# Particle-vibration coupling and the spatial dependence of the pairing interaction

P.F. Bortignon, R.A. Broglia, G. Colo',

L. Micene, A. Pastore , E. Vigezzi

**Milano University and INFN**

F. Barranco

**Sevilla University**

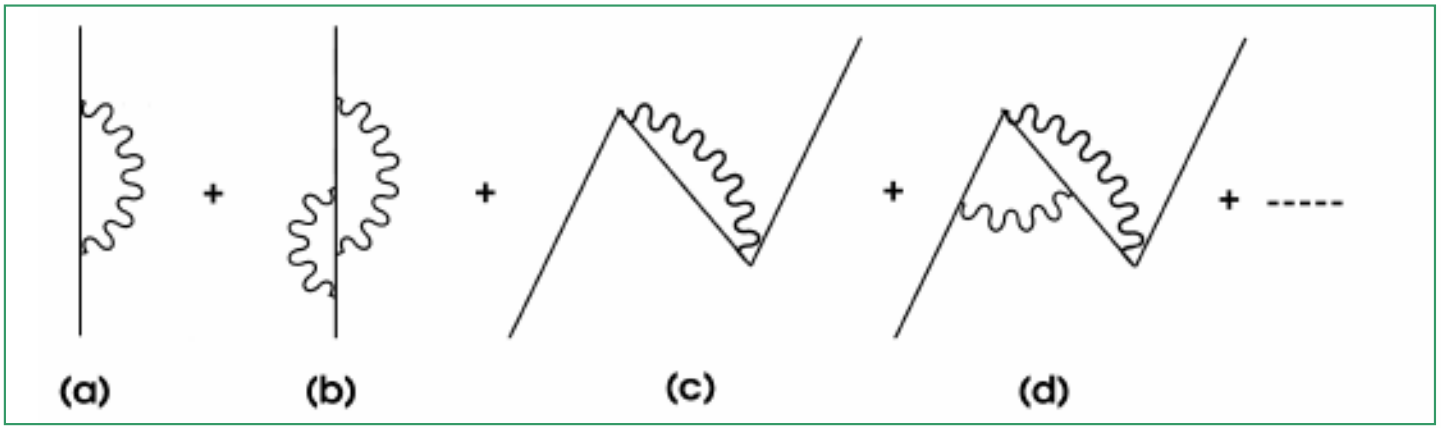
Various effective forces in the pairing channel have been proposed, with different features (finite/zero range, density dependence…)
Unfortunately it is difficult to discriminate among them comparing with available data.

We want to follow a different strategy:

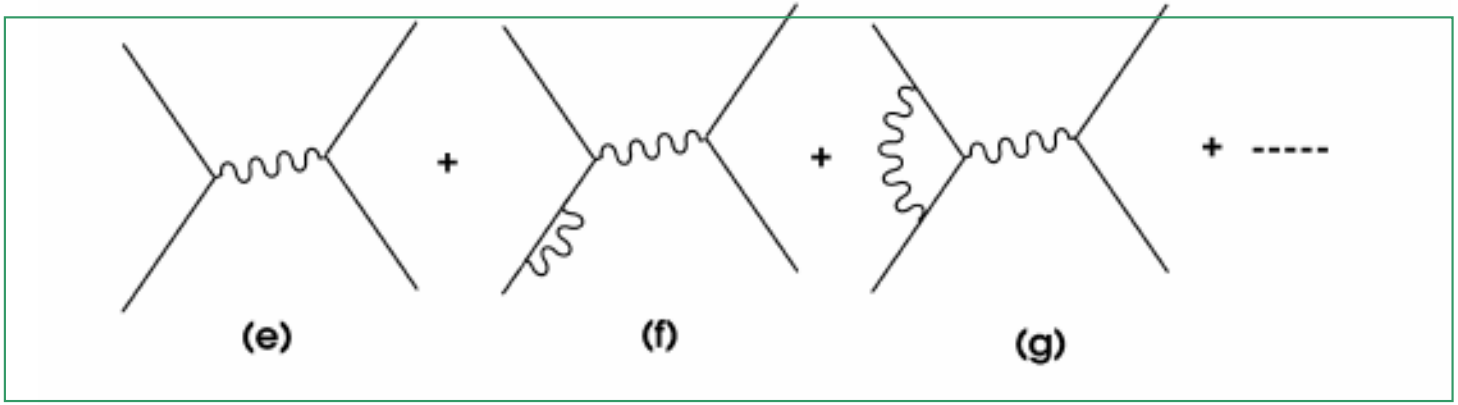
- Start from an Hartree-Fock calculation with a 'reasonable' interaction. Then solve the pairing problem with a bare interaction in the $^1S_0$ channel. And finally add correlations beyond mean field.
We know that these correlations strongly renormalize the density of single-particle levels (effective mass) and their occupation factors (fragmentation), and we expect that they can have a large effect on pairing properties.

# Going beyond mean field: medium polarization effects

Self-energy




Induced interaction (screening)



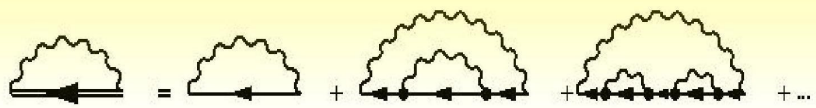
# Going beyond the quasi-particle approximation

J. Terasaki et al., Nucl.Phys. **A697**(2002)126

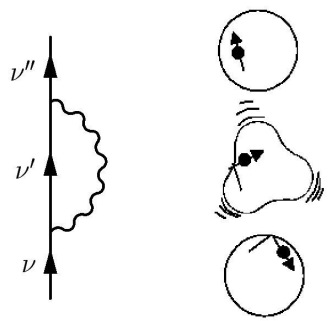
by extending the Dyson equation…

$$G_\mu^{-1} = (G_\mu^o)^{-1} - \Sigma_\mu(\omega)$$

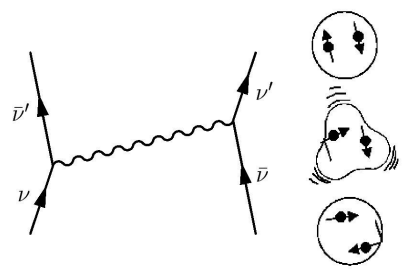
$$\Sigma_\mu(\omega) = \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \sum_{\mu'} \frac{1}{\hbar} G_{\mu'}(\omega') \sum_{\alpha} \frac{1}{\hbar} D_\alpha^o(\omega-\omega') * V^2_{\mu\mu',\alpha}$$



