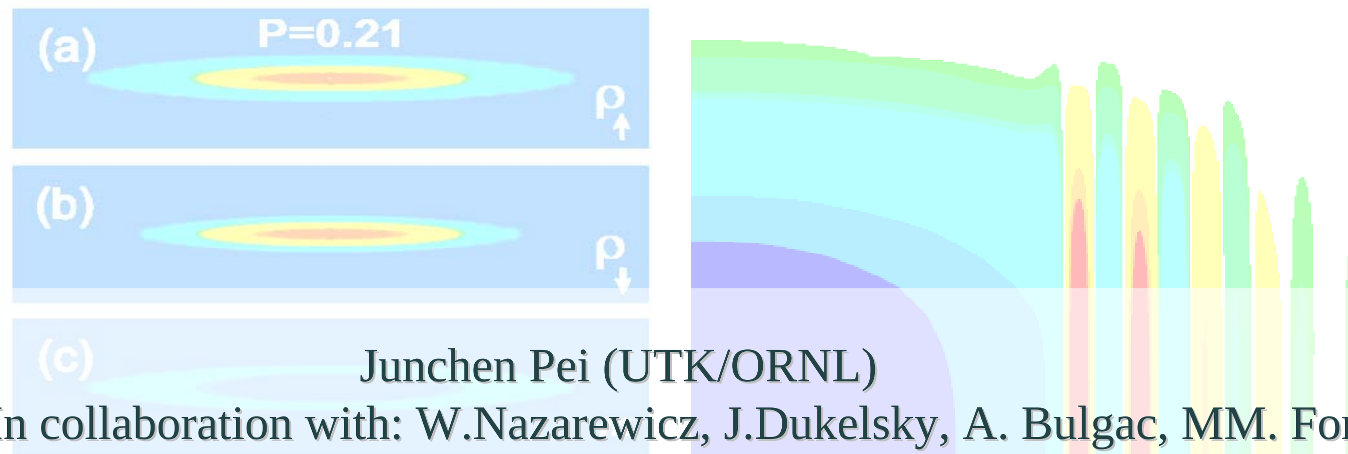
Numerical Study of Exotic Pairing Phases in Imbalanced Fermi Condensates



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4th JUSTIPEN workshop, Oak Ridge, 2010

Oak Ridge National Laboratory

Pairing (superfluidity) in Fermi Systems

- Pairing (superfluidity is produced by forming Cooper pairs of two spin components) is a generic feature in strongly interacting Fermi systems
- Pairing phases are more interesting in imbalanced systems: with mismatched chemical potentials of the two spin states.

Imbalanced (or polarized or asymmetric) systems:

- magnetized superconductors
- Cold dense quark matter in compact stars : $\delta\mu = \frac{1}{2}(\mu_d - \mu_u) = \frac{1}{2}\mu_e$
  (R.Casalbuoni, et al, RMP 76,263, 2004)
- ultracold atomic gas: two hyperfine states of ^{6}Li
- Nuclear physics (odd-nuclei, neutron-skin,n-p pairing, neutron stars, asymmetric nuclear matter)

Exotic pairing phases (temperature and polarization):

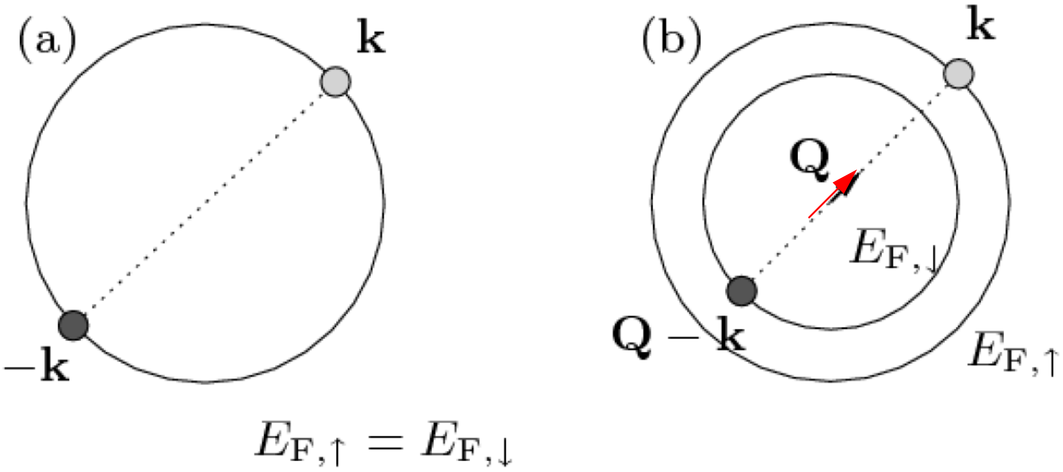
- Gapless excitations. (also happen in nuclear high-spin states: G.Bertsch et al, PRA 79, 043602 (2009))
- Phase separation: a paired superfluid core surrounded by an unpaired shell
  G.B.Partridge,et al., Science 311, 503
- FFLO pairing phase

Expected exotic FFLO pairing

- In imbalanced Fermi systems, pairing with none-zero momentum can happen: Flude-Ferrell-Larkin-Ovchinnikov (**FFLO**)

  Oscillation pairing gap is expected; Modulated densities (crystallized).
- It exists in many theoretical calculations, but difficult to find.
- Some signatures in heavy fermions systems. Radovan, et al. Nature 425, 51, 2003.

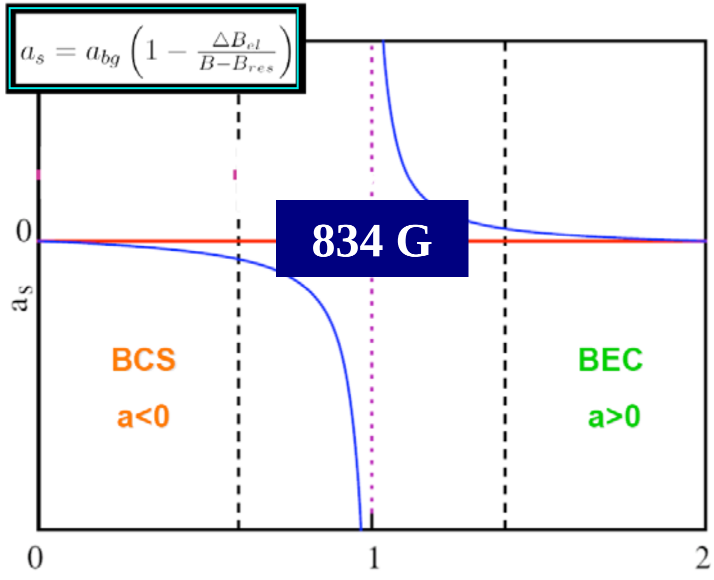
$$\Delta(\vec{x}) = \Delta_0 \sum_n C_n e^{i\vec{q}_n \cdot \vec{x}}$$



Some experiments

- Advantages of using cold atoms:
  interaction is controllable; clear physics; High T_c; implications for other Fermi systems
- **Unitary limit:** two body *s*-wave scattering length diverges: $a_s \to \pm\infty$

