Natural Units in the Nuclear Energy Density Functional Theory

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Skyrme EDF

- Most used EDF in nuclear physics

$$E_t^{\text{even}} = C_t^{\rho}\rho_t^2 + C_t^{\tau}\rho_t\tau_t + C_t^{\Delta\rho}\rho_t\Delta\rho_t^2 + C_t^{\nabla J}\rho_t\nabla J_t + C_t^{J}J_t^2\ ,\ t=0,1$$

$$C_t^{\rho} = C_{t0}^{\rho} + C_{tD}^{\rho}\rho_0^{\sigma}$$

- Uses local density approximation:

$$V(\mathbf{r}_1,\mathbf{r}_2)\propto\delta(\mathbf{r}_1-\mathbf{r}_2) \Rightarrow \rho(\mathbf{r},\mathbf{r}') \rightarrow \rho(\mathbf{r},\mathbf{r}) = \rho(\mathbf{r})$$

- Density dependence included in ρ_t^2 term
- Derivatives of densities up to second order

Effective Lagrangian

- Each term in effective low-energy Lagrangian is written as

$$g\left[\frac{\psi\psi^{\dagger}}{\Lambda f_{\pi}^{2}}\right]^{l}\left[\frac{\nabla}{\Lambda}\right]^{n}\Lambda^{2}f_{\pi}^{2}$$

- f_{π}^{2} is pion-decay constant, g is a dimensionless constant close to unity and Λ characterizes energy scale beyond effective Lagrangian
- Standard Skyrme coupling constants are scaled by a factor of

$$S = f_{\pi}^{2(l-1)}\Lambda^{n+l-2}$$

- Isovector coupling constants in natural units are scaled by a factor of 4
- Continuation of Furnstahl & Hackworth, PRC56, 2875
- Natural units scaling applied to 48 Skyrme functionals

List of functionals

1—2	SkT3, SkT6	a d i j
3	SkM	a c e f g i
4	SkM*	g j
5—6	SGI, SGII	d e j
7	HFB9	a b f h i
8—9	SI, SII	a c d e f i
10	SkA	a c d e g i j
11	HFB16	a b c f h i
12	SkT	a b d e g h i
13—16	Sly4—7	a c d e f i
17—18	SkI1—2	a b c d f g i
19—20	SkI3—4	a b c d f g i k
21	SkI5	a b c d f g i
22—27	Msk1—6	a b f h i
28—29	SIII, SIV	a c i
30—31	SV, SVI	a c i j
32—33	Sly230a, b	a c d e f i
34—39	E, E_σ, Z, Z_σ, R_σ, G_σ	a c d g i
40	SkP	a b c e f h i
41—42	SkO, SkO'	a b c d f g i k
43	SV-min	a b c d g h k
44	$SkO_{T'}$	i j k
45	SkMP	a j
46—47	SkX, SkX_c	a b c d e f
48	RATP	a d e f i

a) masses of double-magic nuclei (includes ^{90}Zr, ^{116}Sn, ^{124}Sn and ^{140}Ce) used in the fit

b) masses of non-double-magic nuclei used in the fit

c) charge radii used in the fit

d) single particle energies used in the fit

e) symmetric infinite nuclear matter constrains considered in the fit

f) asymmetric infinite nuclear matter constrains considered in the fit

g) surface properties (neutron skin, fission barriers etc.) considered in the fit

