Standard Shell Model Effective Interactions from No-Core Shell Model Calculations

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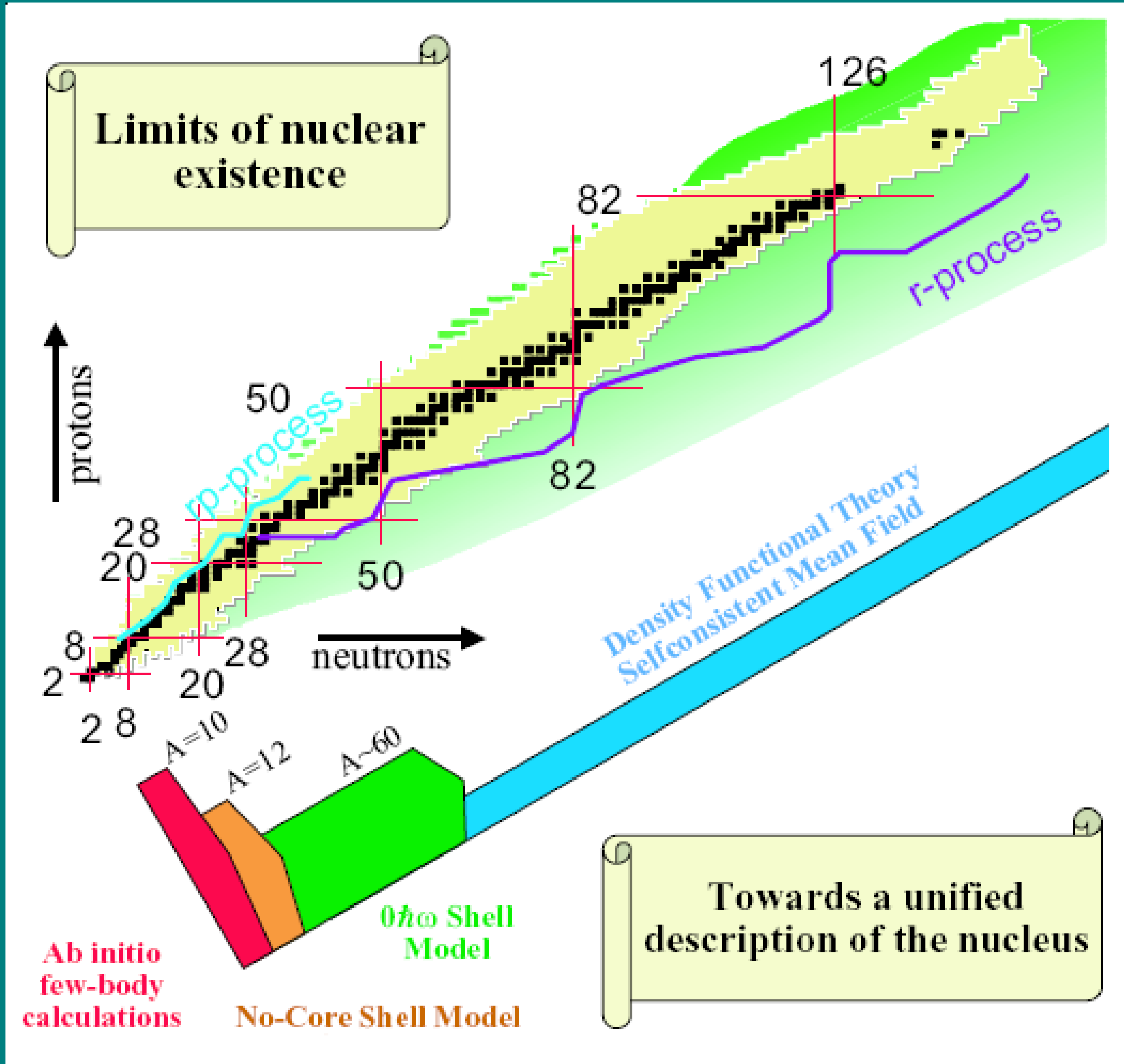


Limits of nuclear existence
protons
neutrons
rp-process
r-process
126
82
50
28
20
8
2
Density Functional Theory
Selfconsistent Mean Field
A=10
A=12
A~60
Ab initio few-body calculations
No-Core Shell Model
0ħω Shell Model
Towards a unified description of the nucleus

No-Core Shell-Model Approach

- Start with the purely intrinsic Hamiltonian

$$H_A = T_{rel} + \mathcal{V} = \frac{1}{A}\sum_{i<j=1}^{A}\frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i<j=1}^{A} V_{NN} \left(+ \sum_{i<j<k}^{A} V_{ijk}^{3b} \right)$$

Note: There are no phenomenological s.p. energies!