  System is strongly correlated and its properties do not dependent on the value of scattering length $\mathbf{a_s}$
- It is realized through Feshbach Resonance: magnetic field at 834 G
- Trapped by optical and magnetic potential



$$U(r,z) = \frac{1}{2} m\omega_B^2 z^2 + U_0[1 - \frac{w_0^2}{w^2(z)} e^{-2r^2/w^2(z)}], w(z) = w_0[1 - (z/z_0)^2]$$

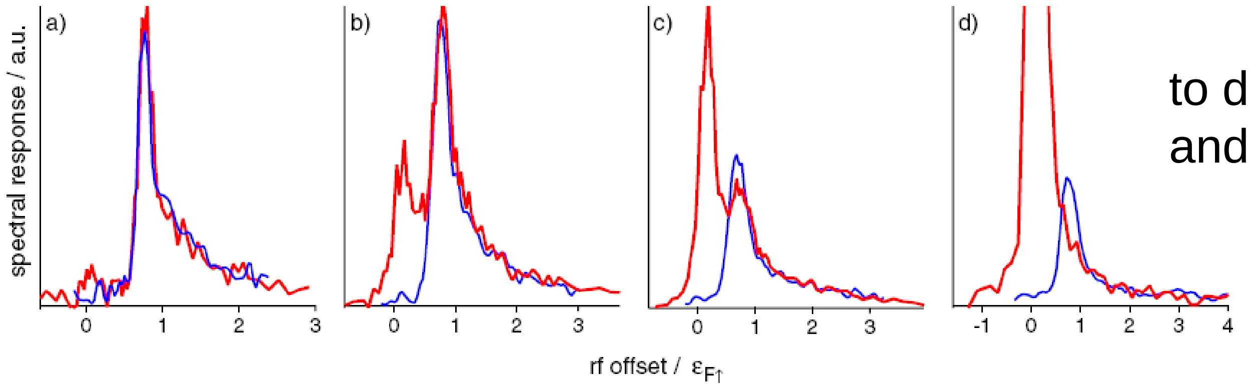
approximate HO potential: $U(r,z) = U_0(1 - \exp(-\frac{1}{2}(w_r^2 r^2 + w_z^2 z^2)/U_0))$

Aspect ratio: $\eta = w_r / w_z$

highly elongate trap is of great interests! (good for looking FFLO pairing)

Some experiments

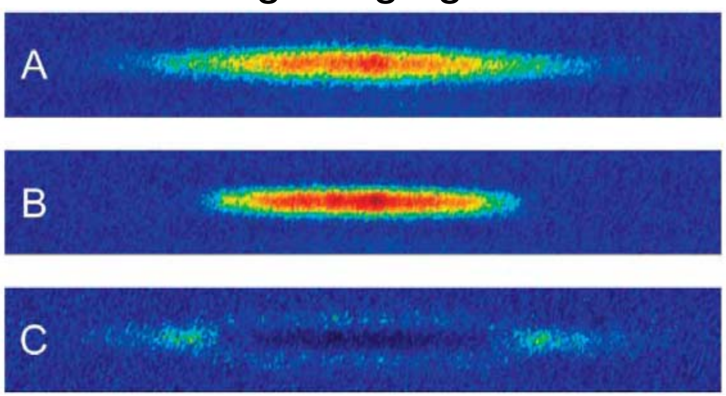
- Radio frequency spectrum



to determine the gap and Hartree energy

Schirotzek, et al. PRL101,140403.

- *In situ* absorbing imaging



Spin-up density

Spin-down density

polarized density

Science 311(2006)503

Experiments(Rice)

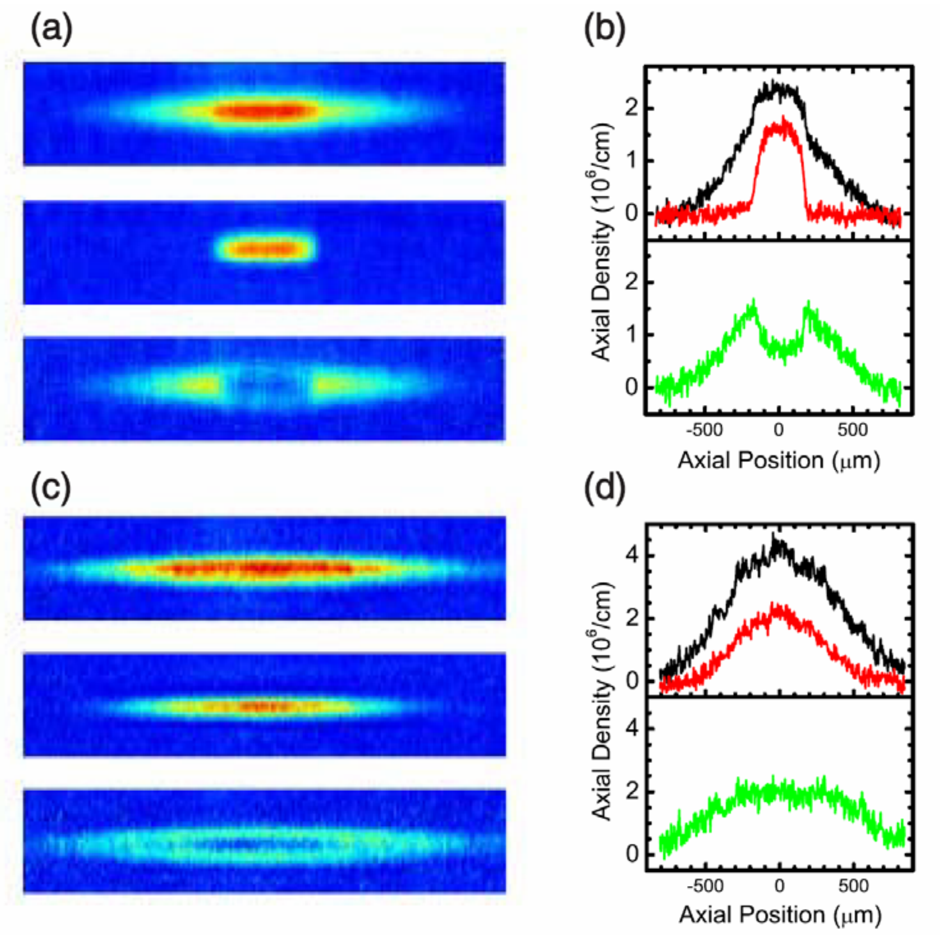
- Phase Separation
- Superfluid Core is deformed from the trap shape

