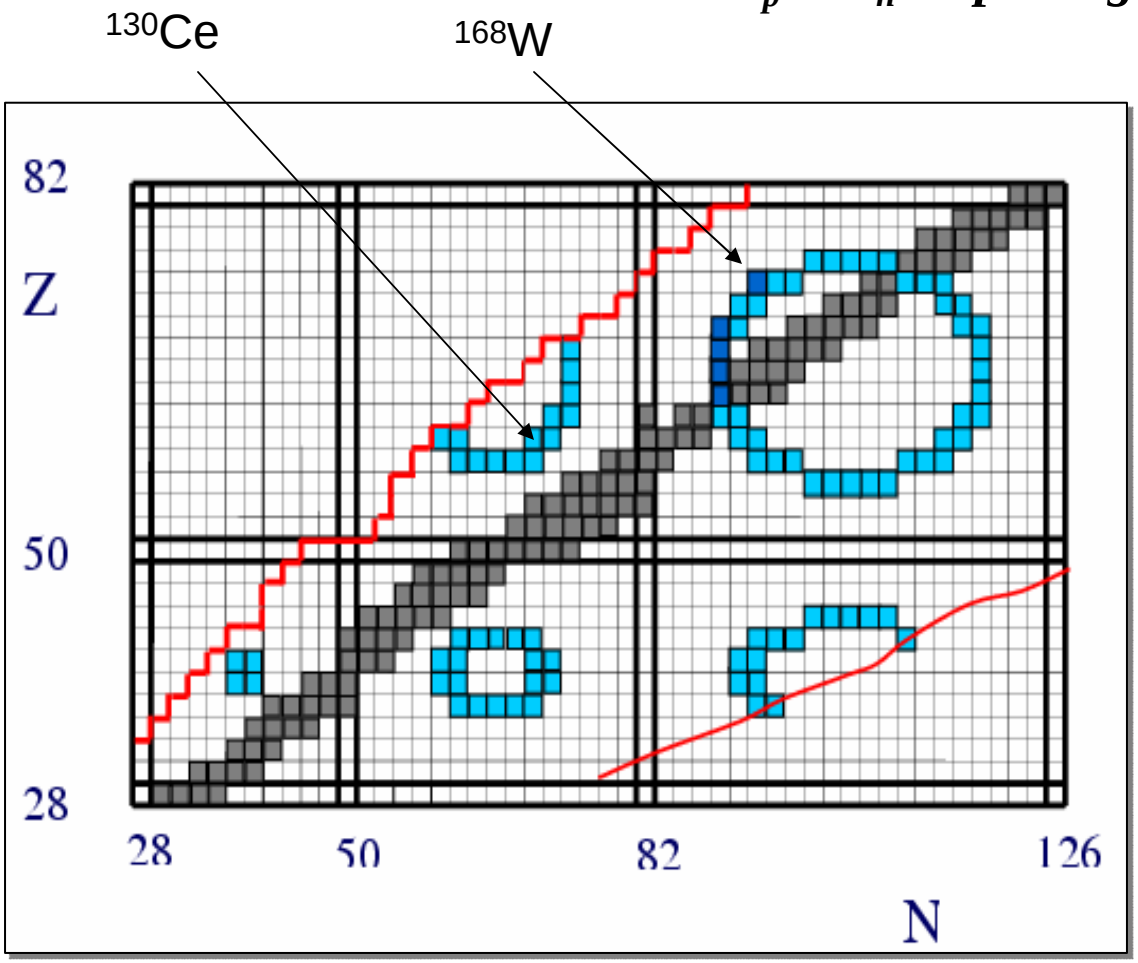

Numbers and Percentage of Transitional and Deformed Nuclei
Percentage
Number
100
90
80
70
60
50
40
30
20
10
0
150
100
50
0
Z=28-50
N=28-50
Z=28-50
N=50-82
Z=50-82
N=82-126
Transitional and Deformed Nuclei
Percentage
Number

Contours define locus of possible X(5) nuclei and enclose regions of deformation

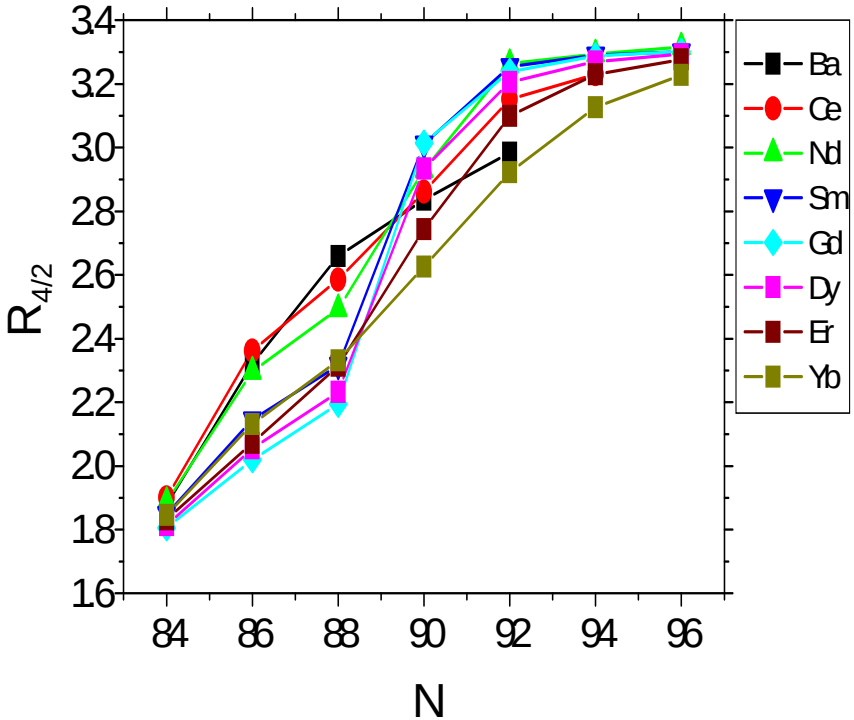
p-n / pairing

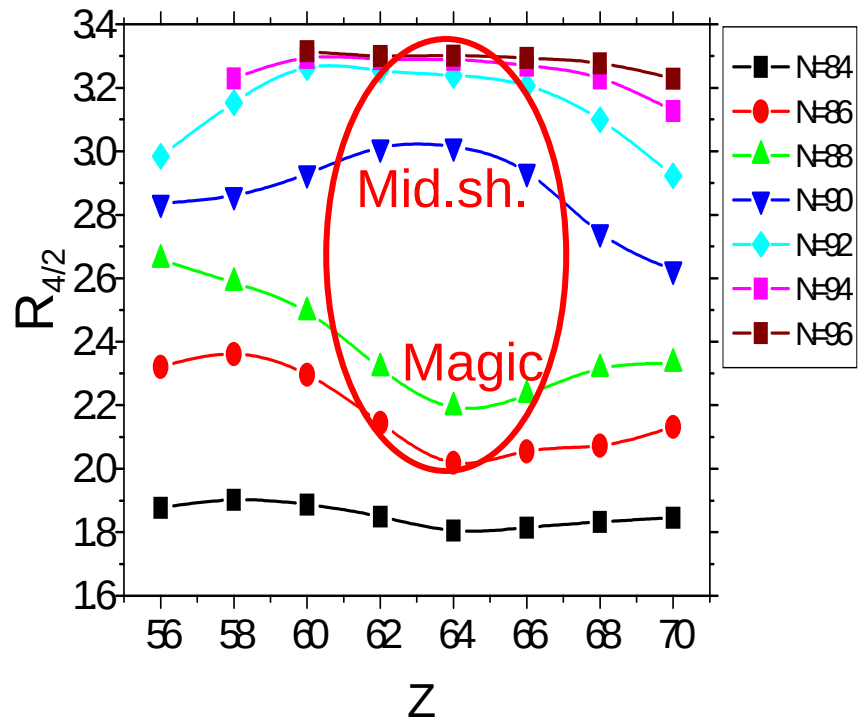
$$P = \frac{N_p N_n}{N_p + N_n} \sim \frac{p-n}{pairing}$$

P ~ 5



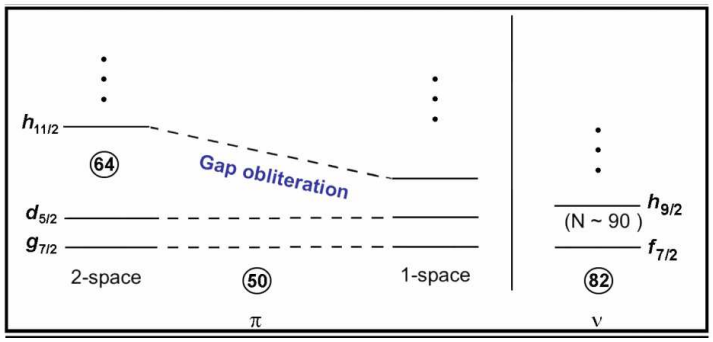
Different perspectives can yield different insights





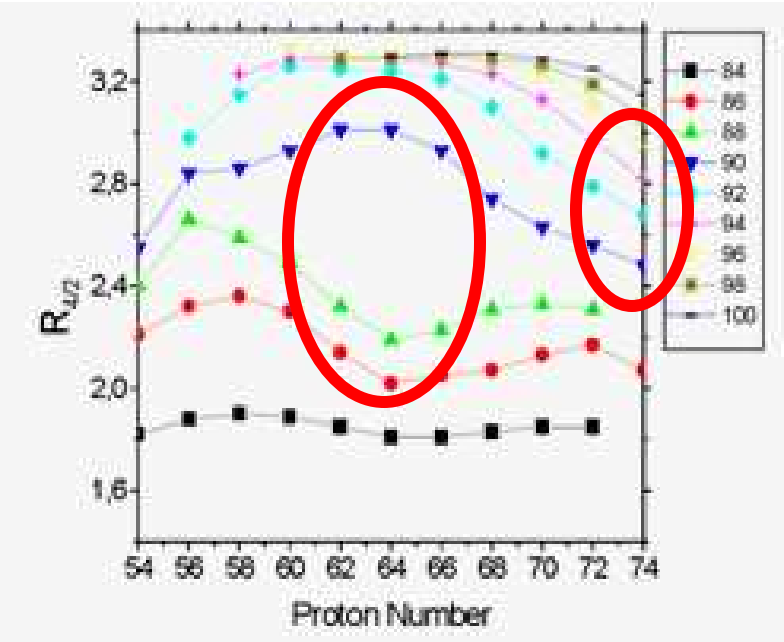
Onset of deformation

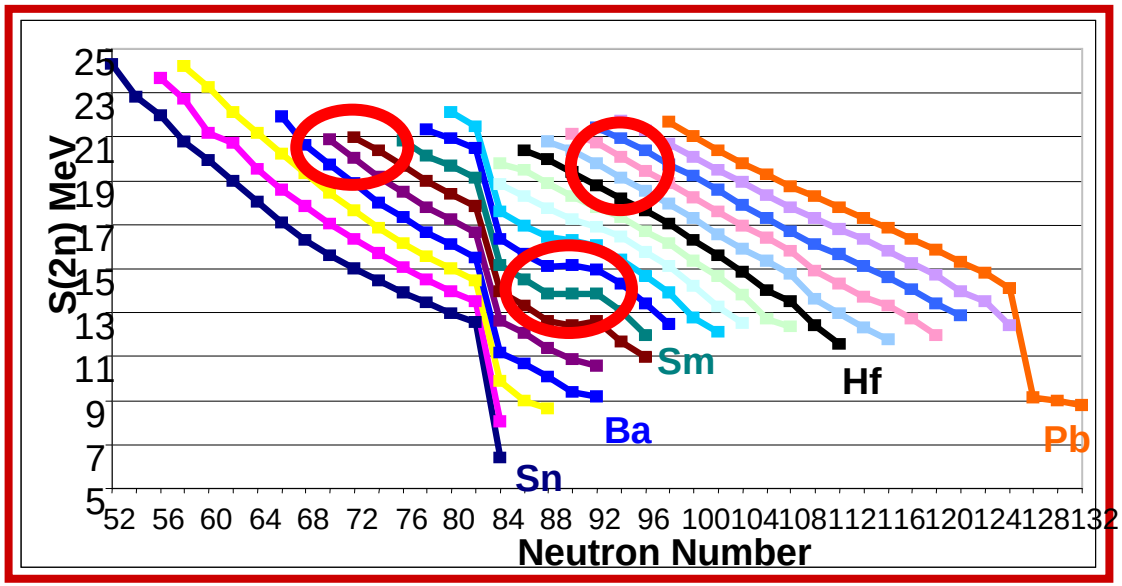
Onset of deformation as a phase transition, and due to a change in shell structure induced by the p-n interaction



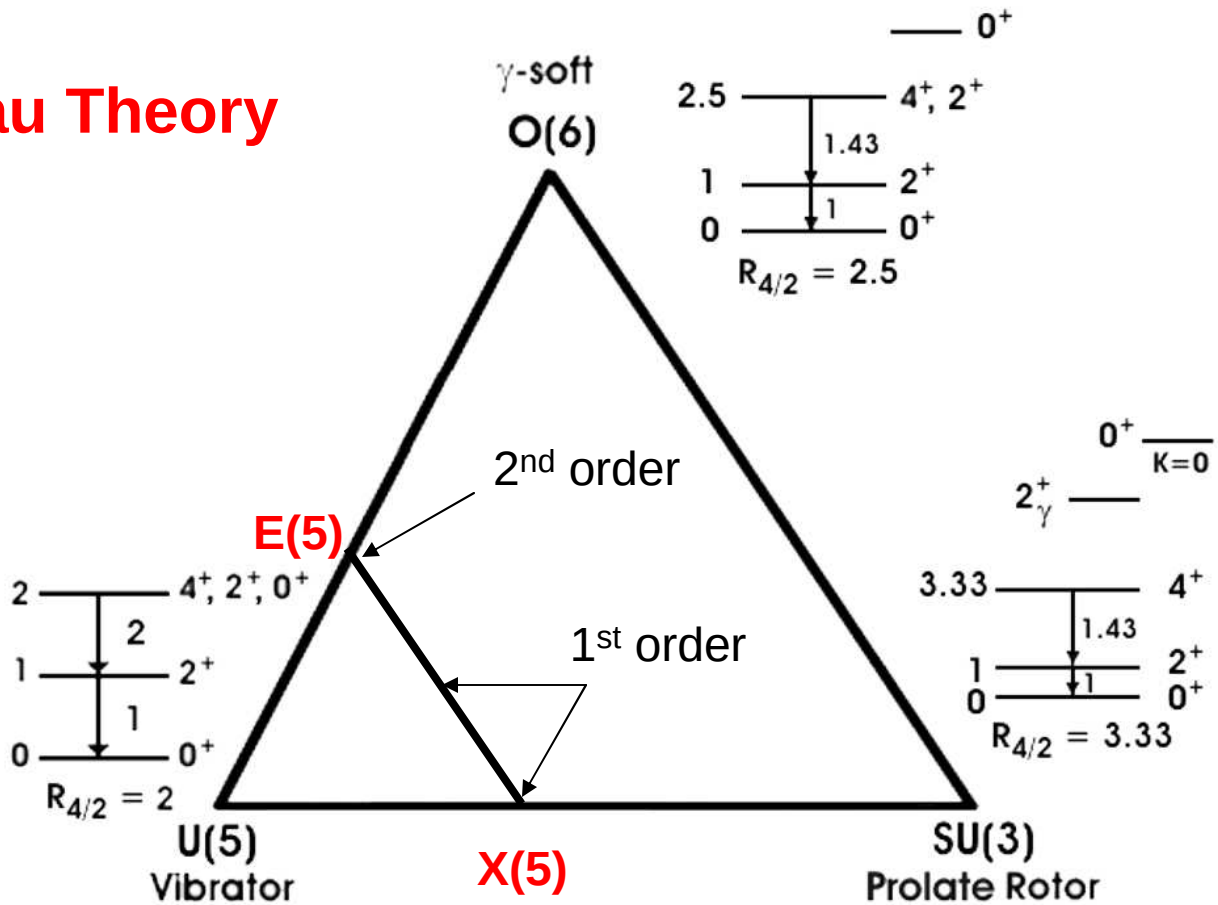
Obliteration of Z=64 proton shell gap as a function of neutron number

Regional perspective on phase transitional behavior





Landau Theory



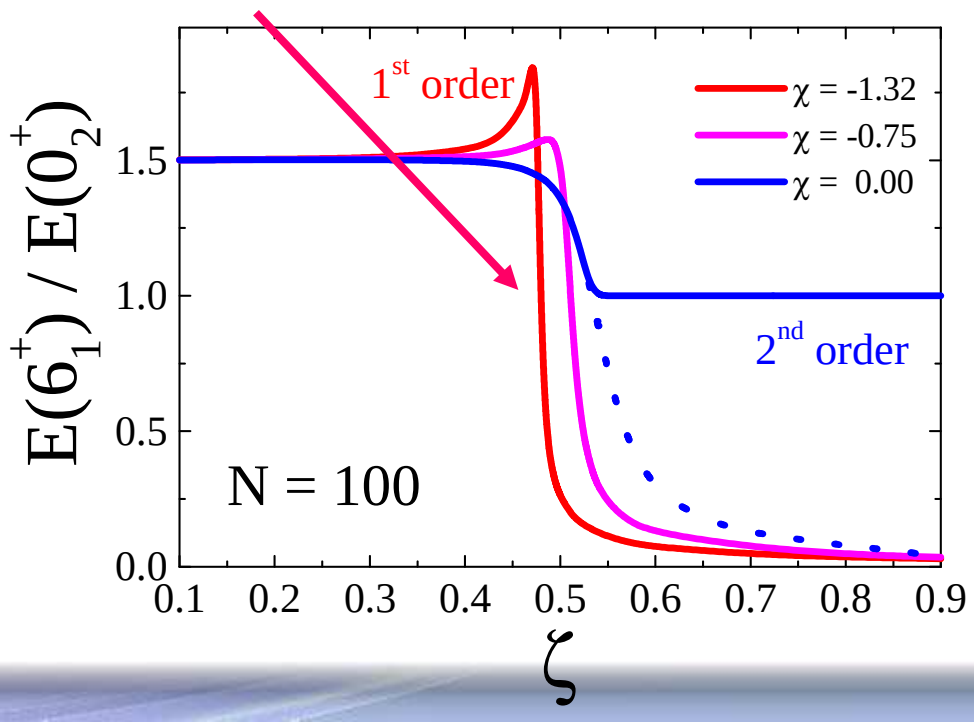
How can we distinguish first and second order quantum phase transitional behavior. Are there sensitive signatures? Are they simple to measure?

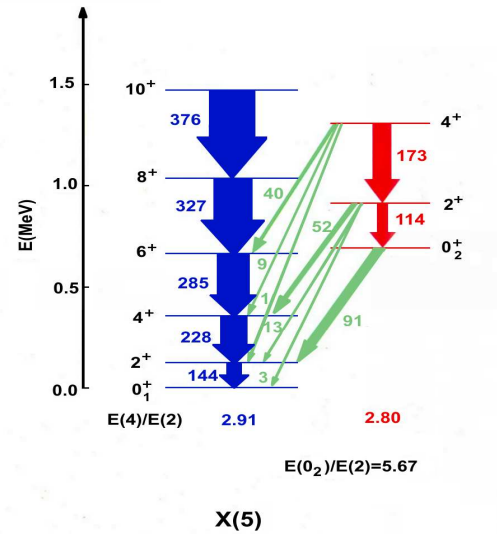
Empirical signature of 1st and 2nd order

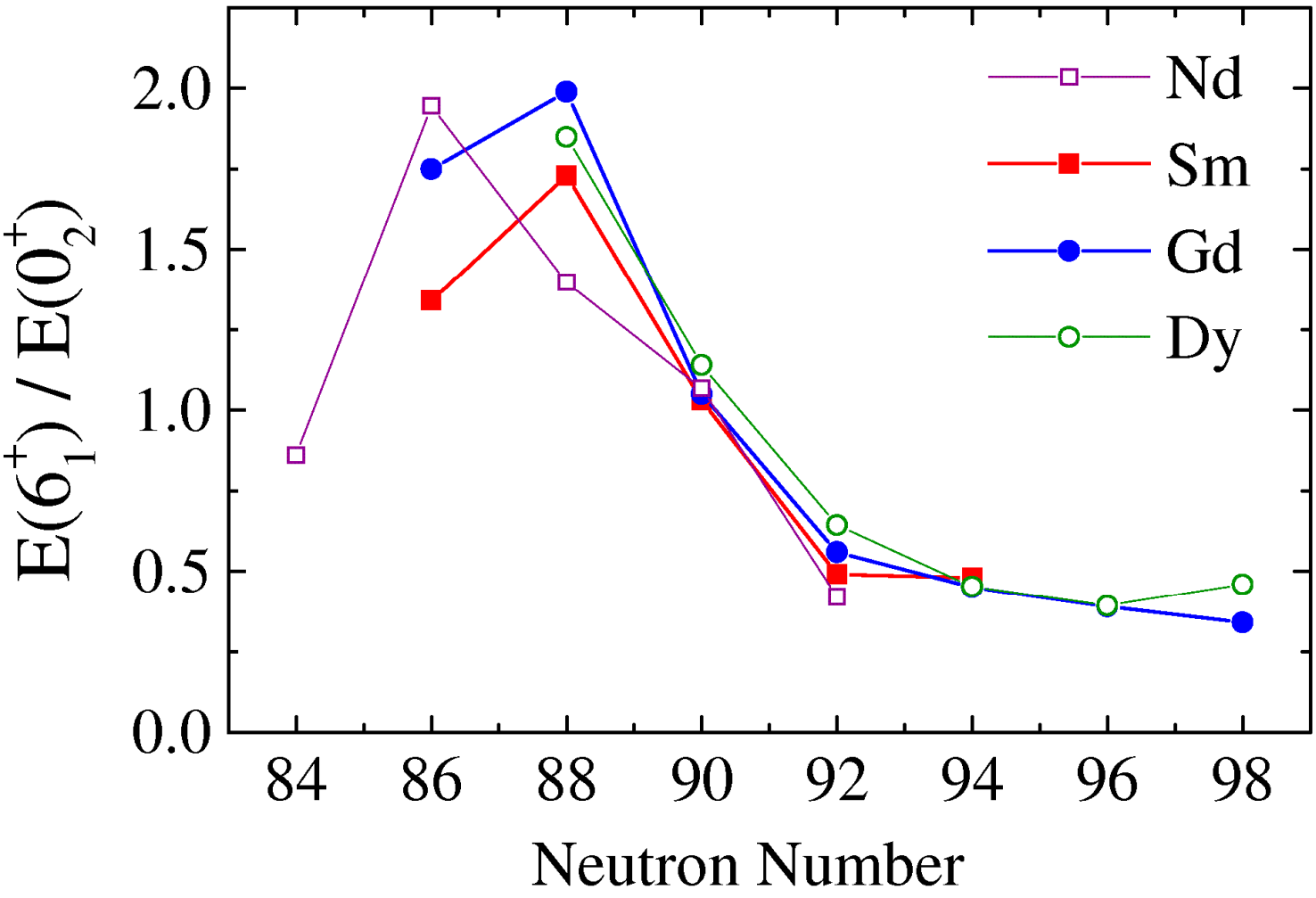
Energy ratio between 6^+ of ground state and first excited 0^+

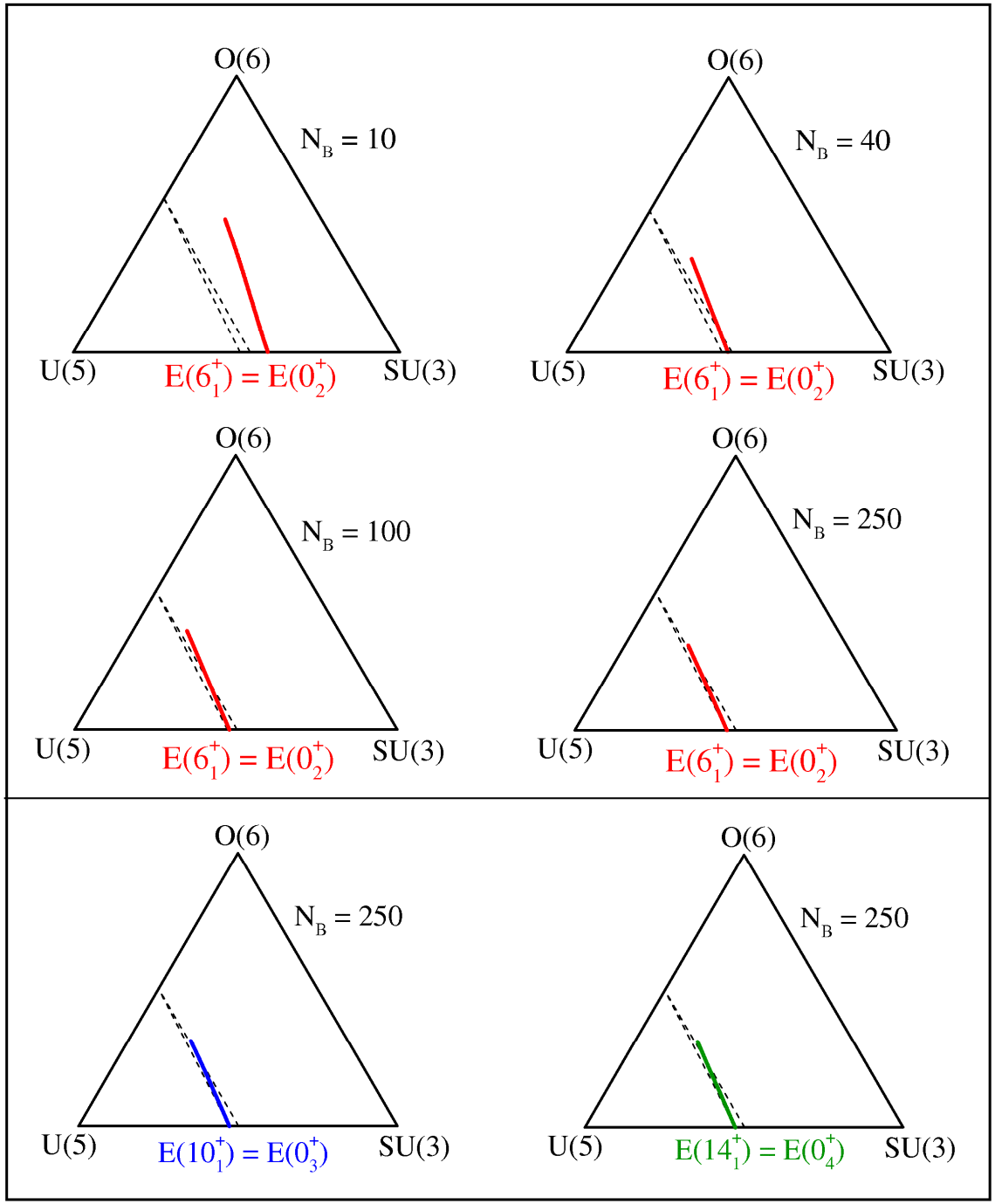
$$\frac{E(6_1^+)}{E(0_2^+)} = \begin{cases} 1.5 & \mathrm{U(5)} \quad \text{Vibrator} \\ \rightarrow 0 & \mathrm{SU(3)} \quad \text{Rotor} \end{cases}$$

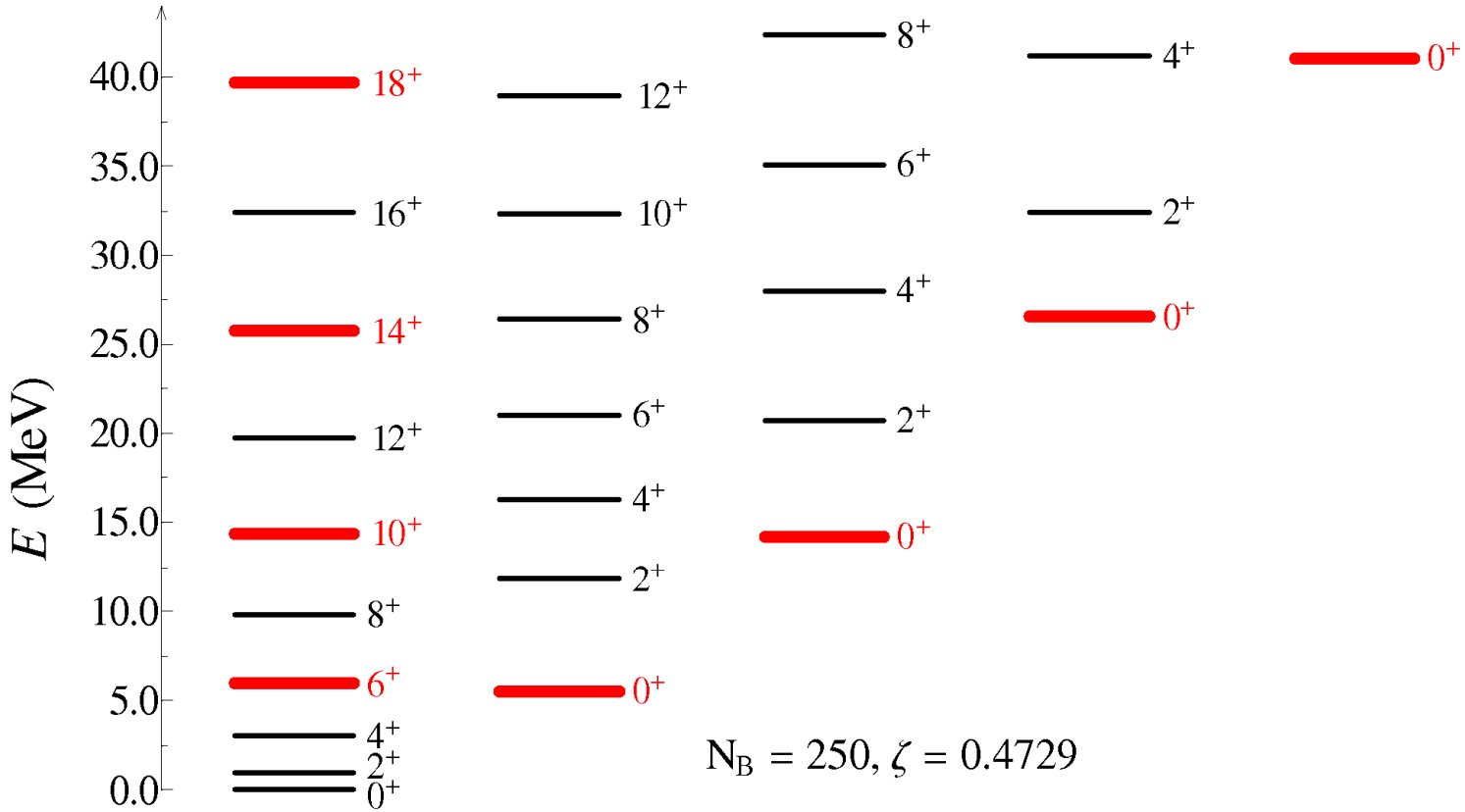
~1 at Ph. Tr ~ X(5)





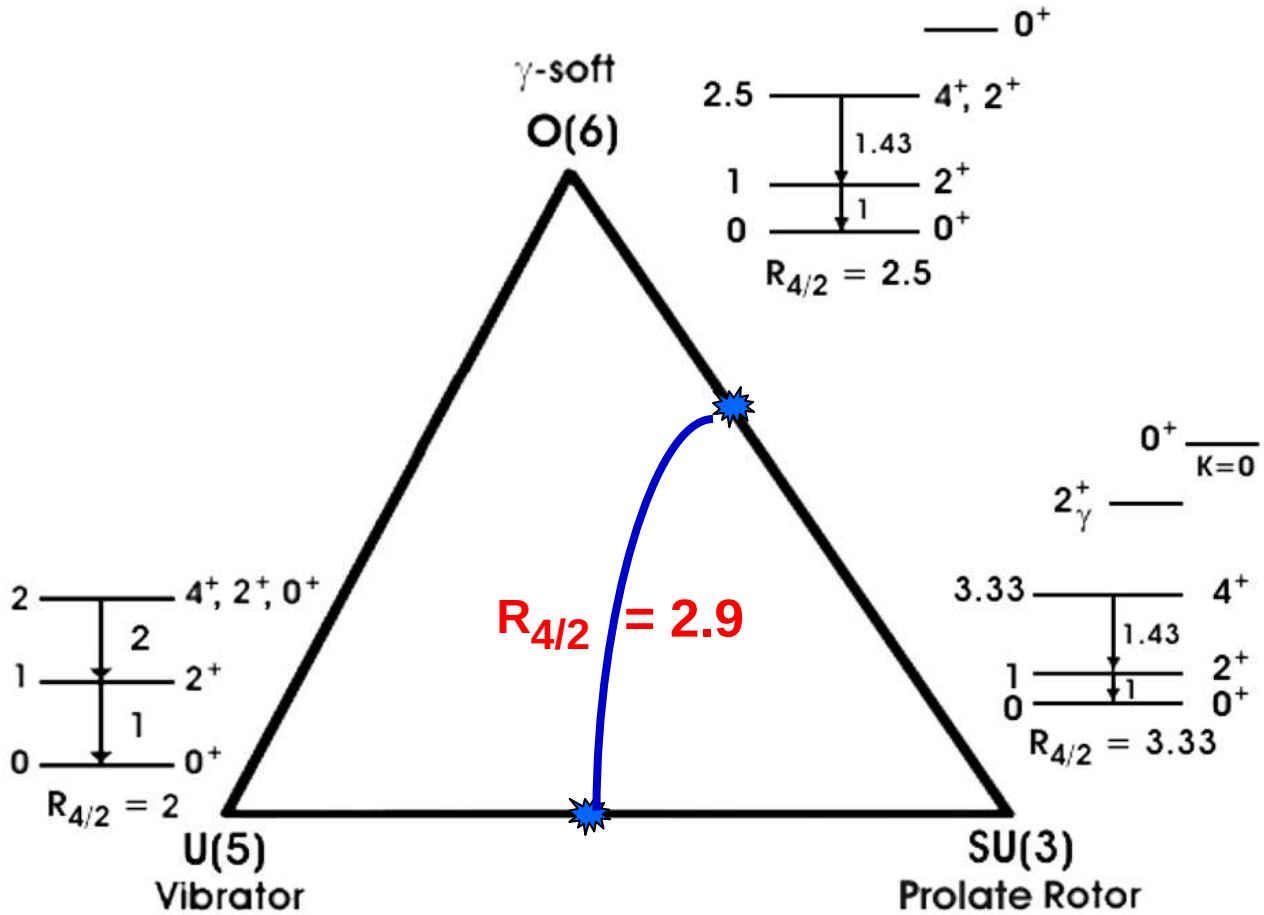

Nd
Sm
Gd
Dy
2.0
1.5
1.0
0.5
0.0
84
86
88
90
92
94
96
98
Neutron Number
E(6$_1^+$) / E(0$_2^+$)


O(6)
$N_B = 10$
U(5)
$E(6_1^+) = E(0_2^+)$
SU(3)
O(6)
$N_B = 40$
U(5)
$E(6_1^+) = E(0_2^+)$
SU(3)
O(6)
$N_B = 100$
U(5)
$E(6_1^+) = E(0_2^+)$
SU(3)
O(6)
$N_B = 250$
U(5)
$E(6_1^+) = E(0_2^+)$
SU(3)
O(6)
$N_B = 250$
U(5)
$E(10_1^+) = E(0_3^+)$
SU(3)
O(6)
$N_B = 250$
U(5)
$E(14_1^+) = E(0_4^+)$
SU(3)



Degeneracies ~ Symmetries ??