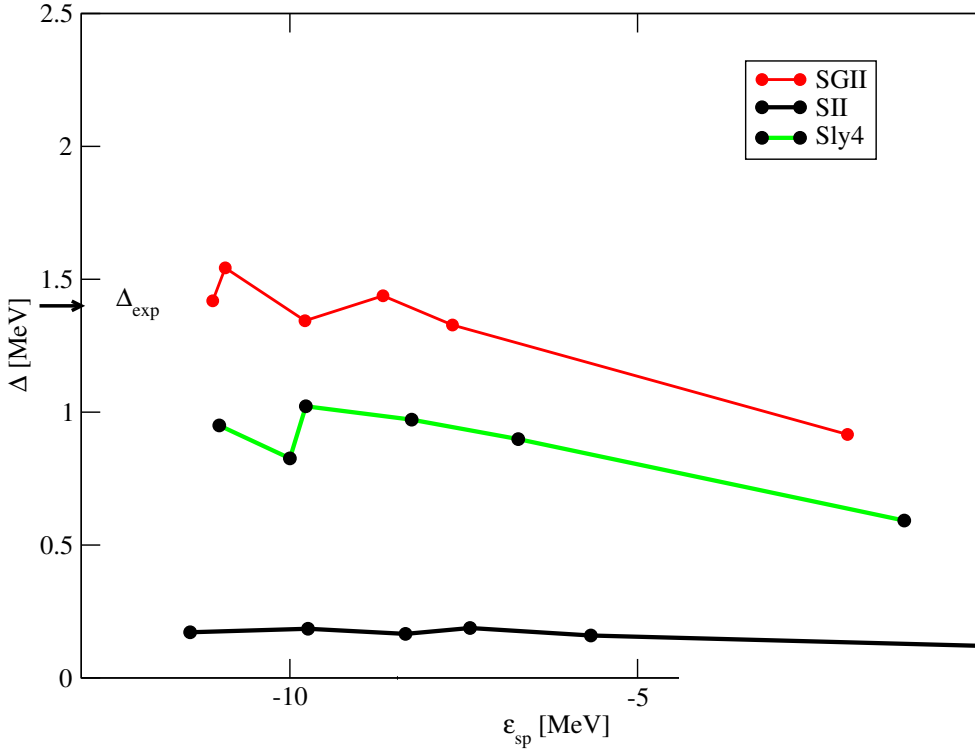
to the case of superfluid nuclei (Nambu-Gor'kov), it is possible to consider both