h) pairing was present in the fit

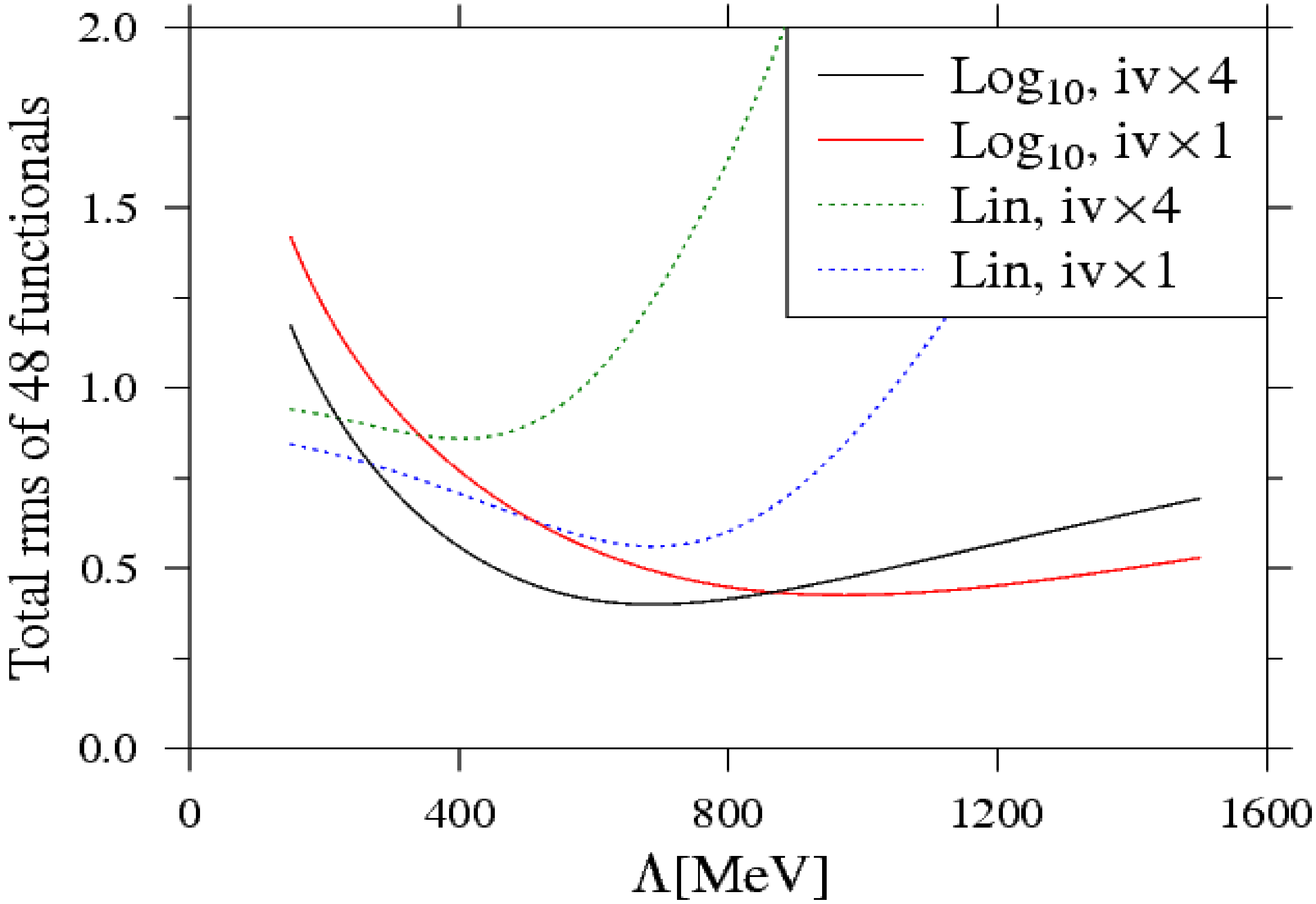
i) some parameters were fixed in the fit