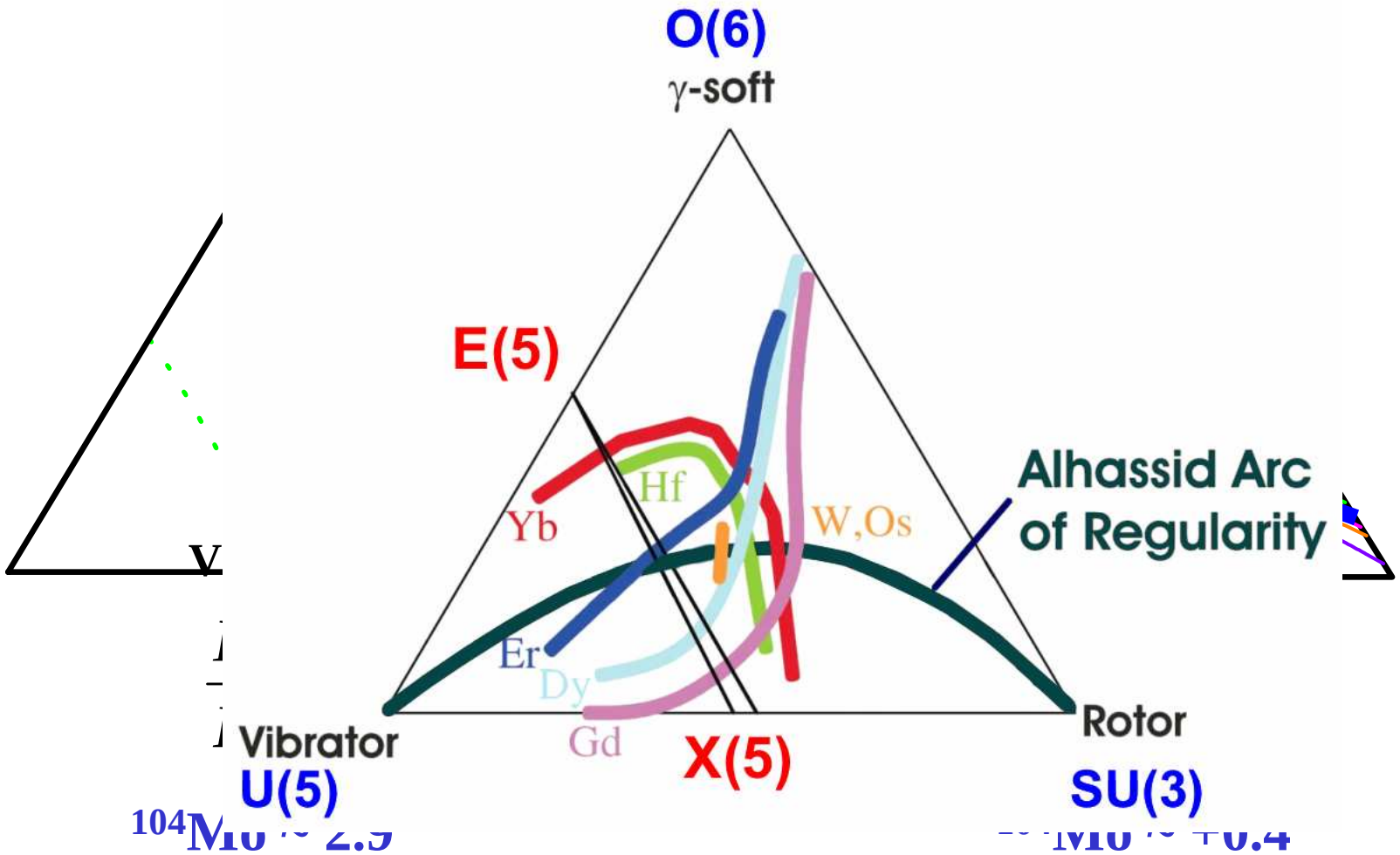
Mapping structure throughout the triangle



- $R_{4/2} = E(4^+)/E(2^+)$ -- Defines locus in triangle
- Need another observable: $H = aH_{sph} + bH_{def} = \varepsilon\, nd - \kappa\, Q \cdot Q$

Evolution of Structure

Mapping Structure with Simple Observables – Technique of Crossing Contours

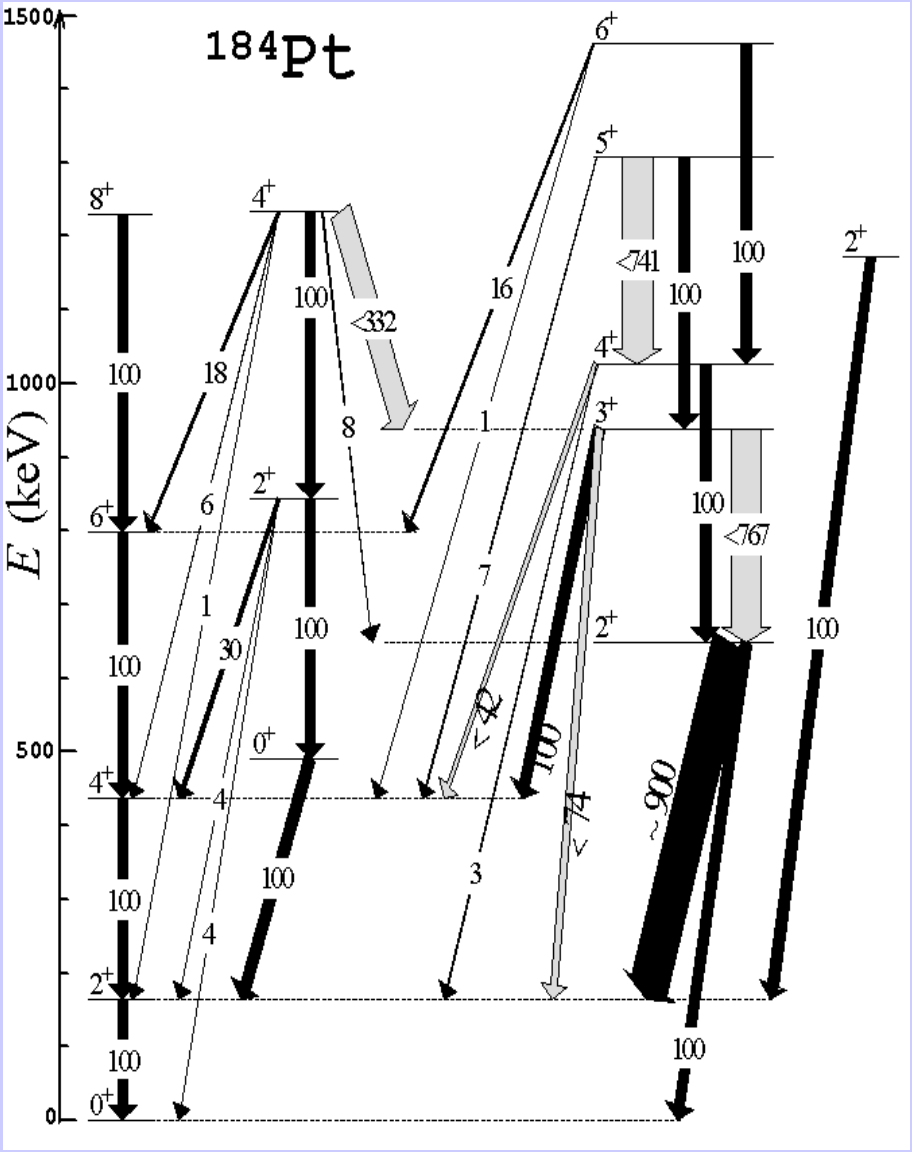


Complementarity of macroscopic and microscopic approaches.

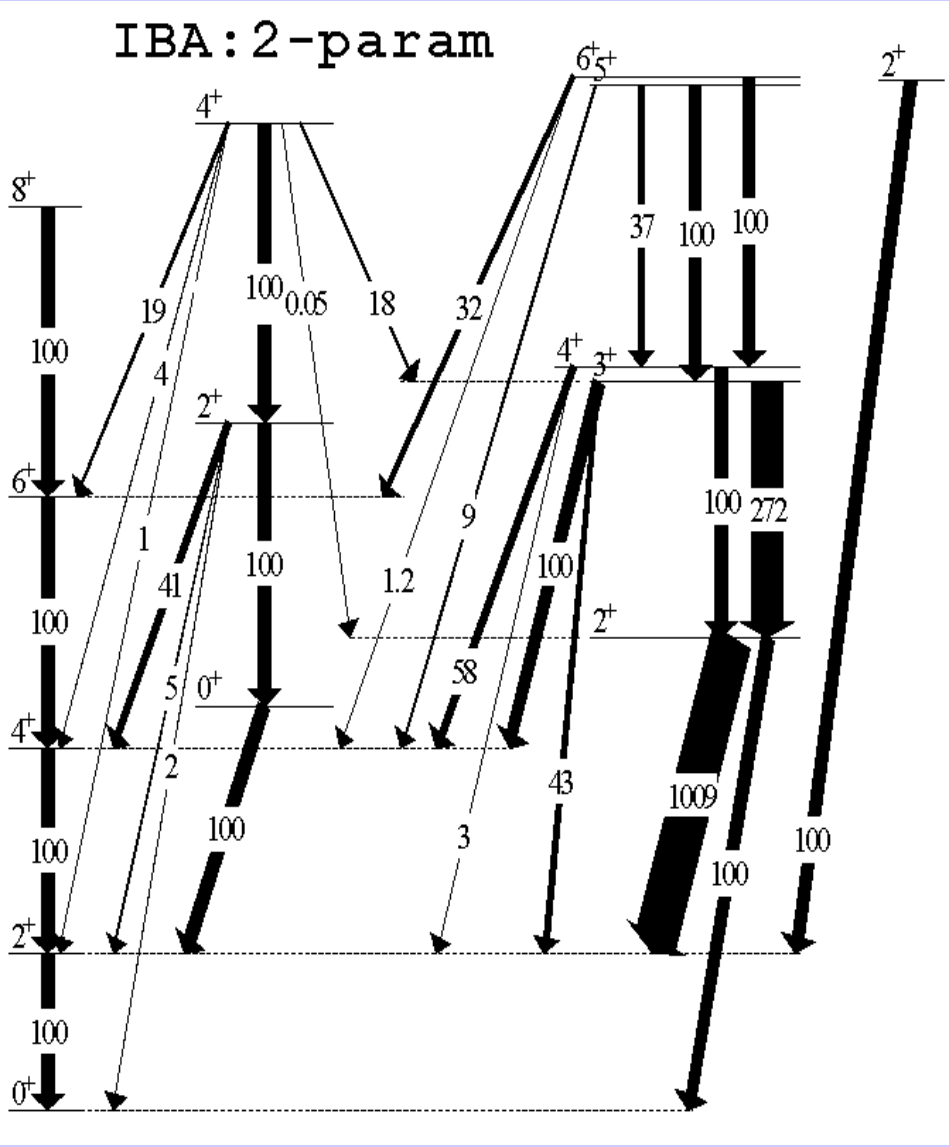
2 parameter – phenomenologically very successful

Why do certain nuclei exhibit specific symmetries? Why these evolutionary trajectories?

What will happen far from stability in regions of proton-neutron asymmetry and/or weak binding?

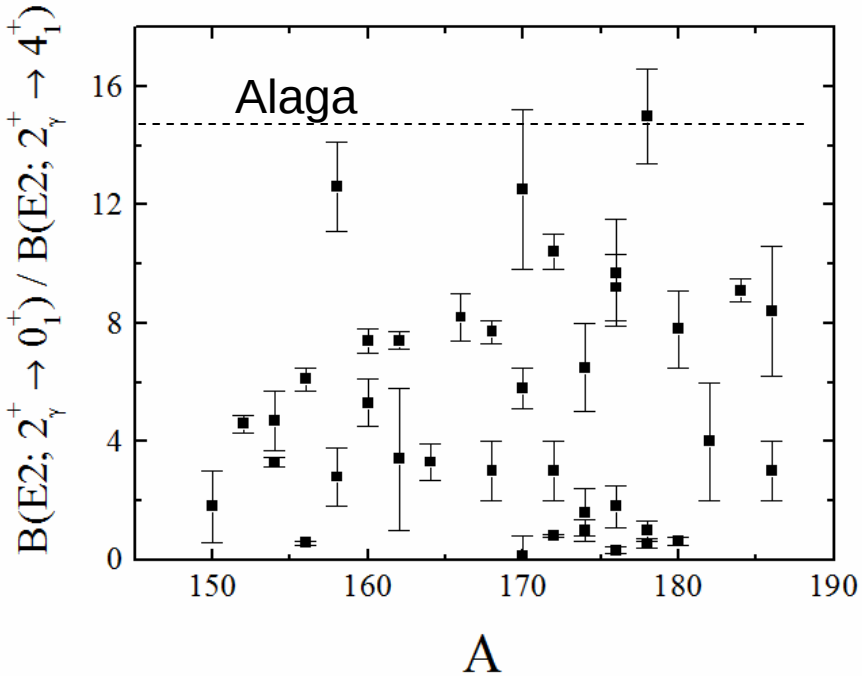


184Pt
E (keV)
1500
1000
500
0
8+
6+
4+
2+
0+
3+
5+
<332
<741
<767
<42
<74
~900

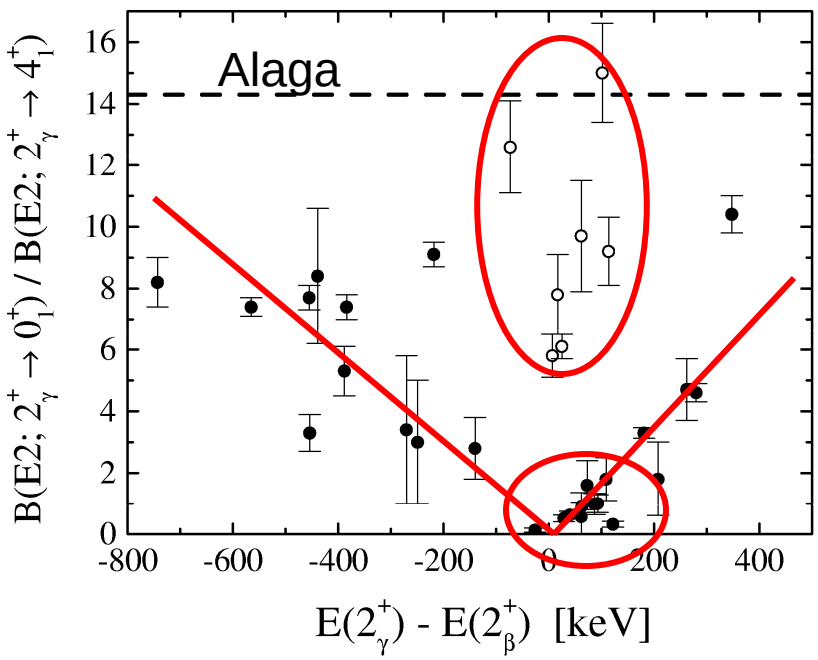


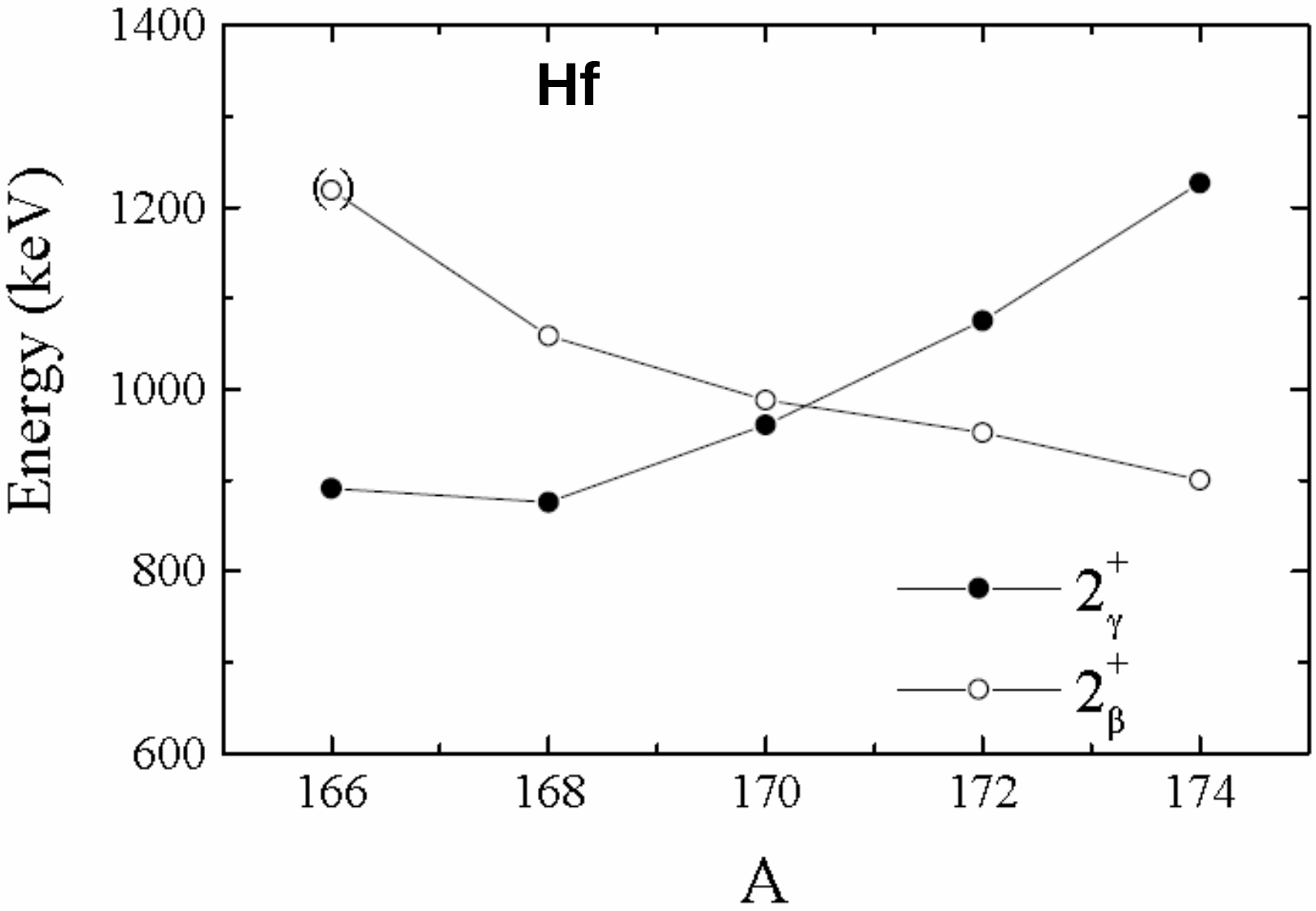
IBA:2-param
8+
6+
4+
2+
0+
3+
5+
1009
272

β - γ Mixing



A different perspective




Hf
Energy (keV)
1400
1200
1000
800
600
166
168
170
172
174
A
2^+_γ
2^+_β

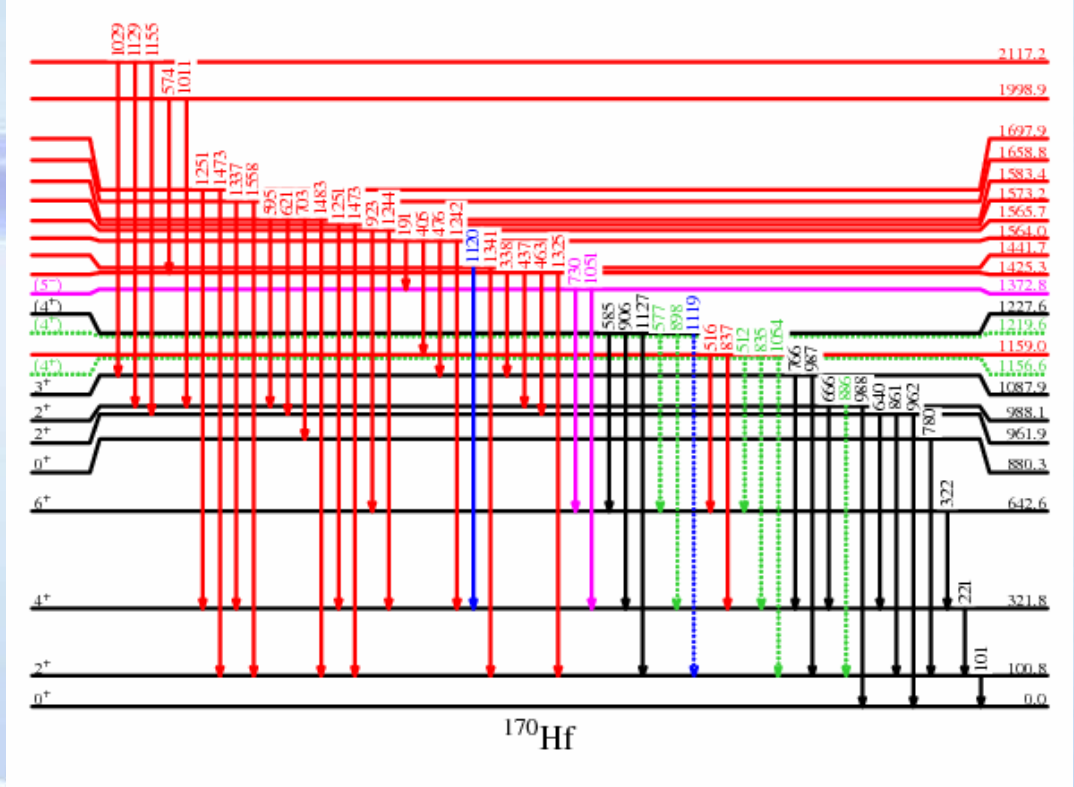
^{170}Hf β-decay Experiment

$$^{159}Tb(^{16}O,5n)^{170}Ta \xrightarrow{\beta} {}^{170}Hf$$

Original Aim: g-factor of 2_1^+ state

➡ Perturbed γ-γ angular correlations

8 ~25% Ge detectors

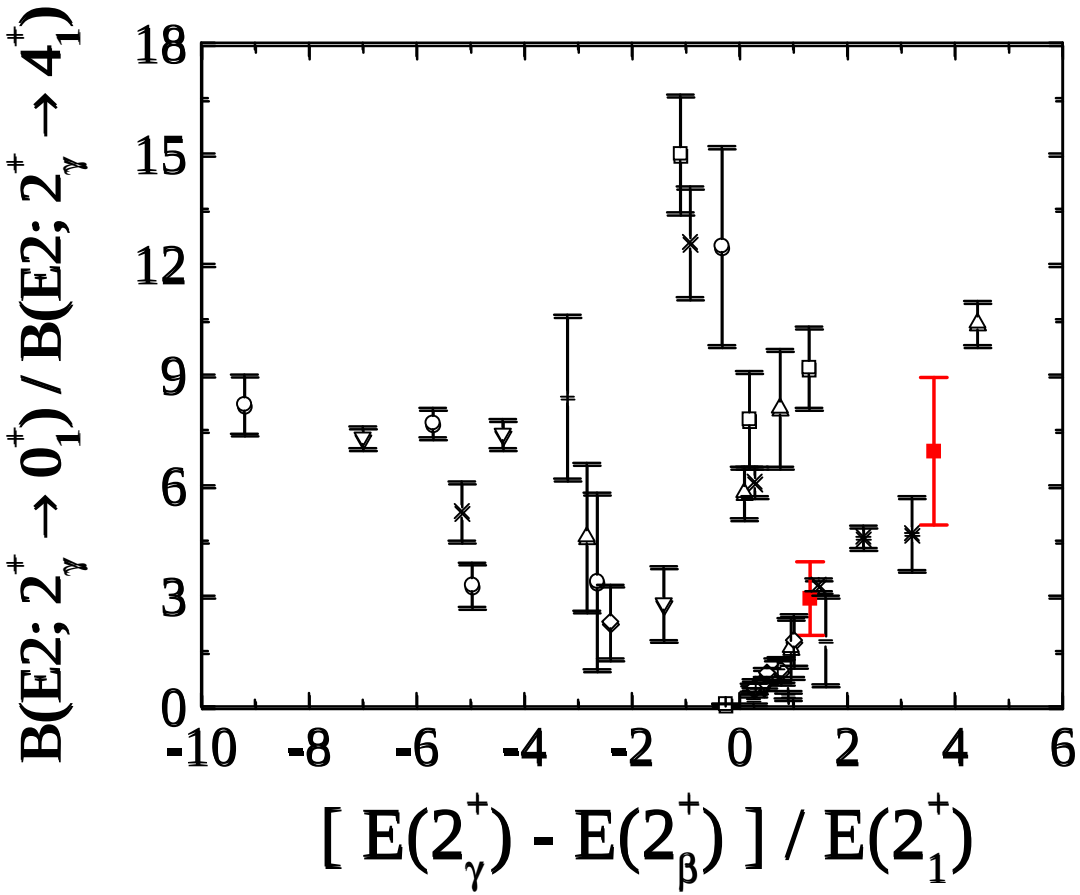


Looking back at systematics

$$\frac{B(E2;2_\gamma^+ \rightarrow 0_1^+)}{B(E2;2_\gamma^+ \rightarrow 4_1^+)}$$

^{172}Hf = 3(1)

^{174}Hf = 7(2)

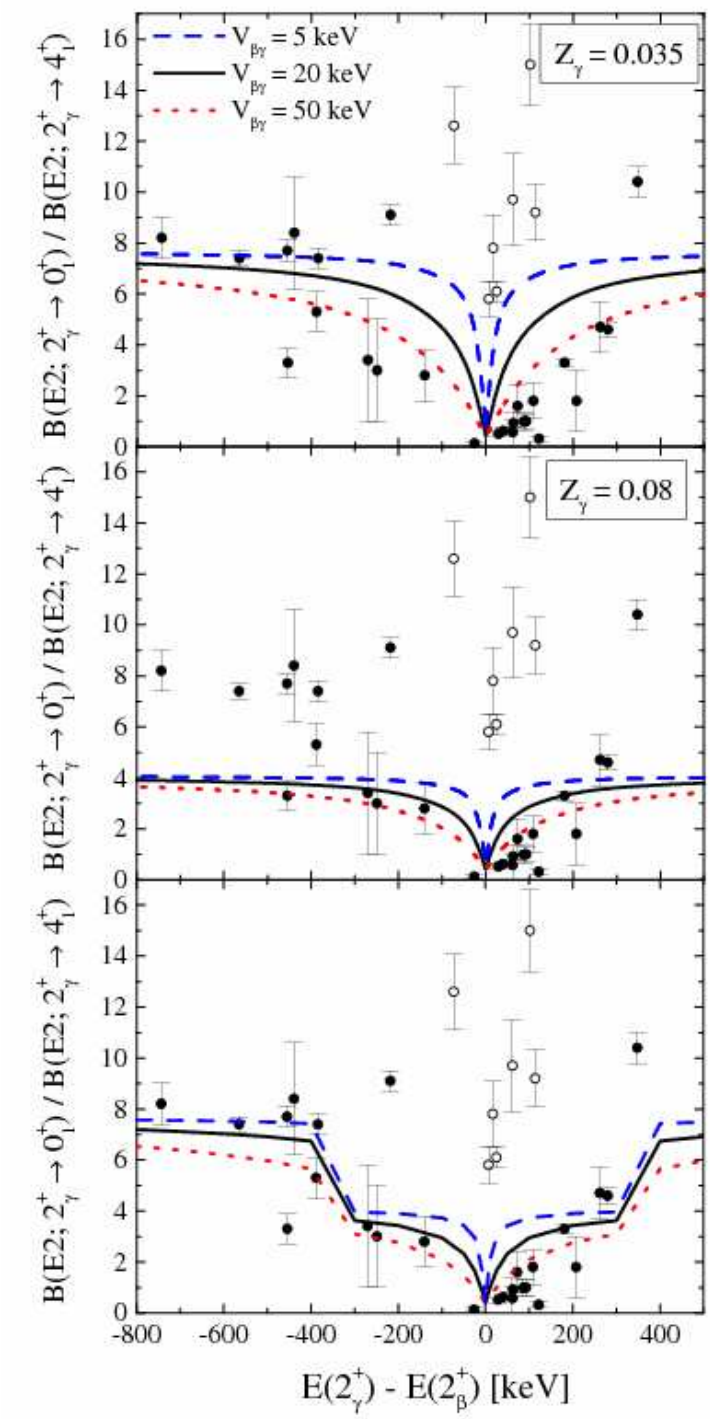


β - γ Mixing

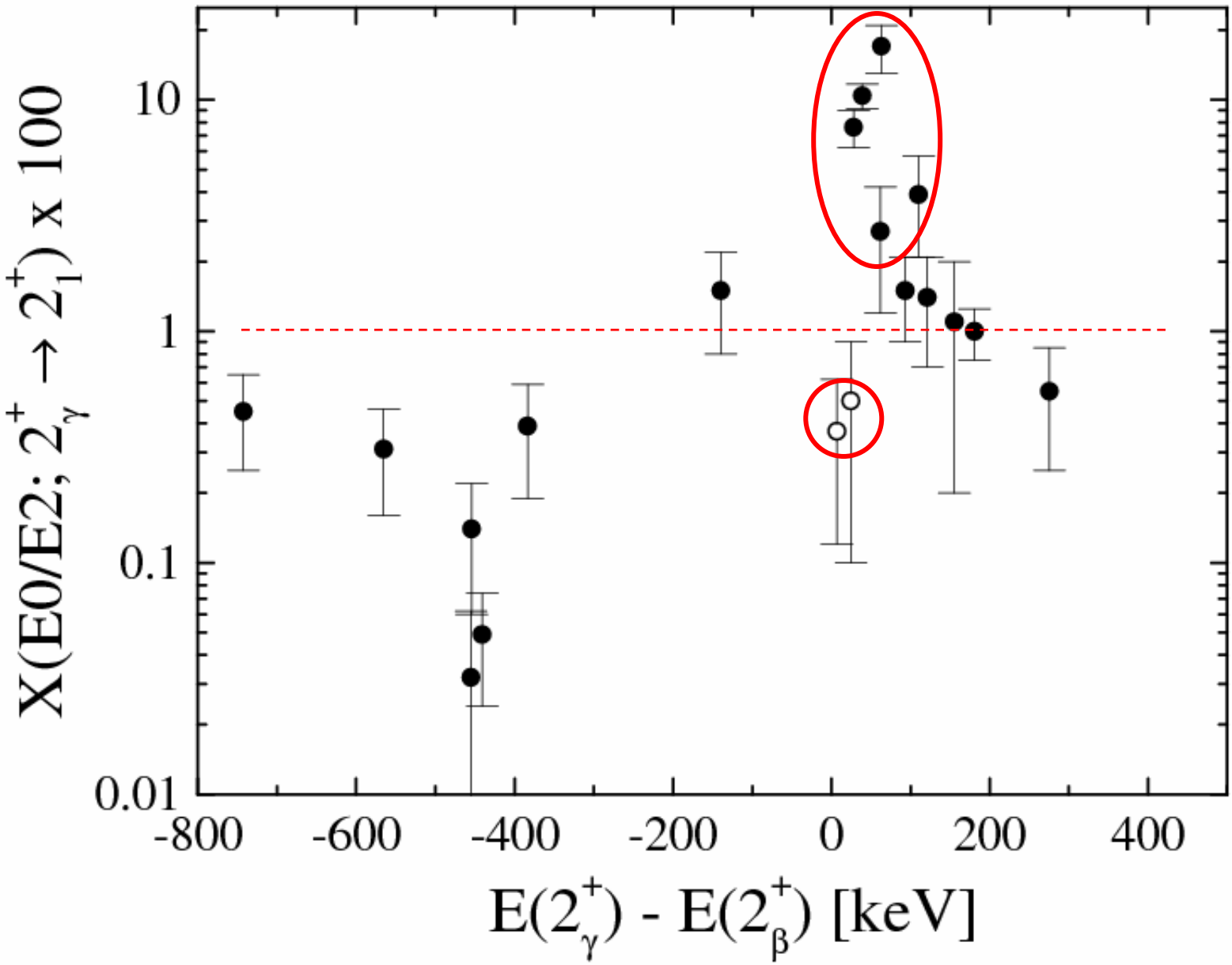
When these excitations are nearly degenerate, they fall into two classes:

1) Nearly pure, SU(3)-like

2) Nearly totally mixed, with very large matrix elements (an order of magnitude larger than when these collective modes are well separated).