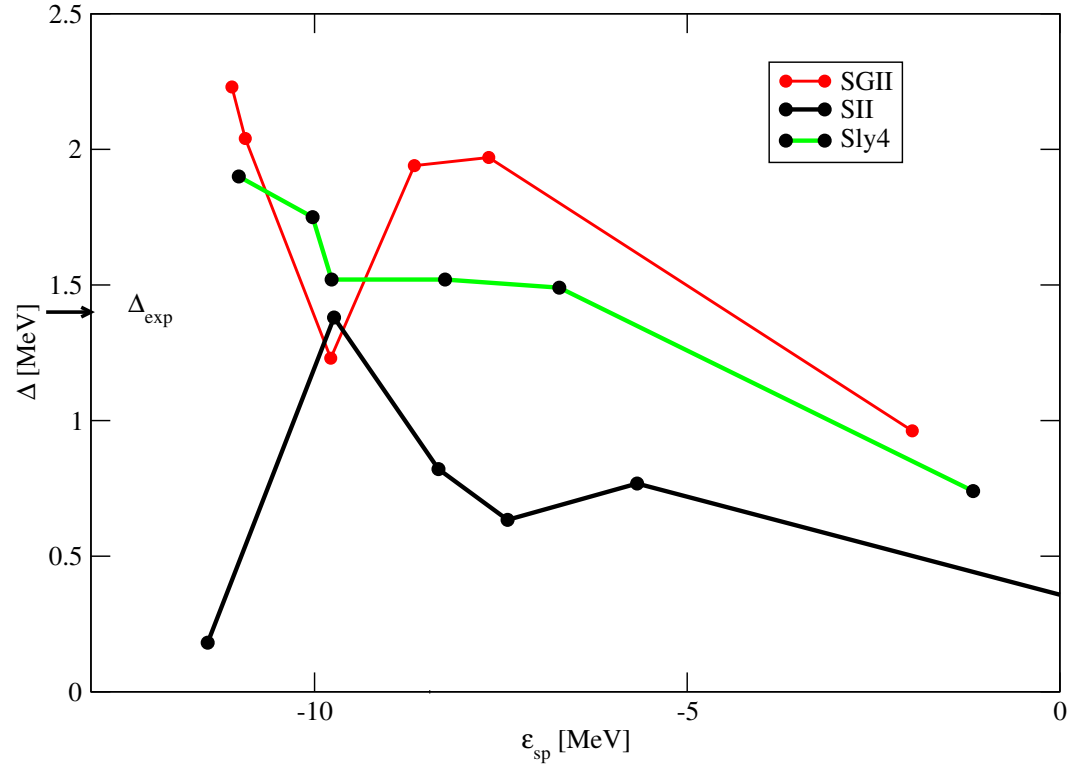

and



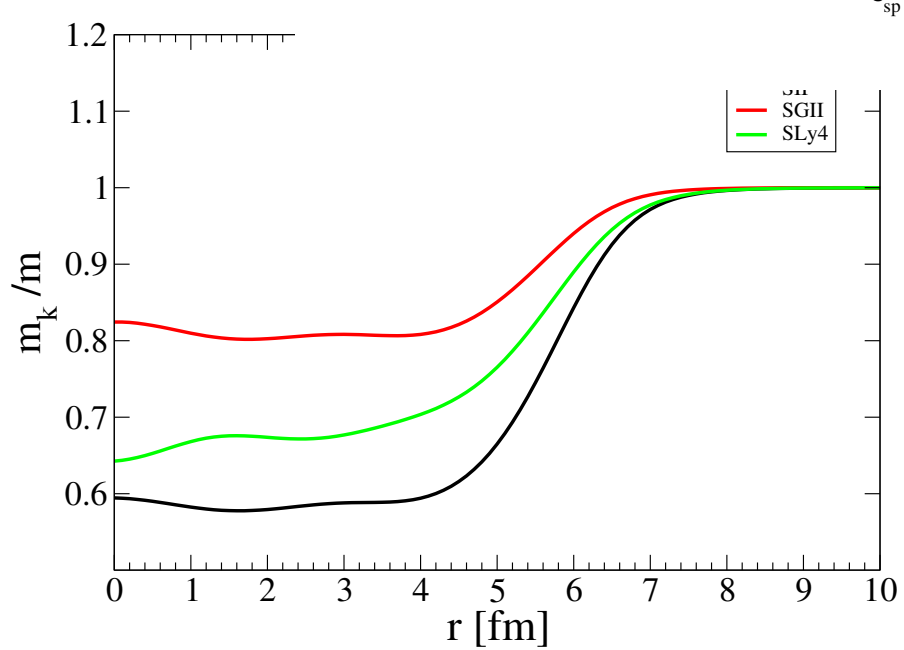
# Bare (Argonne) gaps




# Renormalized gaps

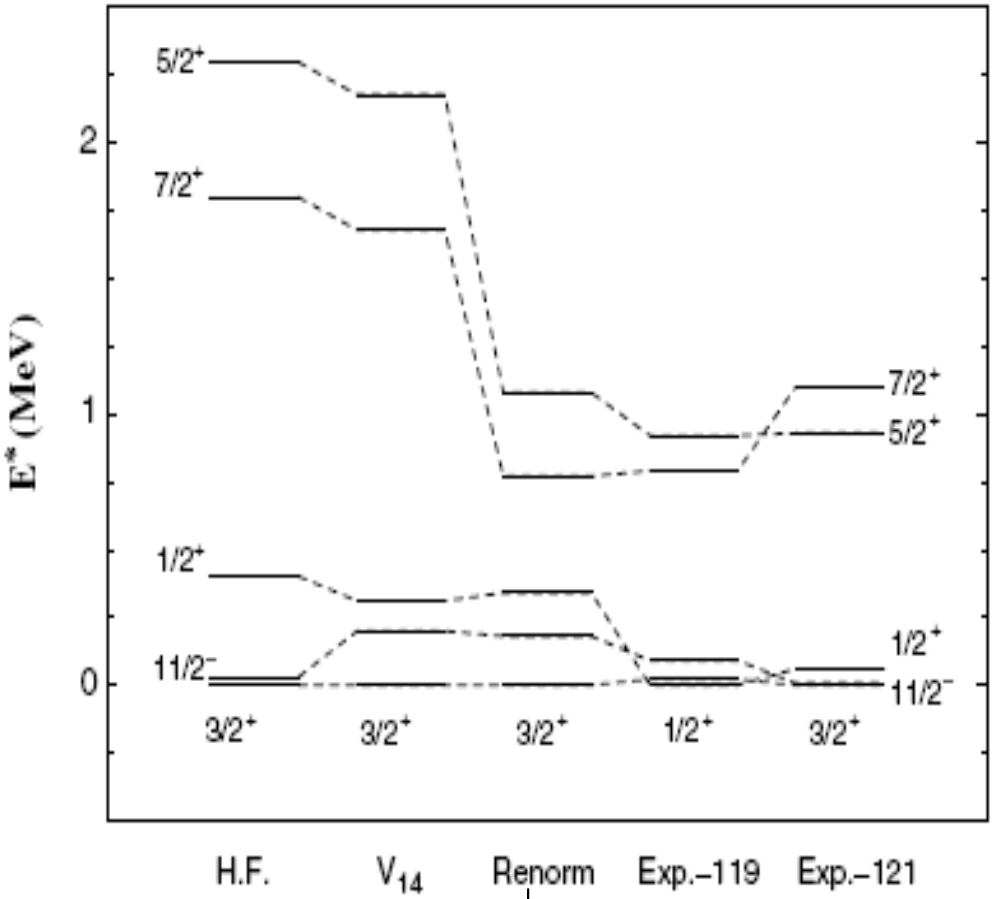



# Effective masses



Renormalization of quasiparticles

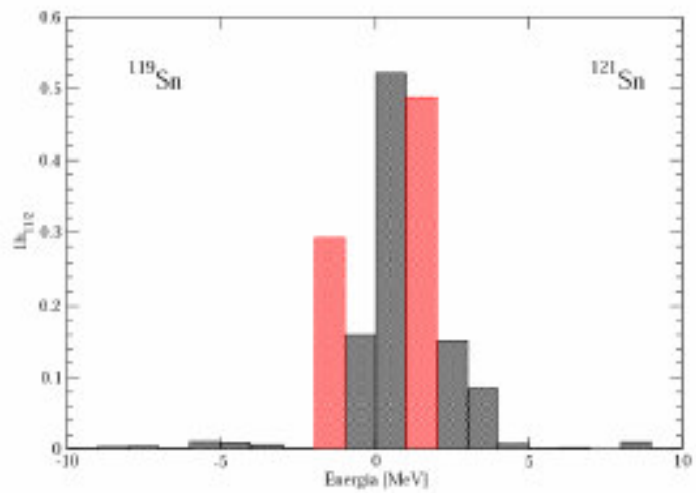
120Sn



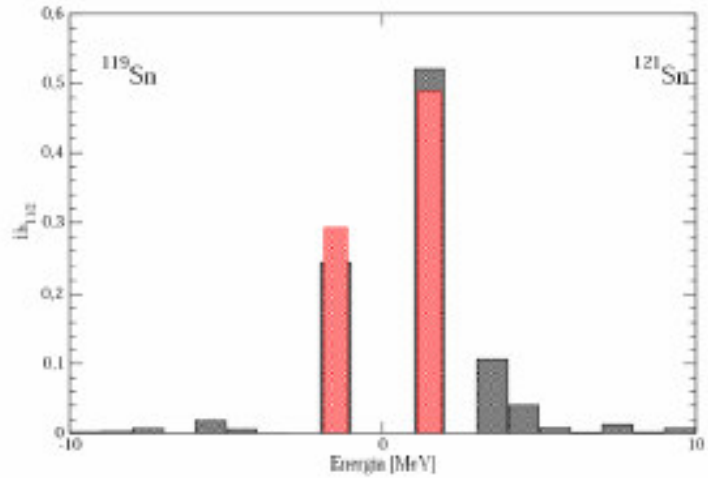

Argonne

Argonne + induced interaction

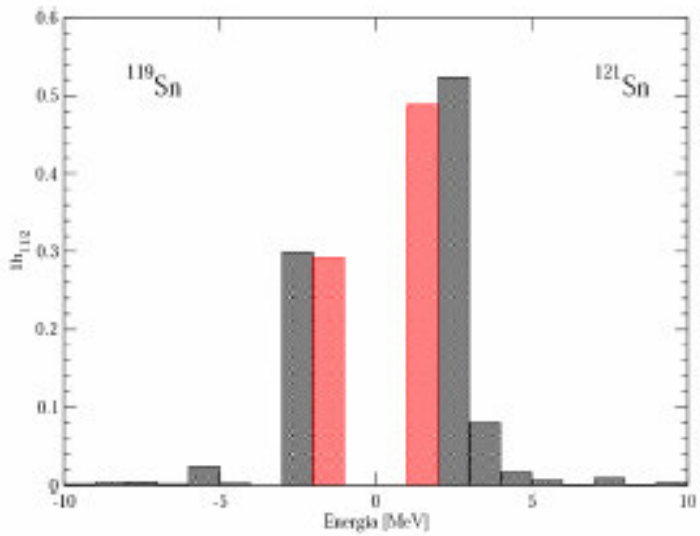
# $h_{11/2}$




SII



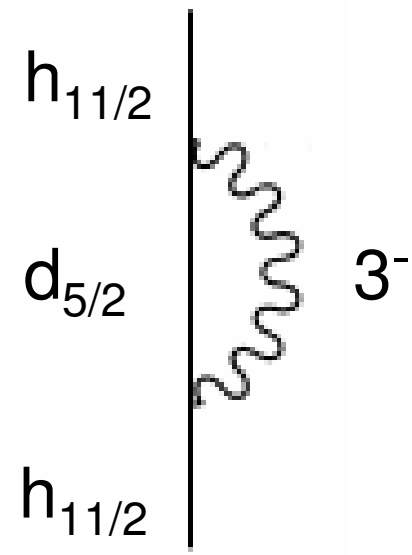

SLy4



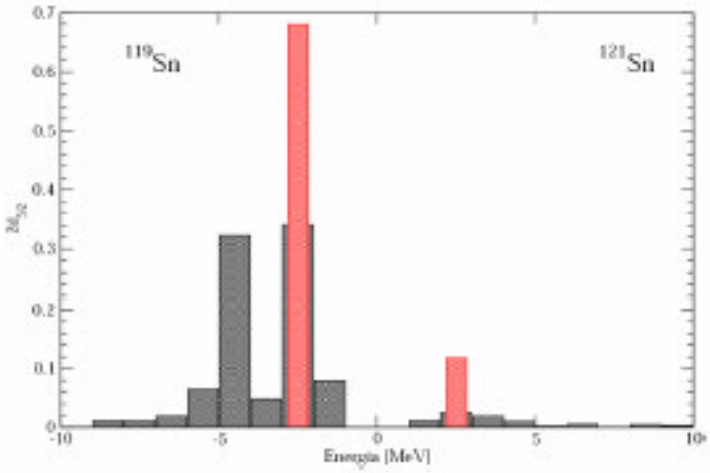

SGII

Exp.