  and such deformation effects disappear at high temperatures

- Trap aspect ratio～**50**: highly elongate
- Particle numbers～10^5

G.B.Partridge, et al, PRL97,190407,2006

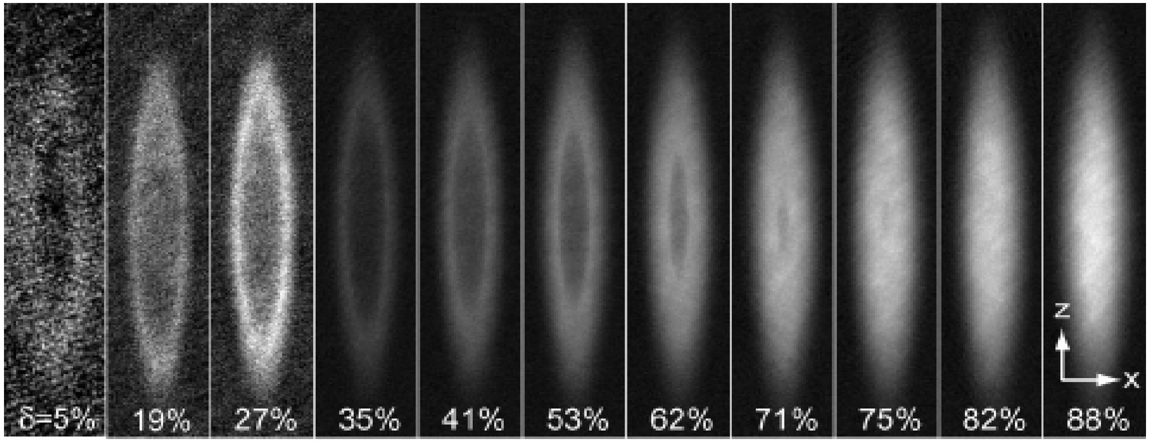
G.B.Partridge, et al, Science,311,503,2006

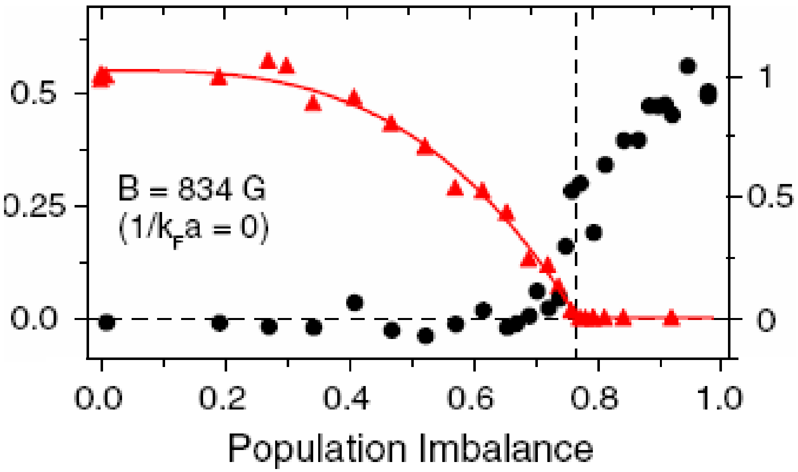


Experiments(MIT)

- Phase separation
- However, no superfluid core deformation

Y.Shin, et al, PRL 97,03401,2006



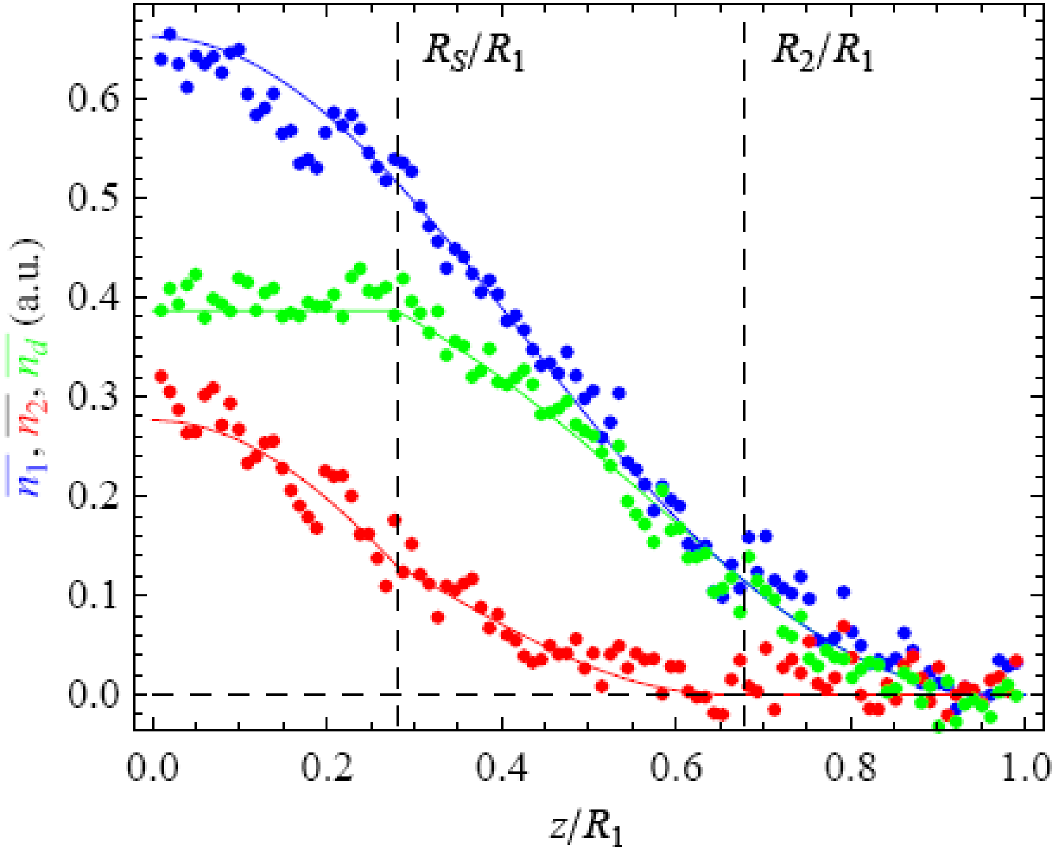


- Clogston-Chandrasekhar limit of superfluidity
- Trap aspect ratio=5, particles=10^6

Experiments-others

- French group: 10^5 particles, aspect ratio=23 (agree with MIT)

No core deformation



C. Salomon, et al, PRL103, 18 (2009) 170402

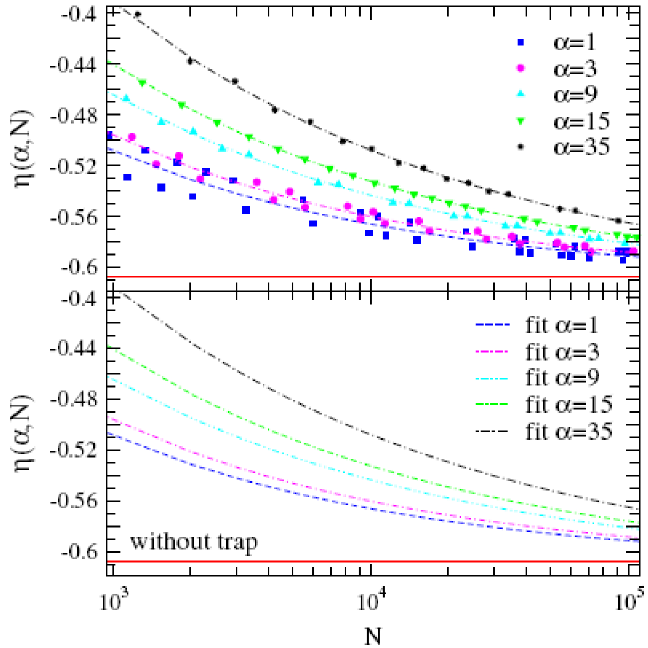
Question???