E0 transitions are a signature of this mixing !



Thanks

BACKUPS

Critical Point Symmetries

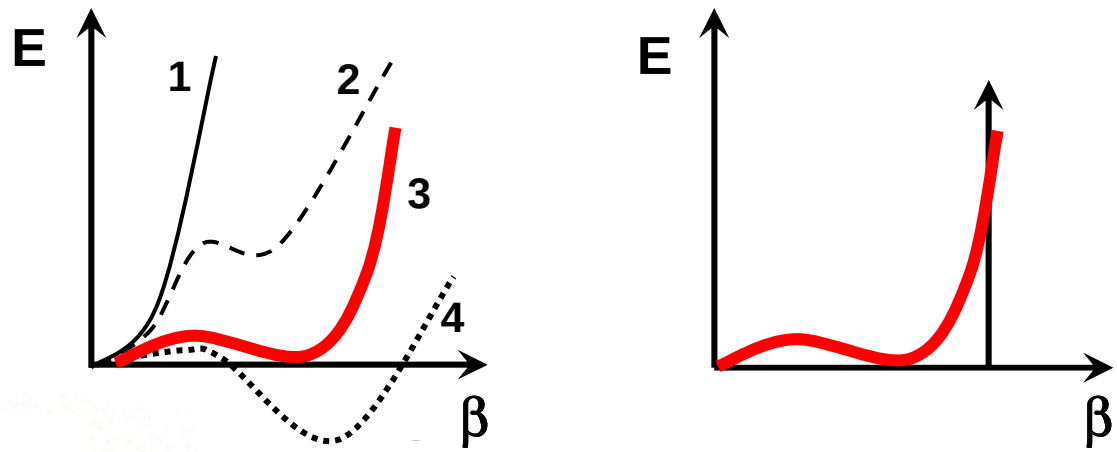
Iachello

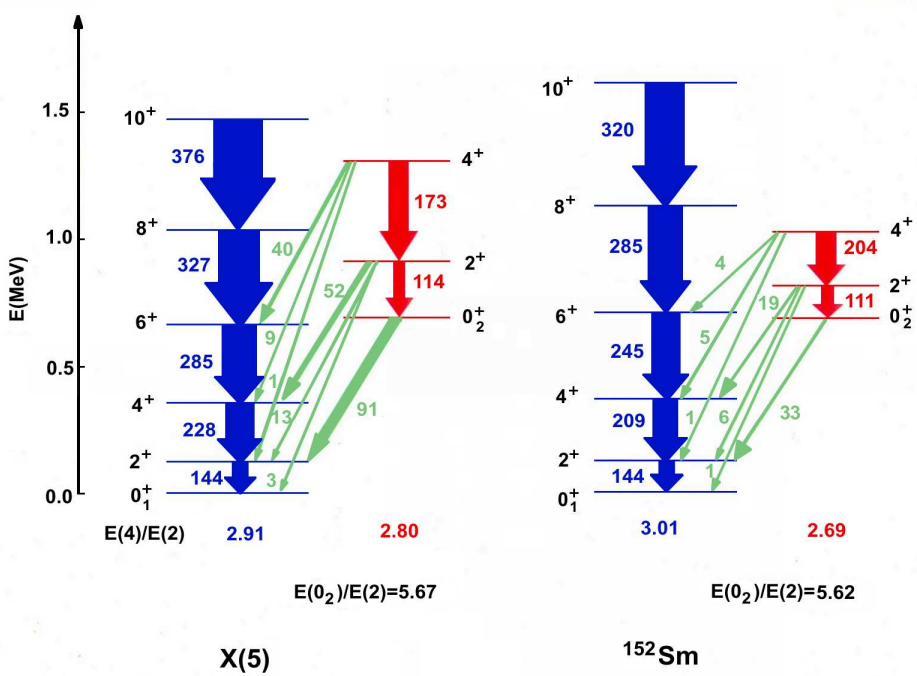
X(5)

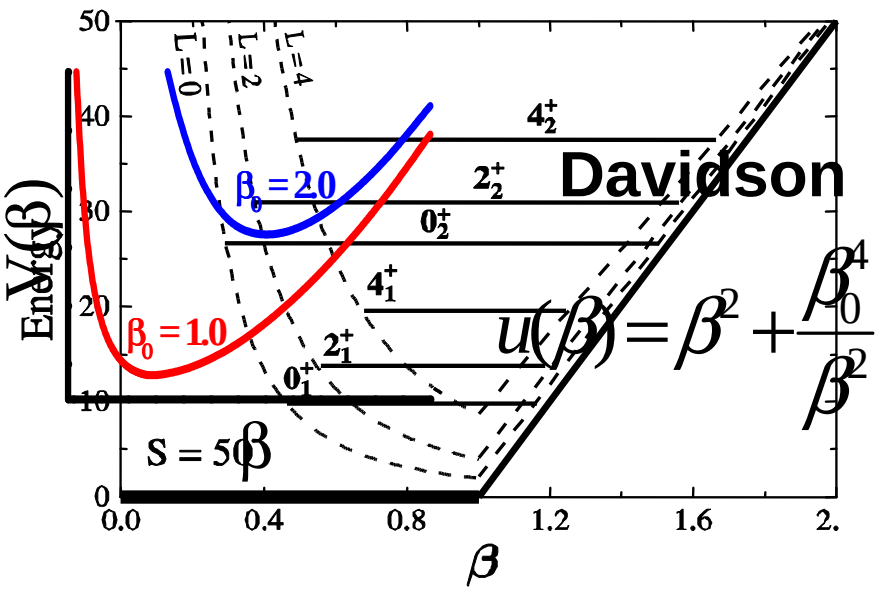
First Order Phase Transition – Phase Coexistence

Energy surface changes with valence nucleon number

Bessel equation







Mapping the Triangle with a minimum of data -- exploiting Ising-type Models

Competition between spherical-driving (pairing – like nucleon) and deformation-driving (esp. p-n) interactions

$$H = aH_{sph} + bH_{def}$$ **Structure ~ a/b**

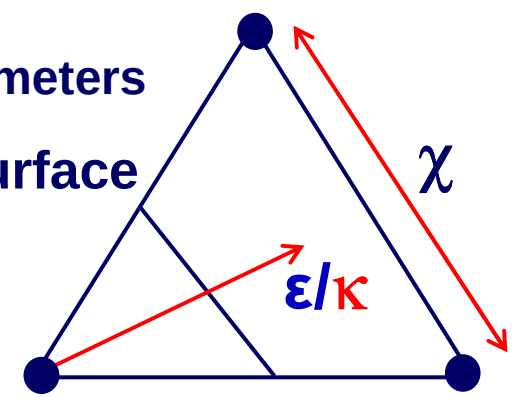
IBA:

$$H = \varepsilon\, n_d - \kappa\, Q \cdot Q$$

Parameters: ε/κ, χ (within Q)

2 parameters

2-D surface

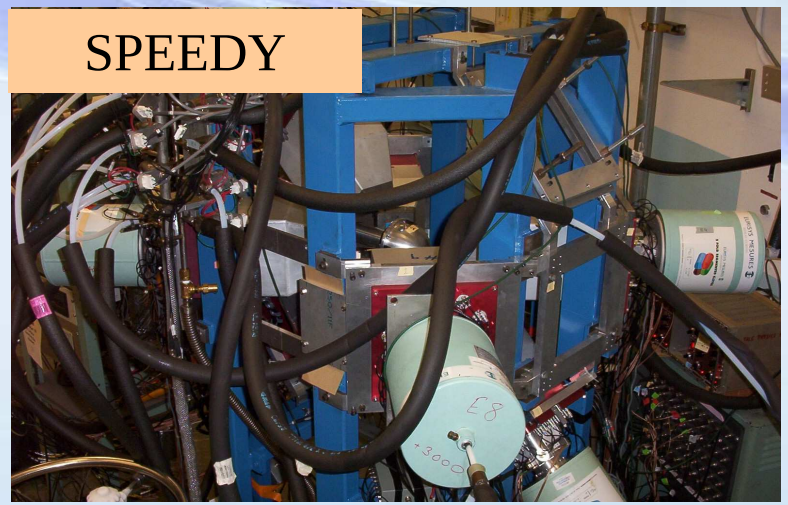


McCutchan and Zamfir

Experiments on ^{172}Hf and ^{174}Hf

^{172}Hf $\quad \dfrac{2^+_\gamma - 2^+_\beta}{2^+_1} = +1.3$

^{174}Hf $\quad \dfrac{2^+_\gamma - 2^+_\beta}{2^+_1} = +3.6$



$$^{165}Ho(^{12}C,5n)^{172}Ta \xrightarrow{\beta} {}^{172}Hf$$

E = 80 MeV

$$^{168}Er(^{11}B,5n)^{174}Ta \xrightarrow{\beta} {}^{174}Hf$$

E = 65 MeV

8 Clover detectors

γ-γ angular correlations

Beam on/beam off

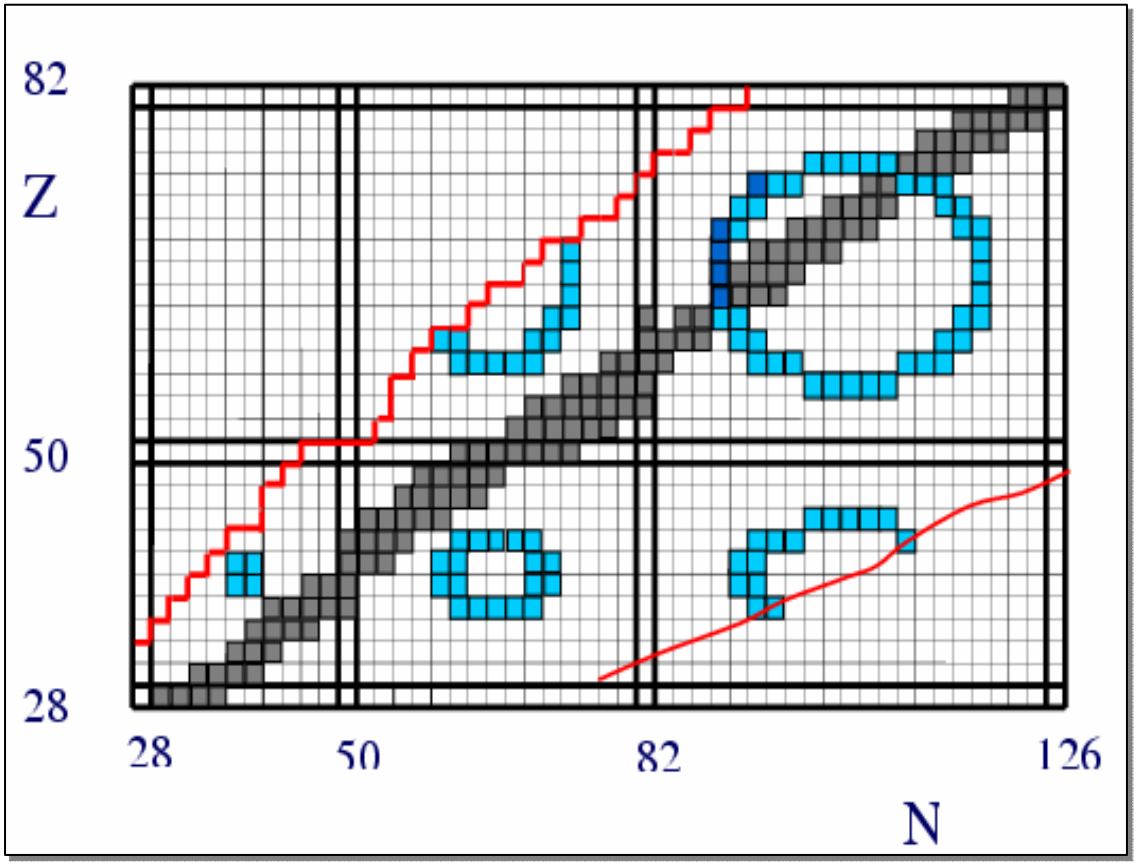
~ 2 days

Contours enclose regions of deformed nuclei

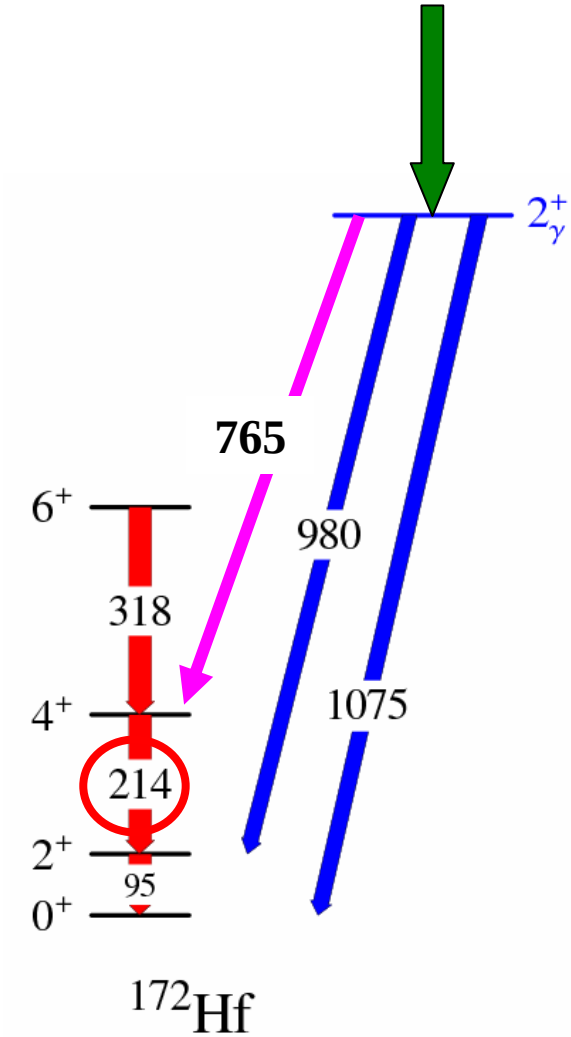
p-n / pairing

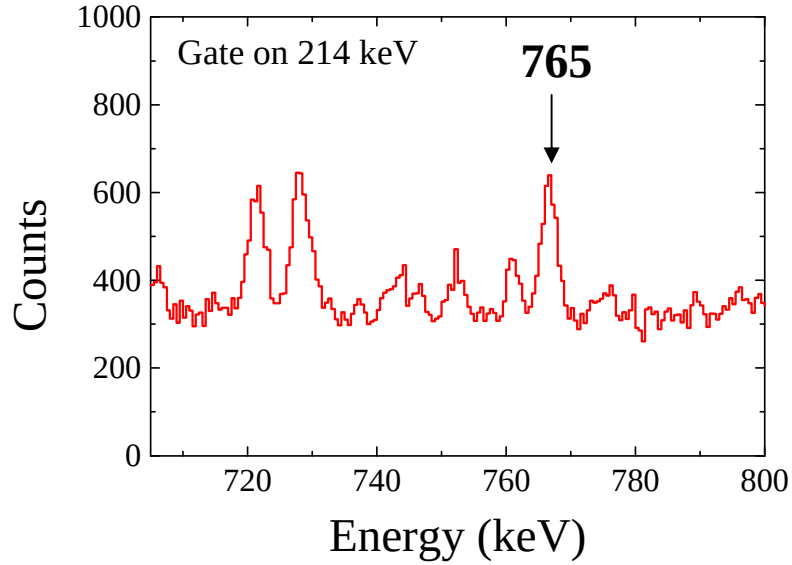
$$P = \frac{N_p N_n}{N_p + N_n} \sim \frac{p - n}{pairing}$$

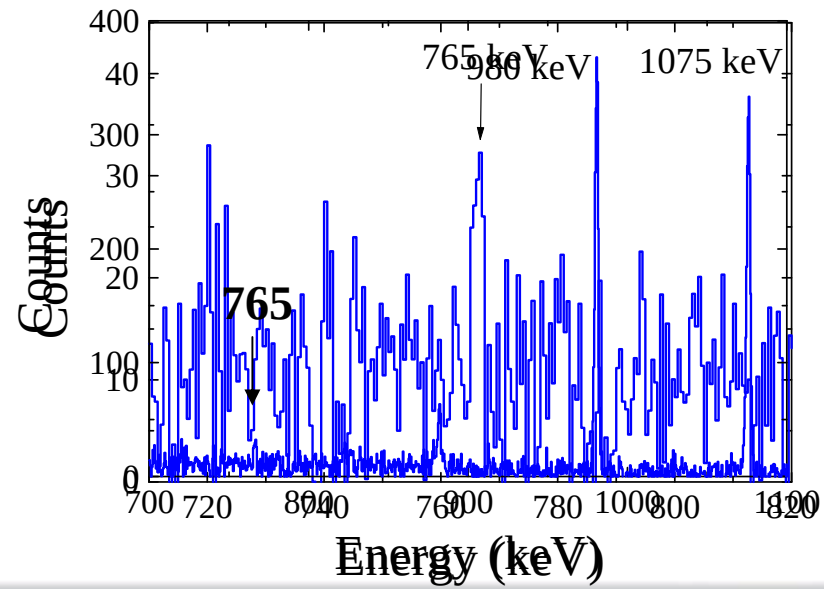
P ~ 5



Search for $2_\gamma \rightarrow 4_1$ transition in ^{172}Hf

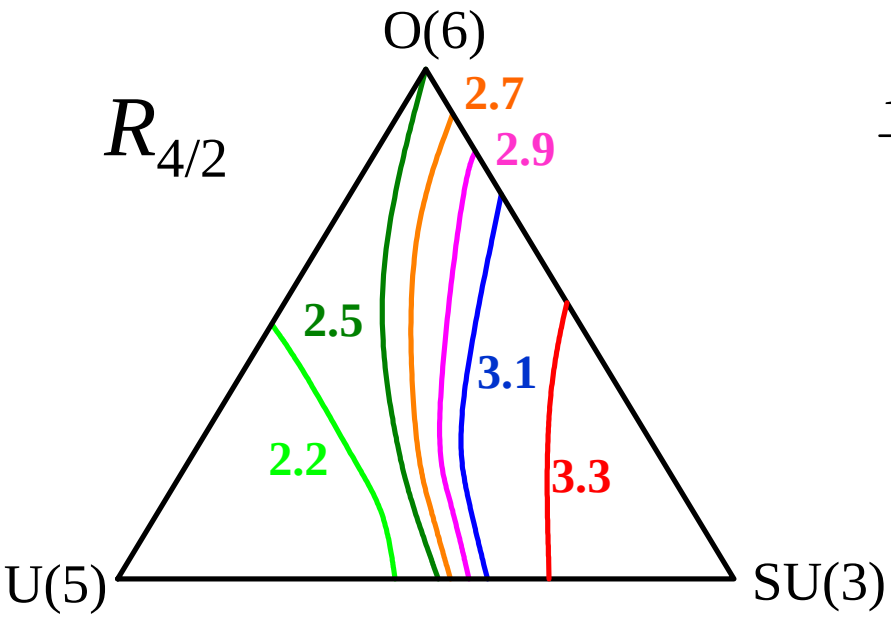


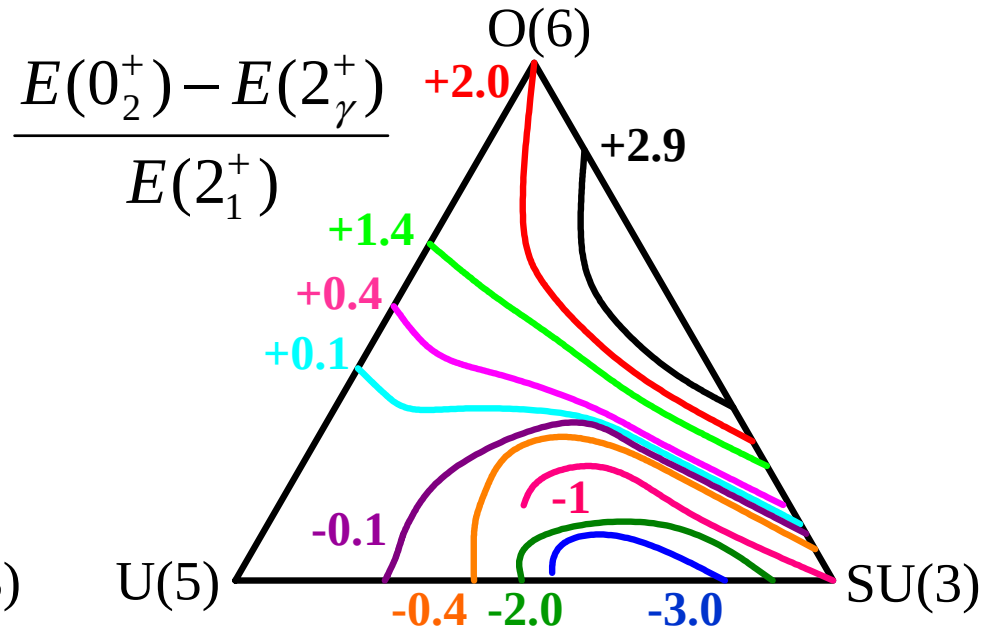


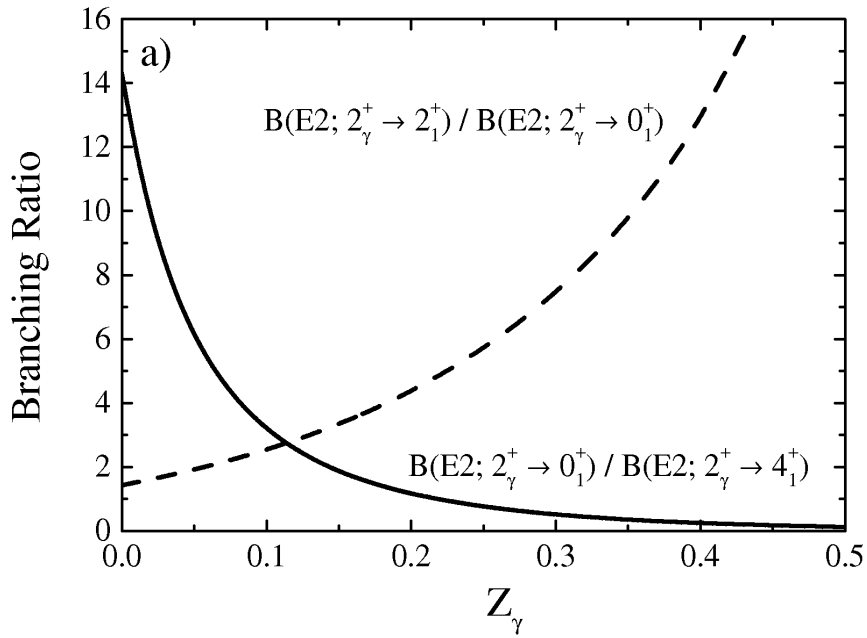


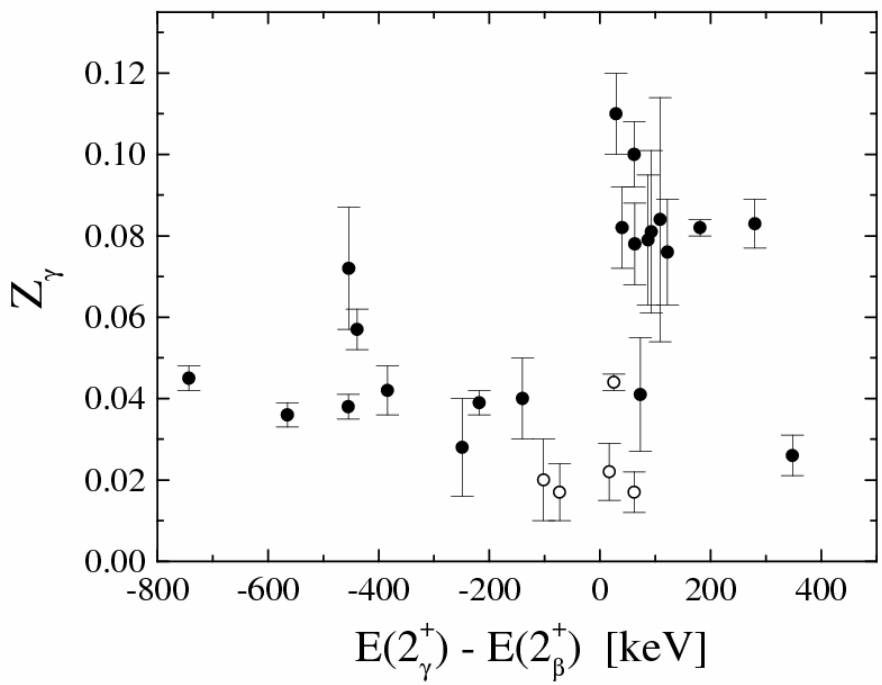
Pinpointing Nuclear Structure

Technique of Orthogonal Crossing Contours (OCC)

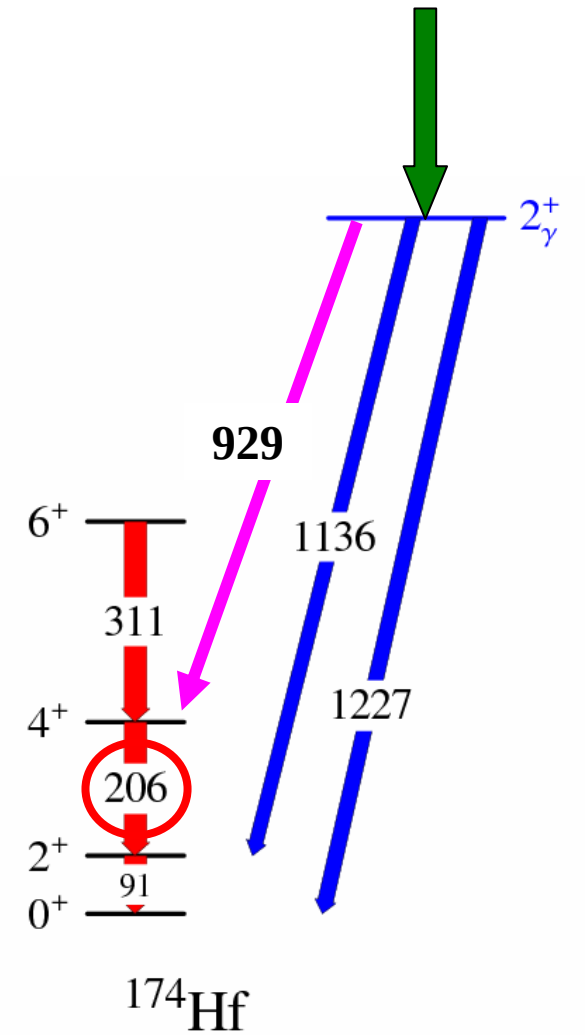


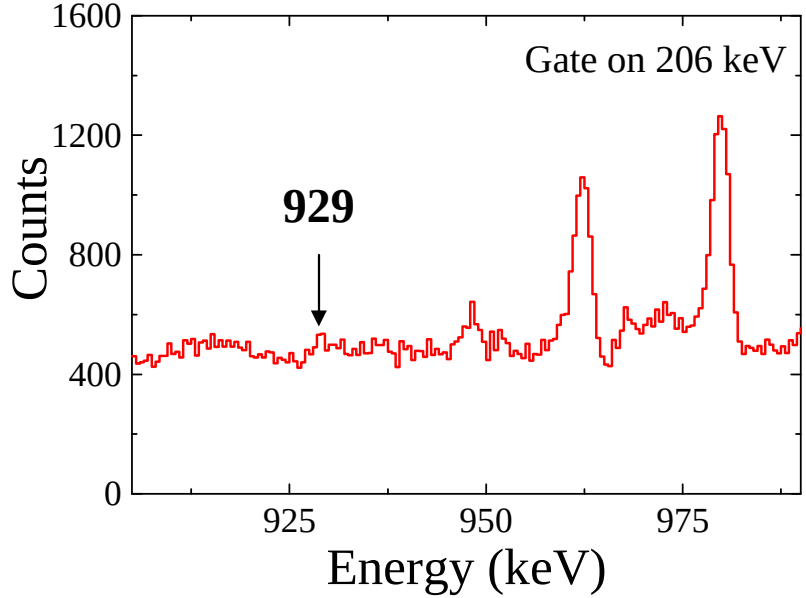


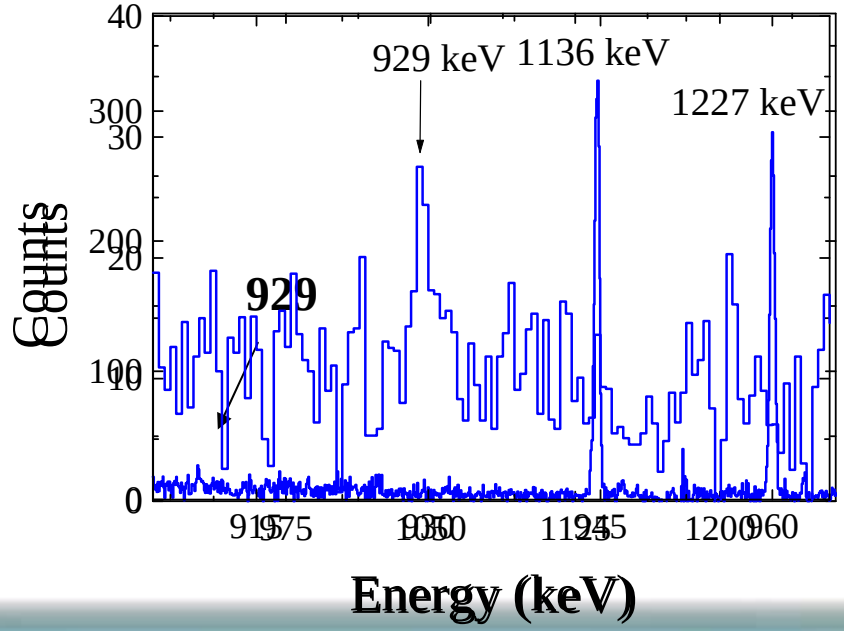

a)
Branching Ratio
B(E2; $2^+_\gamma \rightarrow 2^+_1$) / B(E2; $2^+_\gamma \rightarrow 0^+_1$)
B(E2; $2^+_\gamma \rightarrow 0^+_1$) / B(E2; $2^+_\gamma \rightarrow 4^+_1$)
Z_γ
0
2
4
6
8
10
12
14
16
0.0
0.1
0.2
0.3
0.4
0.5


Z_γ
0.00
0.02
0.04
0.06
0.08
0.10
0.12
-800
-600
-400
-200
0
200
400
$E(2^+_\gamma)$ - $E(2^+_\beta)$ [keV]