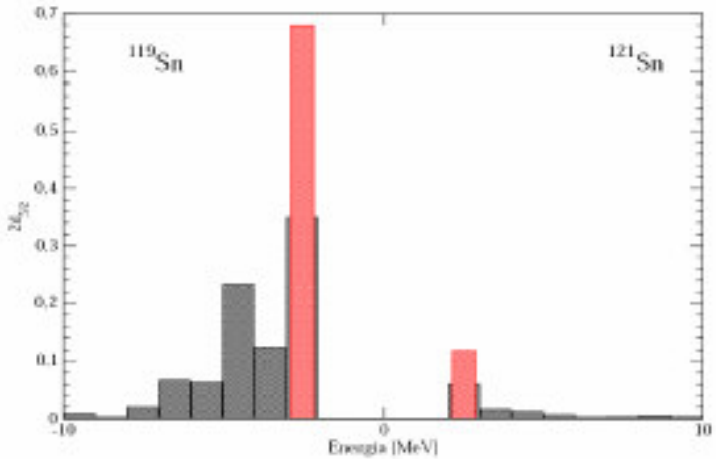
Th.



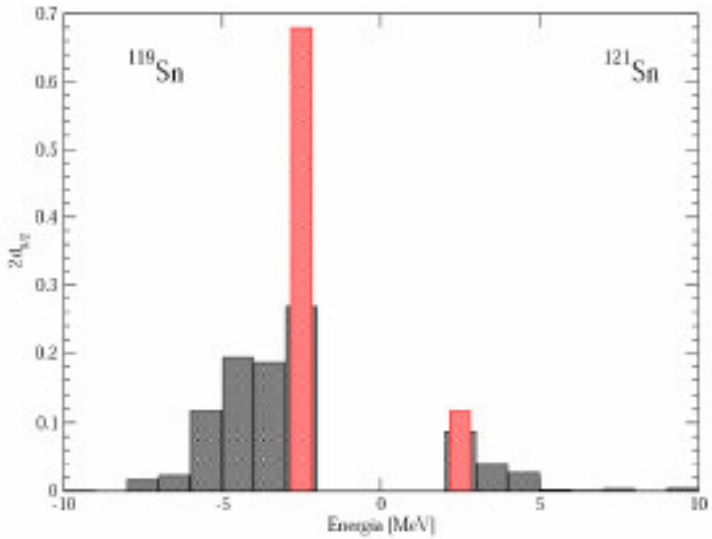
# $d_{5/2}$




SII



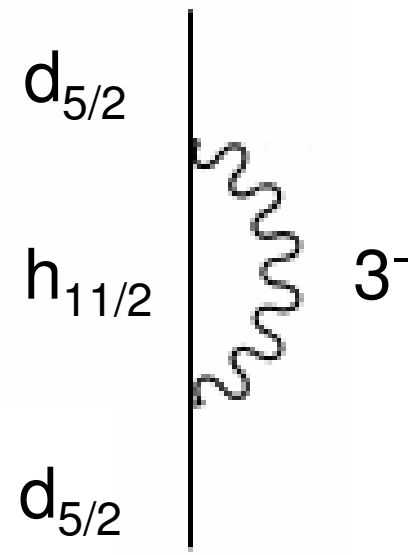

SLy4




SGII

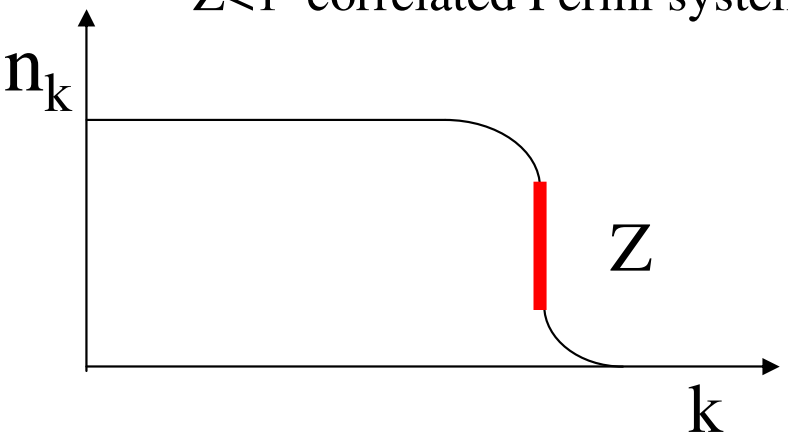
Exp.

Th.



# Generalized Gap Equation (schematic)

Z=1 free Fermi gas
Z<1 correlated Fermi system




quasiparticle strength <1

Induced interaction

$$\Delta_p = -\frac{1}{2}\int d^3p' \frac{Z_p V_{pp'} Z_{p'}}{\sqrt{(\varepsilon_{p'} - \varepsilon_F)^2 + \Delta_{p'}^{\ 2}}}\Delta_{p'}$$

Sensitive to effective mass

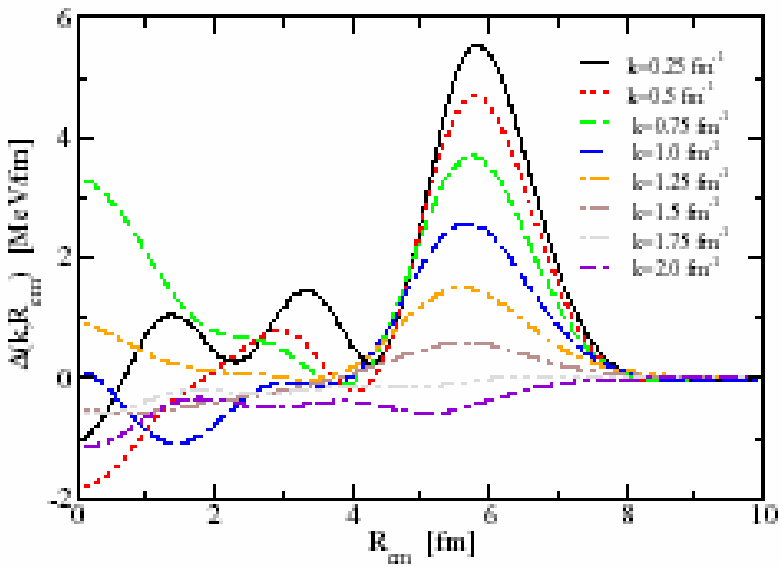
We will use a simple model to take in account the contribution of the exchange of vibrations among particles:

$$\Delta_\nu = \sum_{\nu'} Z(V_{\nu\nu'}^{induced} + V_{\nu\nu'}^{Argonne})\phi_{\nu'}$$

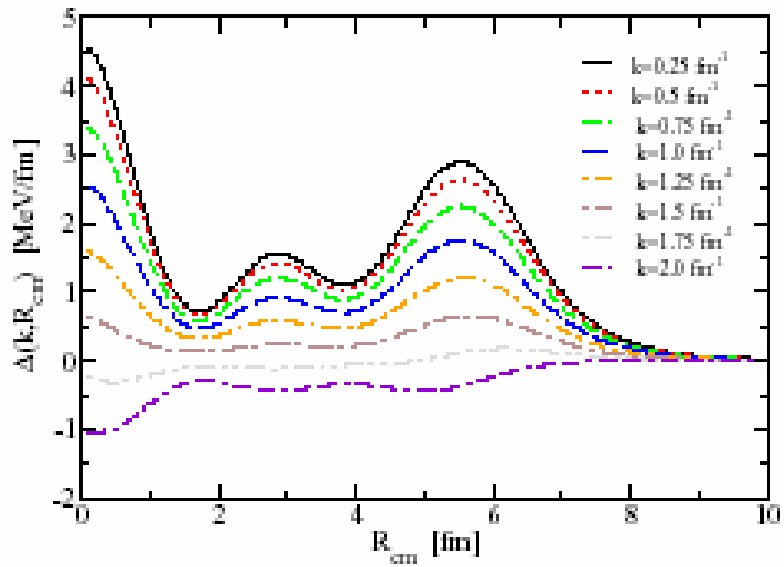
We will simulate the effect of the $m_\omega$ introducing the factor Z=0.7.