Can use any NN potentials

Coordinate space: Argonne V8', AV18, Nijmegen I, II

Momentum space: CD Bonn, EFT Idaho

No-Core Shell-Model Approach

- Next, add CM harmonic-oscillator Hamiltonian

$$H_{CM}^{HO} = \frac{\vec{P}^{\,2}}{2Am} + \frac{1}{2} Am\Omega^2 \vec{R}^{\,2}\,; \quad \vec{R} = \frac{1}{A}\sum_{i=1}^{A} \vec{r}_i\,, \quad \vec{P} = Am\dot{\vec{R}}$$

To H_A, yielding

$$H_A^{\Omega} = \sum_{i=1}^{A}\left[\frac{\vec{p}_i^{\,2}}{2m} + \frac{1}{2} m\Omega^2 \vec{r}_i^{\,2}\right] + \underbrace{\sum_{i<j=1}^{A}\left[V_{NN}(\vec{r}_i - \vec{r}_j) - \frac{m\Omega^2}{2A}(\vec{r}_i - \vec{r}_j)^2\right]}_{V_{ij}}$$

Defines a basis (*i.e.* HO) for evaluating V_{ij}

Effective Interaction

- Must truncate to a finite model space $\quad V_{ij} \longrightarrow V_{ij}^{effective}$
- In general, V_{ij}^{eff} is an A-body interaction
- We want to make an a-body cluster approximation

$$\mathcal{H} = \mathcal{H}^{(1)} + \mathcal{H}^{(A)} \quad \underset{a<A}{\gtrapprox} \quad \mathcal{H}^{(1)} + \mathcal{H}^{(a)}$$

Two-body cluster approximation (a=2)

$$\mathcal{H} \approx \mathcal{H}^{(I)} + \mathcal{H}^{(2)}$$

$$H_2^\Omega = \underbrace{H_{0_2} + H_2^{CM}}_{h_1+h_2} + V_{12} = \frac{\vec{p}^{\,2}}{2m} + \frac{1}{2} m\Omega^2 \vec{r}^{\,2} + H_2^{CM} + V(\sqrt{2}\vec{r}) - \frac{m\Omega^2}{A}\vec{r}^{\,2}$$

Carry out a unitary transformation on H_2^{Ω}

$$\mathcal{H}_2 = e^{-S^{(2)}} H_2^\Omega e^{S^{(2)}} \quad \text{where } S^{(2)} \text{ is anti Hermitian}$$

$S^{(2)}$ is determined from the decoupling condition

$$Q_2 e^{-S^{(2)}} H_2^\Omega e^{S^{(2)}} P_2 = 0$$

P_2 = model space, Q_2 = excluded space, $P_2 + Q_2 = 1$

$$\text{with the restrictions} \quad P_2 S^{(2)} P_2 = Q_2 S^{(2)} Q_2 = 0$$

Two-body cluster approximation (a=2)

It is convenient to write S(2) in terms of another operator omega as:

$$S^{(2)} = \operatorname{arctanh}(\omega - \omega^\dagger) \quad \text{with} \quad Q_2 \omega P_2 = \omega$$

Then the Hermitian effective operator in the P_2 space can be expressed in the form

$$\mathcal{H}_{eff}^{(2)} = P_2 \mathcal{H}_2 P_2 = \frac{P_2 + P_2 \omega^\dagger Q_2}{\sqrt{P_2 + \omega^\dagger \omega}} H_2^\Omega \frac{P_2 + Q_2 \omega P_2}{\sqrt{P_2 + \omega^\dagger \omega}}$$

Analogously, <u>any</u> <u>arbitrary</u> operator can be written in the P_2 space as

$$\mathcal{O}_{eff}^{(2)} = P_2 \mathcal{O}_2 P_2 = \frac{P_2 + P_2 \omega^\dagger Q_2}{\sqrt{P_2 + \omega^\dagger \omega}} O \frac{P_2 + Q_2 \omega P_2}{\sqrt{P_2 + \omega^\dagger \omega}}$$

Exact solution for ω:

Let E_k and $|k\rangle$ be the eigensolutions of $H_{\frac{\Omega}{2}}$,

$$H_{\frac{\Omega}{2}}|k\rangle = E_k|k\rangle$$

Let $|\alpha_P\rangle$ and $|\alpha_Q\rangle$ be HO states belonging to the model space P and the excluded space Q, respectively. Then ω is given by:

$$\langle\alpha_Q|k\rangle = \sum_{\alpha_P}\langle\alpha_Q|\omega|\alpha_P\rangle\langle\alpha_P|k\rangle$$

or

$$\langle\alpha_Q|\omega|\alpha_P\rangle = \sum_{k\in K}\langle\alpha_Q|k\rangle\langle\tilde{k}|\alpha_P\rangle$$