1. different experimental conditions
2. or theory is not precise

Finite-size effect

- Finite-size effect of trap deformations and particle numbers

  small deformation trap doesn't violate LDA solutions



M.Ku, PRL 102, 255301, 2009

- Surface tension is important at large deformations

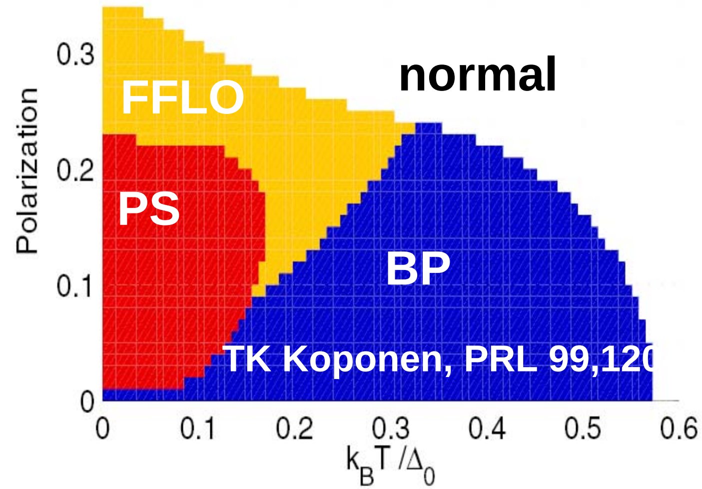
  T.N. De Silva, et al, PRL 97, 070402(2006)

Theoretics

- Quantum Monte Carlo: QMC is very precise but limited to small systems
- Bogoliubov de-Genes equation: Mean Field approximation

$$H(\mathbf{r}) = \sum_{\sigma} \Psi_{\sigma}^{\dagger}(\mathbf{r})[H_0(\mathbf{r}) - \mu_{\sigma}]\Psi_{\sigma}(\mathbf{r}) - g\Psi_{\uparrow}^{\dagger}(\mathbf{r})\Psi_{\downarrow}^{\dagger}(\mathbf{r})\Psi_{\downarrow}(\mathbf{r})\Psi_{\uparrow}(\mathbf{r})$$

$$\begin{pmatrix} H_0(\mathbf{r}) - \mu & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -H_0(\mathbf{r}) + \mu \end{pmatrix} \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix} = E_i \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix}$$



Plenty of calculations, no Hartree potential, and not quantitatively accurate

A contest of computation: Tokyo U:30000 particles; Rice U: 10^5 particles

- DFT: at the unitary limit, the physical properties only depends on the density. It is good for DFT descriptions. Superfluid Local Density Approximation (SLDA) is very precise.

ASLDA

- ASLDA Equations:

$$\begin{bmatrix} h_a(\boldsymbol{r})-\lambda_a & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -h_b(\boldsymbol{r})+\lambda_b \end{bmatrix}\begin{bmatrix} u_i(\boldsymbol{r}) \\ v_i(\boldsymbol{r}) \end{bmatrix} = E_i \begin{bmatrix} u_i(\boldsymbol{r}) \\ v_i(\boldsymbol{r}) \end{bmatrix}$$

$$\rho_a(\boldsymbol{r}) = \sum_i f_i |u_i(\boldsymbol{r})|^2, \ \rho_b(\boldsymbol{r}) = \sum_i (1-f_i)|v_i(\boldsymbol{r})|^2$$

$$f_i = 1/(1+\exp(E_i/kT))$$

$$\kappa(\boldsymbol{r}) = \sum_i f_i u_i(\boldsymbol{r}) v_i^*(\boldsymbol{r}), \ \Delta(\boldsymbol{r}) = g_{eff}(\boldsymbol{r})\kappa(\boldsymbol{r})$$

ASLDA energy density functional:

$$\mathcal{E} = \alpha_a(x)\frac{\tau_a}{2} + \alpha_b(x)\frac{\tau_b}{2} + \frac{(3\pi^2(\rho_a+\rho_b))^{5/3}}{10\pi^2}\beta(x) - \Delta\kappa$$

$$g_{eff}(\boldsymbol{r}) = \frac{\gamma(x)}{(\rho_a+\rho_b)^{1/3} + \Lambda(\boldsymbol{r})\gamma(x)}$$

Pairing regularization

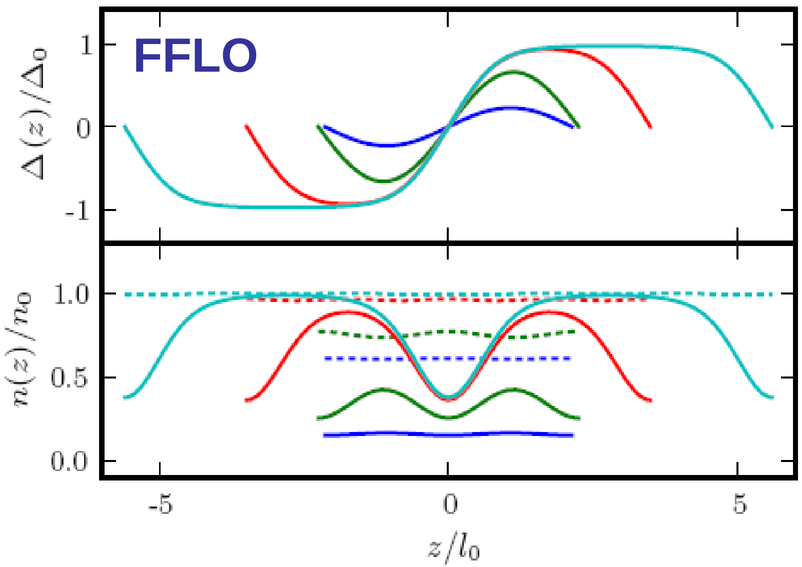
- Becomes SLDA when effective mass of spin-up and spin-down is the same.