# Pairing field in momentum space

Argonne+ induced




Argonne

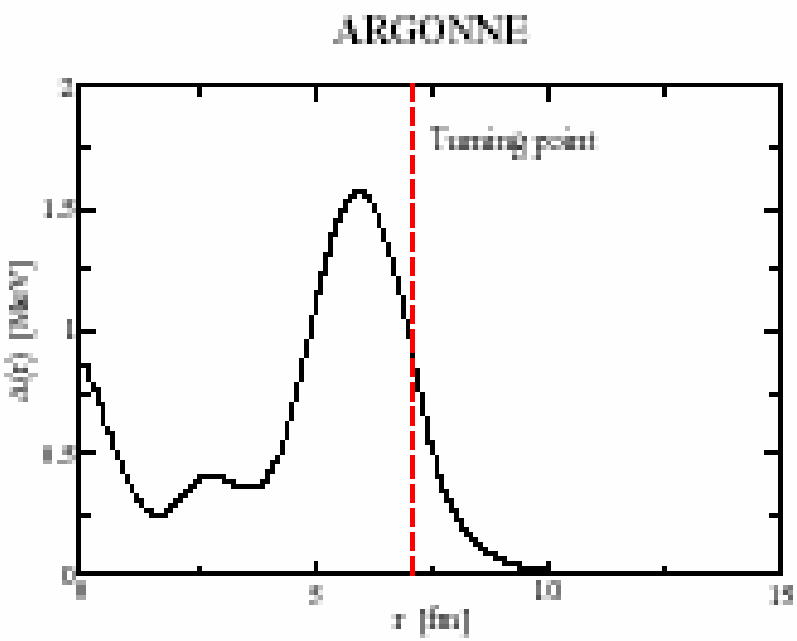



A. Pastore et al., nucl-th/0801.1385

# Local approximation to the pairing field

$$\Delta_{loc}(R_{CM}) \quad = \quad \Delta(R_{CM}, k_F(R_{CM}))$$

$$\hbar^2 k_F(R_{CM})^2 \quad = \quad 2m^*(R_{cm})(E_F - U(R_{CM}))$$




In the forbidden region :

$$\Delta(\vec{R}_{CM}) \quad = \quad \frac{1}{(2\pi)^3} \int d^3 r_{12} \; \Delta(\vec{R}_{CM}, \vec{r}_{12})$$

# Compare the local approximation with the full calculation

With the local pairing field we solve HFB in r-space:

$$\left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + \frac{2m^*}{\hbar^2}[e_F + E_{sqp} - V(r)]\right) u_{lj}(r) + \frac{2m^*}{\hbar^2}\left(\frac{\hbar^2}{2m^*}\right)' \frac{d}{dr} u_{lj}(r) - \frac{2m^*}{\hbar^2}\Delta_{loc}(r) v_{lj}(r) = 0,$$

$$\left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + \frac{2m^*}{\hbar^2}[e_F - E_{sqp} - V(r)]\right) v_{lj}(r) + \frac{2m^*}{\hbar^2}\left(\frac{\hbar^2}{2m^*}\right)' \frac{d}{dr} v_{lj}(r) + \frac{2m^*}{\hbar^2}\Delta_{loc}(r) u_{lj}(r) = 0.$$

| | Argonne | | | |
|---|---|---|---|---|
| $l$ $2j$ | $E_{qp}^{full}$ | $v_{can}^2$ | $E_{sqp}$ | $v_{sqp}^2$ |
| 0 1 | 1.41 | 0.85 | 1.31 | 0.86 |
| 1 1 | 11.95 | 0.99 | 11.93* | 0.96 |
| 1 3 | 8.19 | $6 \cdot 10^{-4}$ | 7.98 | $2 \cdot 10^{-3}$ |
| 2 3 | 1.18 | 0.72 | 1.09 | 0.73 |
| 2 5 | 3.46 | 0.98 | 3.42 | 0.98 |
| 3 5 | 9.27 | $10^{-3}$ | 9.19* | $2 \cdot 10^{-3}$ |
| 3 7 | 6.52 | $3 \cdot 10^{-3}$ | 6.54 | $5 \cdot 10^{-3}$ |
| 4 7 | 2.33 | 0.94 | 2.27 | 0.95 |
| 4 9 | 9.30 | 0.99 | 9.29* | 0.98 |
| 5 11 | 1.37 | 0.15 | 1.47 | 0.17 |
| 6 13 | 10.96 | $10^{-3}$ | 10.99* | $3 \cdot 10^{-3}$ |

Table: The lowest quasiparticle energies associated with the quantum numbers (l,2j) obtained solving the HFB equations with the Argonne are indicated with $E_{HFB}^{qp}$; also listed are the occupation probabilities in the canonical basis $v_{can}^2$. They are compared with the quasiparticle energies $E_{loc}^{qp}$ and occupation probabilities $v_{loc}^2$ obtained solving the HFB equations in coordinate space with the local potentials.

# Density Dependent Delta Interaction

We give a description in terms of density dependent delta force of our local approximation:

$$v(\vec{r}_1, \vec{r}_2) = v_0 \left[ 1 - \eta \left( \frac{\rho\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)}{\rho_0} \right)^{\alpha} \right] \delta(\vec{r}_1 - \vec{r}_2)$$

We fix the parameter $v_0 = -458.4$ MeVfm $^3$ and consequently the cut-off in single-particle energy spectrum $\epsilon_{cut} = 60$ MeV.

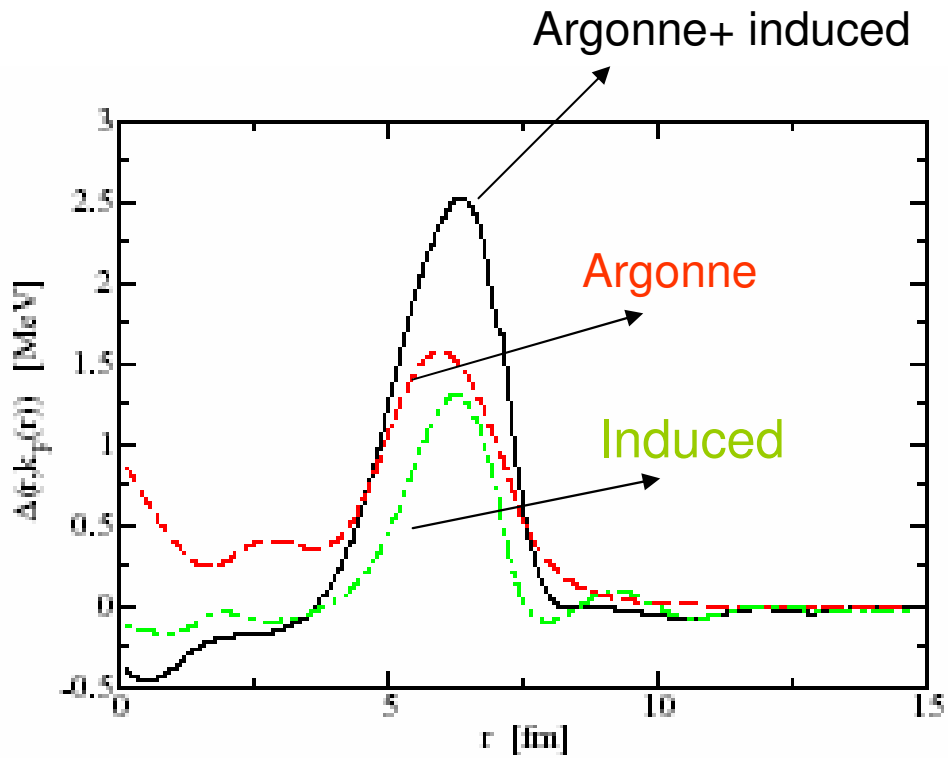
M.Matsuo, Phys. Rev. C 73 (2006), 044309