NCSM ROAD MAP

1. Choose a NN interaction (or NN + NNN interactions)

2. Solve $H_n^{\Omega}|k_n\rangle = E_n|k_n\rangle$ for E_n and $|k_n\rangle$ with n=2,3,...

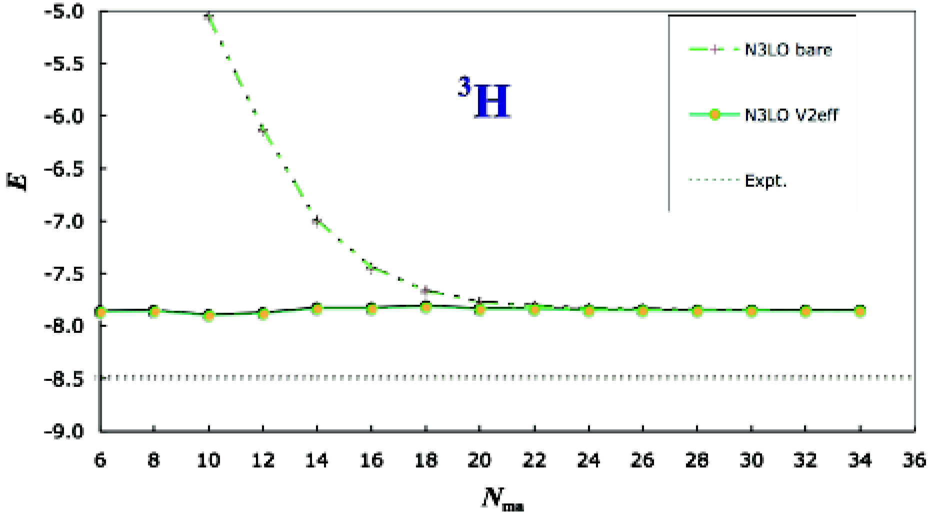
3. Calculate $\langle\alpha_Q^n|\,\omega\,|\alpha_P^n\rangle = \sum_{k\in K}\langle\alpha_Q|k_n\rangle\langle\tilde{k}_n|\alpha_P\rangle$

4. Determine $\mathcal{H}_n^{\text{eff}}$ and $\mathcal{O}_n^{\text{eff}}$ in the given model space

5. Diagonalize $\mathcal{H}_n^{\text{eff}}$ in the given model space, *i.e.*,
   $N_{max}\,\hbar\Omega$ = energy above the ground state

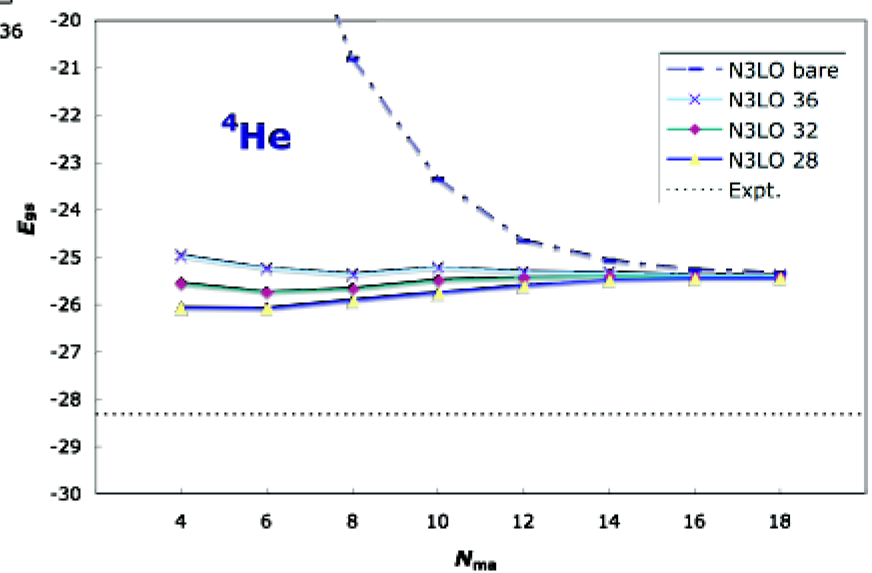
6. To check convergence of results repeat calculations
   for: *i)* increasing N_{max} and/or cluster level
   ii) several values of $\hbar\Omega$