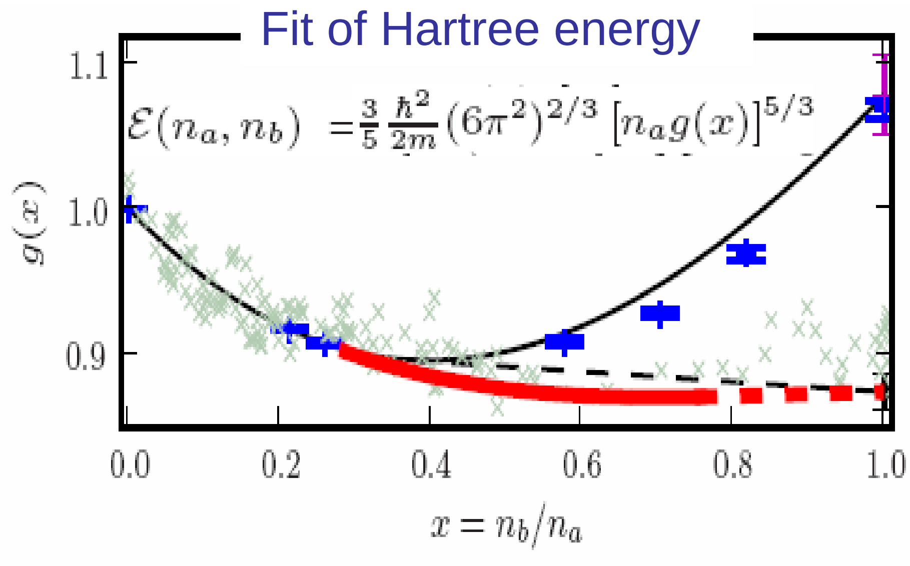
ASLDA

- Parameters are fitted according to experiments and QMC calculations (effective mass; energy density) as functions of polarizations close to SLDA at small polarizations

- ASLDA predicted FFLO phase: periodic boundary condition

  A.Bulgac, M.Forbes, PRL101:215301,2008

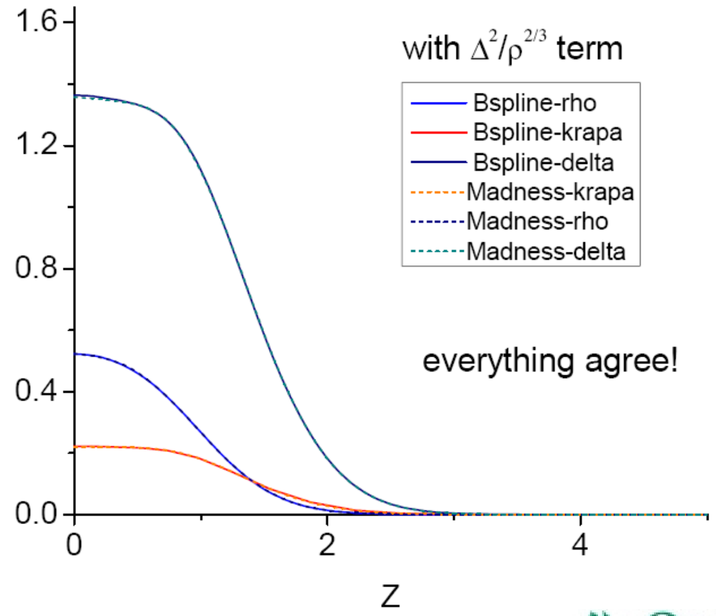




Coordinate-space HFB calculations

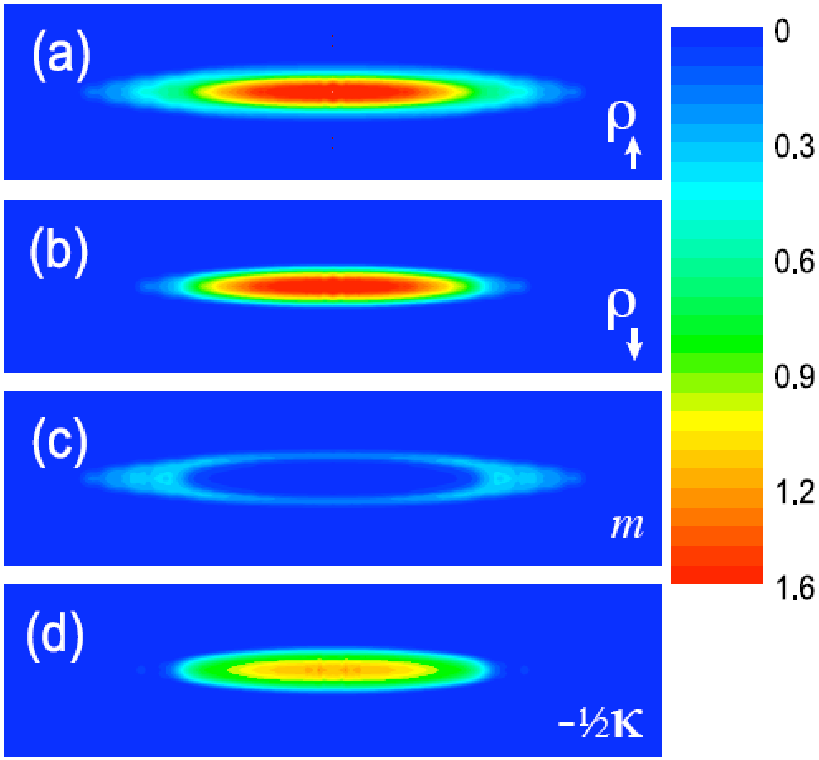
- It is the advantage of coordinate-space HFB to treat elongated potential.
  HFB-AX code: using B-spline techniques; Axially symmetry; Very precise for deformed and weakly bound nuclei.
  1000 atoms employed.
  dimensionless 2D-box: R=25×0.3, Z=(125～300)×0.3 (very dense spectrum)
  the elongation is set to be η=10, 20, 30, 40. ASLDA is very time consuming
  JP et al, PRC 78, 064306, 2007

- 3D HFB-Madness is under development (multi-resolution adaptive wavelet basis):
  SLDA-Madness has been benchmarked.
  ASLDA-Madness is being debug.



Phase separation

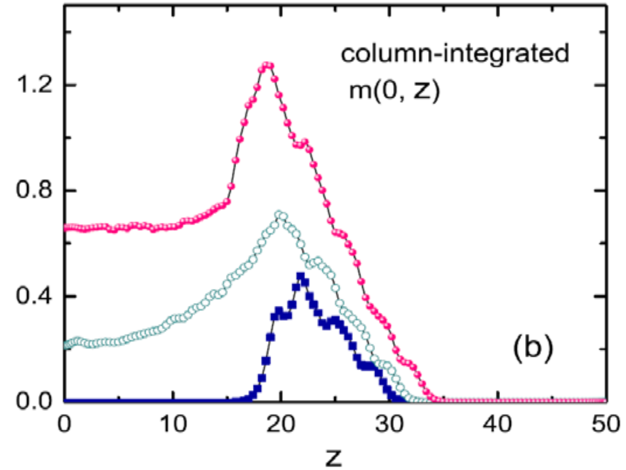
- SLDA calculations(η =10):