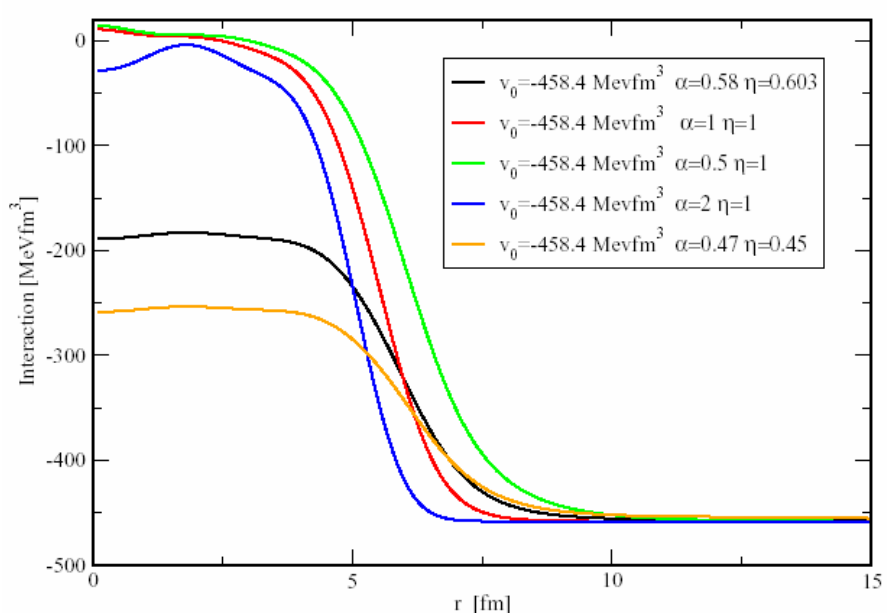
We use the following procedure:

1. I take trial values for $\alpha$ and $\eta$,and I use them to solve HFB equations self-consistently, so I obtain the anomalous density, $\Phi(\vec{r}_1, \vec{r}_2)$
2. I fit $\Delta_{loc}(r)$, obtained with the semiclassical approximation, with the least square method with the function $v(\vec{r}_1, \vec{r}_2) \cdot \Phi(\vec{r}_1, \vec{r}_2)$, and I obtain two new values of the constants $\alpha$ and $\eta$.
3. I insert the new values of $\alpha, \eta$ in the HFB equations and I recalculate the anomalous density $\Phi(\vec{r}_1, \vec{r}_2)$
4. I repeat all this procedure until the values of $\alpha, \eta$ I obtained from the fitting-procedure are close to the ones I used in the previous step to calculate $\Phi(\vec{r}_1, \vec{r}_2)$; the tolerance is $|\alpha_{fit} - \alpha_{previous}| < 0.05$ and $|\eta_{fit} - \eta_{previous}| < 0.05$

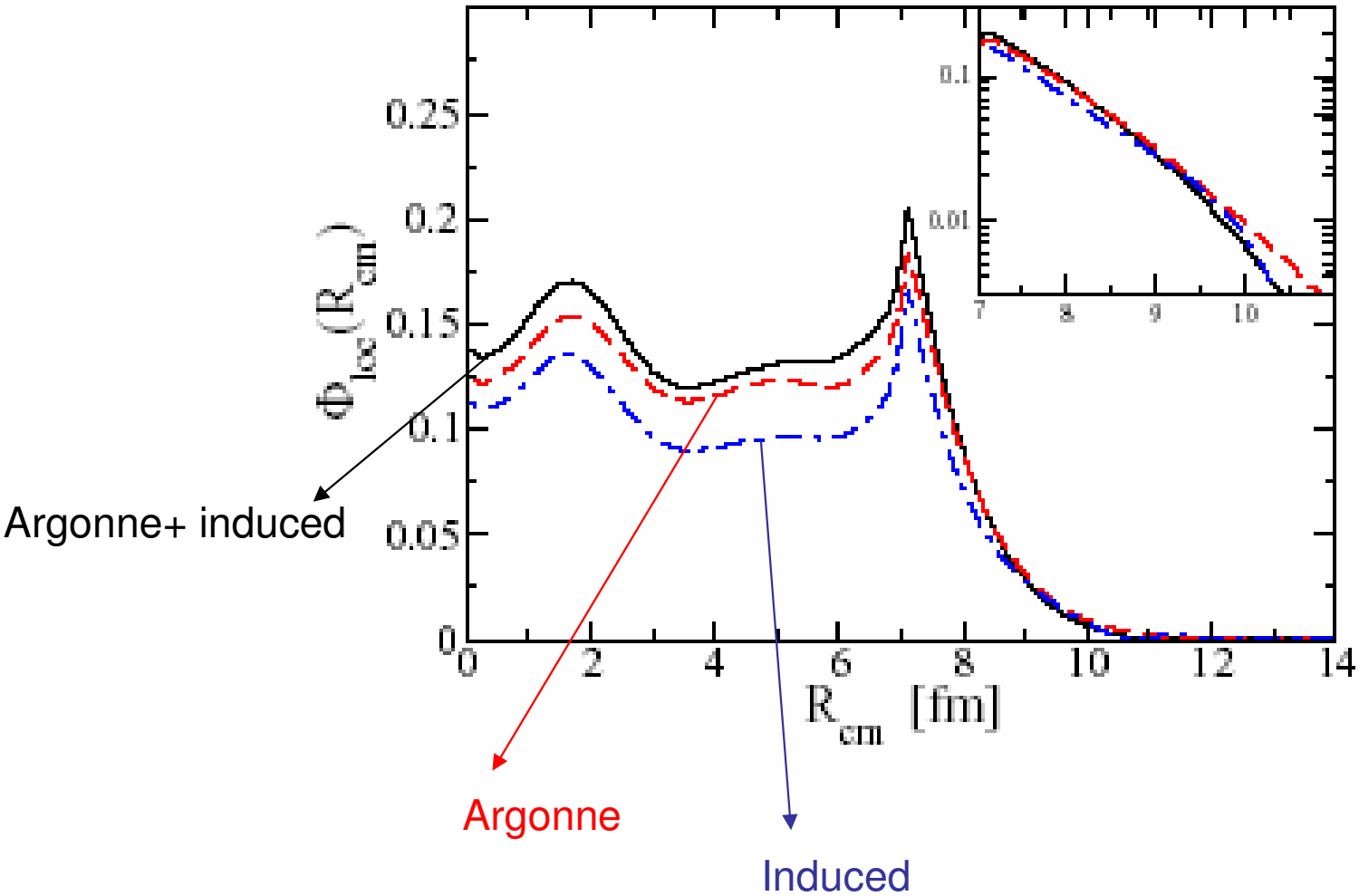
# Local approximation

| Interaction | $\alpha$ | $\eta$ |
|---|---|---|
| Argonne v18 plus Induced | 1.2 | 1.0 |
| Gogny D1 | 0.55 | 0.62 |
| Argonne v18 | 0.66 | 0.84 |
| Induced | 0.85 | 1 |