- NCSM convergence test
  - Comparison to other methods



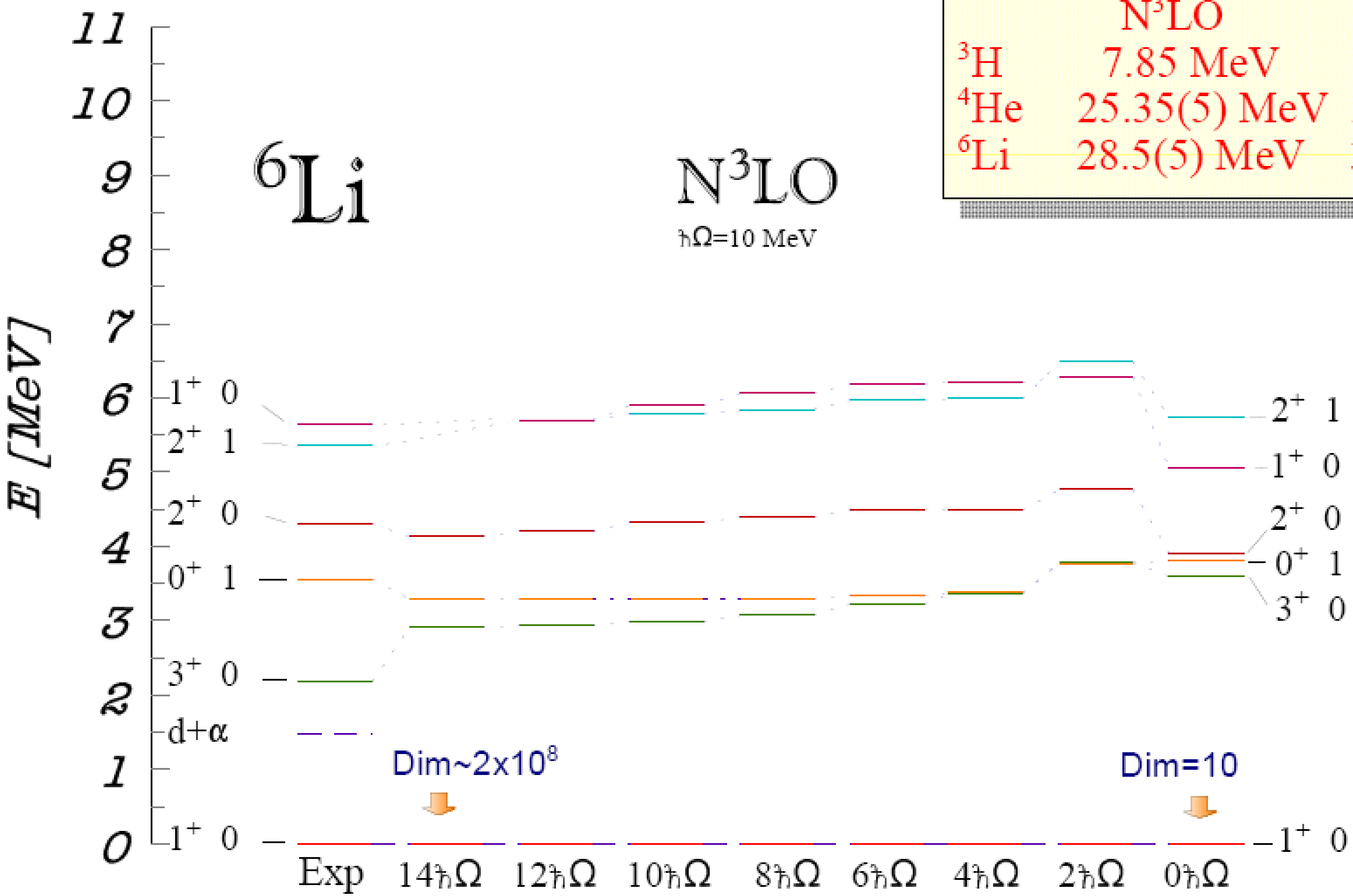
N³LO NN	NCSM	FY	HH
³H	7.852(5)	7.854	7.854
⁴He	25.39(1)	25.37	25.38

- Short-range correlations ⇒ effective interaction
- Medium-range correlations ⇒ multi-$\hbar\Omega$ model space
- Dependence on
  - size of the model space (N_{max})
  - HO frequency ($\hbar\Omega$)
- Not a variational calculation
- Convergence OK
- NN interaction insufficient to reproduce experiment



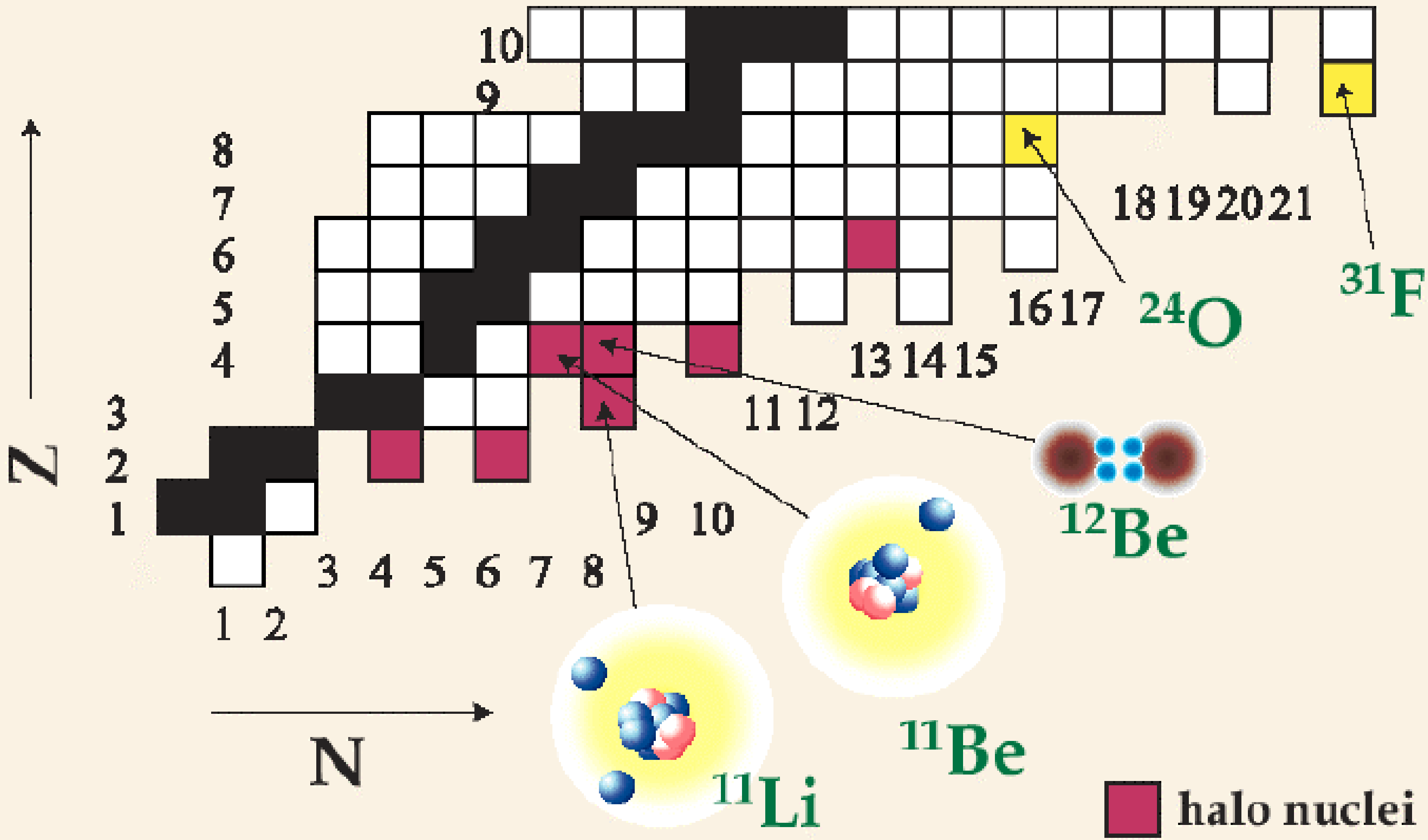
P. Navratil, INT Seminar, November 13, 2007, online