Search for $2_\gamma \rightarrow 4_1$ transition in ^{174}Hf







Alaga rules

For branching ratio, intrinsic matrix element cancels

Depends only on squares of Clebsch-Gordon coefficients

$$\frac{B(E2; J_i \to J_f)}{B(E2; J_i \to J_f^{'})} = \frac{\left\langle J_i K_i 2\Delta K \mid J_f K_f \right\rangle^2}{\left\langle J_i K_i 2\Delta K \mid J_f^{'} K_f \right\rangle^2}$$

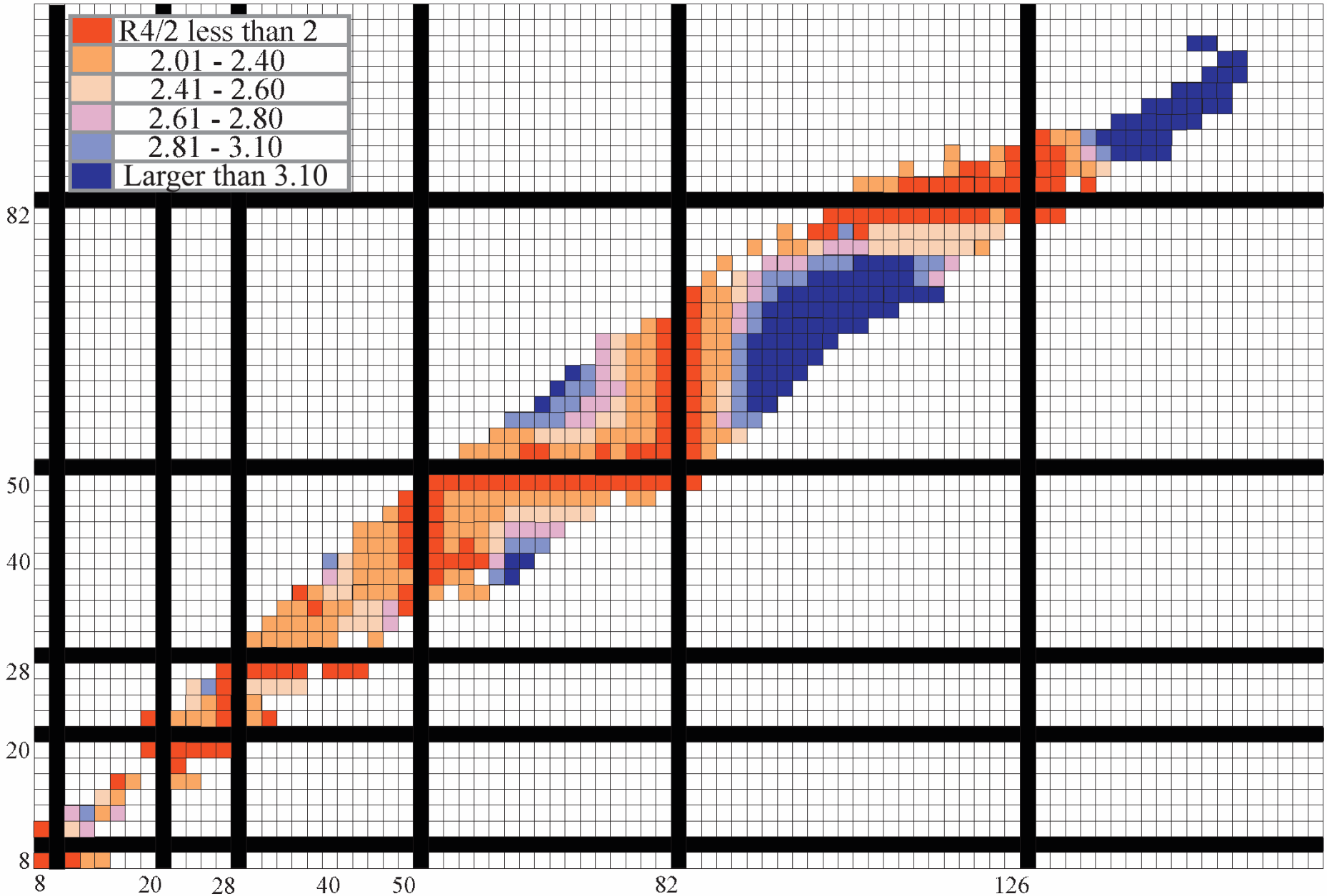
$$\frac{B(E2; 2_\gamma^+ \to 0_1^+)}{B(E2; 2_\gamma^+ \to 4_1^+)}$$

Alaga → 14.3

^{170}Hf → 0.07(1)

$$2_\gamma^+ \sim 2_\beta^+$$

$$\Delta E = 25 keV$$

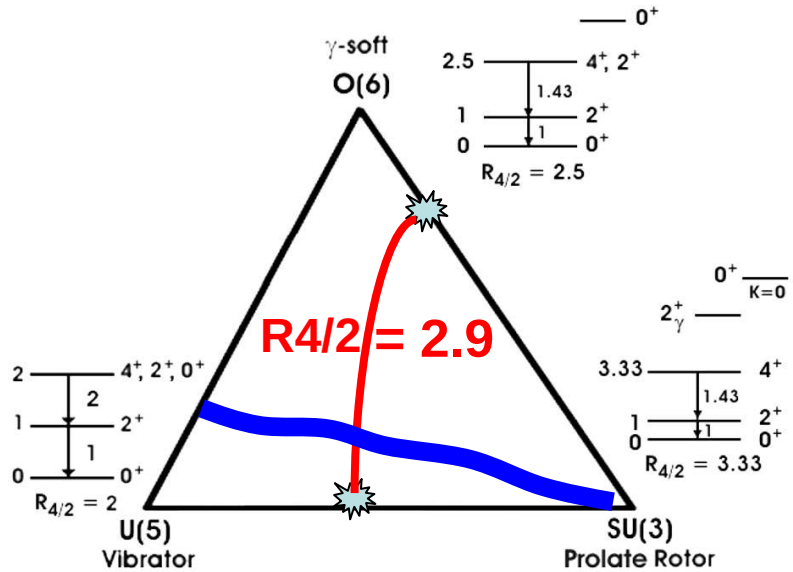

R4/2 less than 2
2.01 - 2.40
2.41 - 2.60
2.61 - 2.80
2.81 - 3.10
Larger than 3.10
82
50
40
28
20
8
8
20
28
40
50
82
126

Signatures of Structure and Phase Transitions

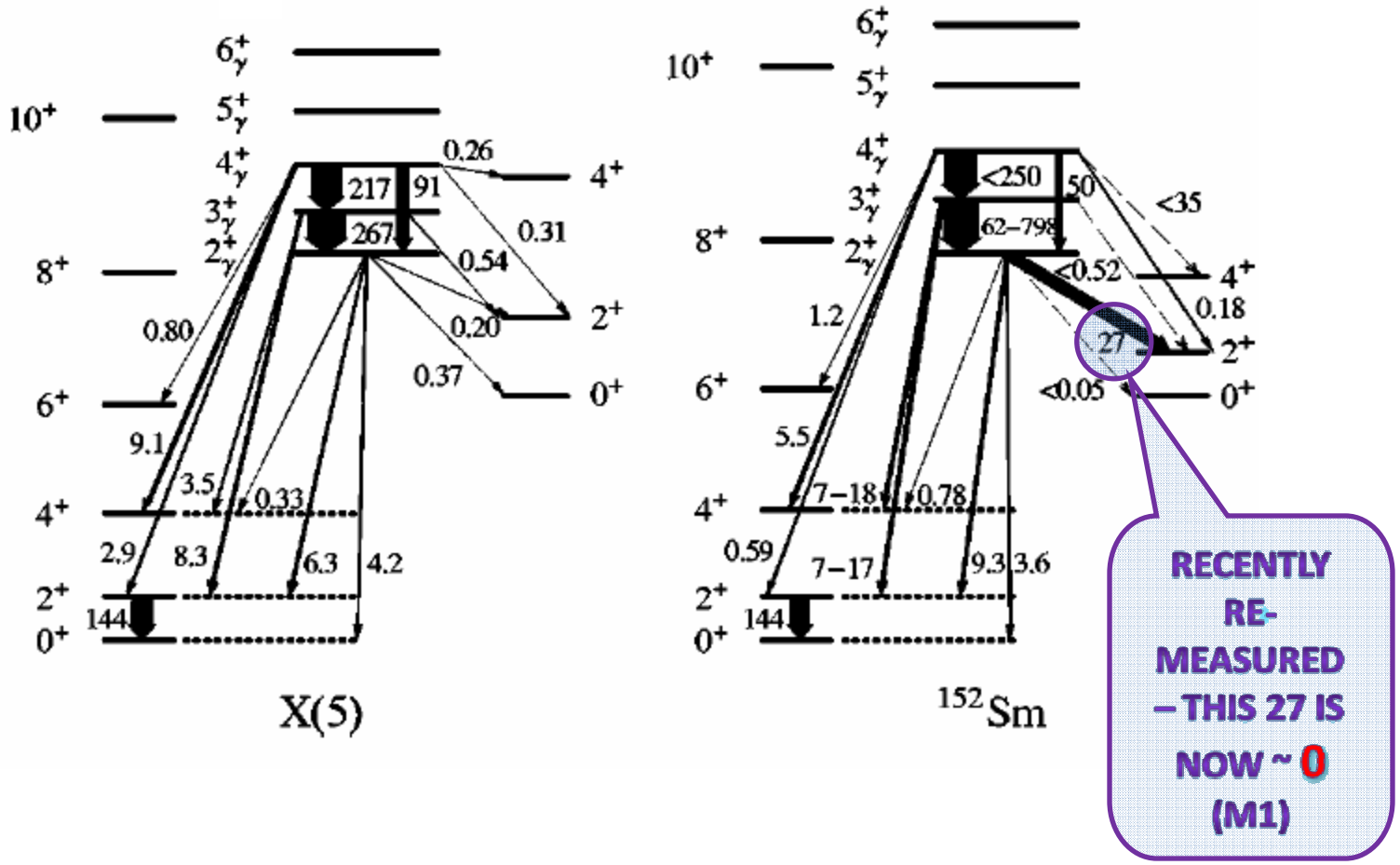
- $R_{4/2} = E(4^+)/E(2^+)$ -- Defines locus in triangle

- Need another observable:

$$\frac{E(0_2^+) - E(2_\gamma^+)}{E(2_1^+)}$$



Gamma Band in X(5)



One glaring discrepancy: $2^+_\gamma \rightarrow 2^+_2$ **transition.**

Measured B(*E*2) = 27(4) W.u. **X(5) predicts 0.20 W.u.**

PHYSICAL REVIEW C **76**, 034319 (2007)

$N = 90$ region: The decays of ^{152}Eu^{m,g} to ^{152}Sm

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(Received 27 June 2006; published 21 September 2007)

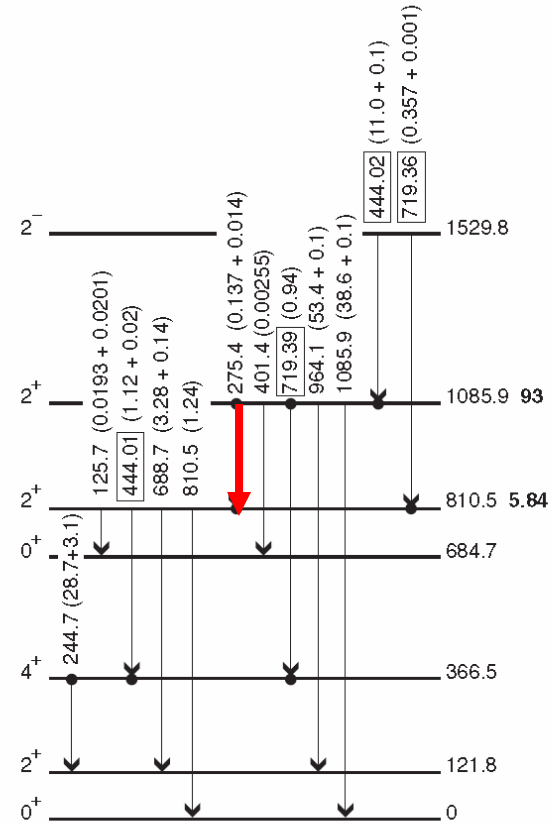


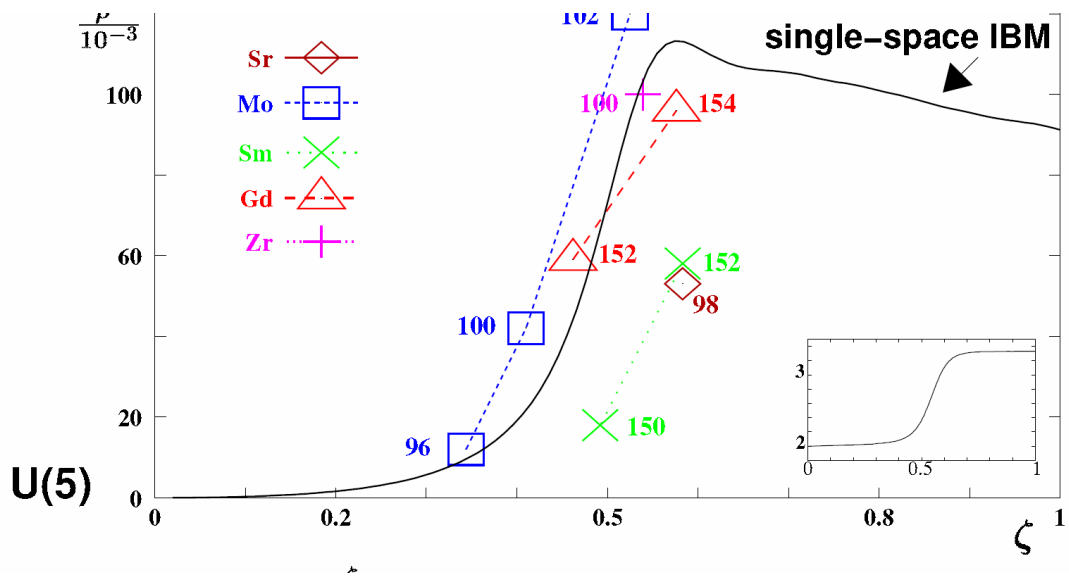
FIG. 2. Partial level scheme for ^{152}Sm which shows the γ-ray and internal conversion intensities required to disentangle the unresolved 444.01-719.36 and 444.02-719.39 doublet pairs. The total intensities I_T out of the 810.5 and 1085.9 keV levels are indicated to the right of the respective levels. The iterative approach adopted to determine the intensities of the 444.01 and 719.39 keV γ rays is discussed in the text. Note that the key 275 keV γ-ray transition is between the 444.01 and 444.02 γ rays; the impact of this is also discussed in Sec. V A.

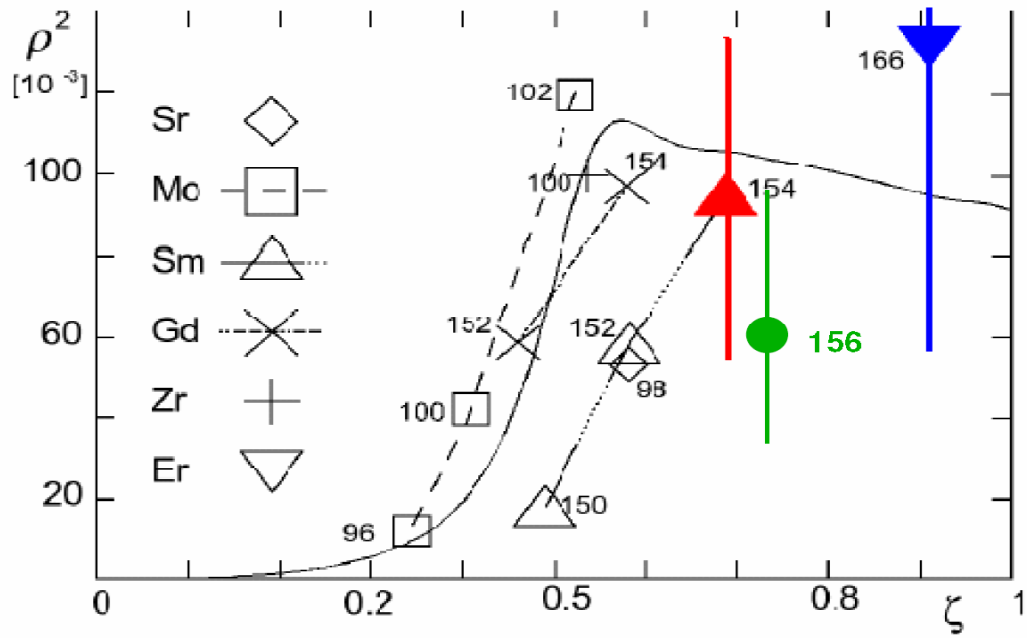
The outcome of this is that the 275 keV *g*–ray intensity, when combined with the conversion electron intensities of Goswamy *et al.* [19] yields *αK(275)* = 0.097 (8) (expt.), cf. *αK*(275,M1) = 0.0862 (12) (theory) [15]. Therefore, the 275 keV transition is (nearly) pure M1 and is noncollective.

E0 Transitions between 0^+ states

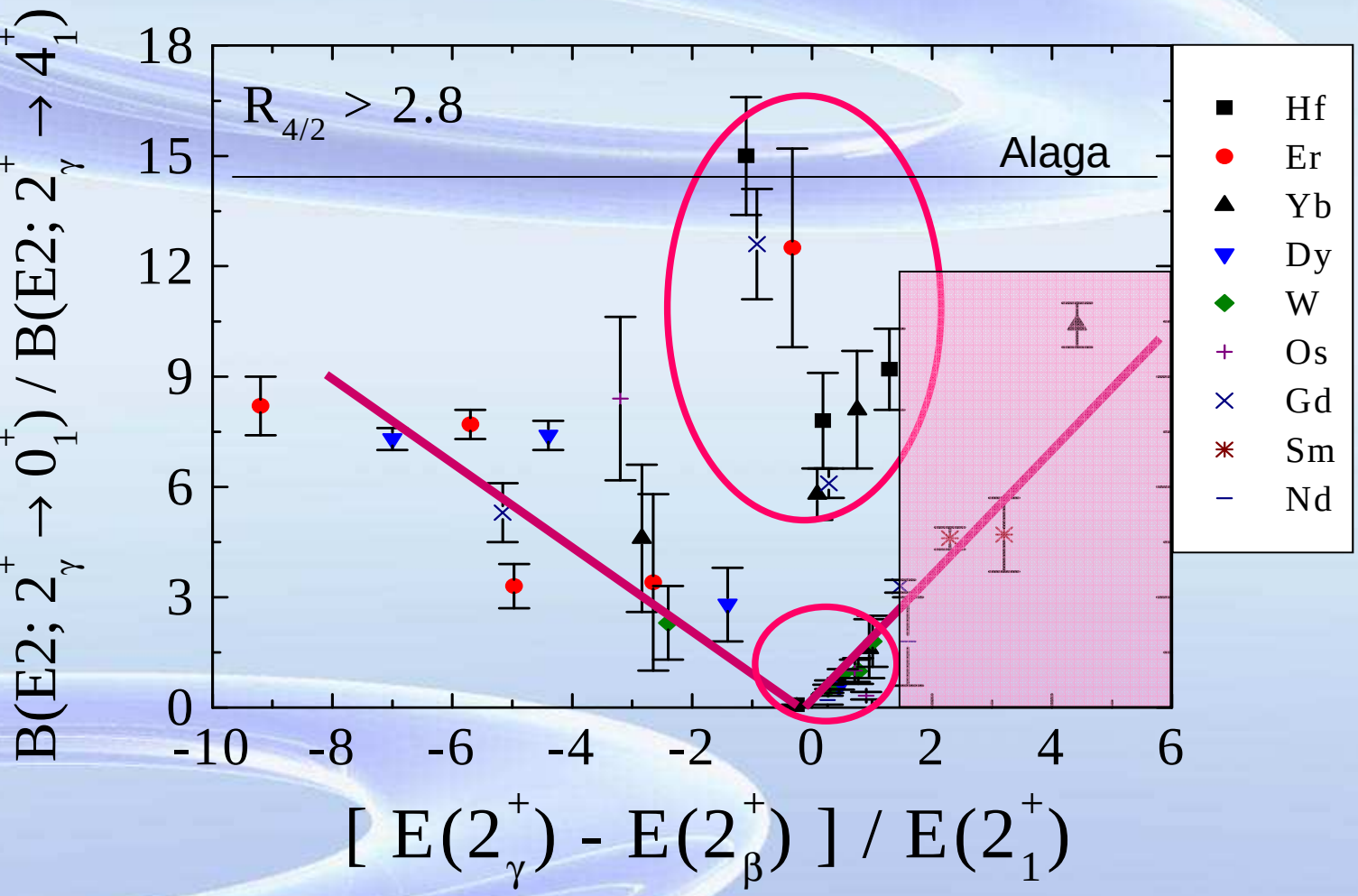
Large values in deformed nuclei are a fundamental prediction of most collective models – [In IBA, they directly sample the n_d boson structure]

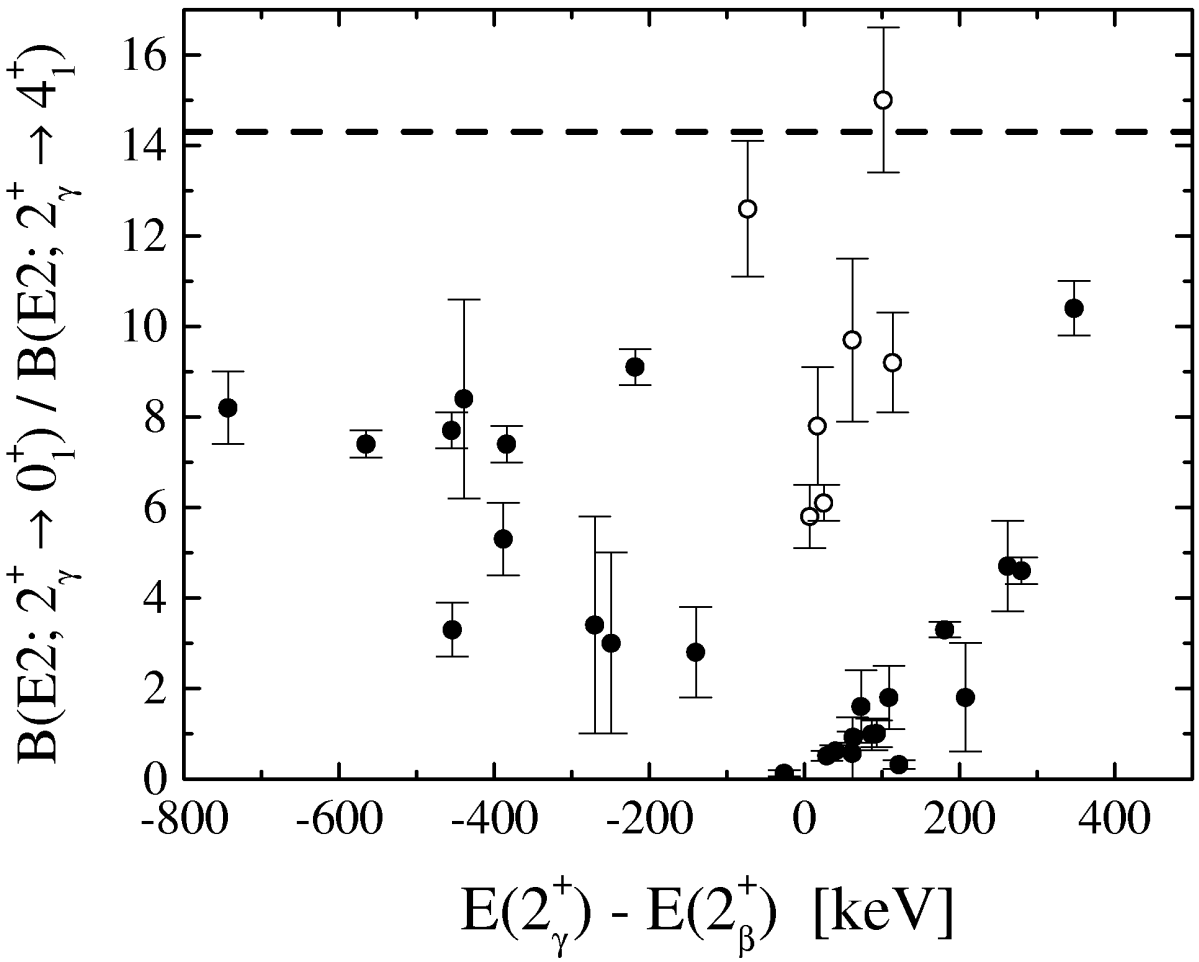
Hence these and other new measurements are important results.

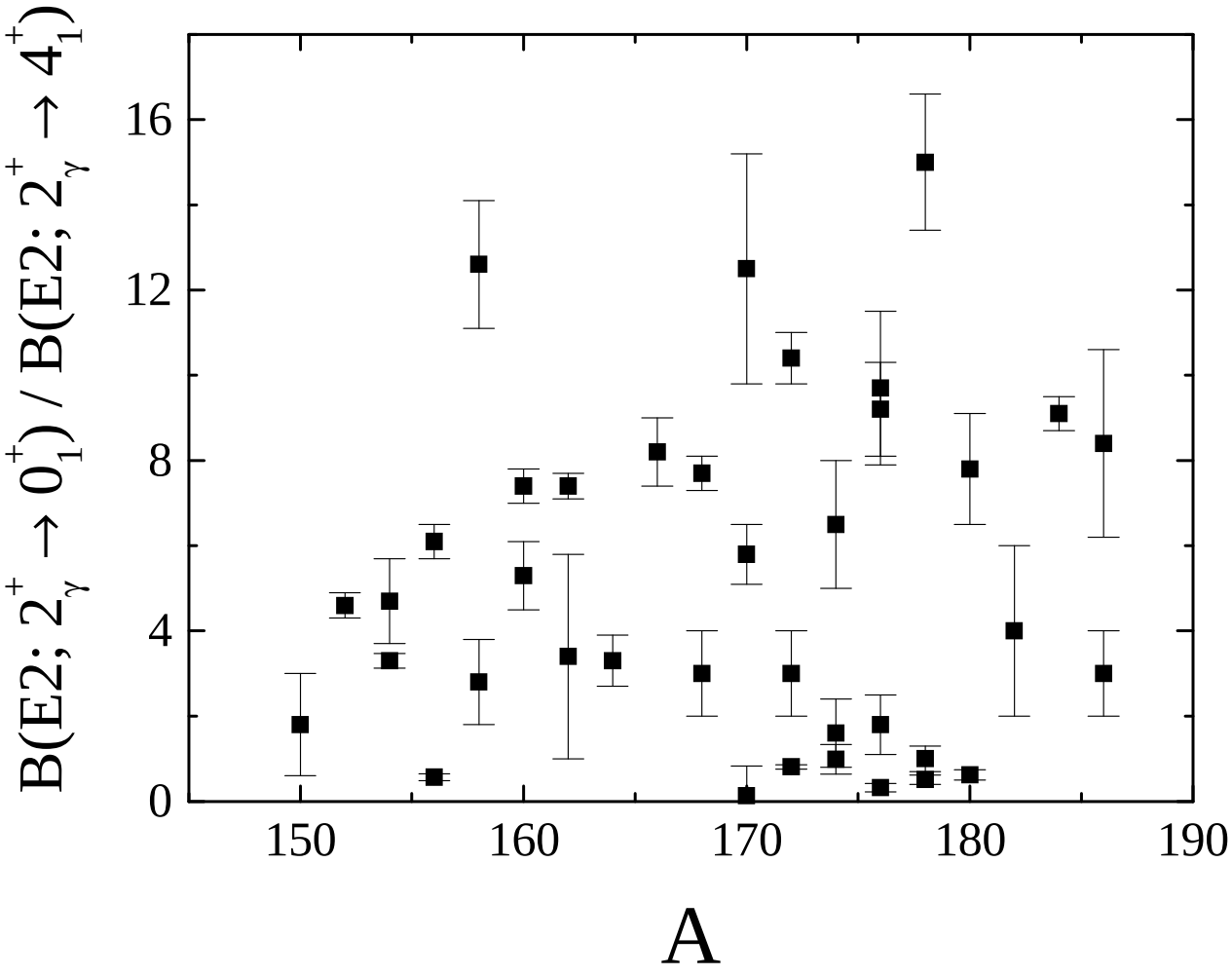


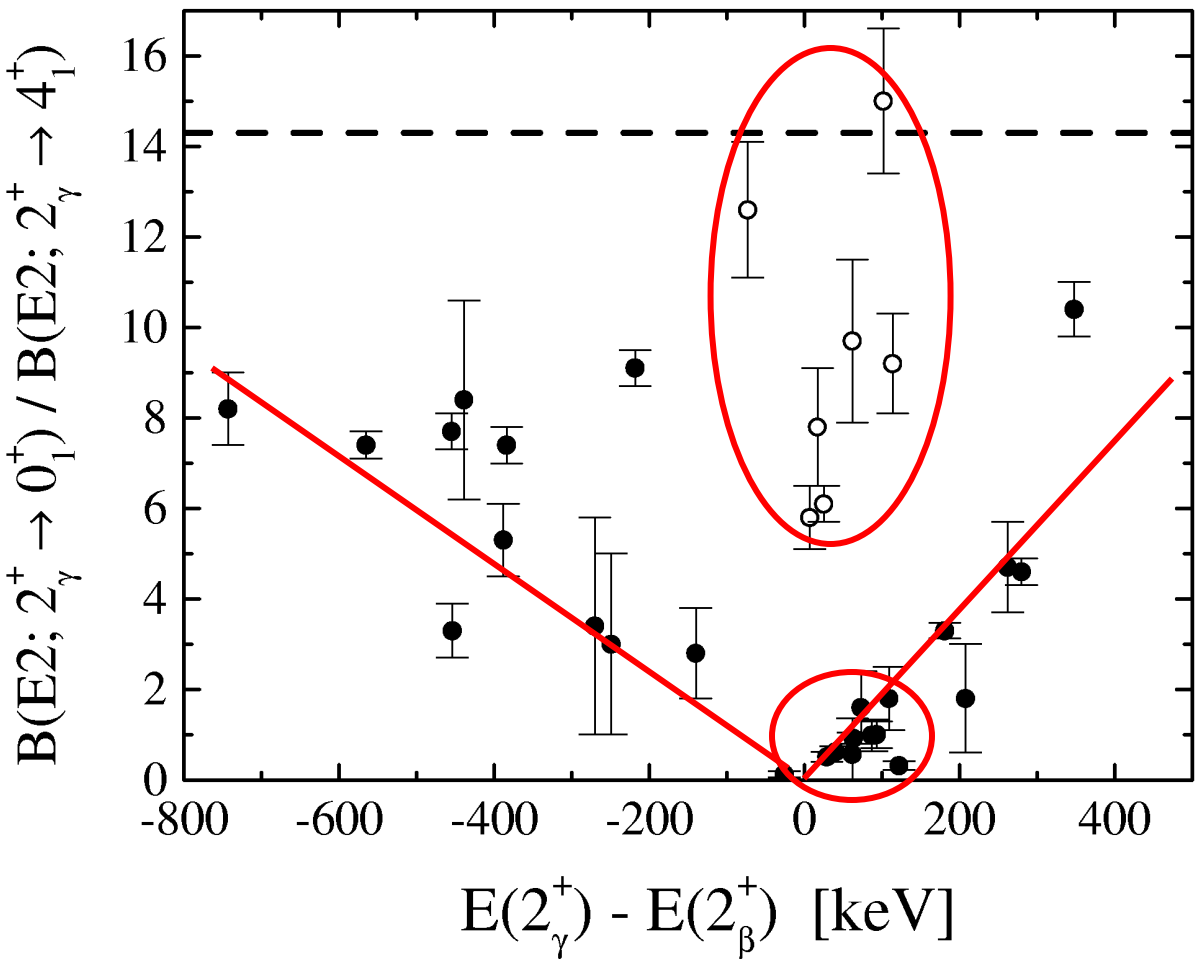


Looking more systematically

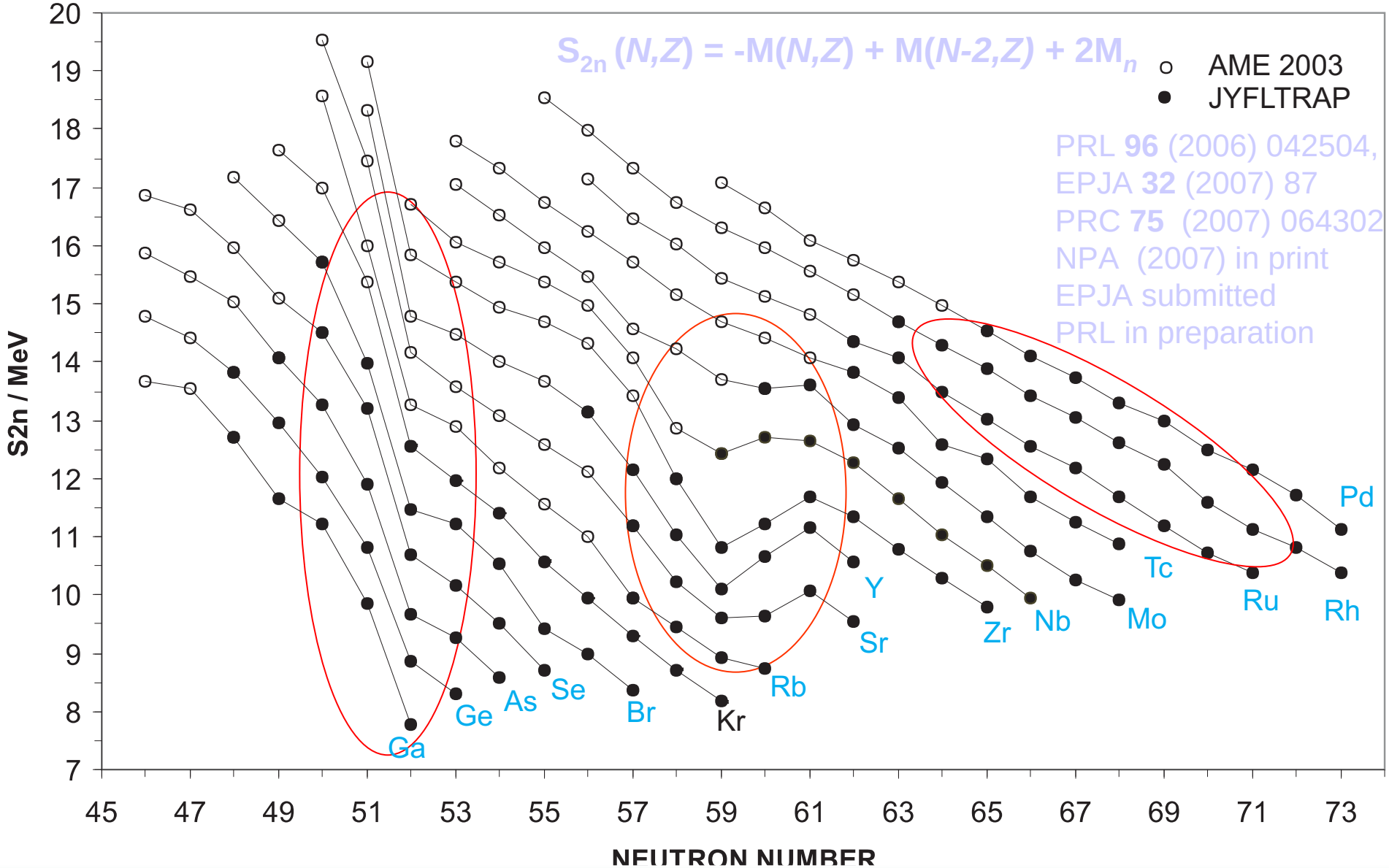



B(E2; 2+γ → 0+1) / B(E2; 2+γ → 4+1)
16
14
12
10
8
6
4
2
0
-800
-600
-400
-200
0
200
400
E(2+γ) - E(2+β) [keV]