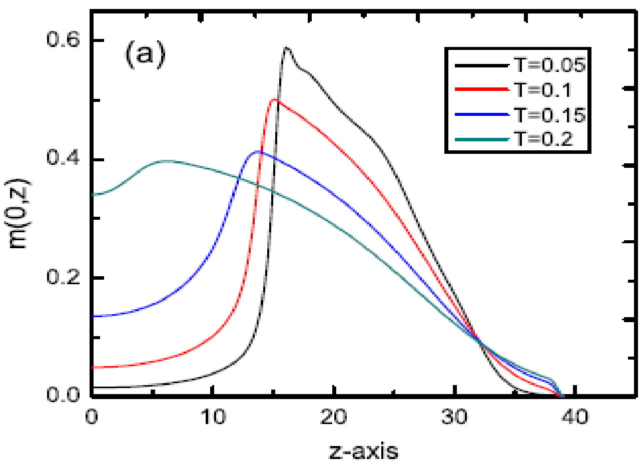
$$P = (N_a - N_b)/(N_a + N_b)$$

JP, W. Nazarewicz, M. Stoitsov, EPJA42,595 2009

Polarization effects

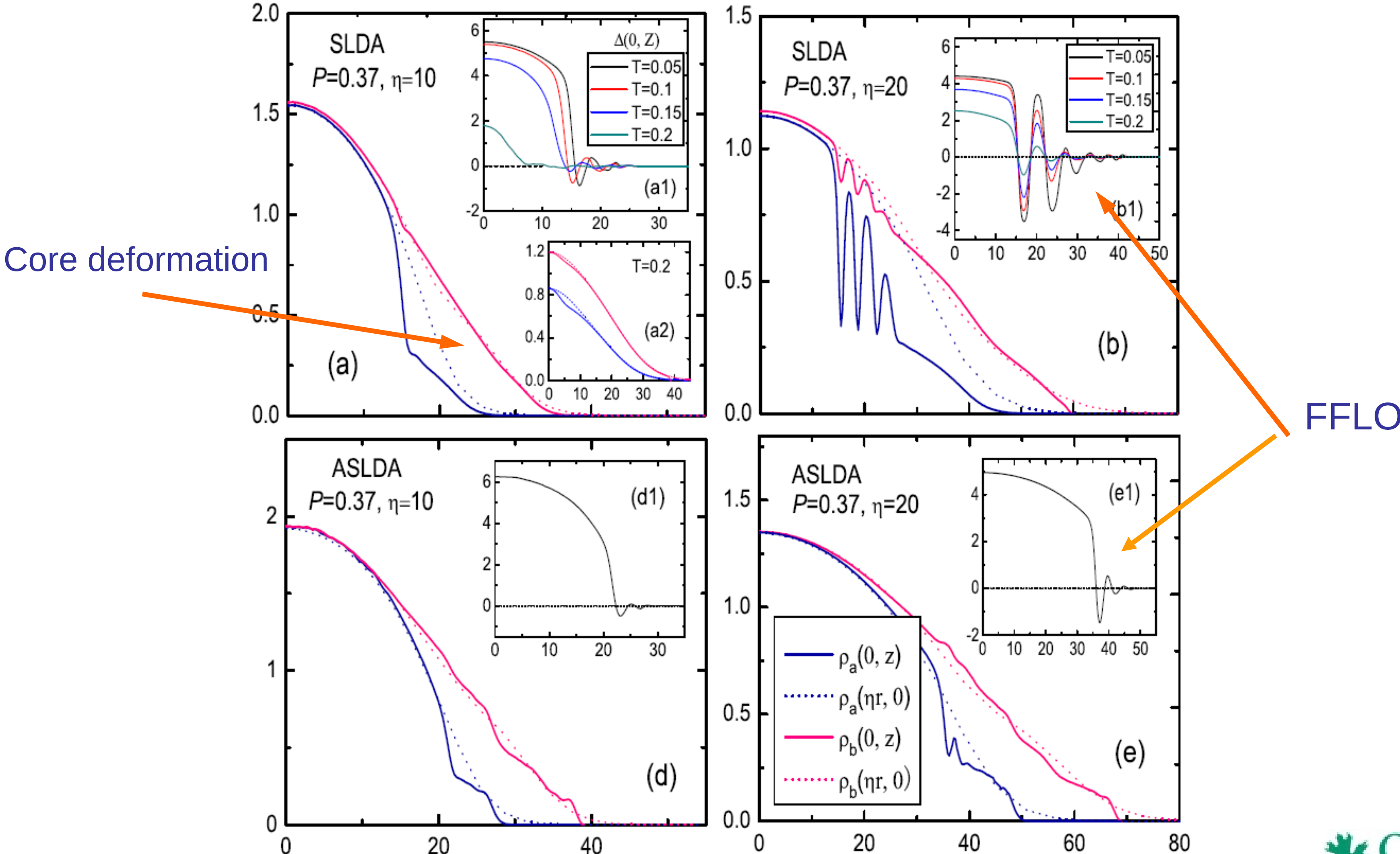


Temperature effects



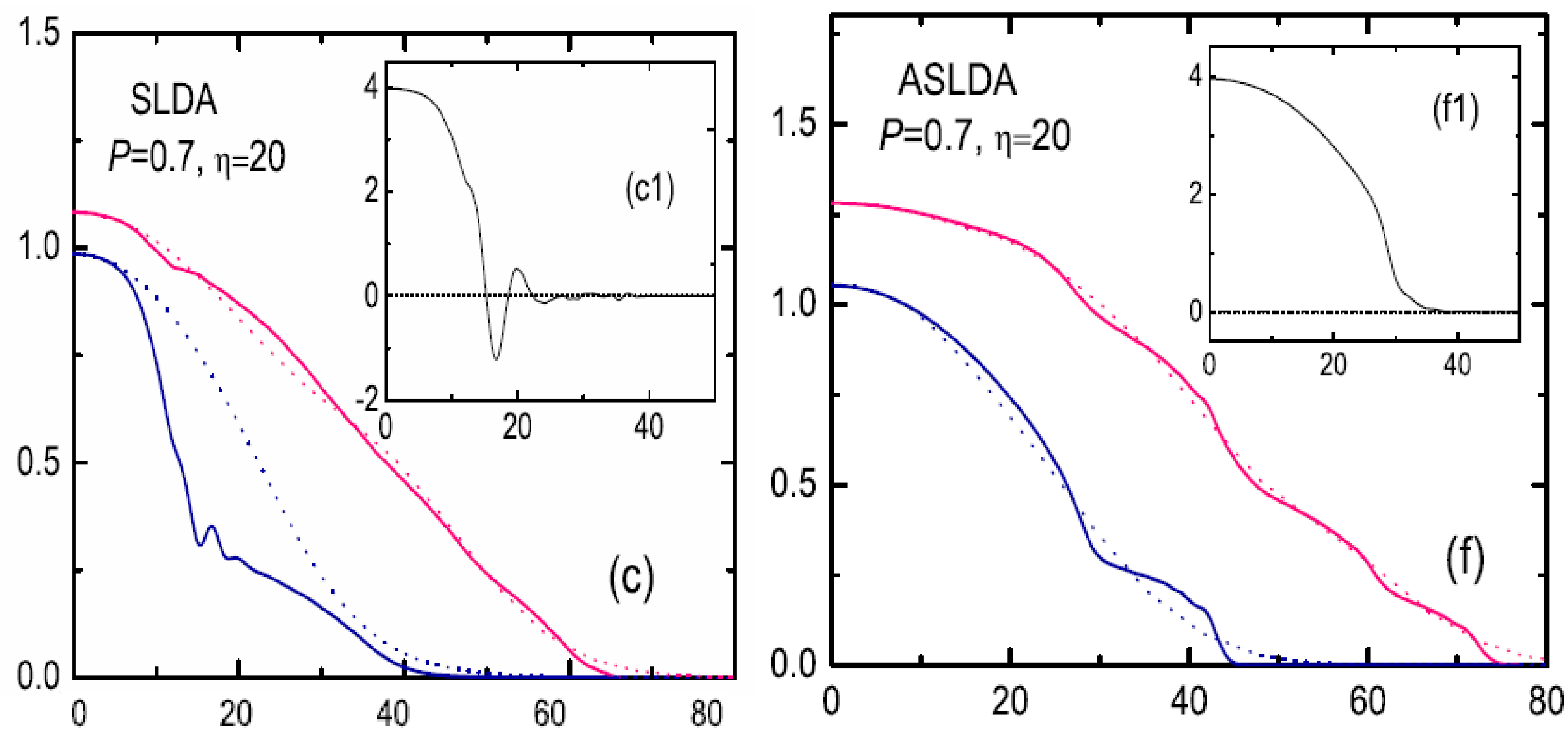
Traps with small elongation

- Core deformation is predicted in a small deformed trap by SLDA



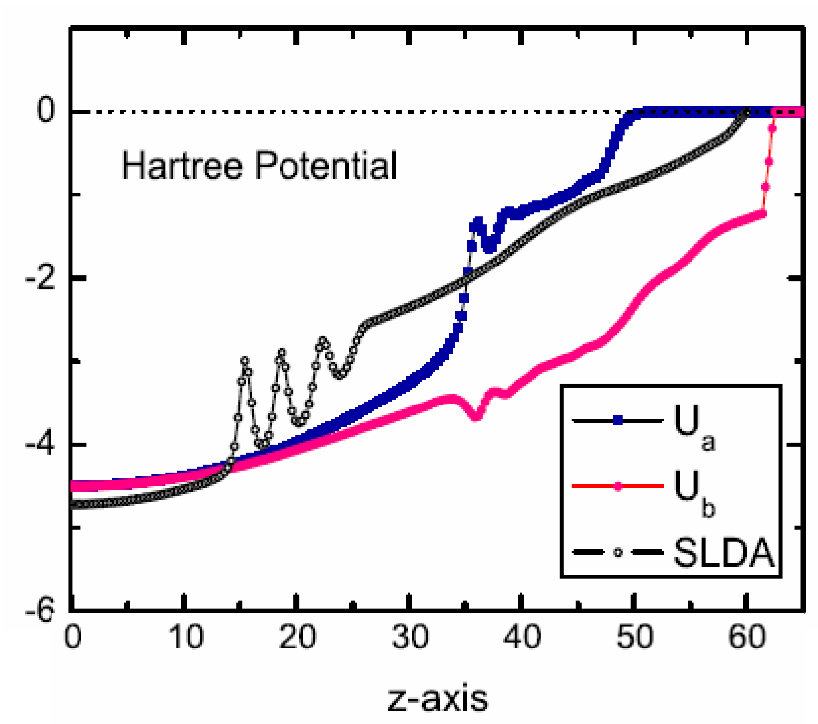