	N^{3}LO	Exp
^{3}H	7.85 MeV	8.48 MeV
^{4}He	25.35(5) MeV	28.30 MeV
^{6}Li	28.5(5) MeV	31.99 MeV



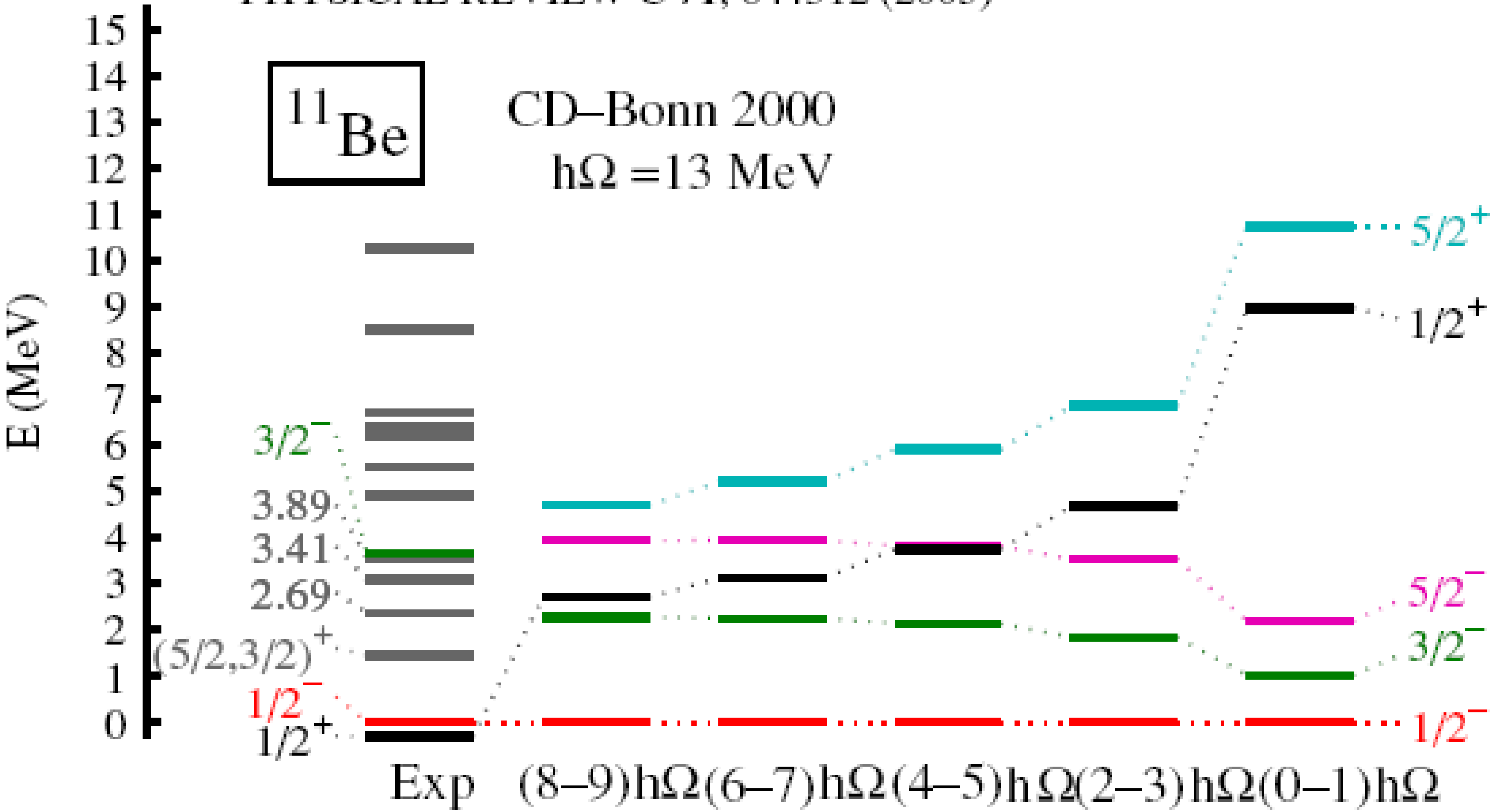
P. Navrátil and E. Caurier, Phys. Rev. C **69**, 014311 (2004)