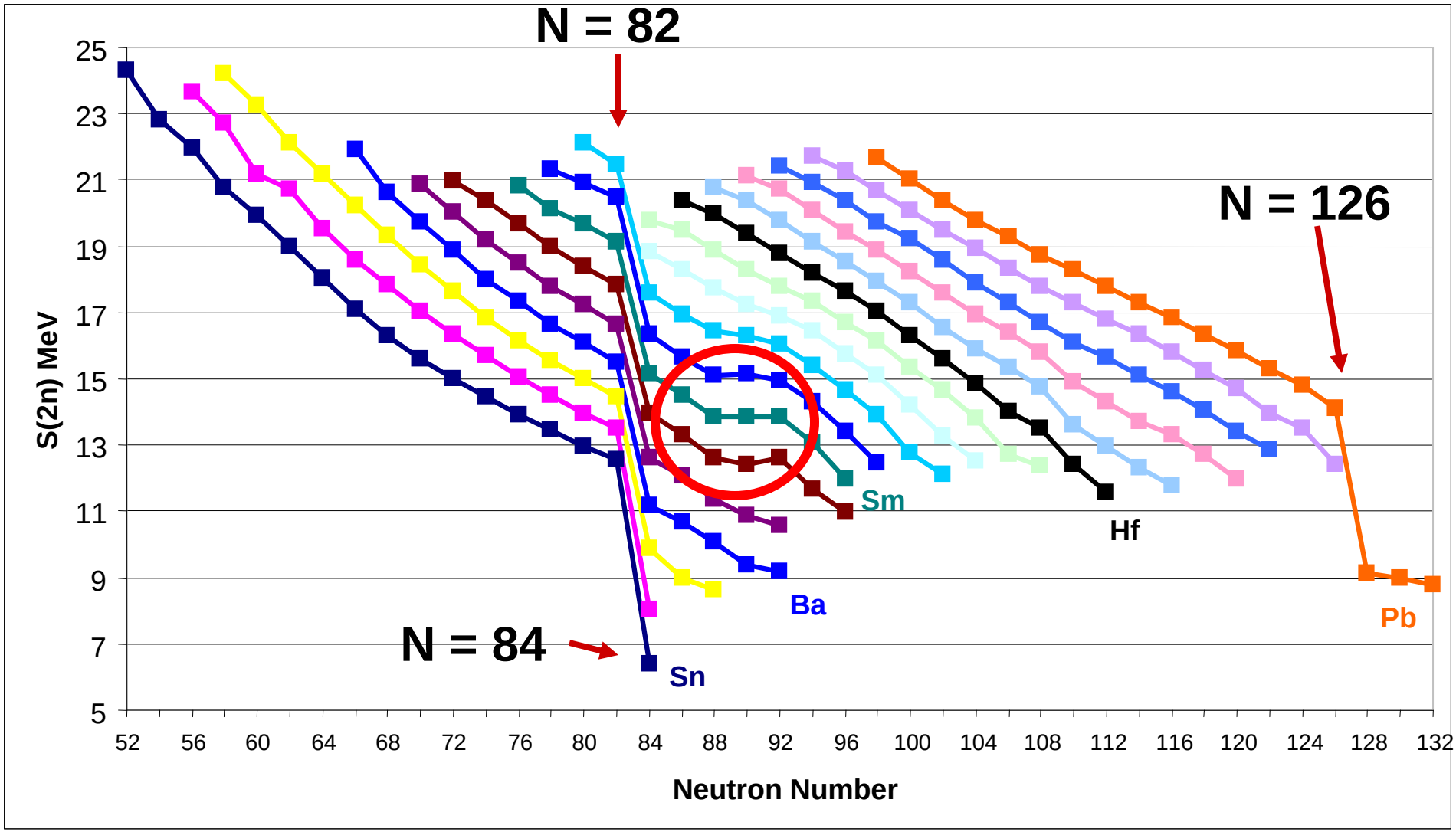

B(E2; 2$^+_\gamma$ → 0$^+_1$) / B(E2; 2$^+_\gamma$ → 4$^+_1$)
0
4
8
12
16
150
160
170
180
190
A

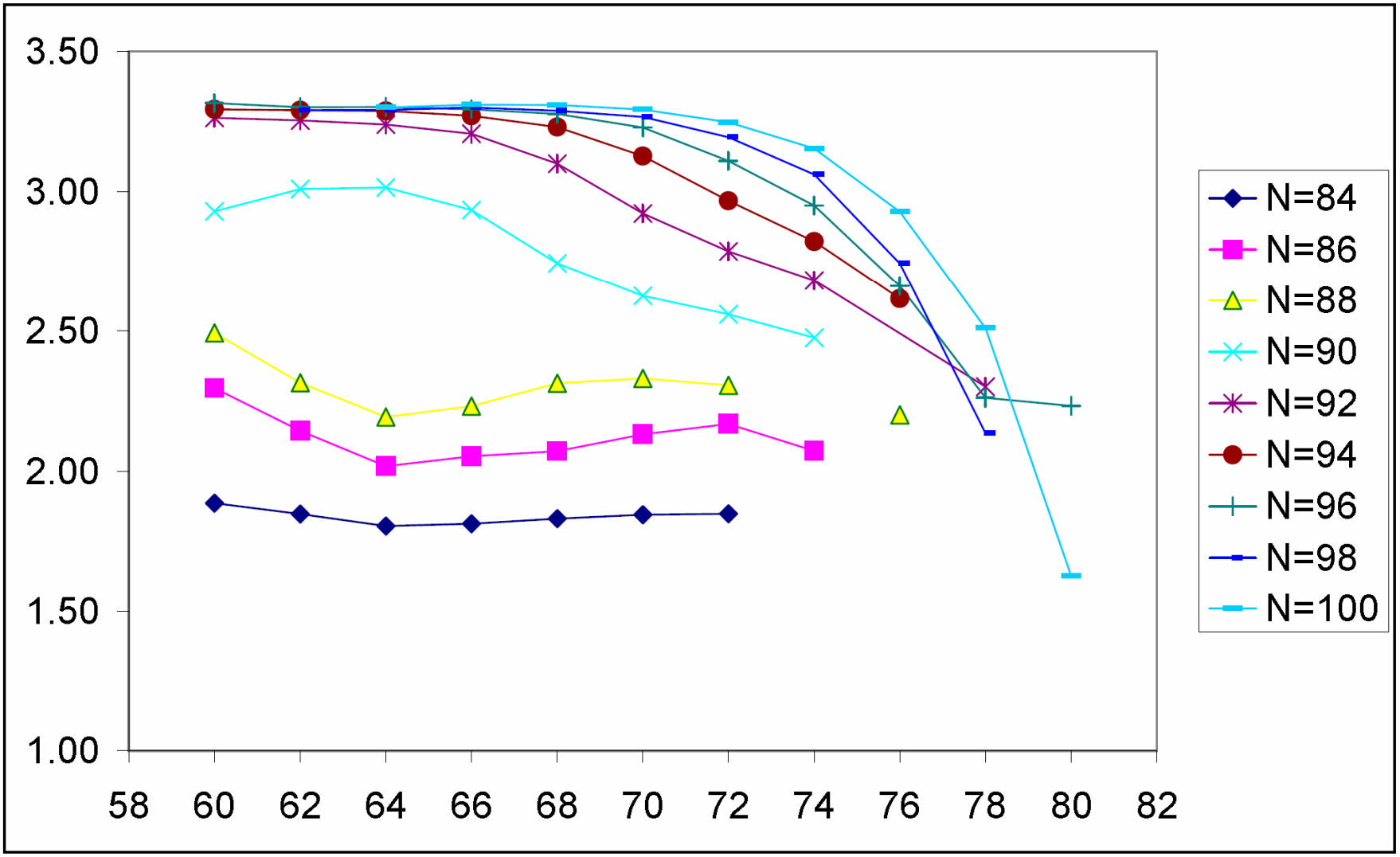

B(E2; 2+γ → 0+1) / B(E2; 2+γ → 4+1)
16
14
12
10
8
6
4
2
0
-800
-600
-400
-200
0
200
400
E(2+γ) - E(2+β) [keV]

Some results



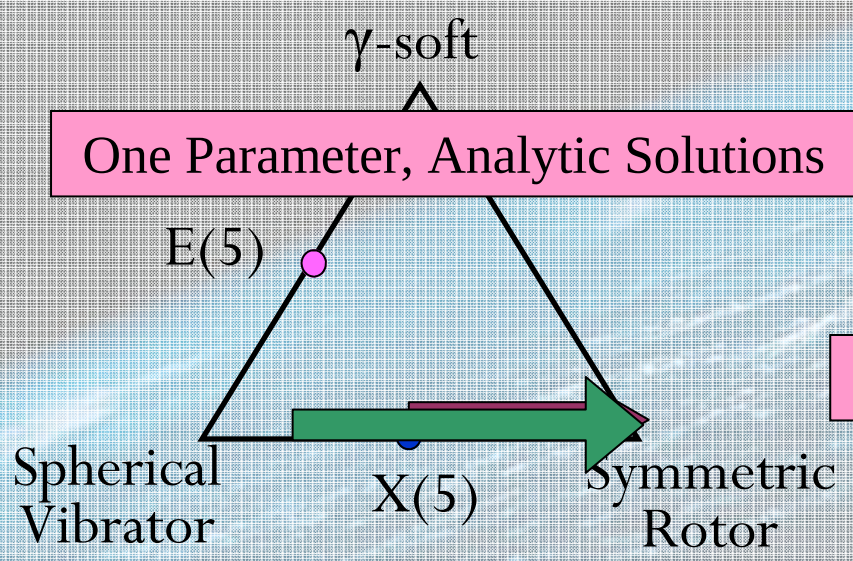
Energy required to remove two neutrons from nuclei




3.50
3.00
2.50
2.00
1.50
1.00
58
60
62
64
66
68
70
72
74
76
78
80
82
N=84
N=86
N=88
N=90
N=92
N=94
N=96
N=98
N=100

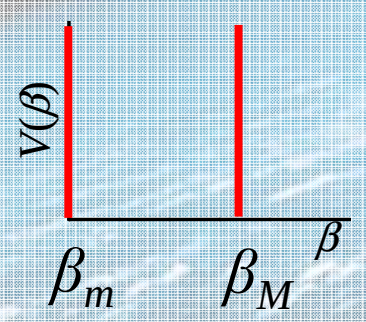
Structure Beyond the Phase Transition

New simple geometrical models to describe a wider range of structures



Confined Beta Soft (CBS)

N. Pietralla and O.M. Gorbachenko, Phys. Rev. C **70**, 011304(R) (2004).

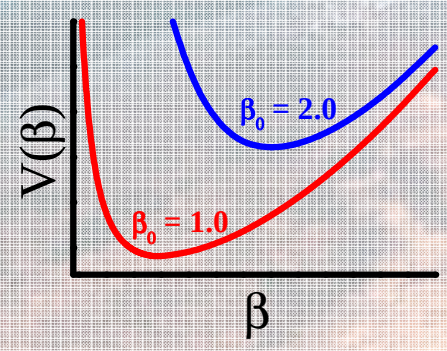


$$r_\beta = \frac{\beta_m}{\beta_M}$$

$$r_\beta : 0 \rightarrow 1$$

Davidson Potential

Dennis Bonatsos *et al.*, Phys. Rev. C **70**, 024305 (2004).



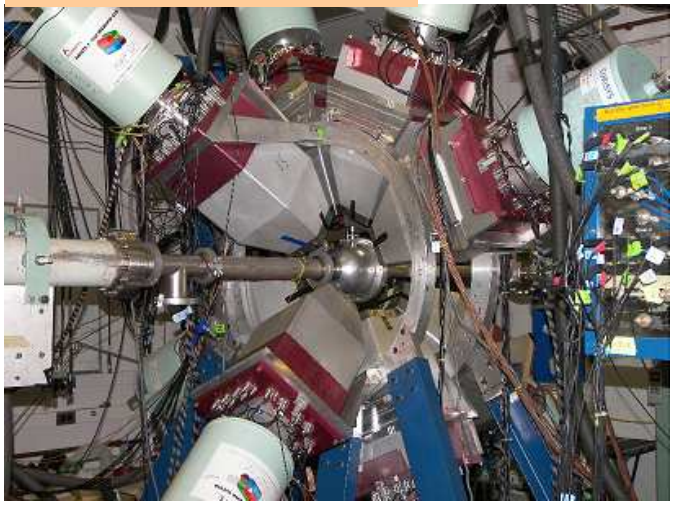
$$u(\beta) = \beta^2 + \frac{\beta_0^4}{\beta^2}$$

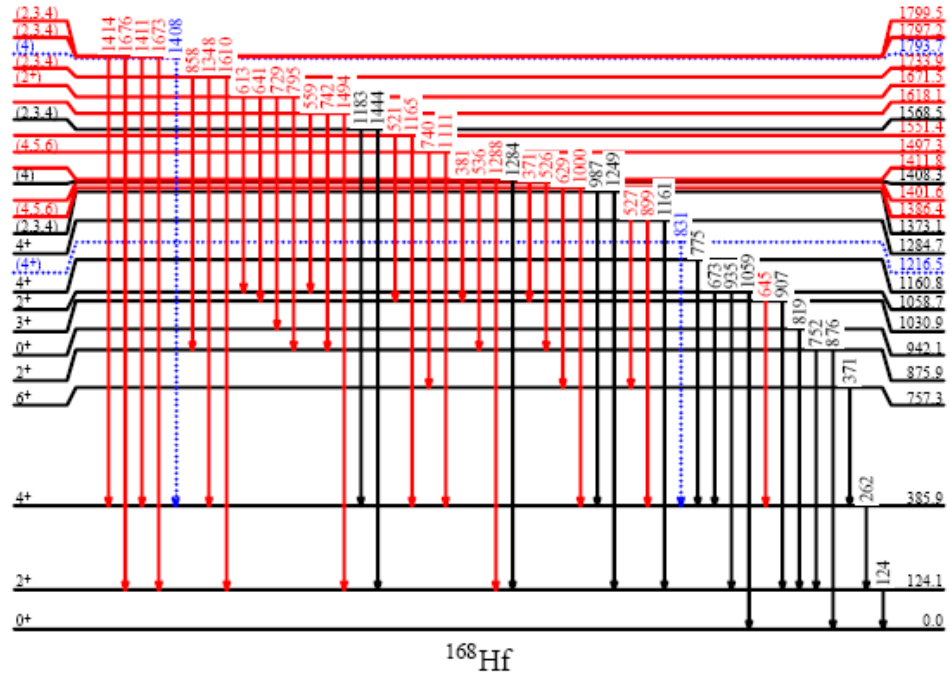
$$\beta_o : 0 \rightarrow \infty$$

^{168}Hf Experiment ($R_{4/2}$ = 3.11)

$$^{159}Tb(^{16}O,7n)^{168}Ta \xrightarrow{\beta} {}^{168}Hf$$

YRAST BALL



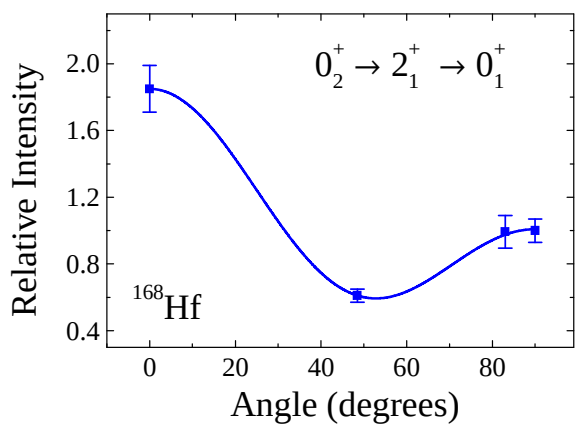


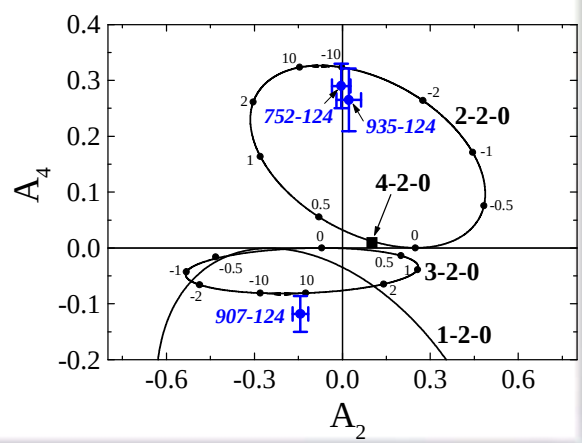
8 Clover detectors

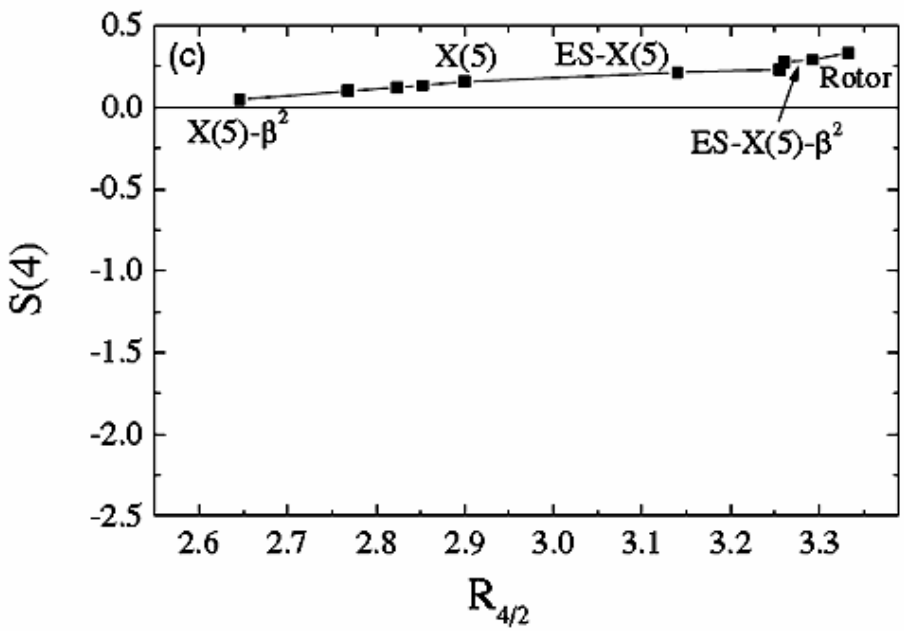
γ-γ angular correlations

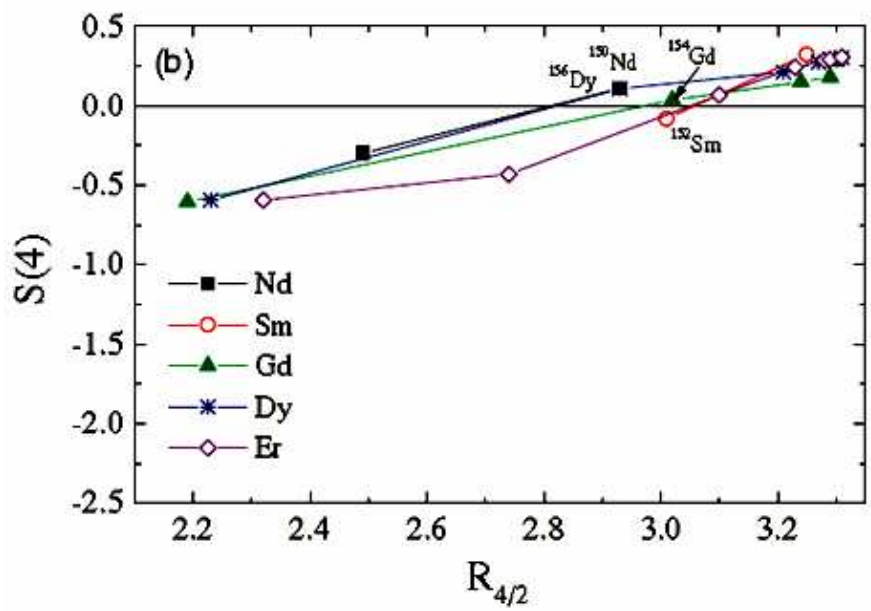
Beam on/beam off

~ 2 days






(c)
X(5)
ES-X(5)
Rotor
X(5)-β²
ES-X(5)-β²
S(4)
R4/2
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
2.6
2.7
2.8
2.9
3.0
3.1
3.2
3.3

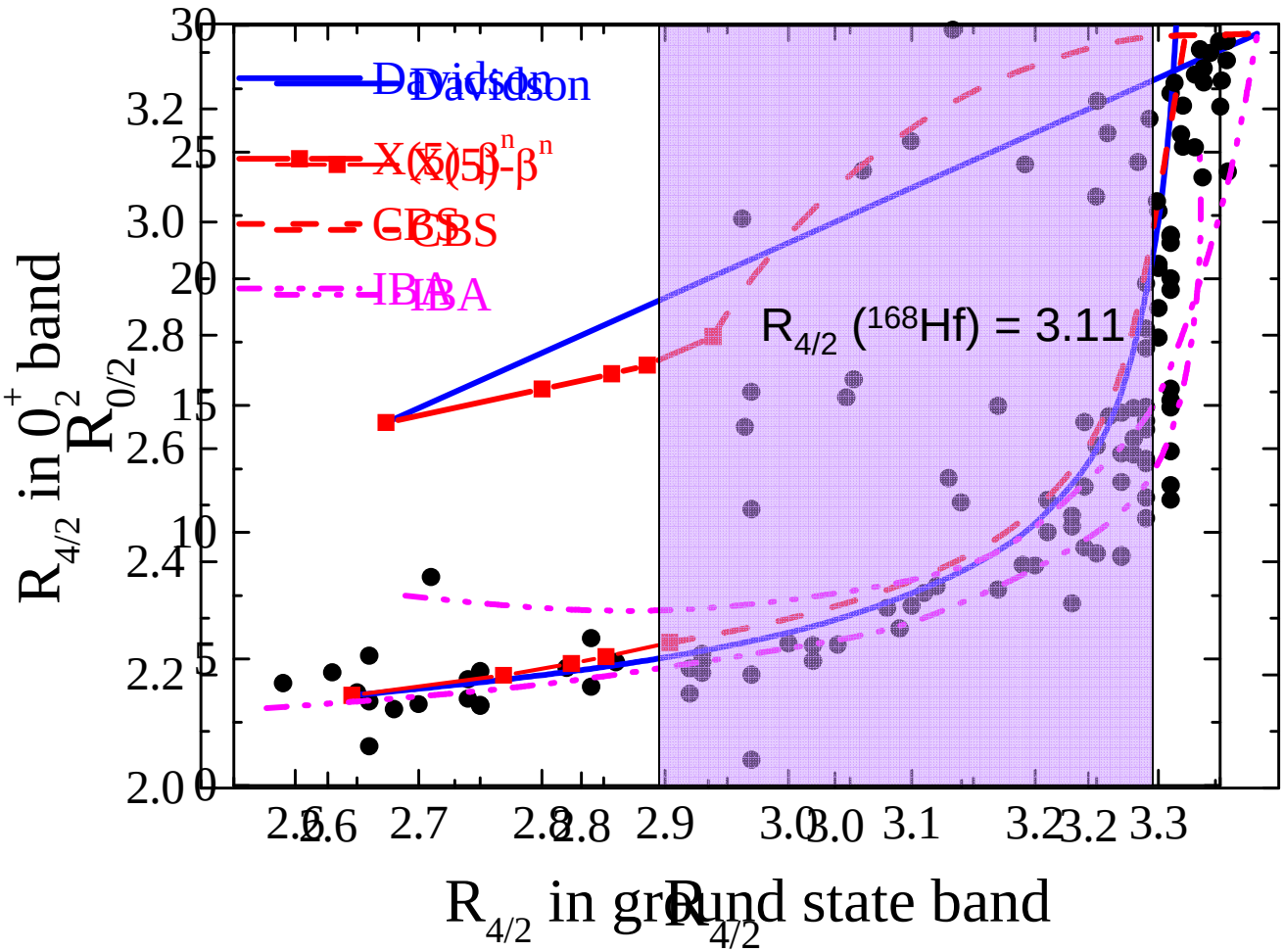

(b)
156Dy
150Nd
154Gd
152Sm
Nd
Sm
Gd
Dy
Er
S(4)
R4/2
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
2.2
2.4
2.6
2.8
3.0
3.2

Staggering indices for gamma softness, triaxiality

$$S(J) = \frac{\{E(J_\gamma^+) - E[(J-1)_\gamma^+]\} - \{E[(J-1)_\gamma^+] - E[(J-2)_\gamma^+]\}}{E(2_1^+)}$$

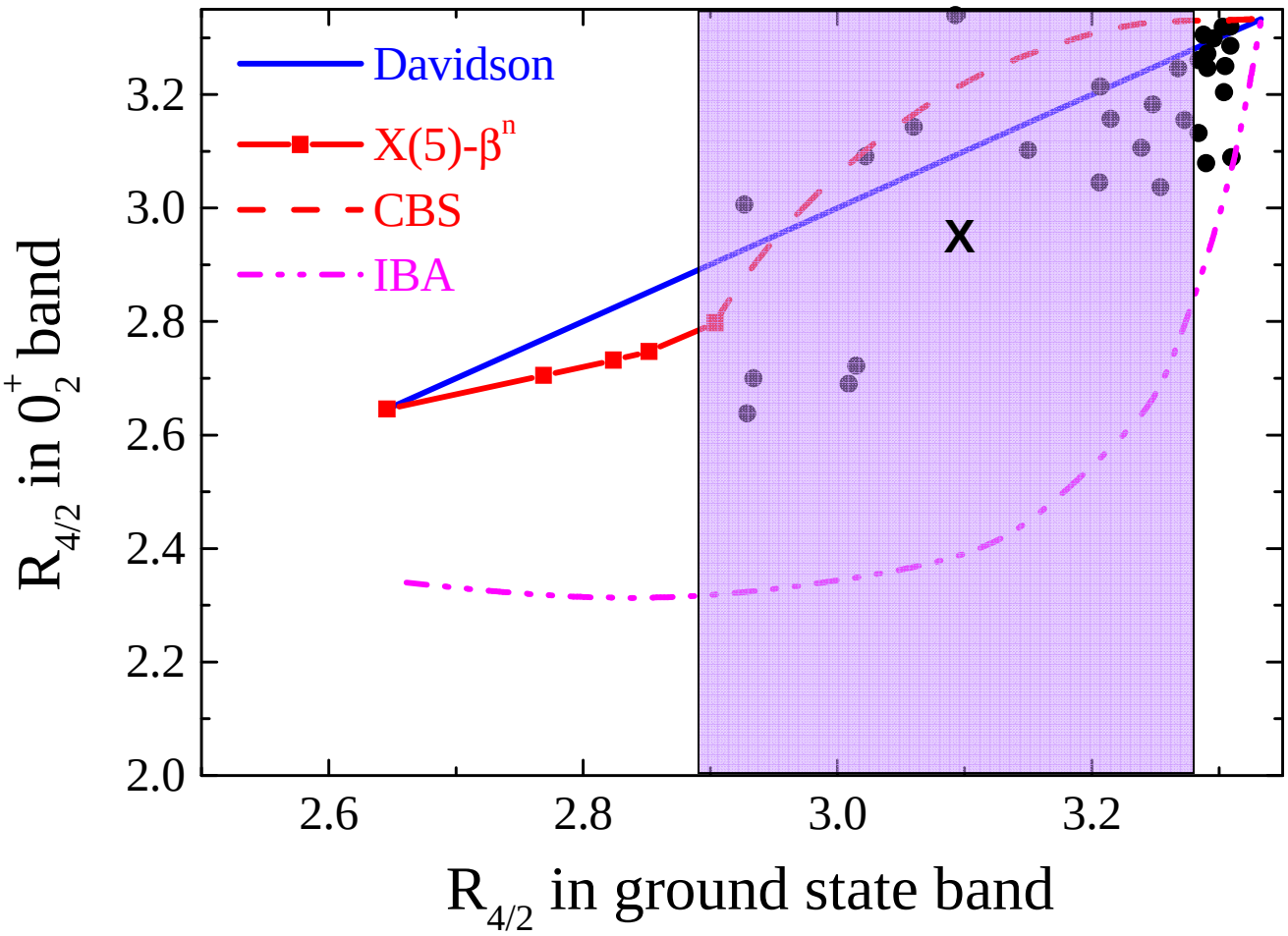
$$S(4) = \frac{[E(4_\gamma^+) - E(3_\gamma^+)] - [E(3_\gamma^+) - E(2_\gamma^+)]}{E(2_1^+)}$$

Testing new geometrical models



Significantly different predictions, particularly in transitional regions

Testing new geometrical models

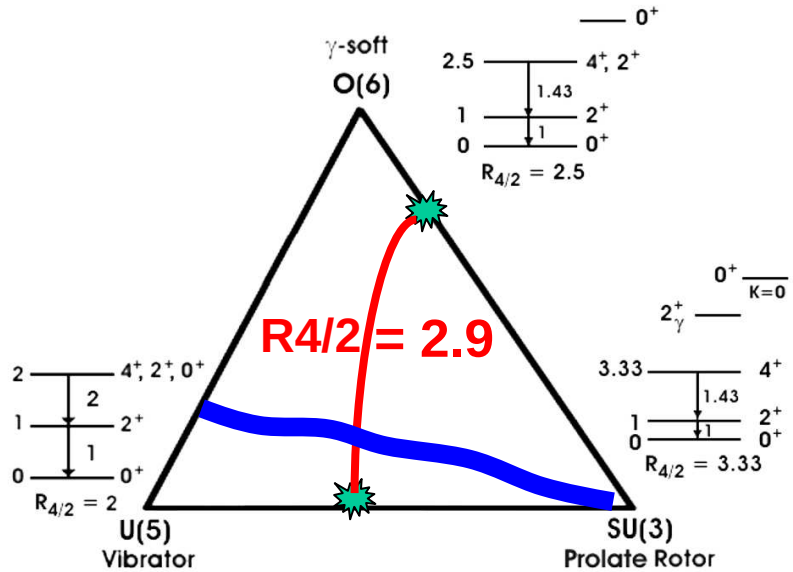


Signatures of Structure and Phase Transitions

- $R_{4/2} = E(4^+)/E(2^+)$ -- Defines locus in triangle

- Need another observable:

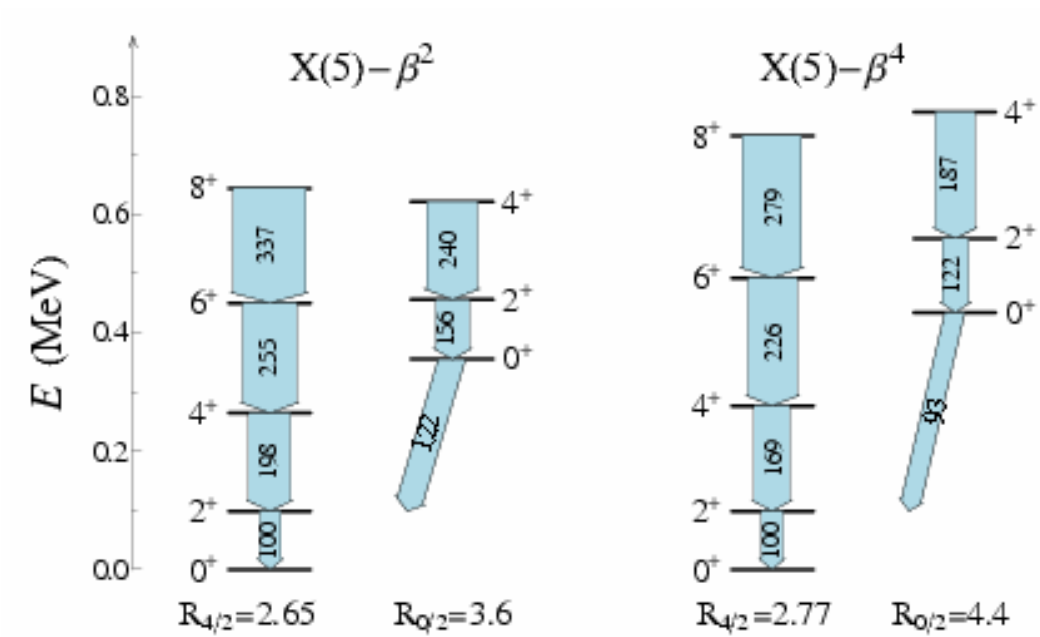
$$\frac{E(0_2^+) - E(2_\gamma^+)}{E(2_1^+)}$$

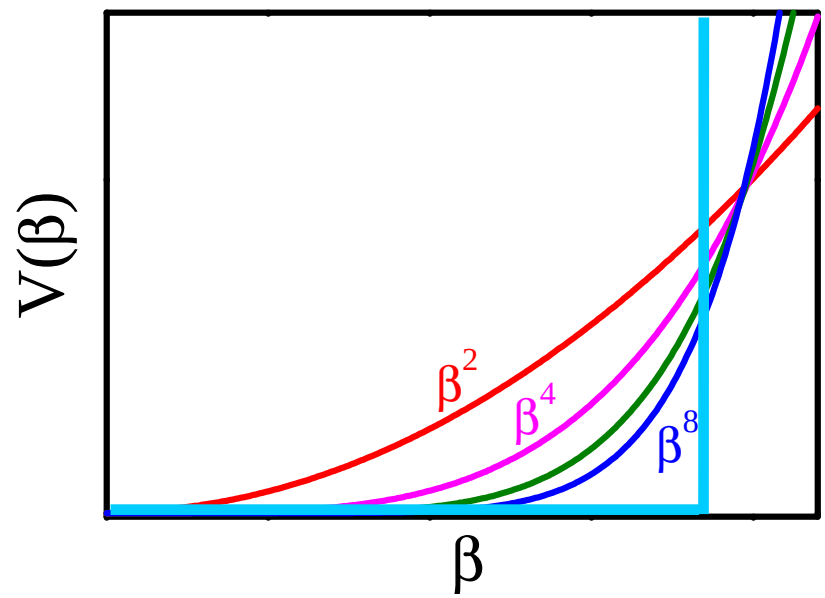


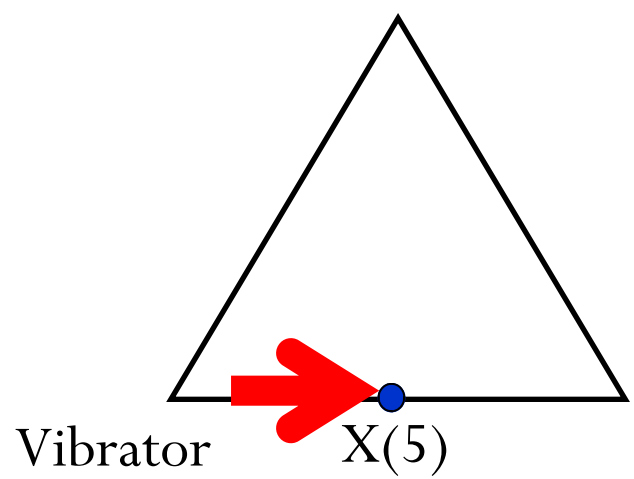
- Good but not enough: a regional perspective is also valuable

X(5)-β^n model

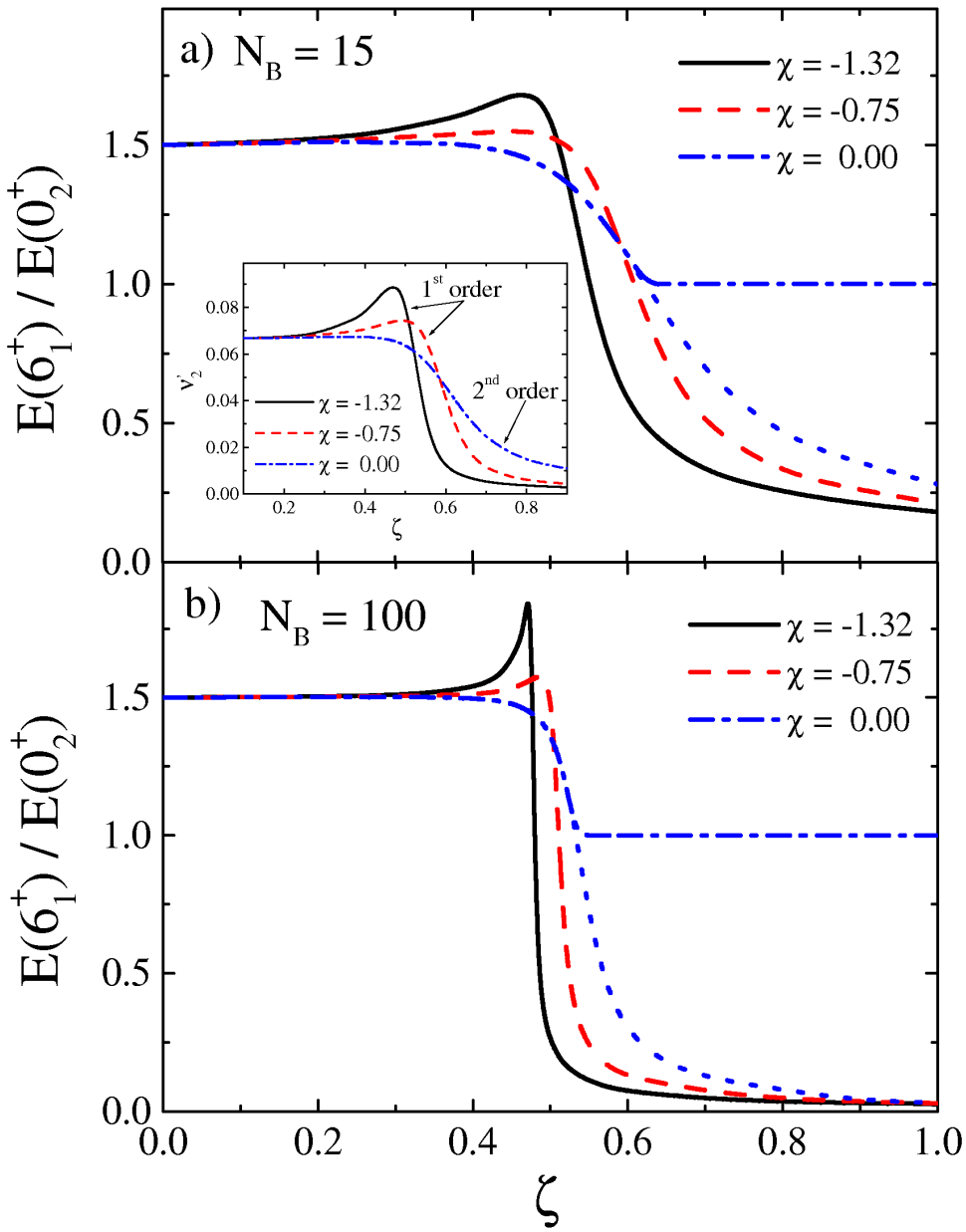
$$V(\beta) = \frac{\beta^{2n}}{2}$$



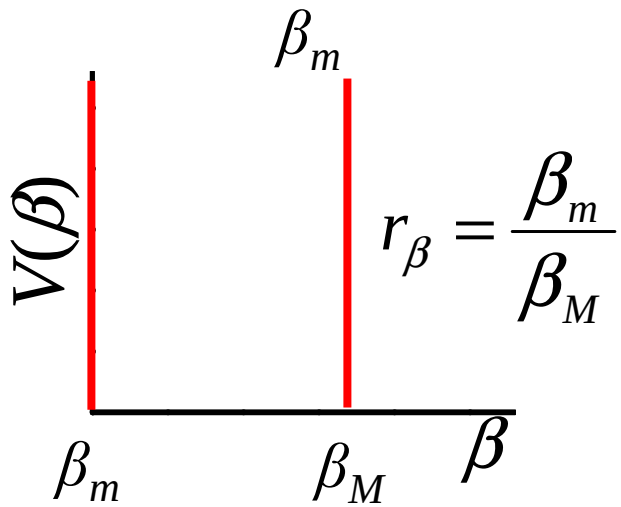


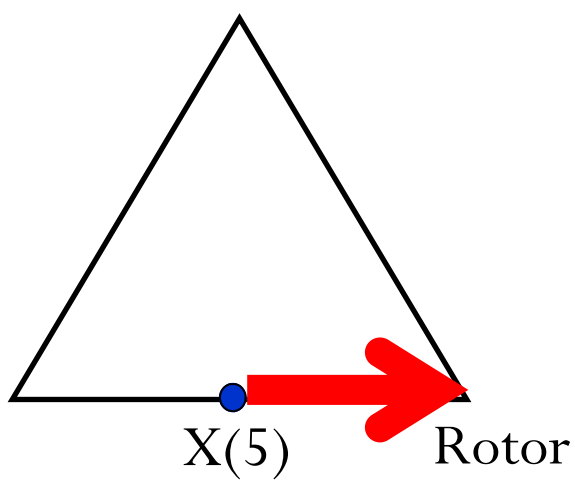


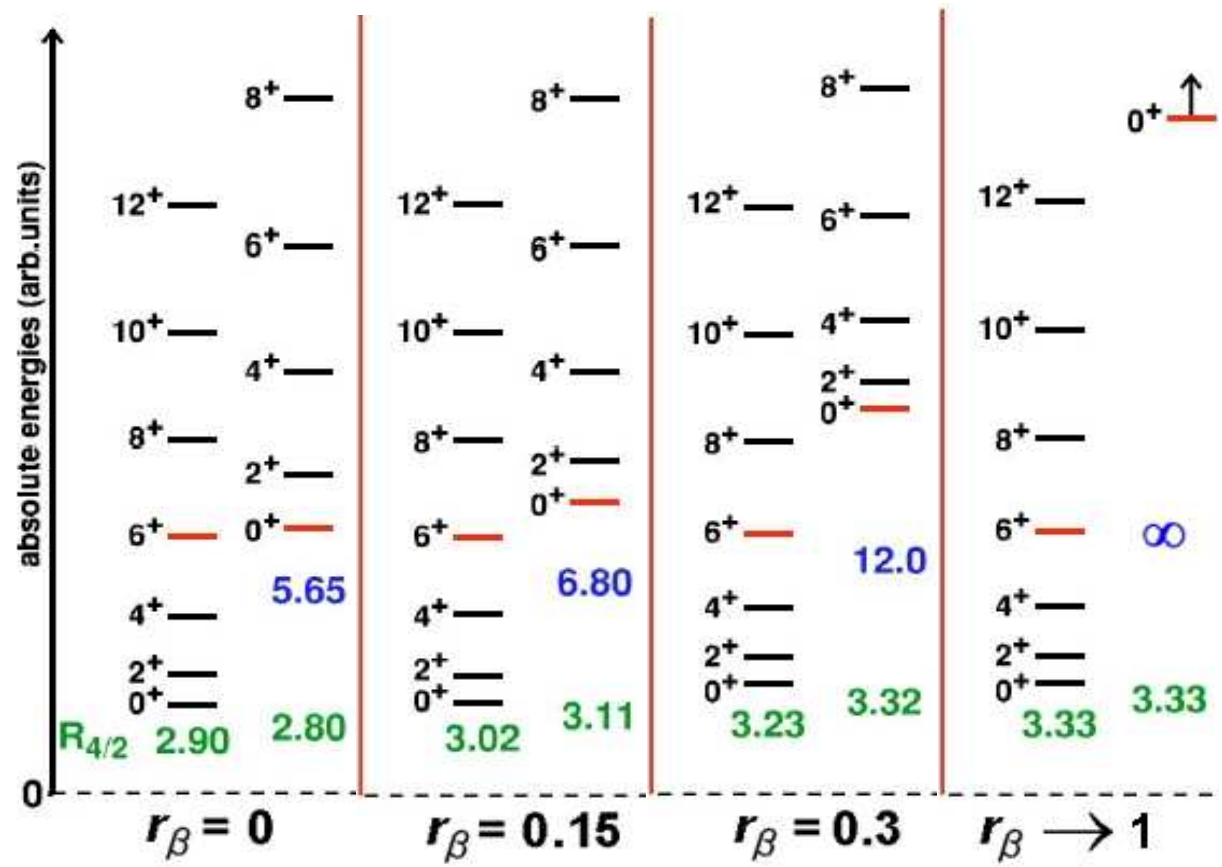
Dennis Bonatsos *et al.*, Phys. Rev. C **69**, 014302 (2004).


a) $N_B = 15$
$\chi = -1.32$
$\chi = -0.75$
$\chi = 0.00$
$E(6_1^+) / E(0_2^+)$
1.5
1.0
0.5
0.0
1st order
2nd order
v_2'
0.08
0.06
0.04
0.02
0.00
0.2
0.4
0.6
0.8
ζ
b) $N_B = 100$
$\chi = -1.32$
$\chi = -0.75$
$\chi = 0.00$
$E(6_1^+) / E(0_2^+)$
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
ζ

Confined Beta-Soft (CBS)

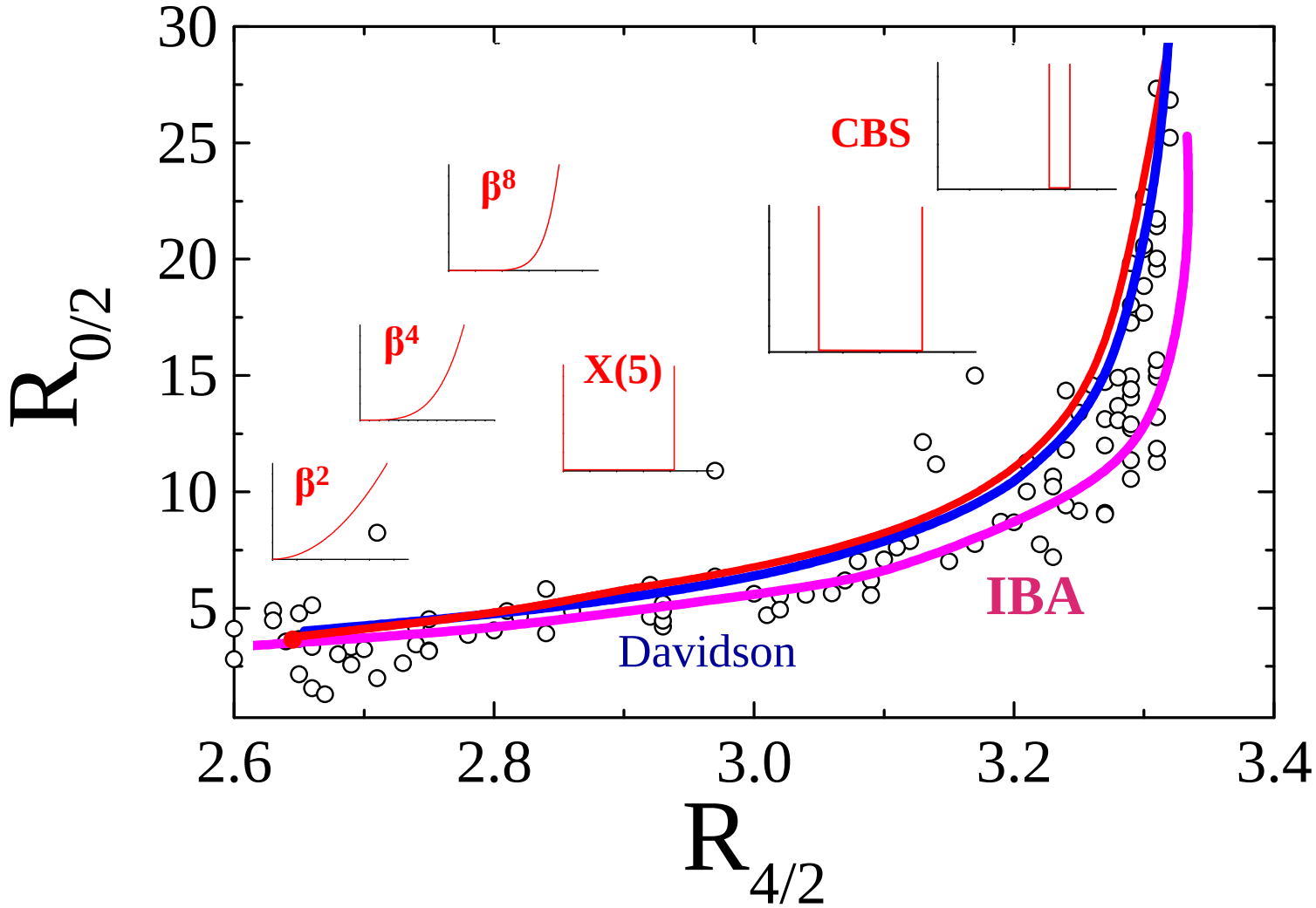


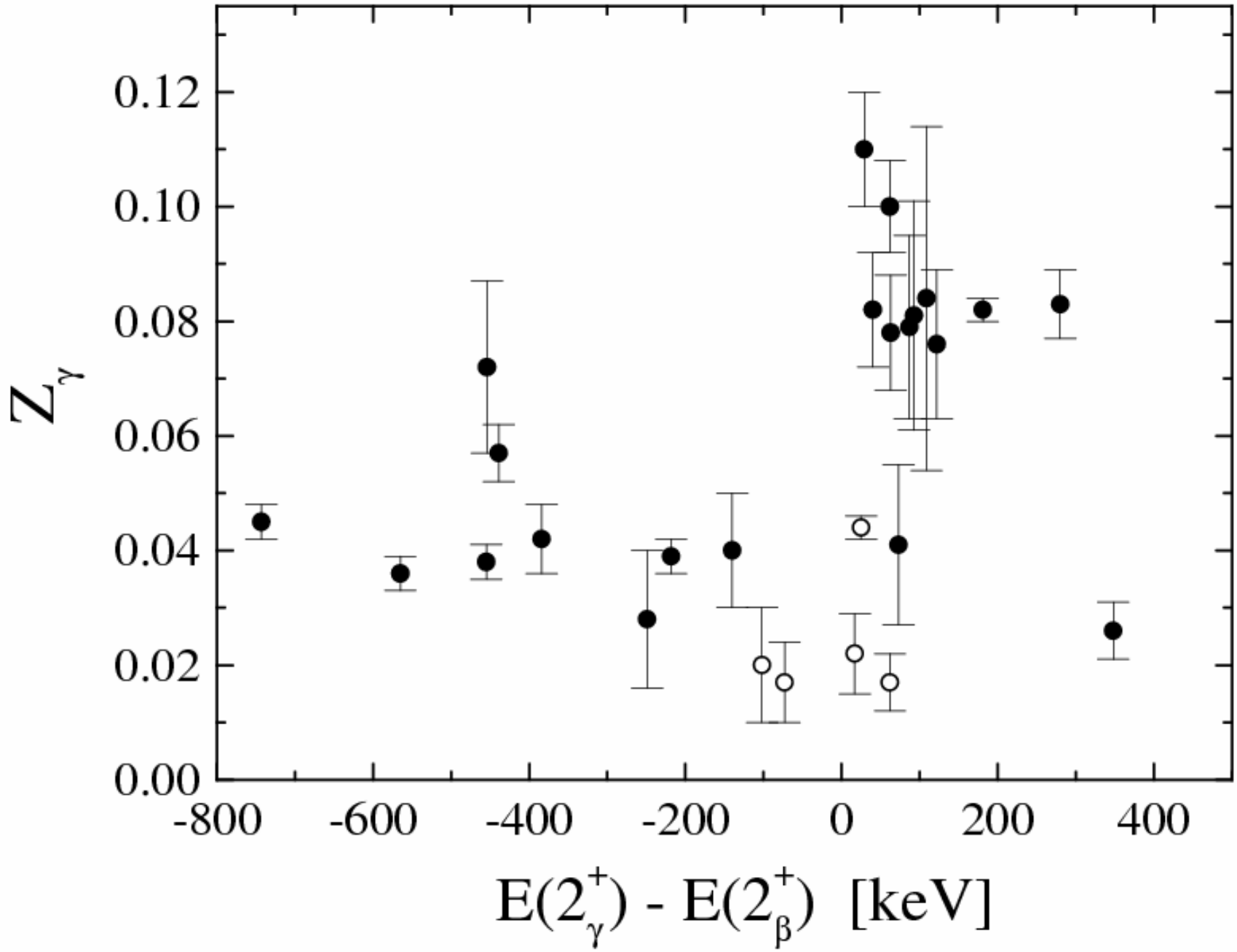


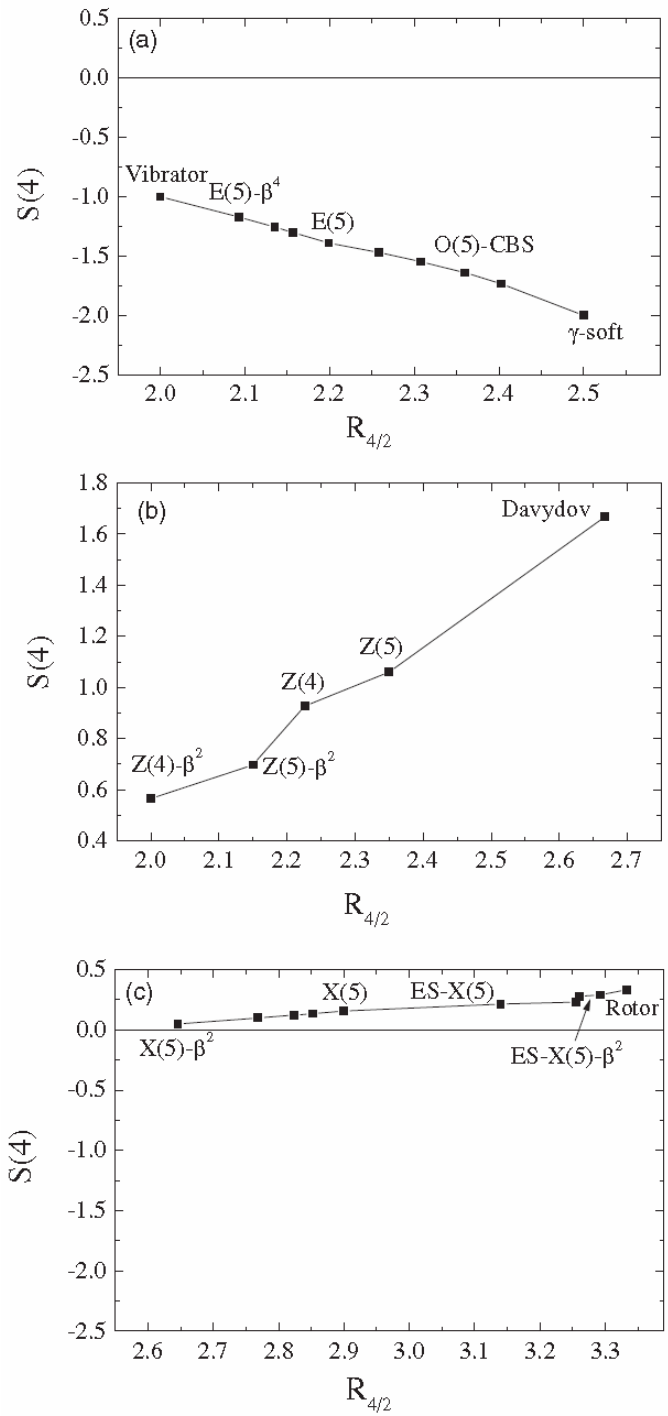


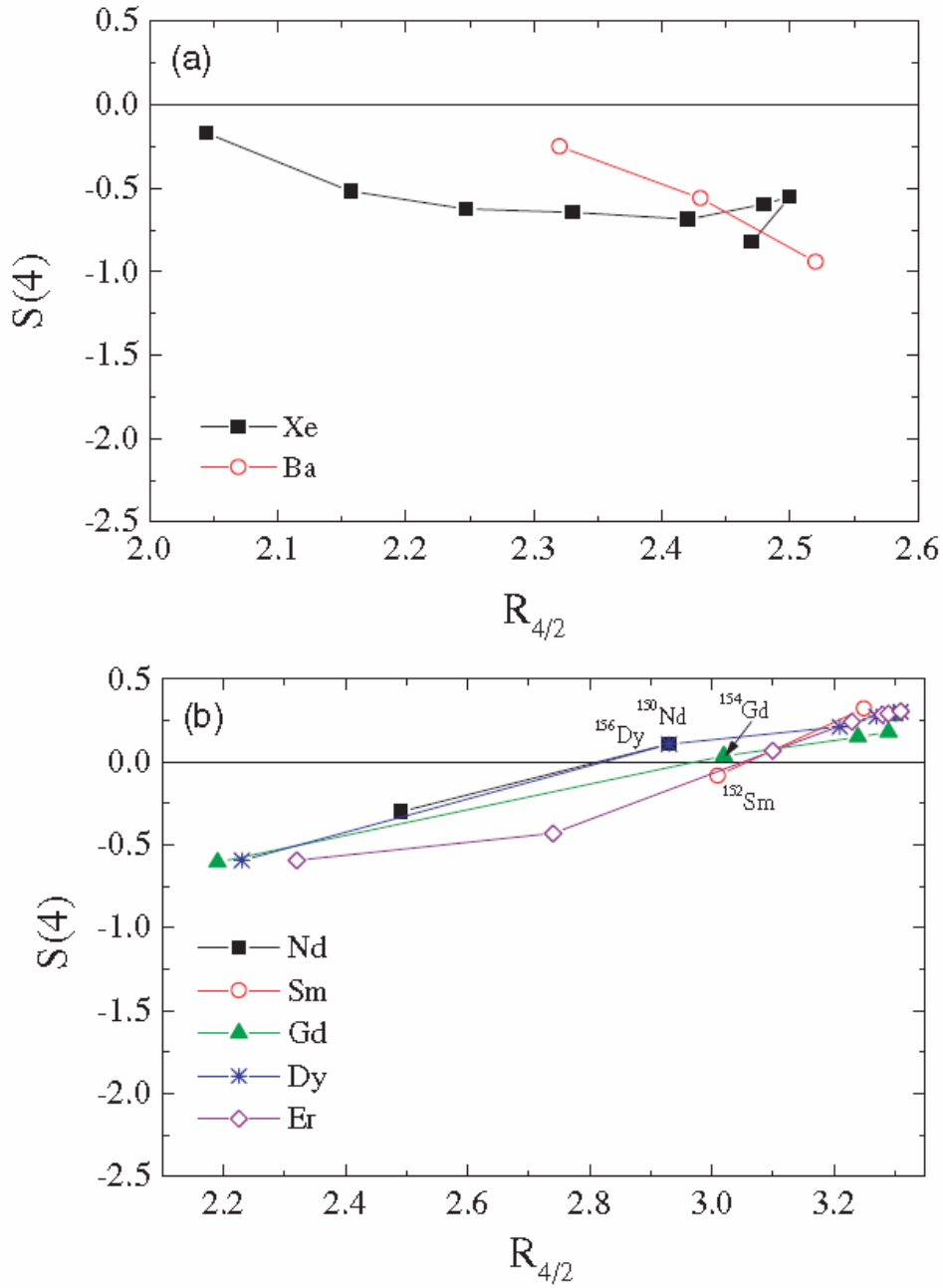
N. Pietralla and O.M. Gorbachenko, Phys. Rev. C **70**, 011304(R) (2004).