Large polarization

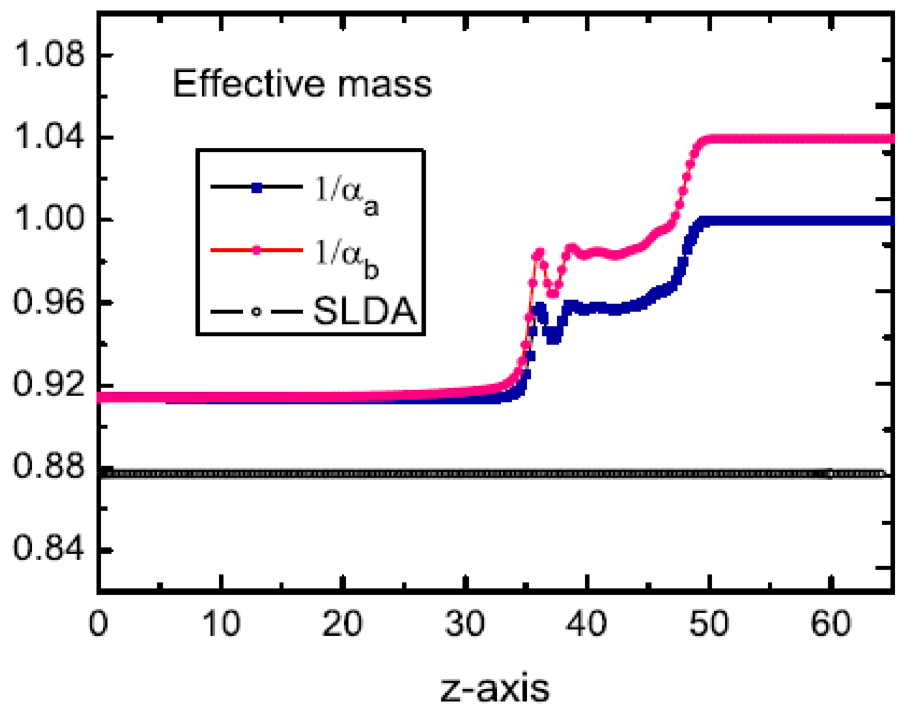
- FFLO and phase separation fades away
- ASLDA predicts less significant FFLO than SLDA



SLDA and ASLDA difference

- Very different Hartree potentials in ASLDA
- Different effective mass





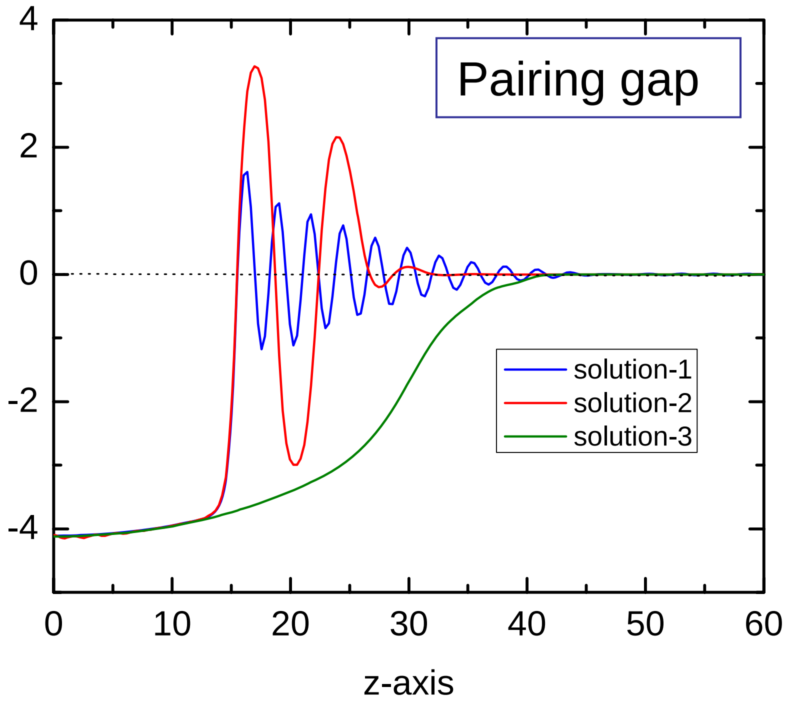
Multi-solutions in SLDA

- Different initial conditions lead to different solutions(η =20).