Light drip line nuclei



C. FORSSÉN, P. NAVRÁTIL, W. E. ORMAND, AND E. CAURIER

PHYSICAL REVIEW C **71**, 044312 (2005)



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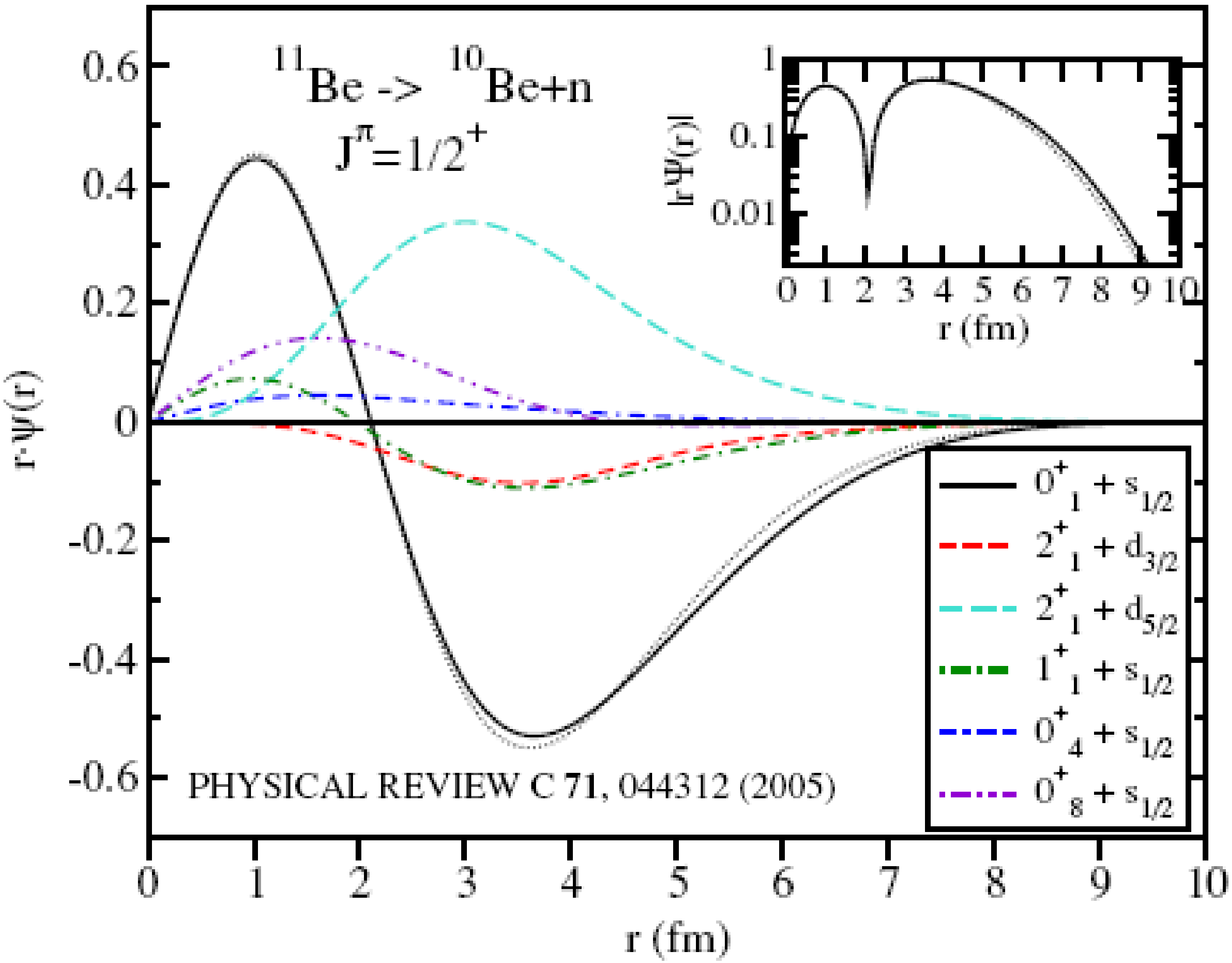


From 4hΩ NCSM to sd CSM for ^{18}F

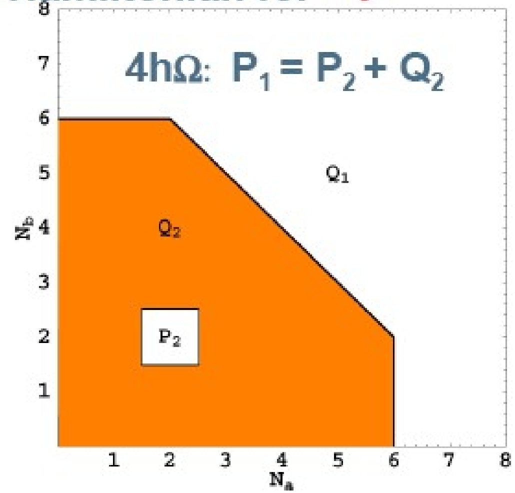
Petr Navrátil, Michael Thoresen, and Bruce R. Barrett, Phys. Rev. C 55, R573 (1997)

