

Three-Nucleon Forces and Nuclear Structure in Neutron-Rich Calcium Isotopes

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Outline

⊕ **Microscopic Theory for Valence Shell Effective NN Interactions**

- ◆ Many-body perturbation theory: methods and interactions
- ◆ Deficiencies: revealed in monopole interaction, Ca properties

⊕ **3N forces**

- ◆ Chiral Effective Field Theory
- ◆ Inclusion in valence-shell interactions

⊕ **Impact on Nuclear Structure**

- ◆ 3N monopole part of interaction
- ◆ Prediction of binding energies and evolution of shell structure

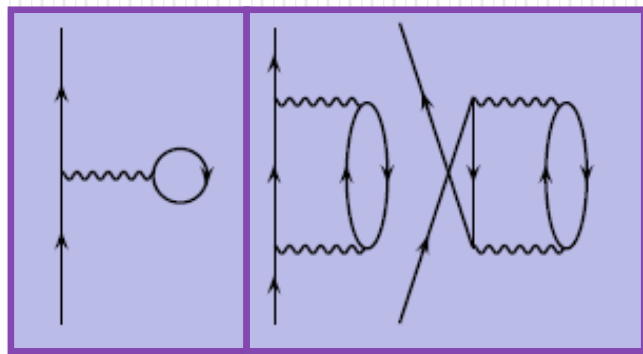
Many-body Problem for Finite Nuclei

- Various methods to solve many-body problem: Coupled Cluster, NCSM, In-medium SRG – we use many-body perturbation theory (MBPT)
- Solve the many-body Schrödinger equation for nuclear systems: $H\psi = E\psi$
- Impossible to solve in heavy systems in complete Hilbert space
- Consider problem in truncated (model) space defined by projection operator P , where

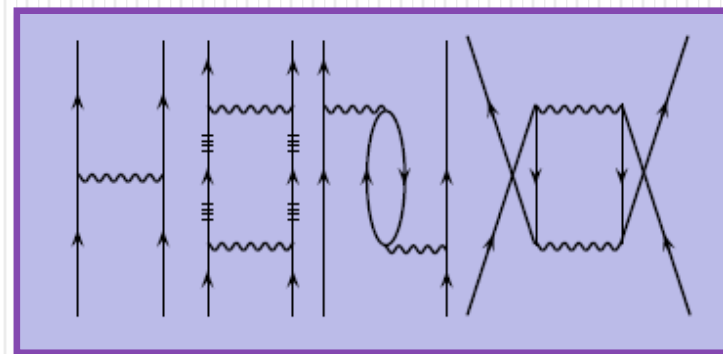
$$PH_{eff}P\psi = EP\psi; \quad H_{eff} = H_0 + V_{eff}$$

and V_{eff} acts in the model space given by P

Folded-diagrams: method to construct effective interaction perturbatively



1-body to 2nd order



2-body to 2nd order

Monopole Part of Interaction

- **Microscopic MBPT** typically works for few particles/holes away from closed shell: deteriorates beyond this
- Deficiencies in microscopic interactions can be improved by adjusting a particular set of two-body matrix elements (TBME):

Angular average of interaction
Determines interaction of orbit a
with b : evolution of orbitals

$$V_{ab}^T = \frac{\sum_J (2J+1) V_{abab}^{JT} [1 - (-1)^{J+T} \delta_{ab}]}{\sum_J (2J+1) [1 - (-1)^{J+T} \delta_{ab}]}$$

Phenomenological shell model interactions typically start from MBPT results then exploit importance of monopoles:

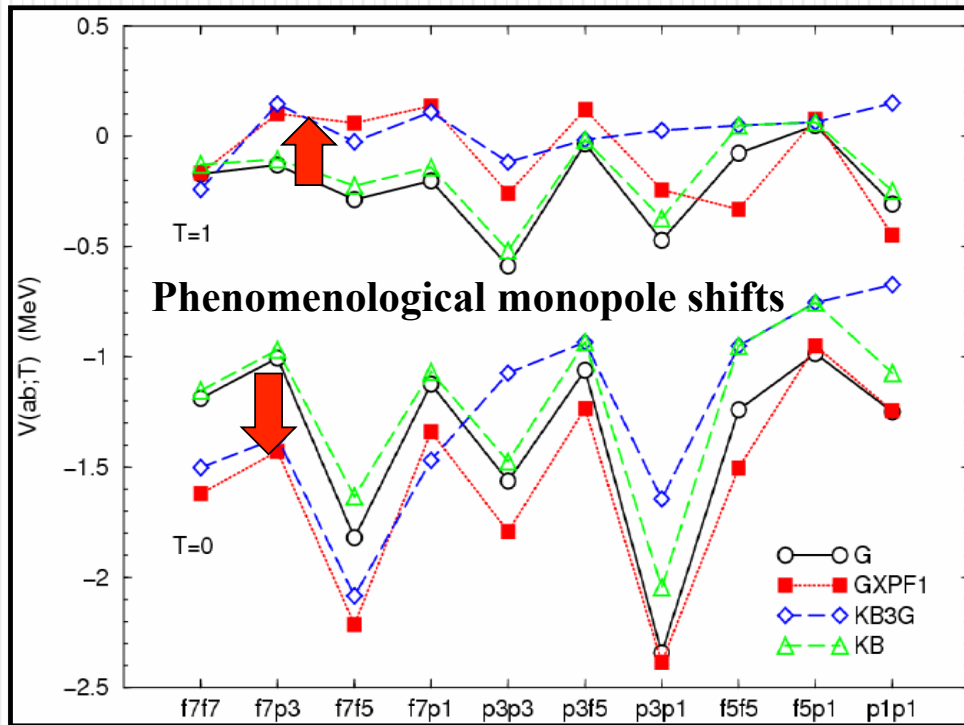
***sd*-shell:** 63 TBME - USD (1984), USDa, USDb (2006)

- global fit of single particle energies (SPEs) and two-body matrix elements; monopoles most important

***pf*-shell:** 195 TBME

- GXPF1 (2004): quasi-global fit; monopoles most important
- KB3G(2001): modification of monopole part only

Phenomenological vs. Microscopic



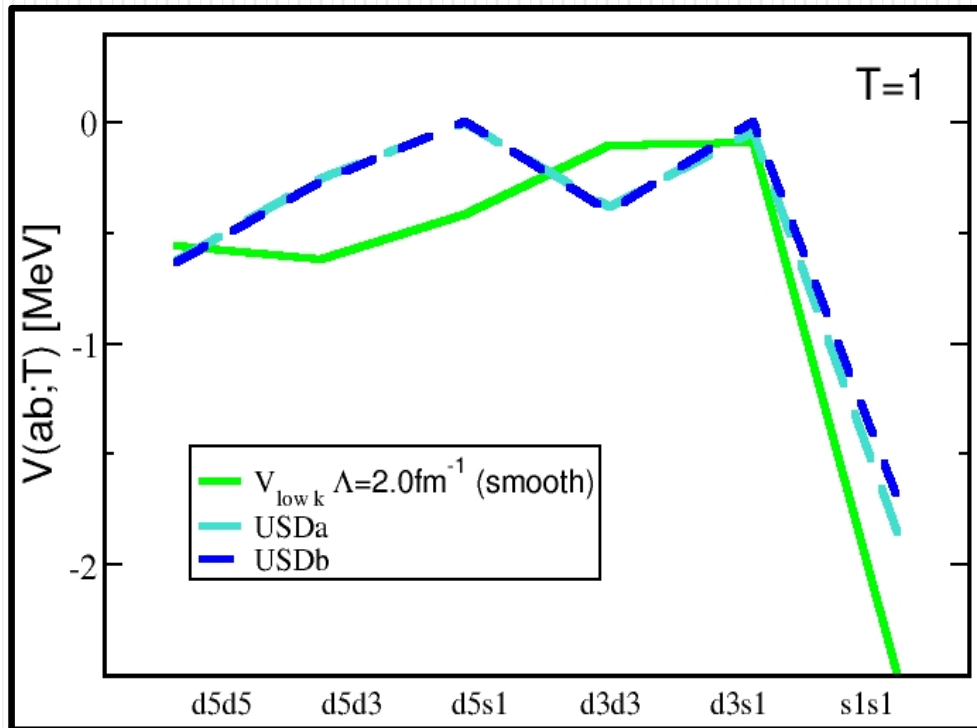
Compare monopoles from:

- *Microscopic G-matrix*, **Kuo-Brown** interactions
- *Phenomenological GXPF1*, **KB3G** interactions.