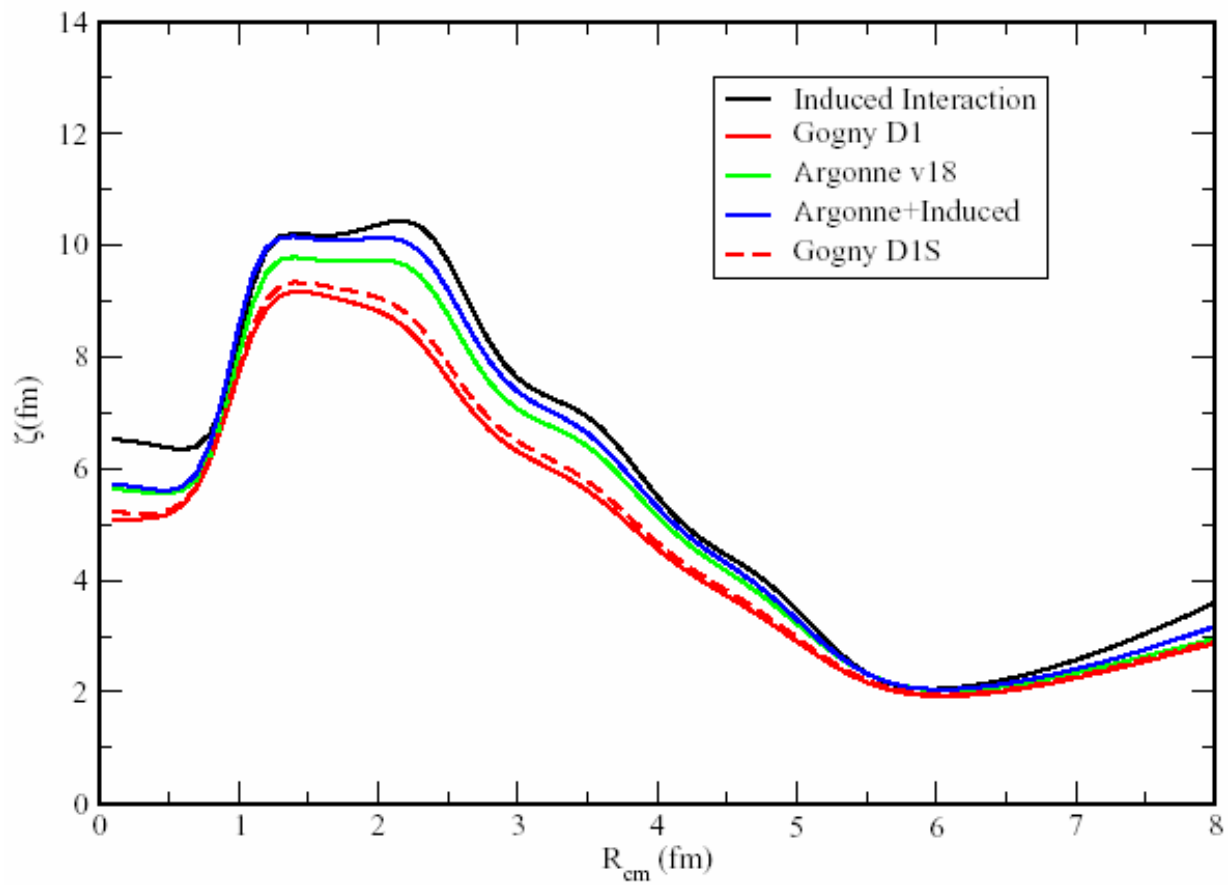



# Abnormal density



# “Local” coherence length

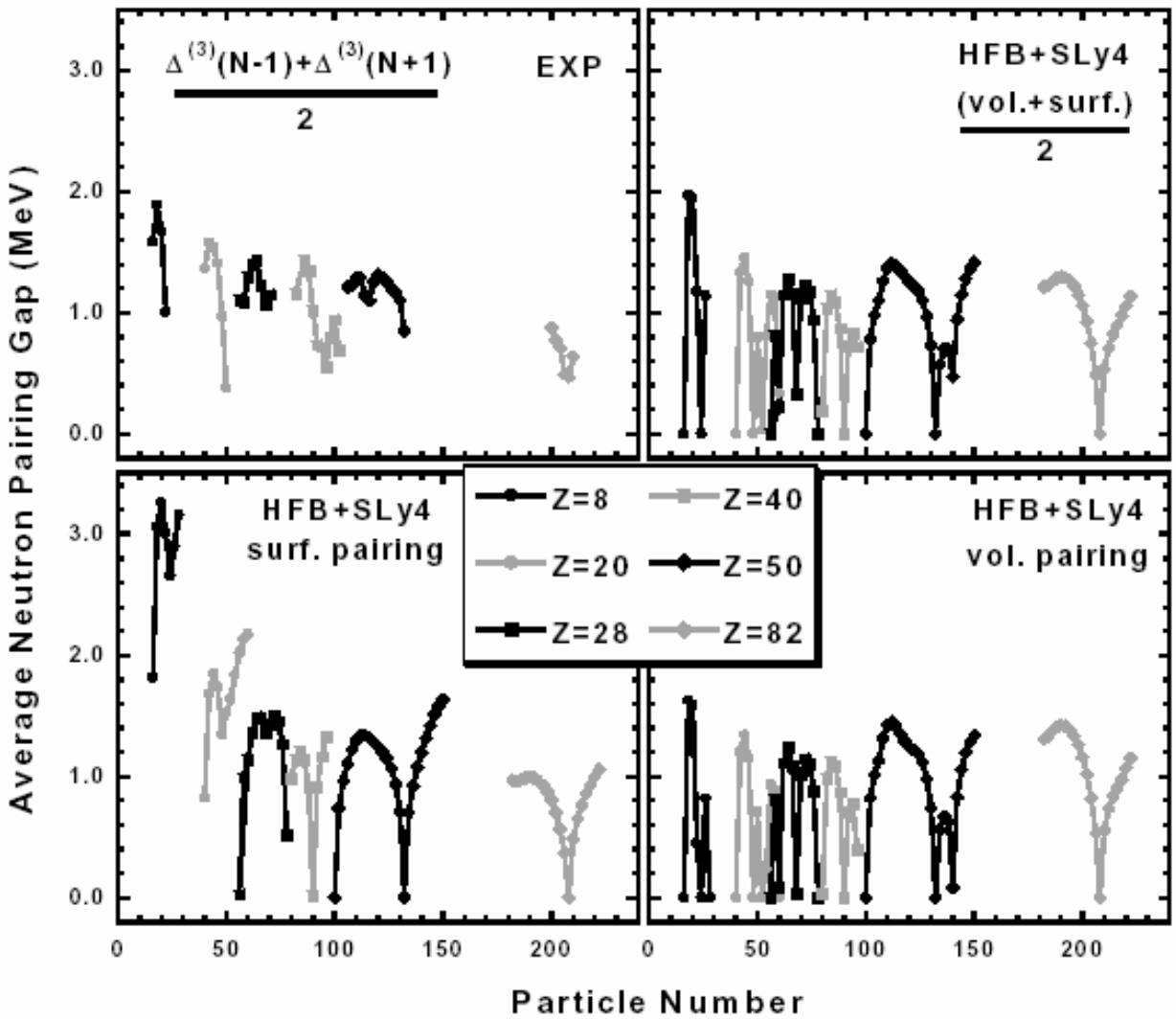
$$\xi(R_{CM}) = \left( \frac{\int d^3 r_{12} \, | \kappa(R_{CM}, r_{12}) |^2 \, r_{12}^{\;2}}{\int d^3 r_{12} | \kappa(R_{CM}, r_{12}) |^2} \right)^{1/2}$$





M. Matsuo, Phys. Rev. C56 (2007) 3054
N. Pillet, N. Sandulescu, P. Schuck Phys. Rev. C76 (2007)24310
K. Hagino, H. Sagawa, J. Carbonell, P. Schuck, PRL 99 (2007)22506

Connection with observables?