Step 2: Projection of 18-body 4hΩ Hamiltonian onto 0hΩ 2-body Hamiltonian for ^{18}F

$$H_{eff}([sd]^2) = \Sigma_k\ |k,N_{max}=4,A=18> E_k\ (A=18) < \overline{k,N_{max}=4,A=18)}|$$

$$|k,N_{max}=4,A=18> = U_{k,kp2}|k_{p2}[0h\Omega,18]> + U_{k,kq2}|k_{q2}[2+4h\Omega,18]>$$

$\dim(P_1) = 6\ 706\ 870$ $\dim(P_2)=28$ $\dim(Q_2) = 6\ 706\ 842$



$$H_{\rm diag} = UHU^\dagger$$

$$U = \begin{pmatrix} U_{PP} & U_{PQ} \\ U_{QP} & U_{QQ} \end{pmatrix}$$

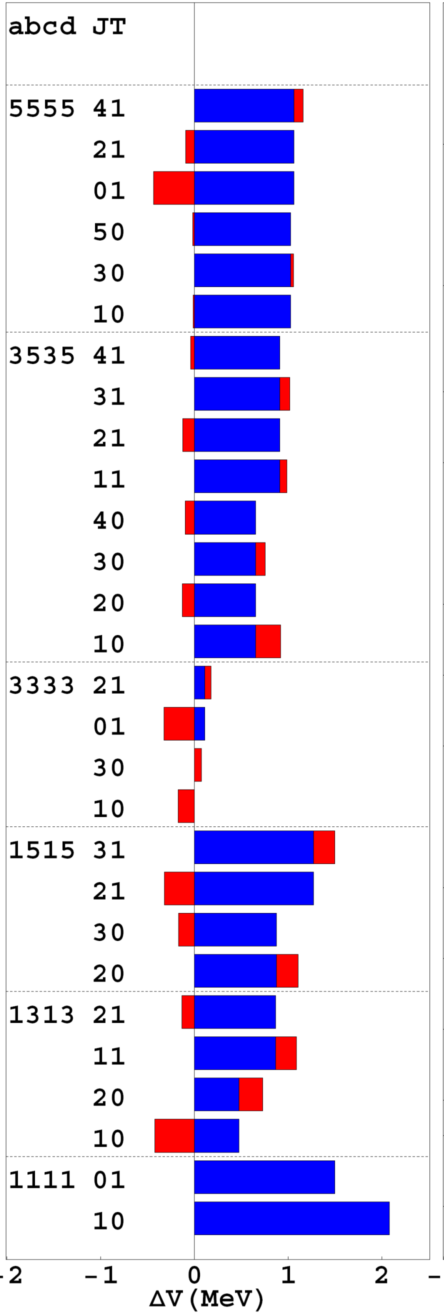
$E_k(A=18)$ $H(N_{max}=4,A=18)$

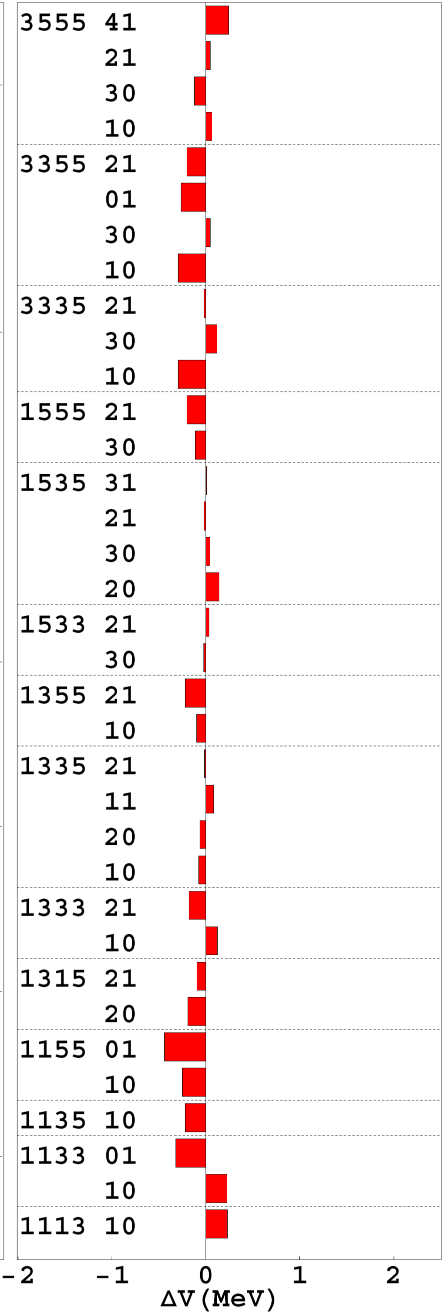
$D(4h\Omega) = 6\ \ 706\ \ 870$

$D(6h\Omega) = 425\ \ 883\ 168$

$$H_{\rm eff} = \frac{U_p^\dagger}{\sqrt{U_p^\dagger U_p}} H^p_{\rm diag} \frac{U_p}{\sqrt{U_p^\dagger U_p}}.$$