solution3- start from flat density distributions

solution2- start from less deformed solution η =10

solution1- start from 2 times frequency of solution-2 (artificial)

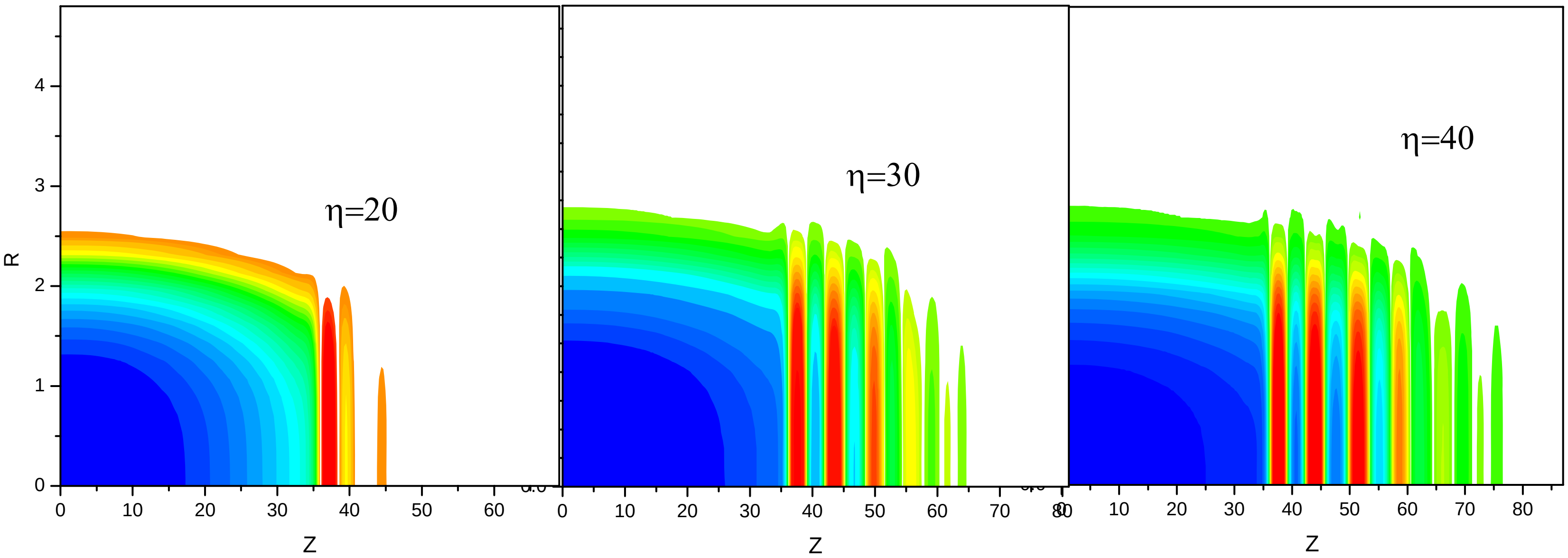


solutions	energy
1	3250
2	3305
3(G.S)	3150

Oscillation amplitudes decrease as frequency increase

Increase the trap aspect ratio

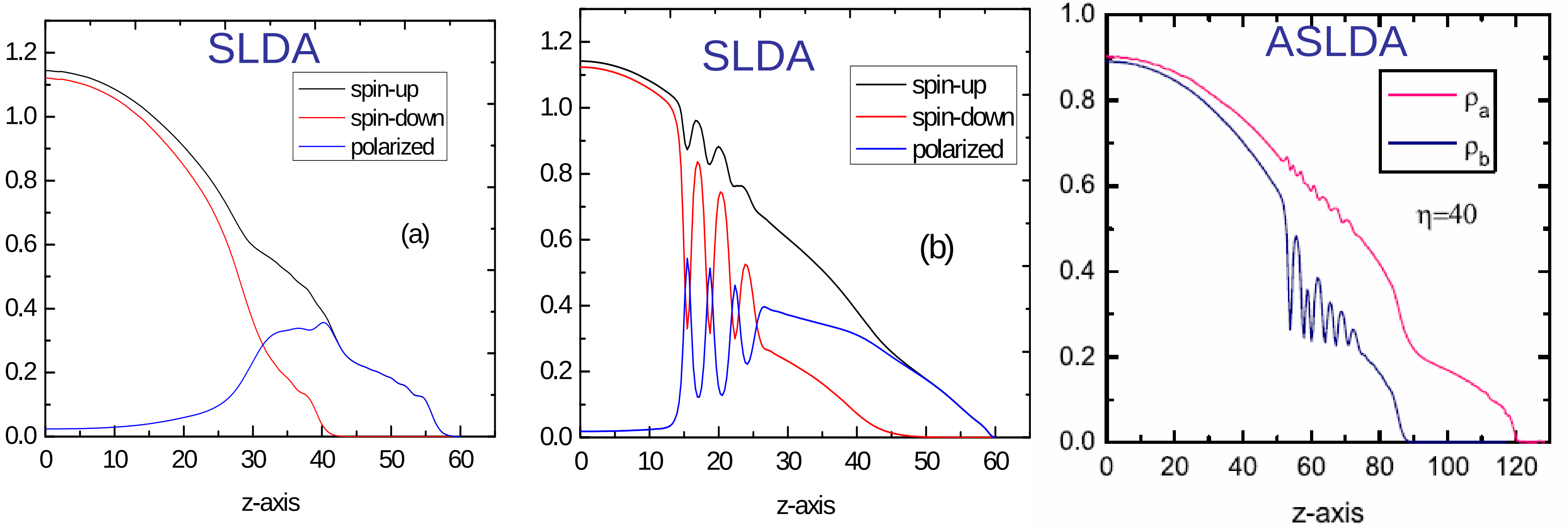
- Pairing oscillations become remarkable as trap aspect ratio increase
- The oscillations are perpendicular to the long axis
- Oscillation periods are the same



ASLDA calculations of polarization=0.37

FFLO detection

- Different densities of the LDA and FFLO solutions should be distinguished by *in situ* imaging.



- How to access the FFLO in experiments (excitation state): we suggest increase the trap elongation after the condensation obtained

Conclusions

- Coordinate-space HFB is good for Fermi condensates study.
- Phase separation is demonstrated in both SLDA and ASLDA
- Phase separation disappears as temperature becomes higher or polarization becomes larger.
- Superfluid Core deformation is not shown in ASLDA. Still ambiguous
- The FFLO is predicted in highly elongated trap, both by SLDA and ASLDA. However, It has a higher energy than LDA solution. Difficult for experiments to access. Calculation still continues.
- A future fully precise DFT study of 10^5 particles is very valuable