j) parameters extrapolated or fine-tuned from existing force or functional

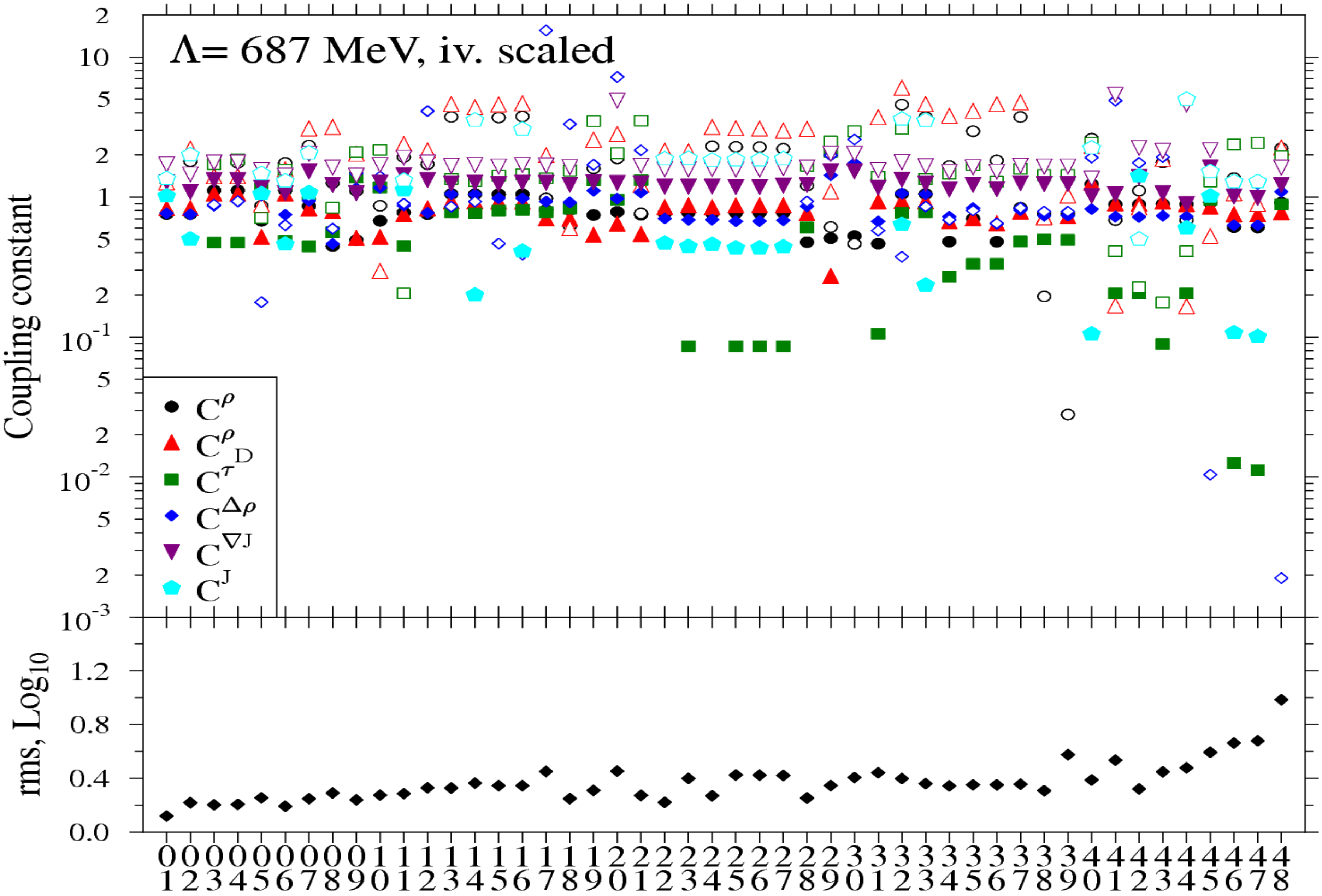
k) functional is an extended functional