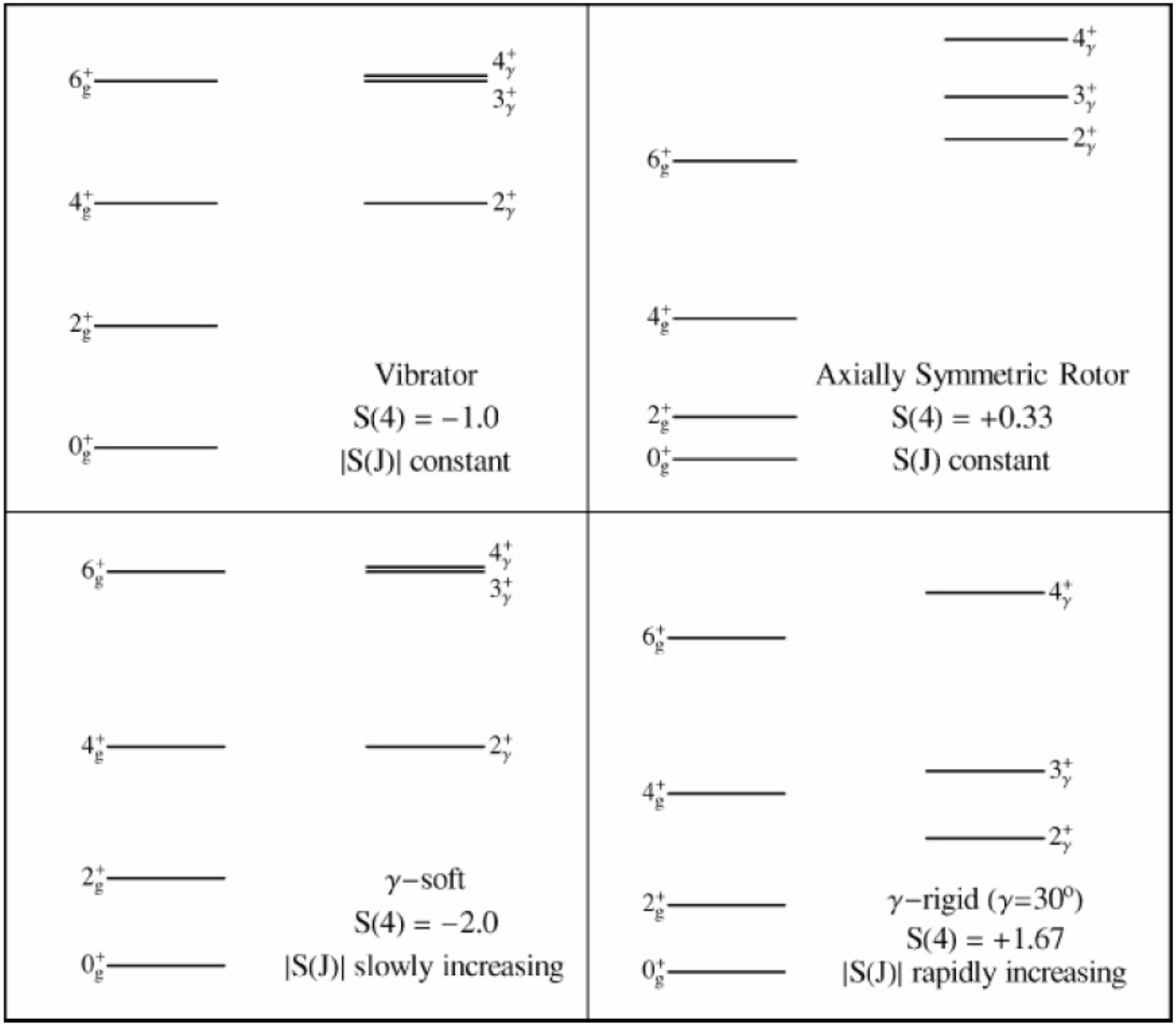
Spanning structure from vibrator to rotor

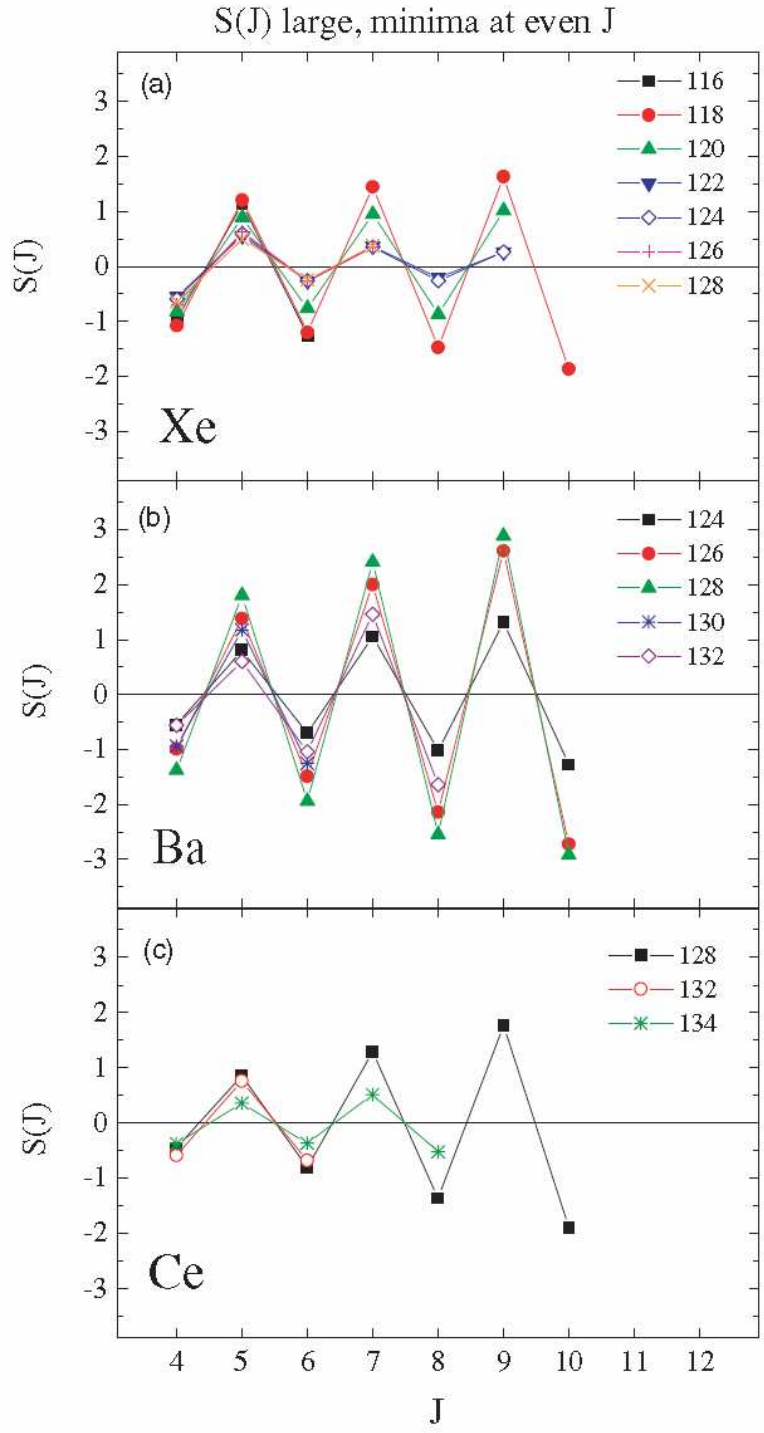



0.12
0.10
0.08
0.06
0.04
0.02
0.00
Z_γ
-800
-600
-400
-200
0
200
400
$E(2^+_\gamma) - E(2^+_\beta)$ [keV]

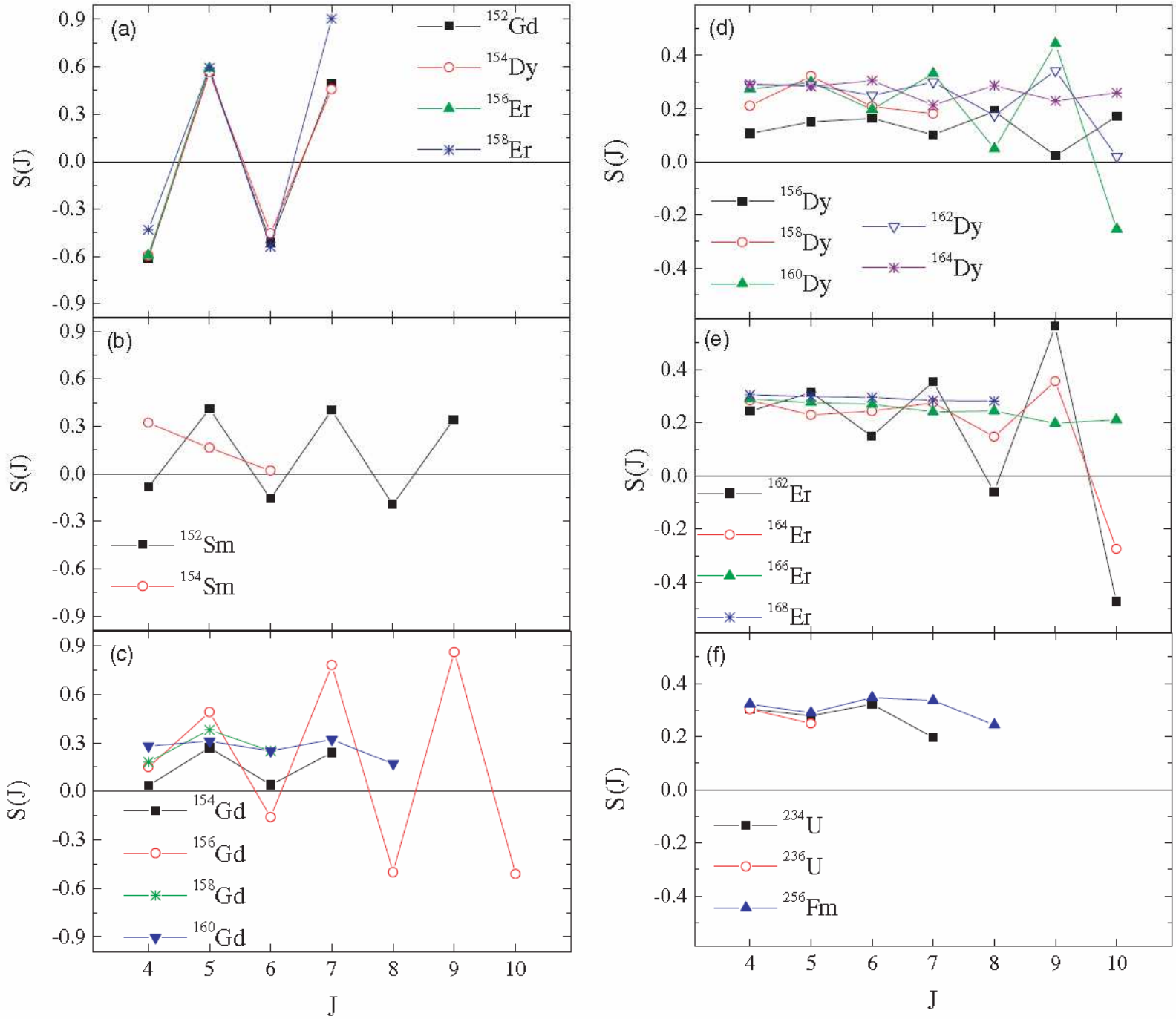

(a)
Vibrator
E(5)-β4
E(5)
O(5)-CBS
γ-soft
S(4)
R4/2
(b)
Davydov
Z(5)
Z(4)
Z(4)-β2
Z(5)-β2
S(4)
R4/2
(c)
X(5)
ES-X(5)
Rotor
X(5)-β2
ES-X(5)-β2
S(4)
R4/2

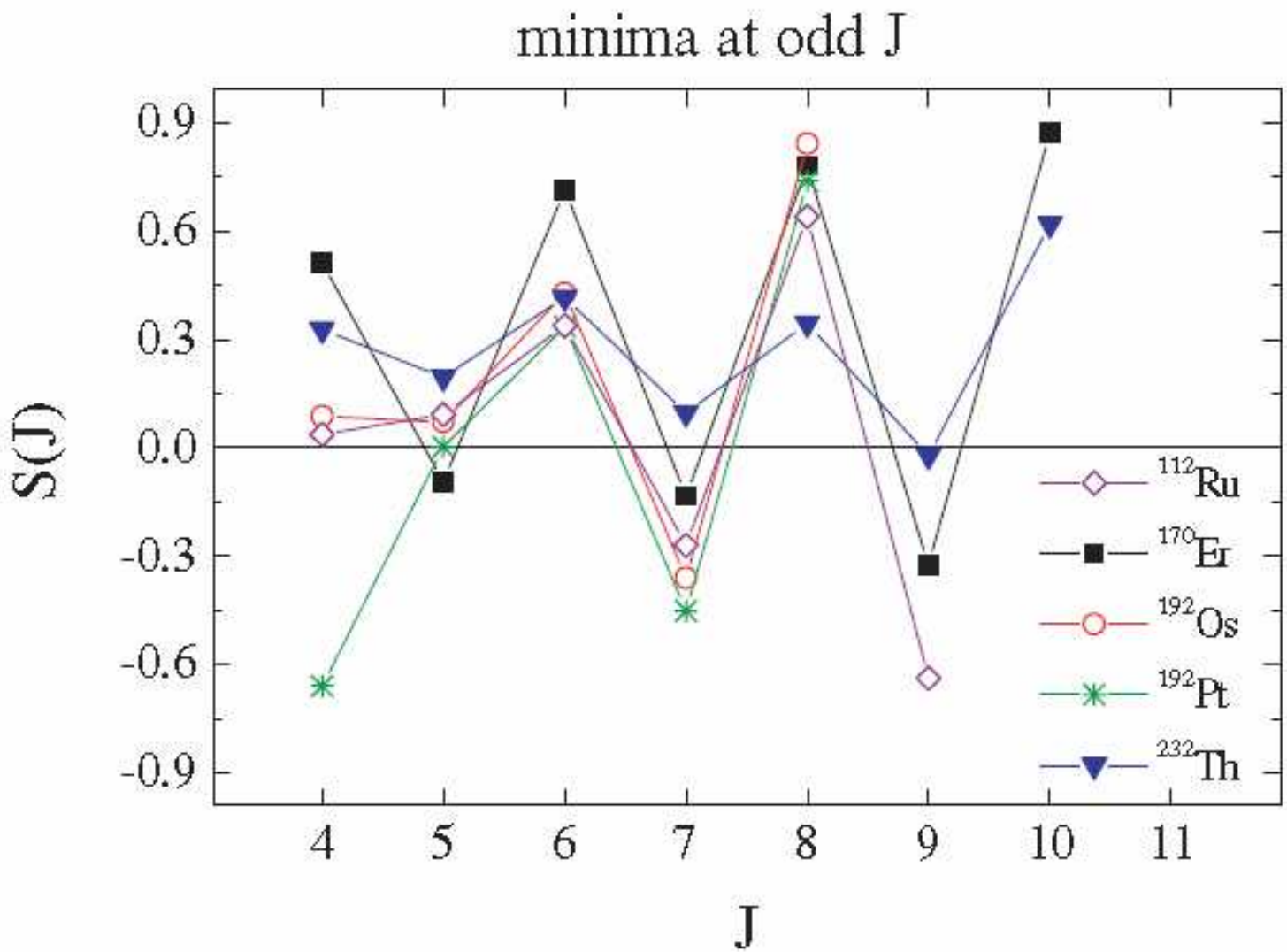

(a)
Xe
Ba
S(4)
R4/2
(b)
156Dy
150Nd
154Gd
152Sm
Nd
Sm
Gd
Dy
Er
S(4)
R4/2

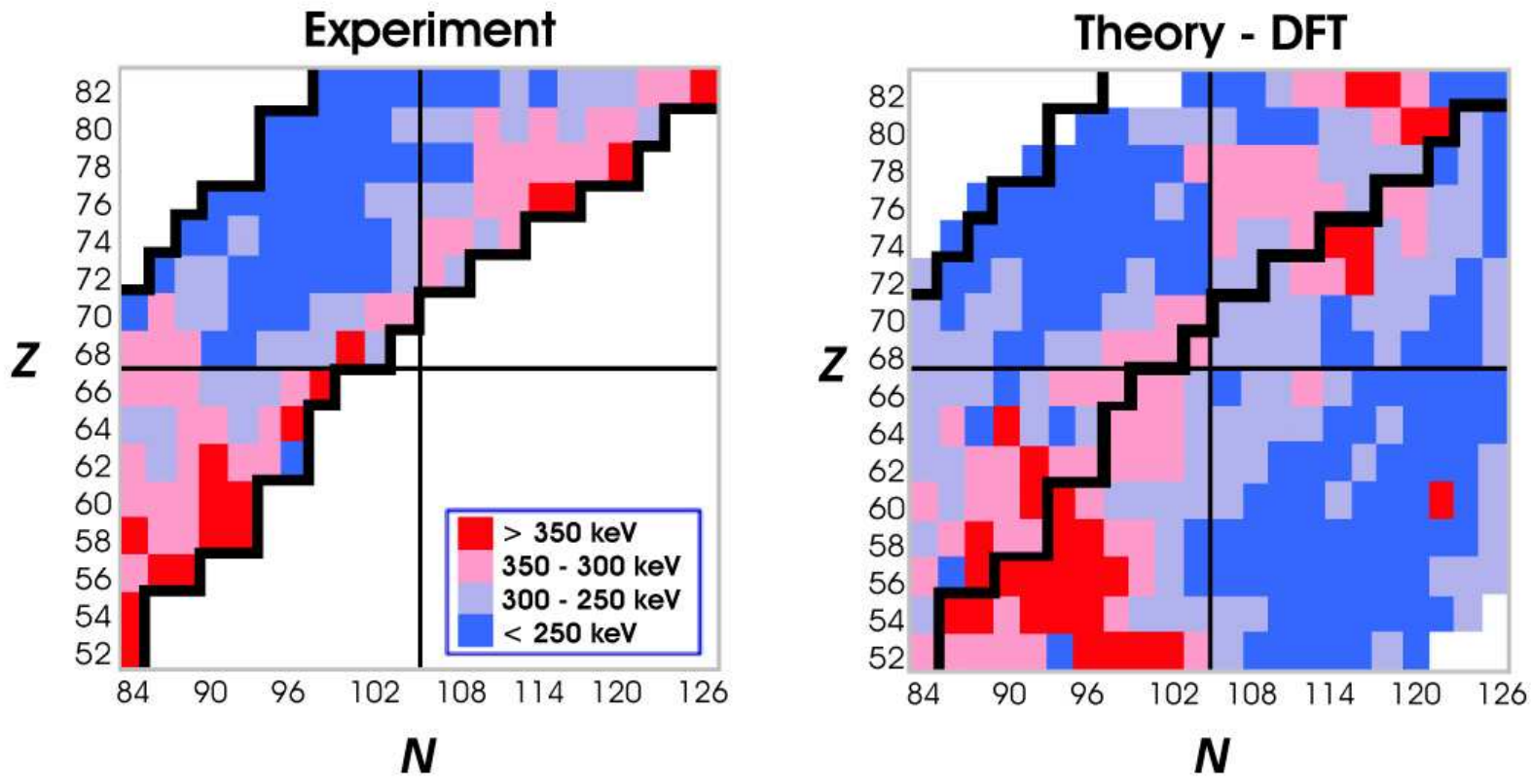

6_g^+
4_g^+
2_g^+
0_g^+
4_γ^+
3_γ^+
2_γ^+
Vibrator
S(4) = −1.0
|S(J)| constant
4_γ^+
3_γ^+
2_γ^+
6_g^+
4_g^+
2_g^+
0_g^+
Axially Symmetric Rotor
S(4) = +0.33
S(J) constant
6_g^+
4_g^+
2_g^+
0_g^+
4_γ^+
3_γ^+
2_γ^+
γ−soft
S(4) = −2.0
|S(J)| slowly increasing
6_g^+
4_g^+
2_g^+
0_g^+
4_γ^+
3_γ^+
2_γ^+
γ−rigid (γ=30°)
S(4) = +1.67
|S(J)| rapidly increasing


S(J) large, minima at even J
(a)
116
118
120
122
124
126
128
Xe
(b)
124
126
128
130
132
Ba
(c)
128
132
134
Ce
S(J)
J

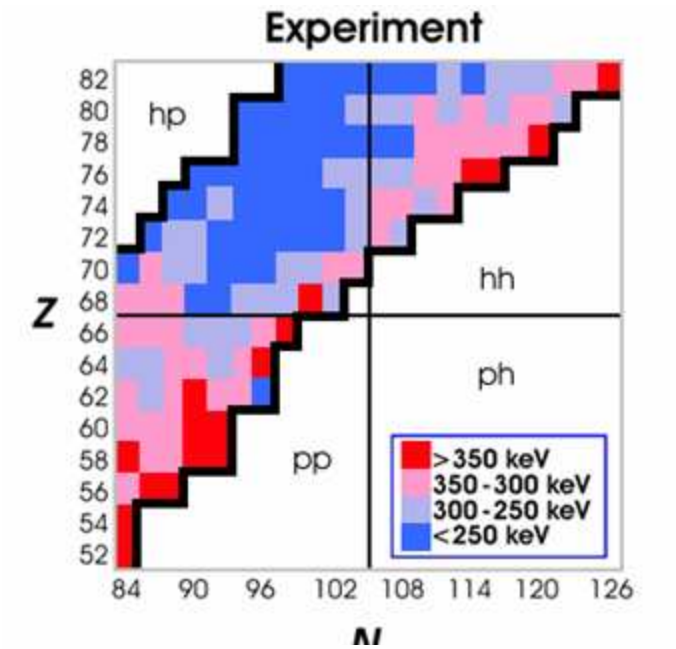
S(J) small: weak minima mostly at even J

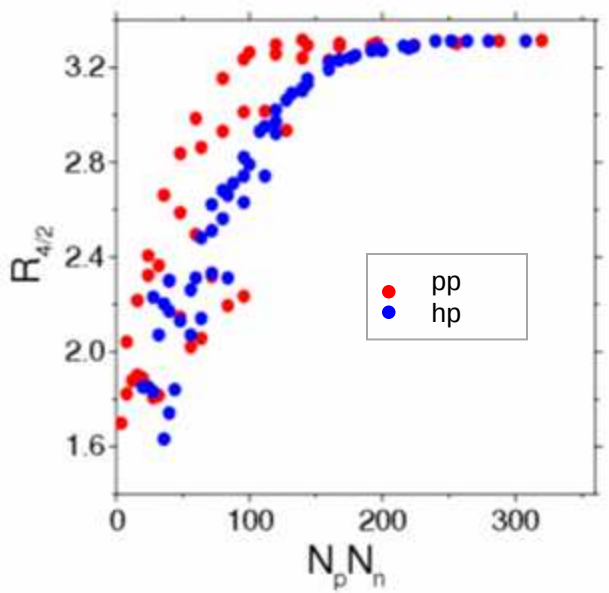



minima at odd J
0.9
0.6
0.3
0.0
-0.3
-0.6
-0.9
S(J)
4
5
6
7
8
9
10
11
J
112Ru
170Er
192Os
192Pt
232Th


Experiment
Theory - DFT
Z
N
82
80
78
76
74
72
70
68
66
64
62
60
58
56
54
52
84
90
96
102
108
114
120
126
> 350 keV
350 - 300 keV
300 - 250 keV
< 250 keV

The microscopic foundations
Empirical p-n interactions





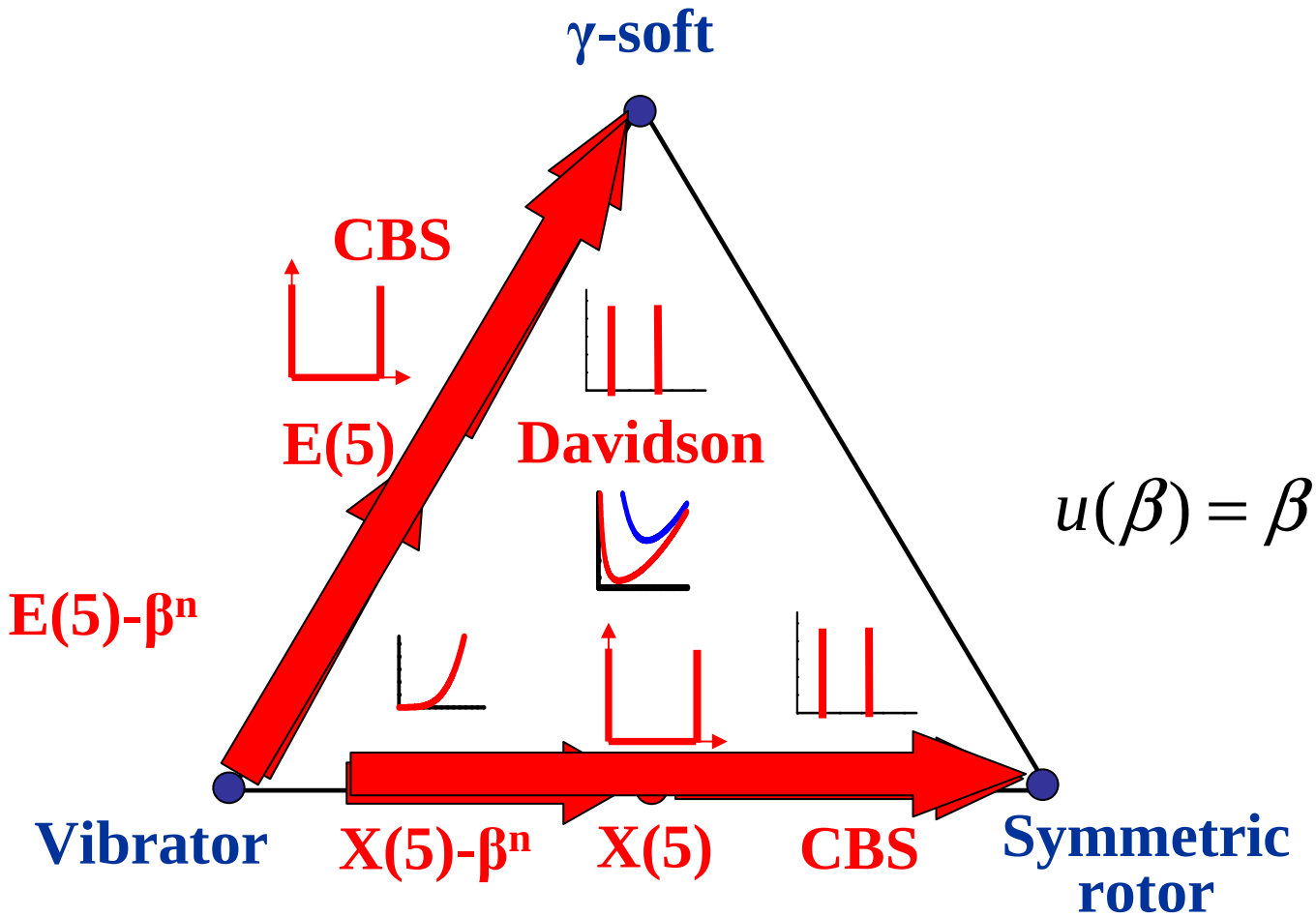
Link to collective structural evolution

- **p-n interactions are stronger in lower left quadrant.**

- **Collectivity (as measured by $R_{4/2}$) grows faster (red points in lower plot) in lower left quadrant too.**

- **This is the first direct correlation of observed growth rates of collectivity with empirical p-n interaction strengths**

New solvable models

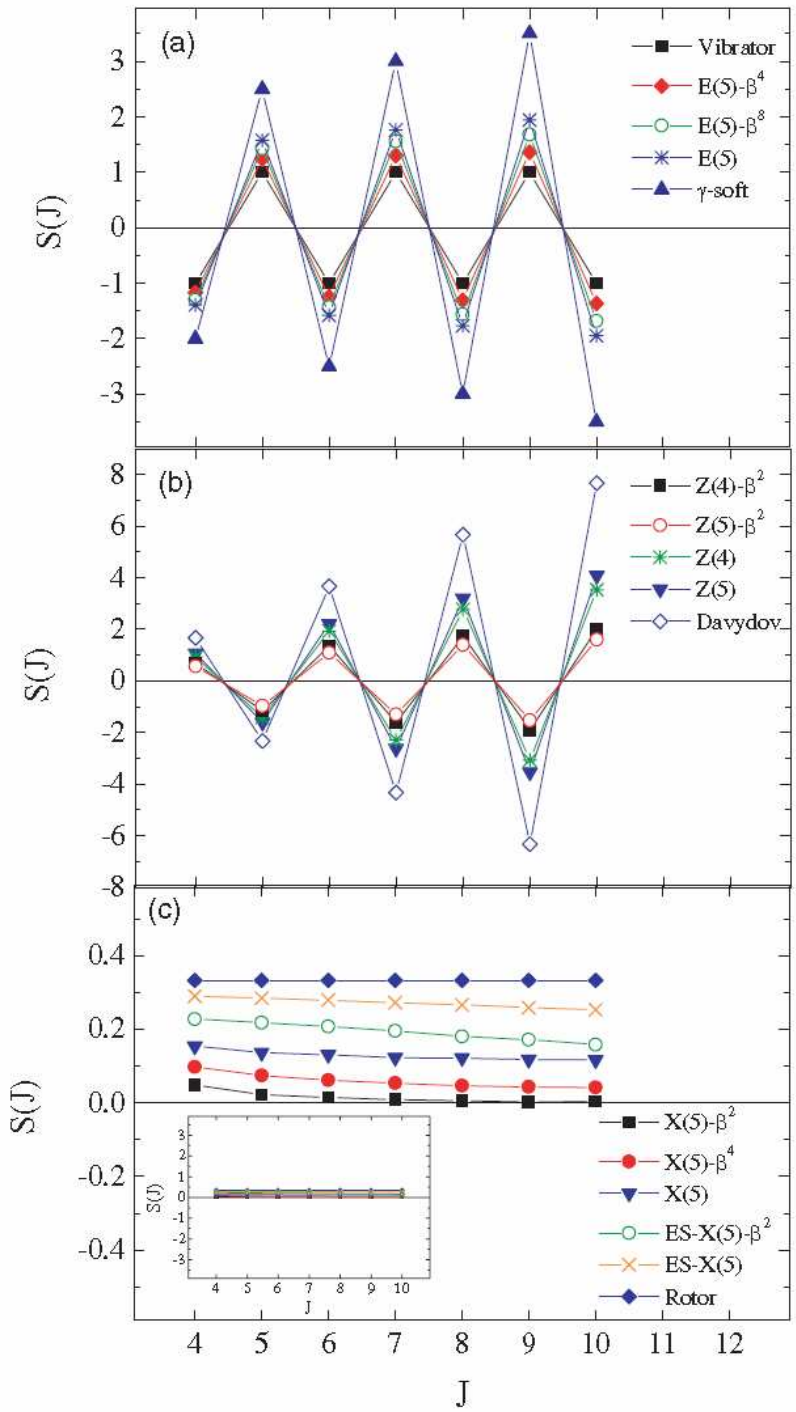


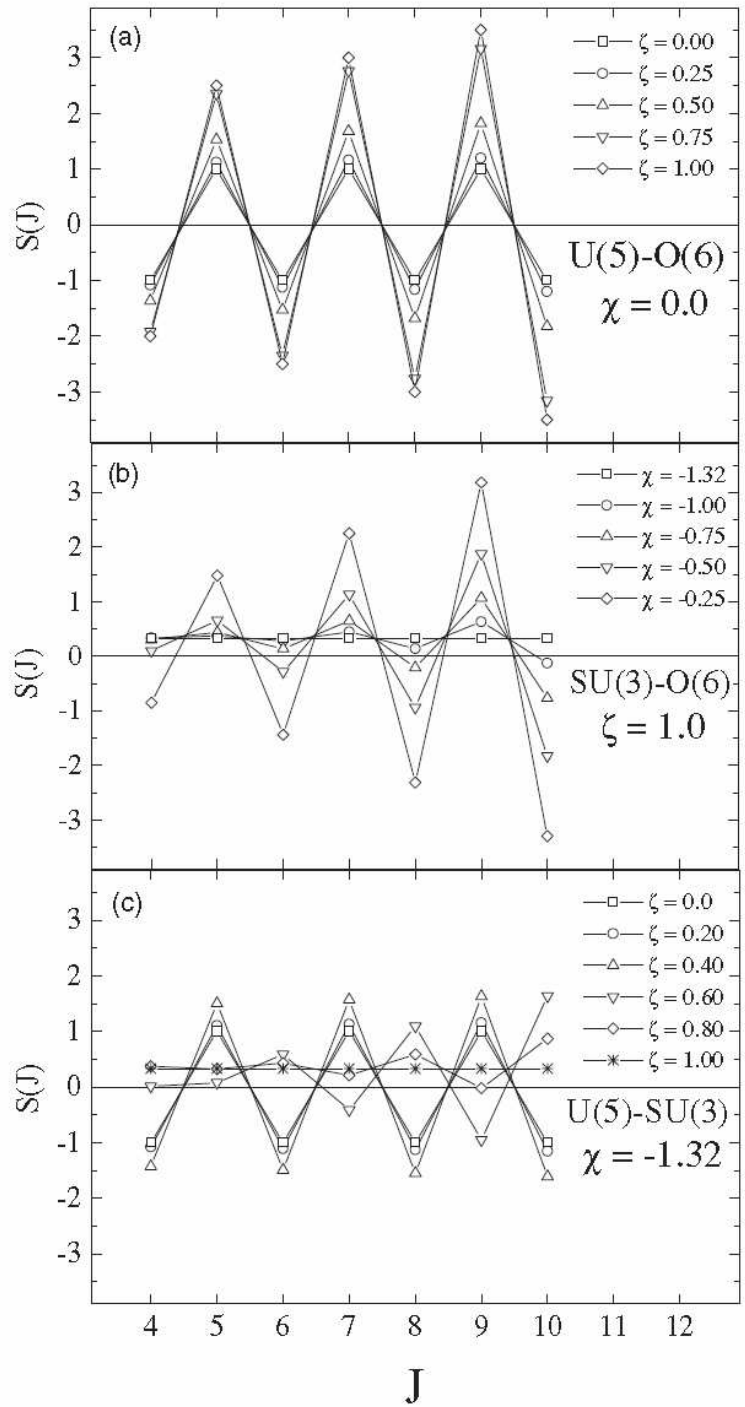
$$u(\beta) = \beta^2 + \frac{\beta_0^4}{\beta^2}$$

Exactly separable

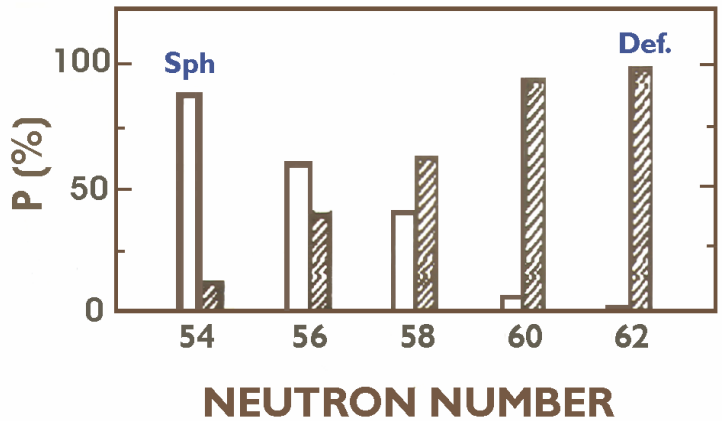
$$V(\beta,\gamma) = u(\beta) + \frac{u(\gamma)}{\beta^2}$$