$$H_{eff} = H_{eff}(1b) + H_{eff}(2b) + H_{eff}(3b) + H_{eff}(4b) + \ldots$$





Comparison to 0hΩ interaction

Effective two-body sd-shell interaction reproduces energies of the 6hΩ

NCSM calculation for ^{18}F (CD-Bonn)

$$H_{eff}(sd;N_{max}=6) = H(N_{max}=0) + \Delta V$$

$$+\Delta E(\text{core}) + \Delta E(\text{one-body})$$

$$\Delta V(abcd;JT) = \Delta V_{mon}(ab;T) + \Delta V_{res}(abcd;JT)$$

$$-\Delta V_{mon}(33;T=0)$$

$$abcd = 2j_a\, 2j_b\, 2j_c\, 2j_d$$

Next steps

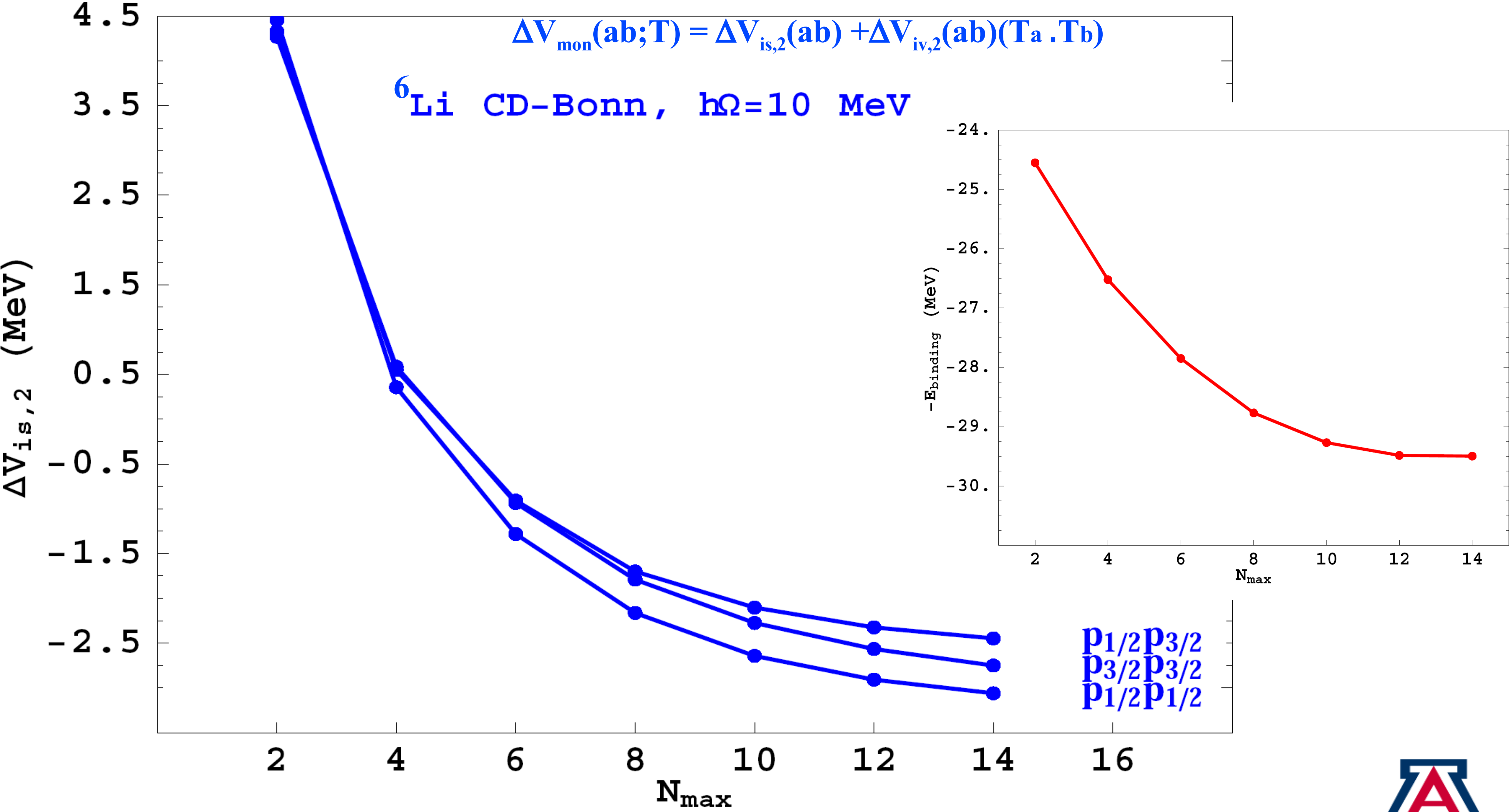
Testing different methods to derive interactions directly:

renormalization

regularities for p- and sd-shells

Testing 6hΩ-sd interaction for other sd-shell nuclei

Systematic for p-shell effective interactions



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