Choice of optimal Λ

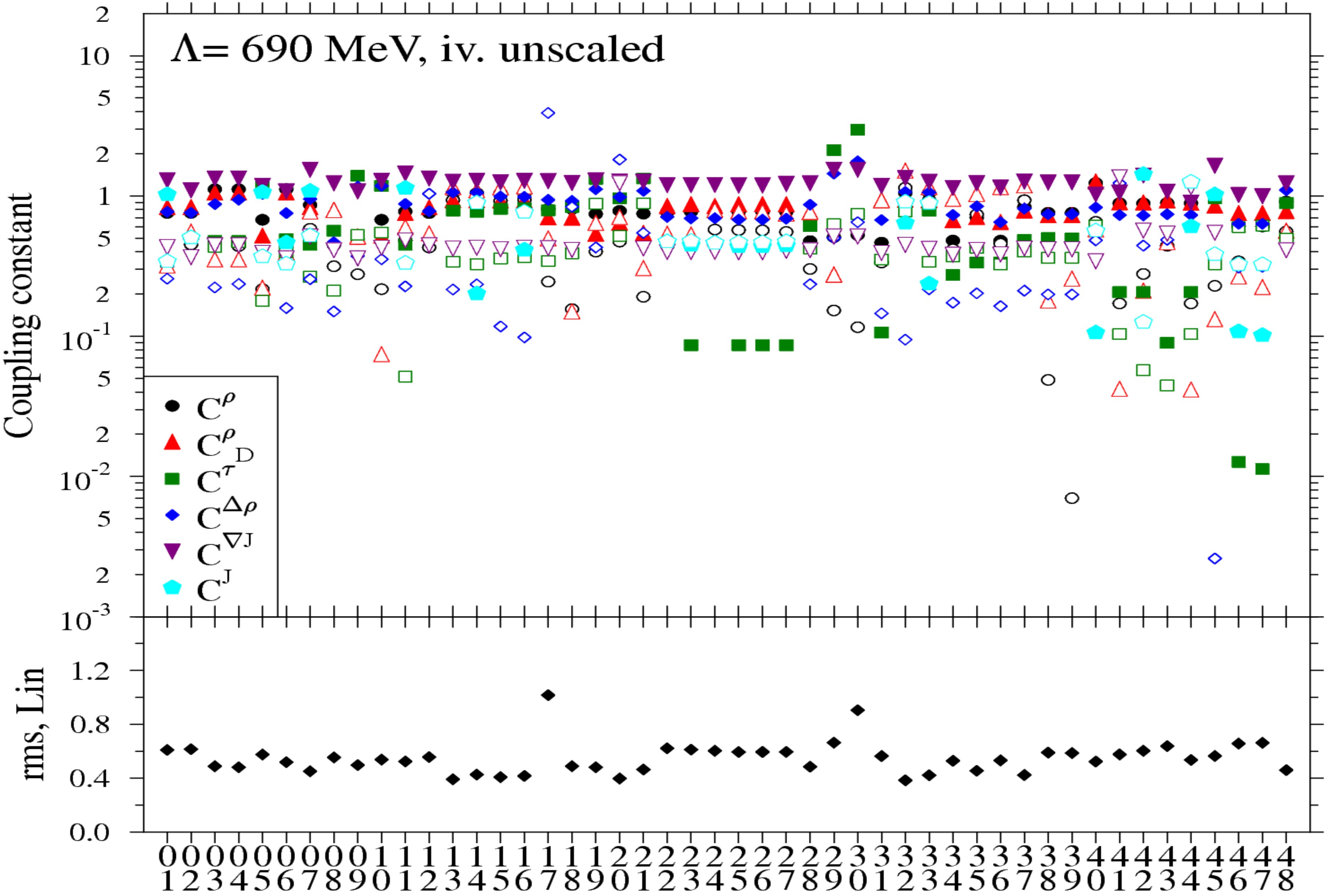
- One way to obtain optimal Λ is to minimize the deviation of coupling constants from unity ⇒ rms fit