J. Dobaczewski et al., nucl-th/0109073

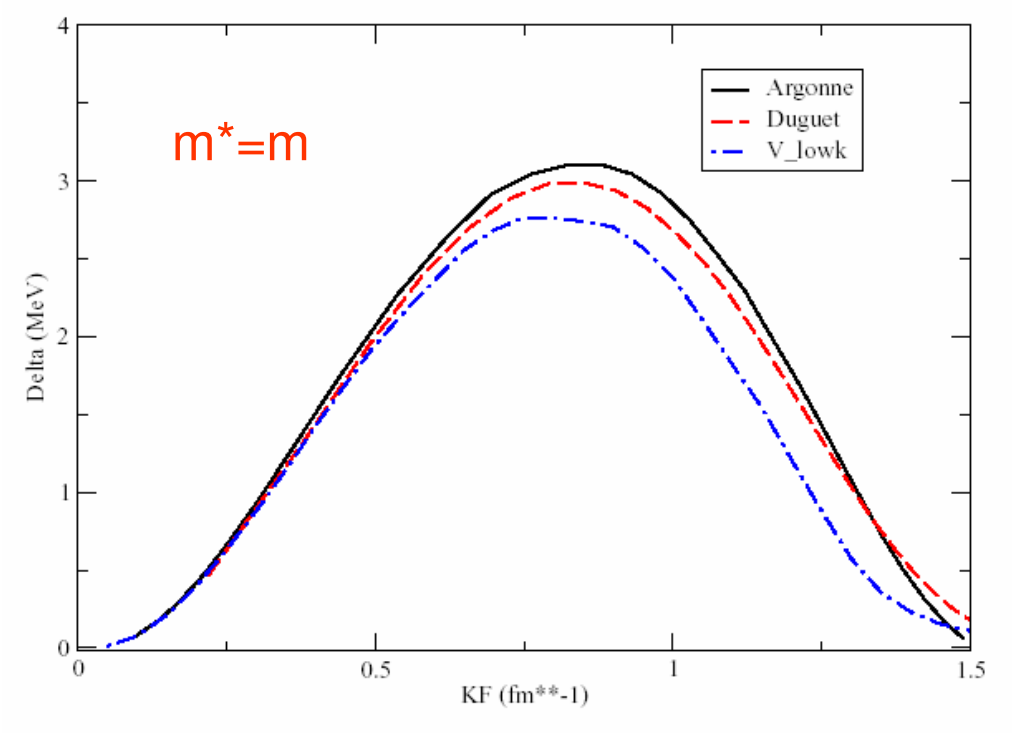
# Conclusions

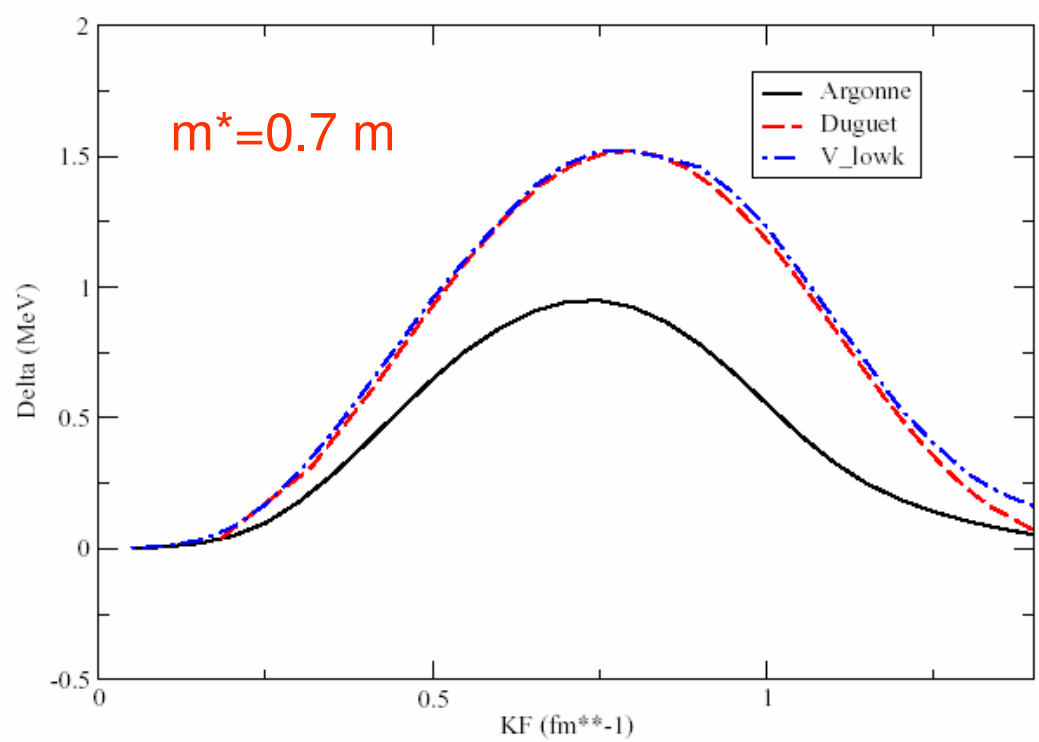
Pairing correlations in the 1S0 channel can be calculated microscopically, adding the bare nucleon-nucleon interaction and the interaction induced by the exchange of surfce vibrations.
At the same time, it is possible to take into account self-energy effects (renormalization and fragmentation of quasi-particle levels), obtaining spectroscopic factors.

It is found that the exchange of collective surface vibrations enhances the pairing gap considerably, although the effect is largely diminished by fragmentation.

These effects can be treated in approximate way introducing a simple, density dependent local interaction which turns out to be very surface-peaked. The abnormal density is less affected.

The consequences on observables remain to be explored.
All the calculations refer to the single case of $^{120}$Sn.


m*=m
Argonne
Duguet
V_lowk
Delta (MeV)
KF (fm**-1)
0
1
2
3
4
0
0.5
1
1.5


m*=0.7 m
Argonne
Duguet
V_lowk
Delta (MeV)
KF (fm**-1)
-0.5
0
0.5
1
1.5
2
0
0.5
1

## Gaussian interaction

We approximate the energy-dependent and non-local induced interaction with a gaussian function:

$$v_{ind}^{G}(R_{cm}, r_{12}) \equiv -b \cdot \exp\left(-((r_{12} - c)/a)^2\right)$$

$$a \simeq 2\ fm \qquad c = 2|R_{nucl} - R_{cm}| \qquad R_{nucl} = 6.49\ fm$$

the parameter b is obtained by a least square method using:

$$\Delta(R_{cm}, r_{12})^{Full} = -v_{ind}^{G}(R_{cm}, r_{12}) \cdot \Phi_{J=0}^{S=0}(R_{cm}, r_{12})^{Full}$$

in the interval $r_{12} = c \pm 2$ fm.