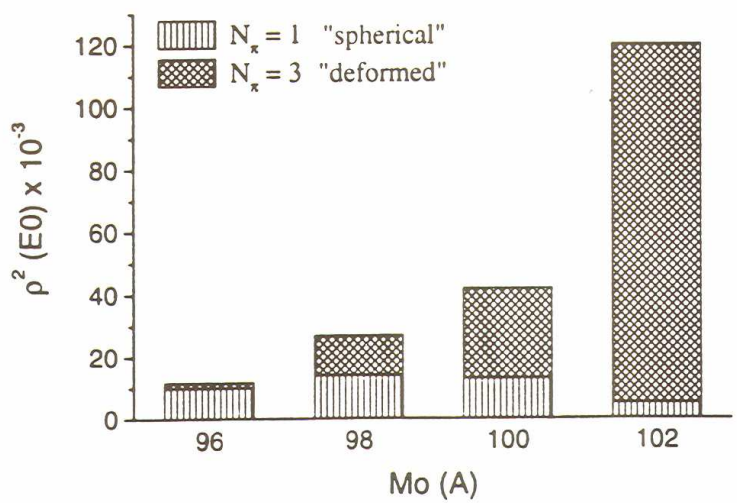
Bonatsos et al, McCutchan


(a)
Vibrator
E(5)-β4
E(5)-β8
E(5)
γ-soft
(b)
Z(4)-β2
Z(5)-β2
Z(4)
Z(5)
Davydov
(c)
X(5)-β2
X(5)-β4
X(5)
ES-X(5)-β2
ES-X(5)
Rotor
S(J)
J


(a)
ζ = 0.00
ζ = 0.25
ζ = 0.50
ζ = 0.75
ζ = 1.00
U(5)-O(6)
χ = 0.0
(b)
χ = -1.32
χ = -1.00
χ = -0.75
χ = -0.50
χ = -0.25
SU(3)-O(6)
ζ = 1.0
(c)
ζ = 0.0
ζ = 0.20
ζ = 0.40
ζ = 0.60
ζ = 0.80
ζ = 1.00
U(5)-SU(3)
χ = -1.32
S(J)
J

Contributions to ρ^2 $\left(E0; 0_2^+ \rightarrow 0_1^+\right)$ from different IBA configurations (Based on Sambataro and Molnar)





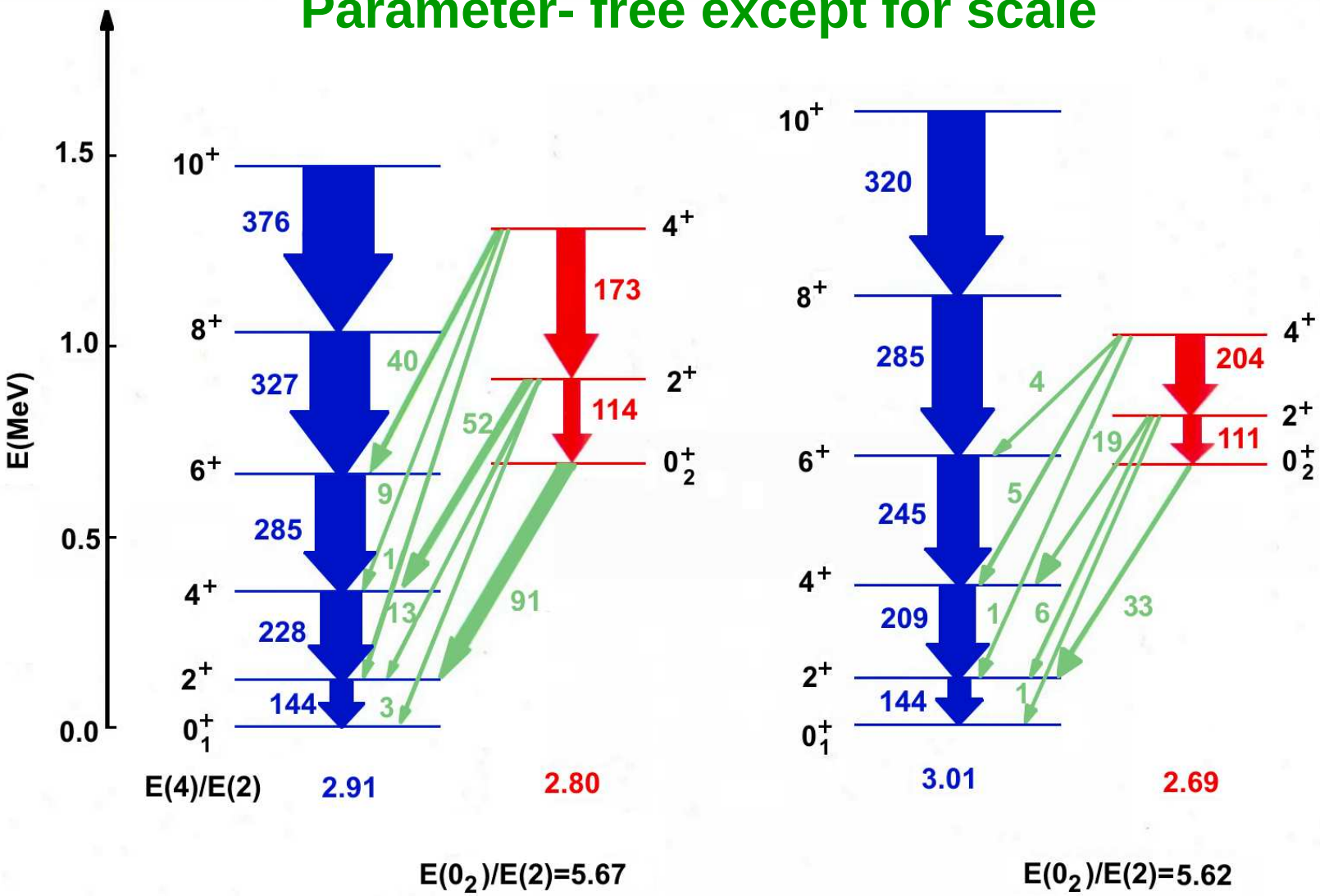
Virtually no mixing in precisely the nucleus that shows the largest ρ^2 ($E0$)

Large ρ^2 ($E0$) is not from mixing, but rather originates within a single set of configurations

New Paradigm for Nuclear Structure

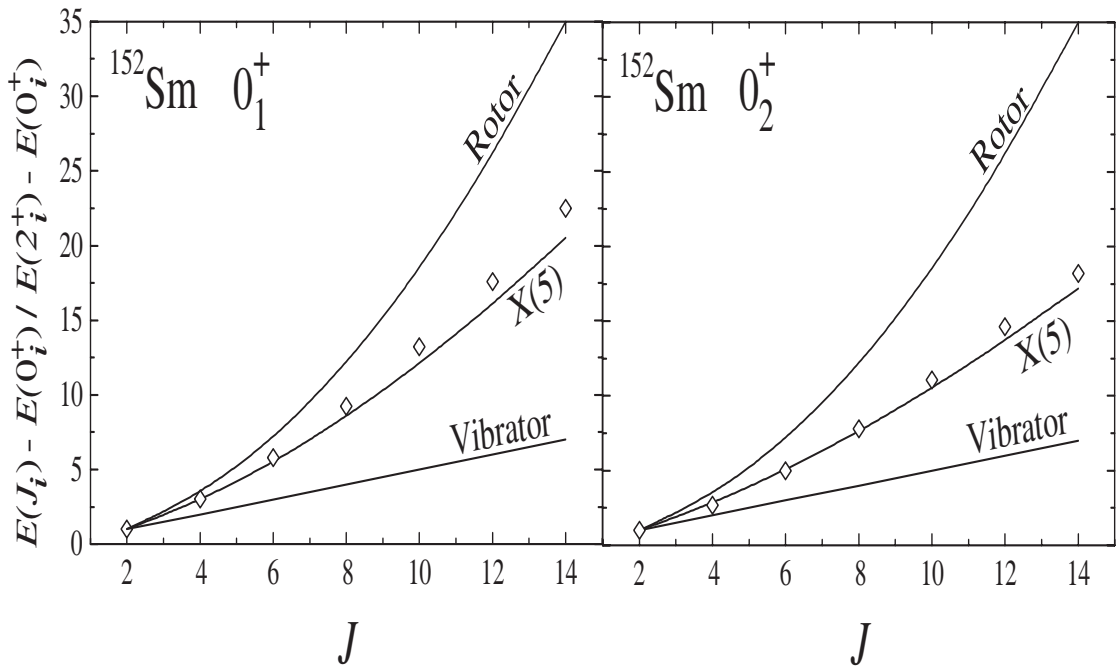
Critical Point Phase Transitional Nuclei

Parameter- free except for scale

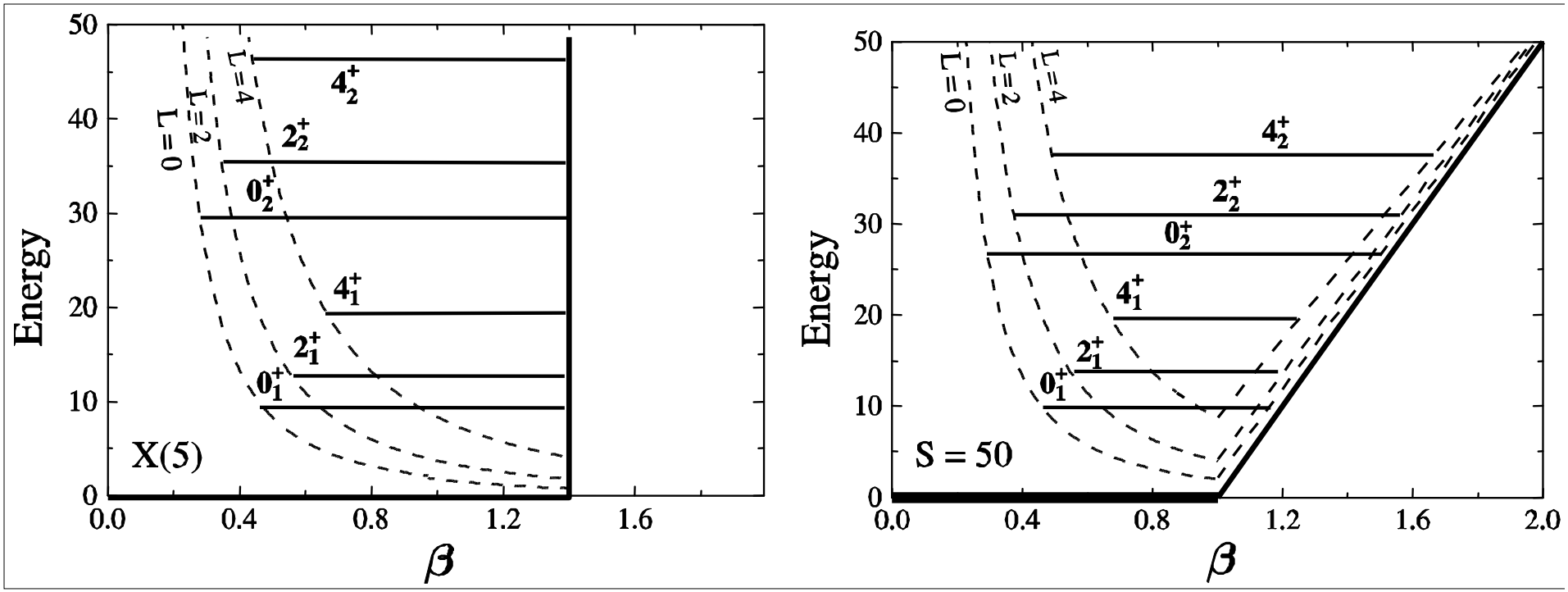


Casten and Zamfir **X(5)** ^{152}**Sm**

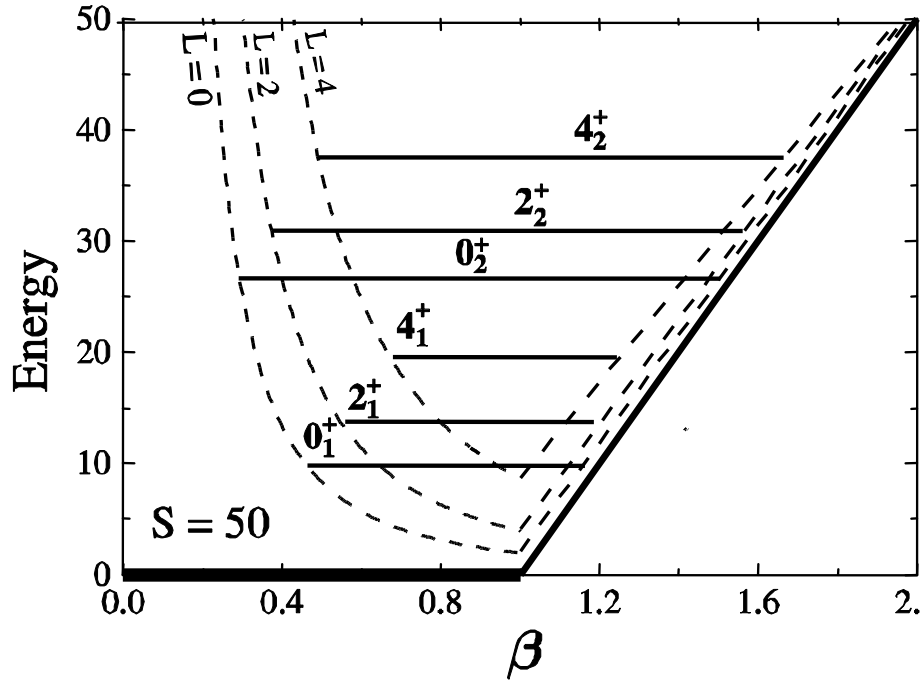
Comparison of relative energies with X(5)

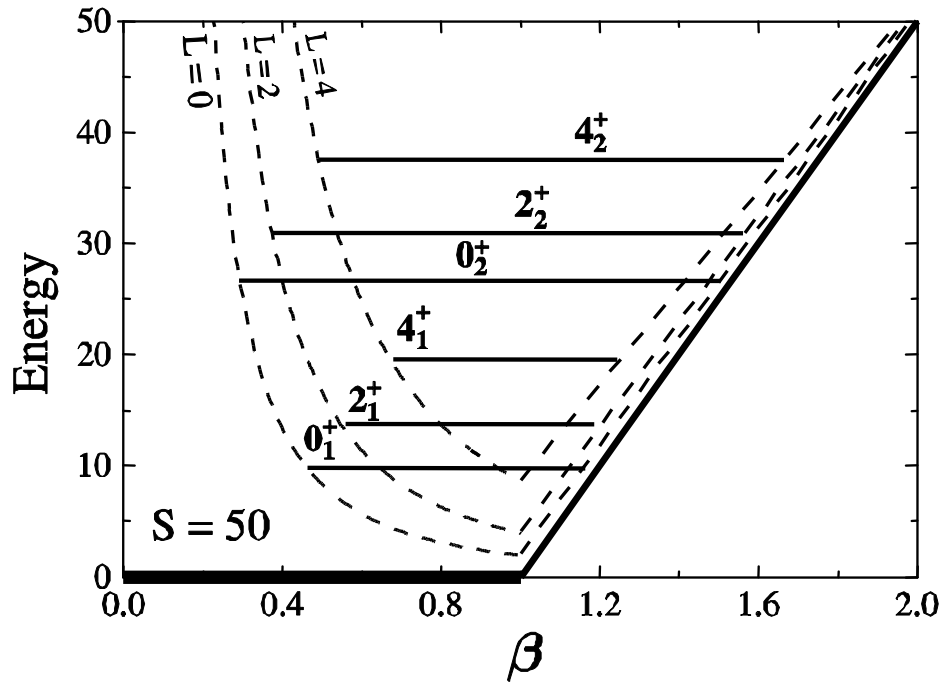


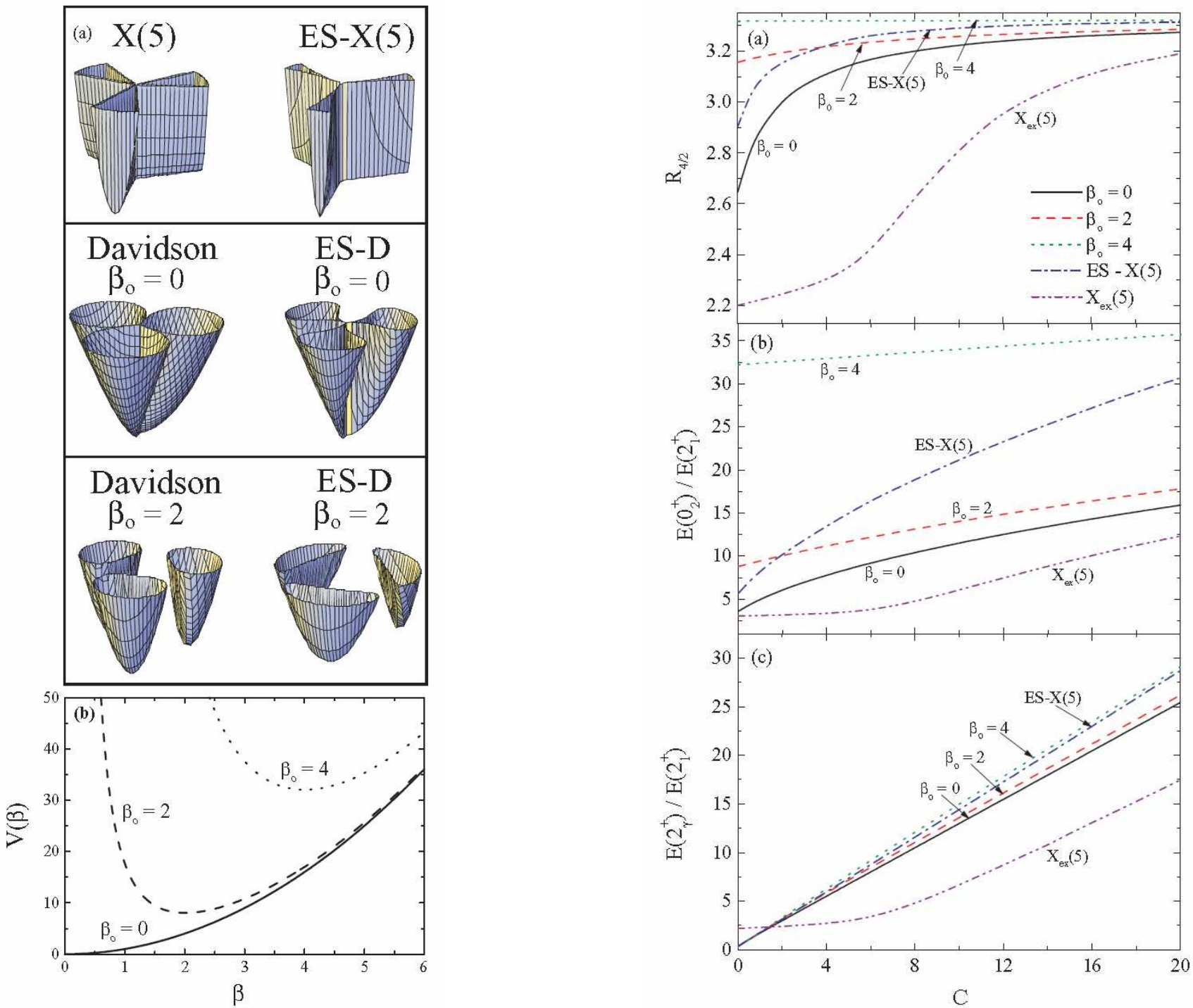
Absolute energy spacings in the 0^+_2 sequence – effects of a sloped wall

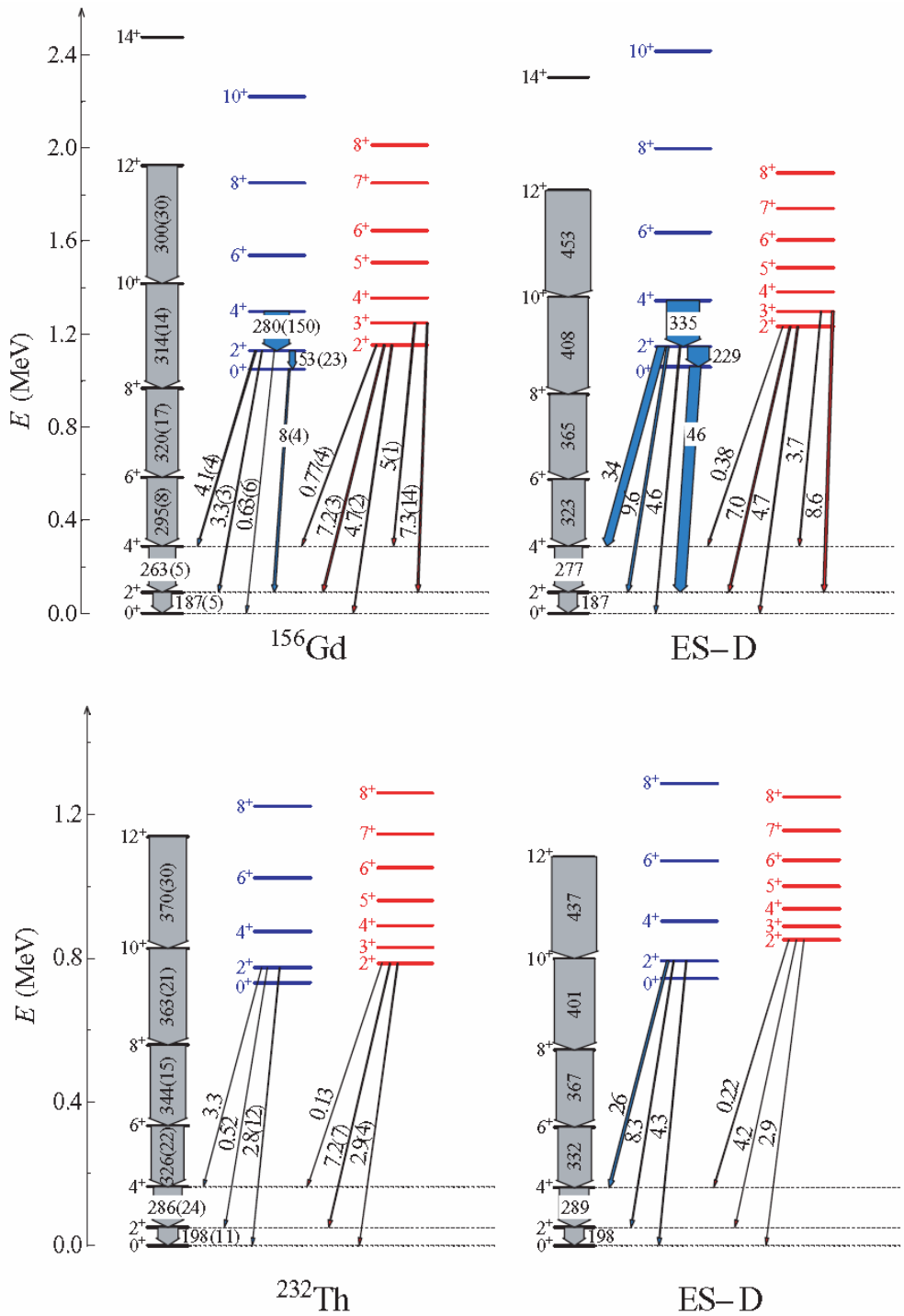


Caprio


Energy
β
S = 50
L = 0
L = 2
L = 4
4_2^+
2_2^+
0_2^+
4_1^+
2_1^+
0_1^+
0
10
20
30
40
50
0.0
0.4
0.8
1.2
1.6
2.


Energy
β
S = 50
L = 0
L = 2
L = 4
4_2^+
2_2^+
0_2^+
4_1^+
2_1^+
0_1^+
0
10
20
30
40
50
0.0
0.4
0.8
1.2
1.6
2.0


(a)
X(5)
ES-X(5)
Davidson
$\beta_o = 0$
ES-D
$\beta_o = 0$
Davidson
$\beta_o = 2$
ES-D
$\beta_o = 2$
(b)
$\beta_o = 4$
$\beta_o = 2$
$\beta_o = 0$
V(β)
β
(a)
$\beta_0 = 4$
ES-X(5)
$\beta_0 = 2$
$\beta_0 = 0$
$X_{ex}(5)$
$R_{4/2}$
$\beta_o = 0$
$\beta_o = 2$
$\beta_o = 4$
ES - X(5)
$X_{ex}(5)$
(b)
$\beta_o = 4$
ES-X(5)
$\beta_o = 2$
$\beta_o = 0$
$X_{ex}(5)$
$E(0_2^+) / E(2_1^+)$
(c)
ES-X(5)
$\beta_o = 4$
$\beta_o = 2$
$\beta_o = 0$
$X_{ex}(5)$
$E(2_\gamma^+) / E(2_1^+)$
C


E (MeV)
156Gd
ES–D
232Th
ES–D

Masses and Nucleonic Interactions

Masses:

Total mass/binding energy: **Sum of all interactions**

Mass differences: **Separation energies**
shell structure, phase transitions

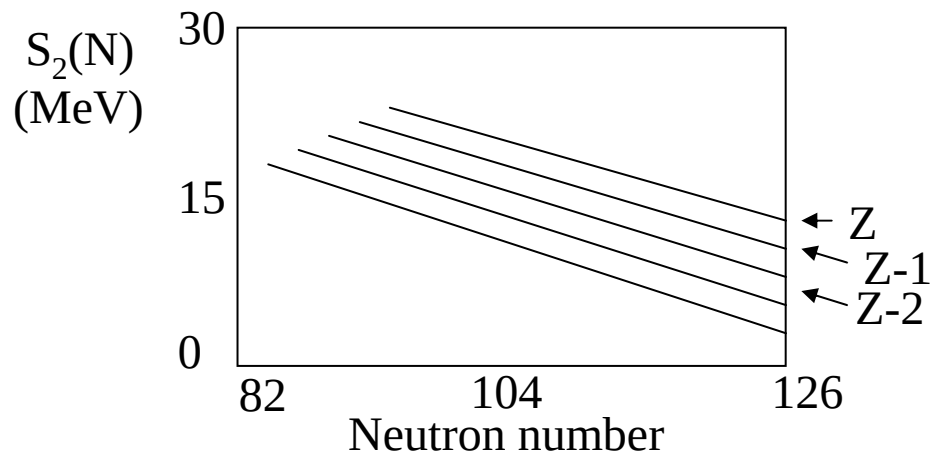
Double differences of masses: **Interaction filters**

		Needed accuracy
	• Shell structure:	100-300 keV
Macro →	• **Quantum 1st order phase trans:**	**~ 100 keV**
Micro →	• **Interaction filters (e.g., p-n)**	**~10-15 keV**

2-neutron separation energies in phase transitional regions

$S_2(N)$ versus N:
IBA gives a straight line in normal regions.

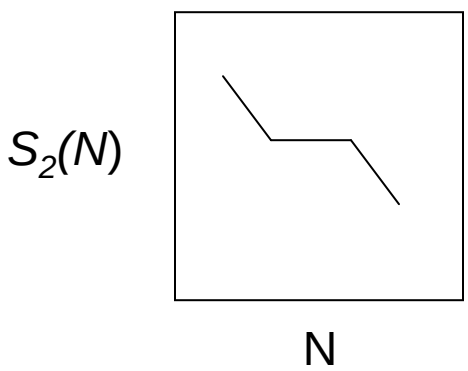
$$S_2(N) = A + BN$$



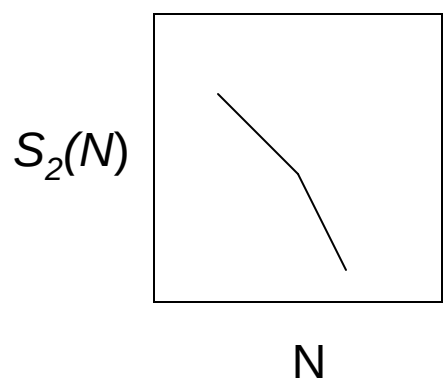
First order shape phase transitions, discontinuities in $S_2(N)$

Second order transitions, discontinuities in $\frac{\Delta S_2}{\Delta N} = S_2(N+1) - S_2(N)$

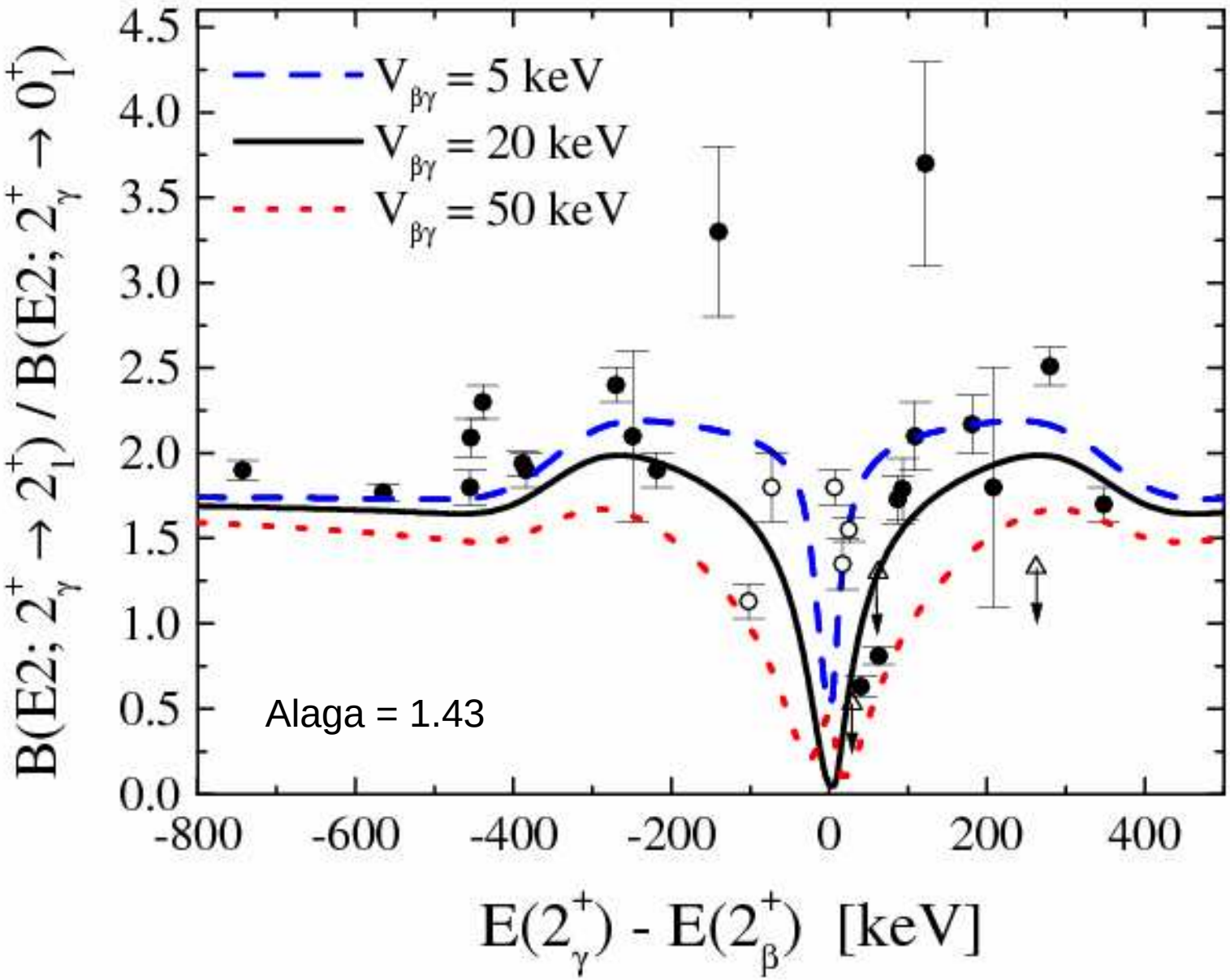
1st order

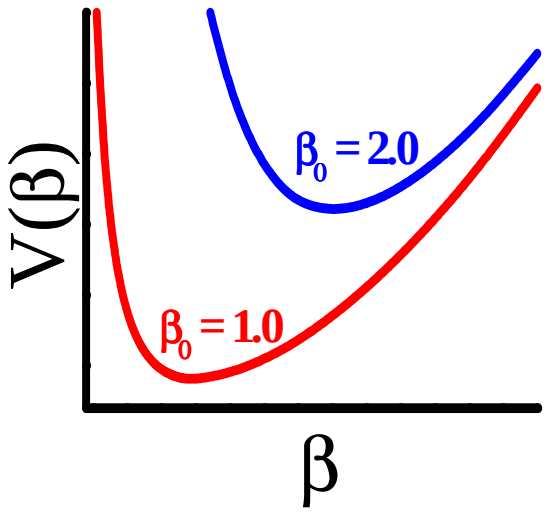


2nd order



Slide based on Iachello


$V_{\beta\gamma}$ = 5 keV
$V_{\beta\gamma}$ = 20 keV
$V_{\beta\gamma}$ = 50 keV
Alaga = 1.43
$B(E2; 2^+_\gamma \rightarrow 2^+_1) / B(E2; 2^+_\gamma \rightarrow 0^+_1)$
$E(2^+_\gamma) - E(2^+_\beta)$ [keV]
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-800
-600
-400
-200
0
200
400



Davidson

$$u(\beta) = \beta^2 + \frac{\beta_0^4}{\beta^2}$$