Coupling constants in natural units



Coupling constants in natural units

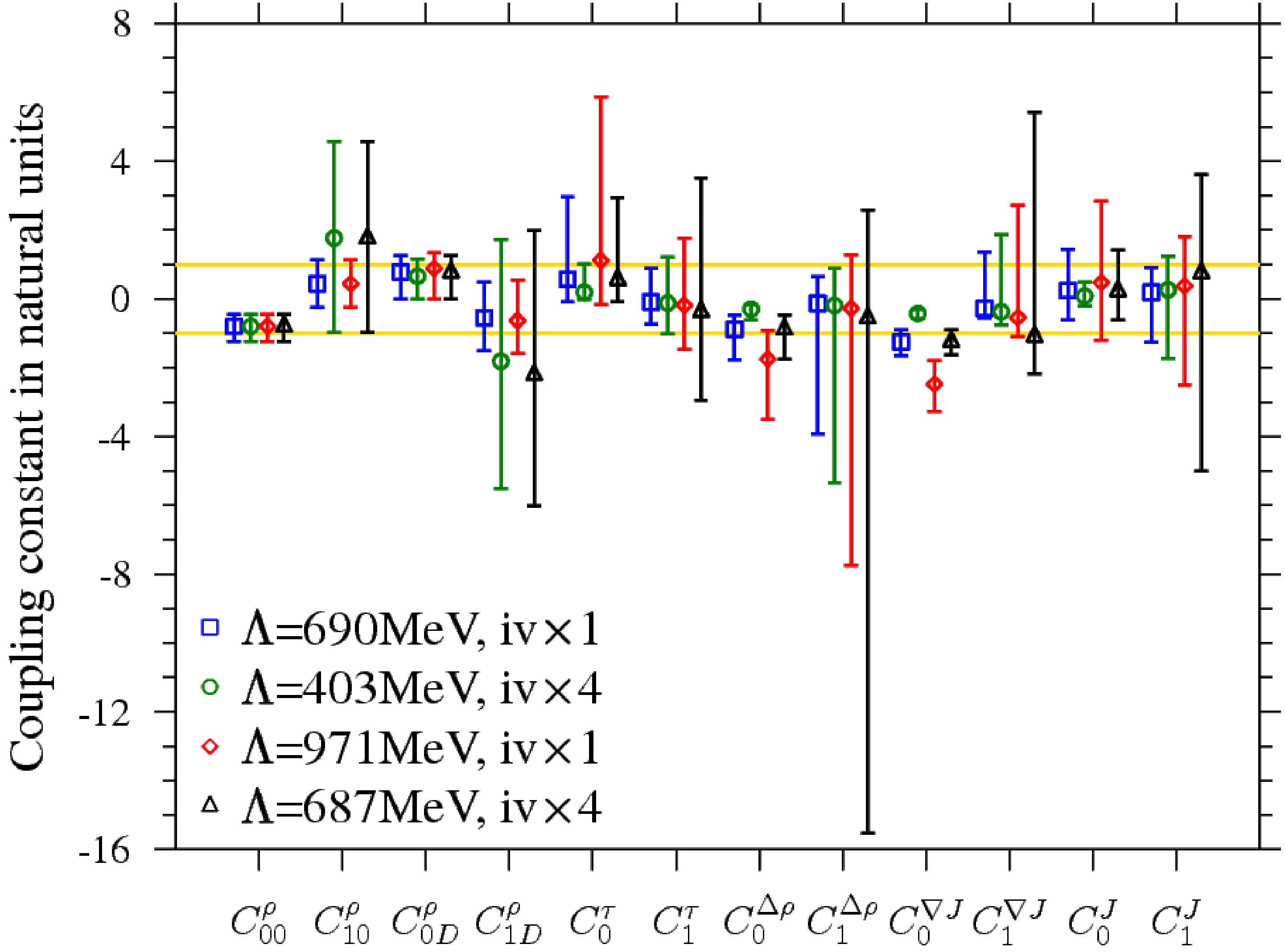


Deviation from unity

- Deviation from unity linked in many cases to uncomplete optimization procedure
- Anomalously small $C^{\Delta\rho}_1$ in RATP explained by a focus on volume part of the functional for astrophysical purposes
- Small $C^{\Delta\rho}_1$ in SkMP comes from mixing SkM* and SkP functionals with only minor adjustment to volume part
- Large $C^{\Delta\rho}_1$ in SkI1 compared to other SkI functionals due to the lack of isotopic shift data in optimization
- Large C^{τ}_0 in SV due to artificially imposed zero density dependence
- Small C^{τ}_0 in SkX results from strong emphasis on single particle data which favors effective mass close to one

Range of coupling constants in natural units

- Naturalness can be used as a guide in future fitting attempts
- Range and average of cc's in 48 functionals:



Conclusions

- Scaling to natural units brings coupling constants close to unity
- Parameter Λ around expected value for optimal rms
- Large deviations from unity can linked to uncomplete optimization procedure in many cases
- Natural units can be used as a guiding principle in future fitting attempts
- Natural units also included in massexplorer.org

People involved

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