Clear shifts for low-lying orbitals:

- T=1 repulsive shift
- T=0 attractive shift

Phenomenological vs. Microscopic



Cause of the shifts?

Compare monopoles from:

- *Microscopic G-matrix*, **Kuo-Brown** interactions
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Clear shifts for low-lying orbitals:

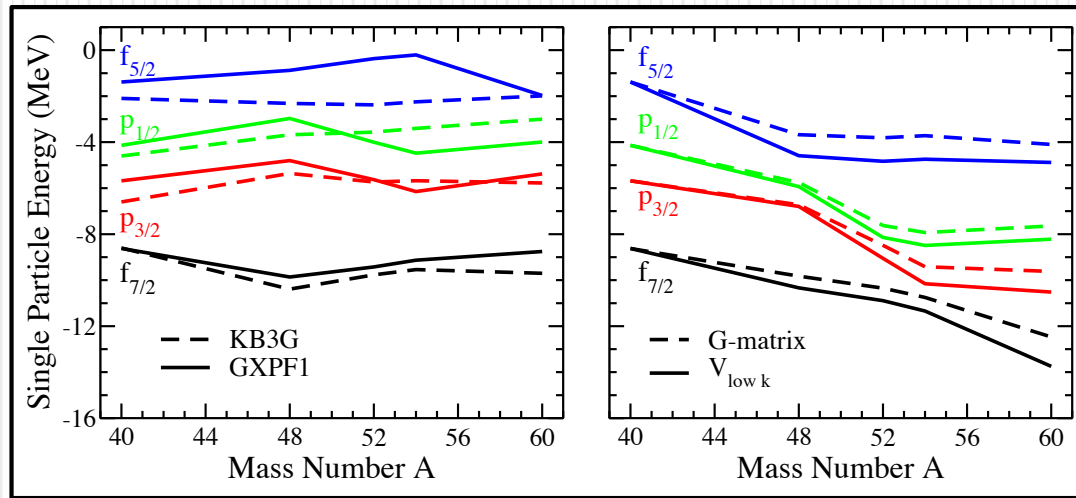
- $T=1$ repulsive shift
- $T=0$ attractive shift

Similar for sd-shell: USDa, USDb interactions: $T=1$ repulsive

** Can 3N forces explain these shifts? -- **Zuker (2003)**

Physical Implications in Calcium

Monopoles drive *evolution* of single particle energies



Phenomenological

Large gap at ^{48}Ca
Trend across
Shell gap at ^{54}Ca ?

Microscopic

Smaller gap at ^{48}Ca

→ Compressed spectra

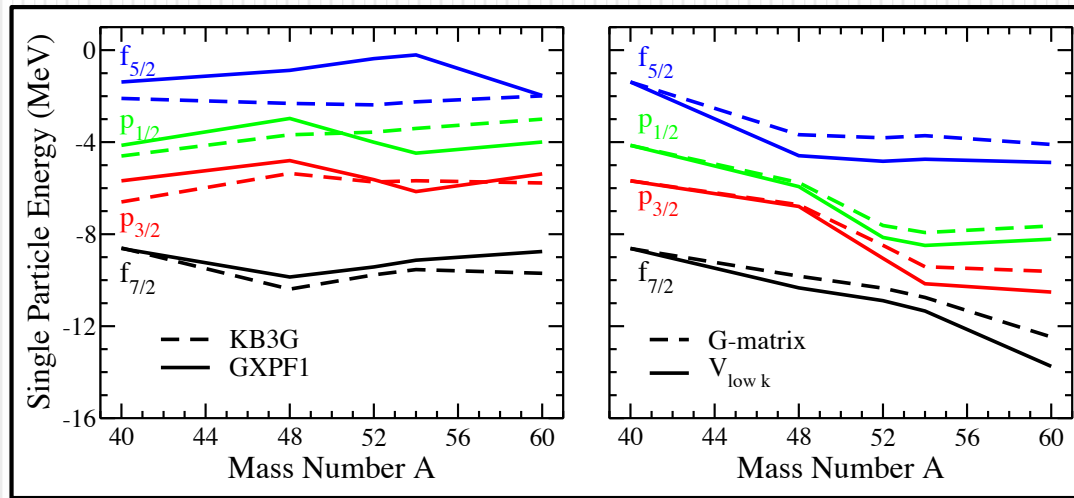
Trend down

$f_{5/2}$ orbit is strongly bound

→ Implications for gs energy

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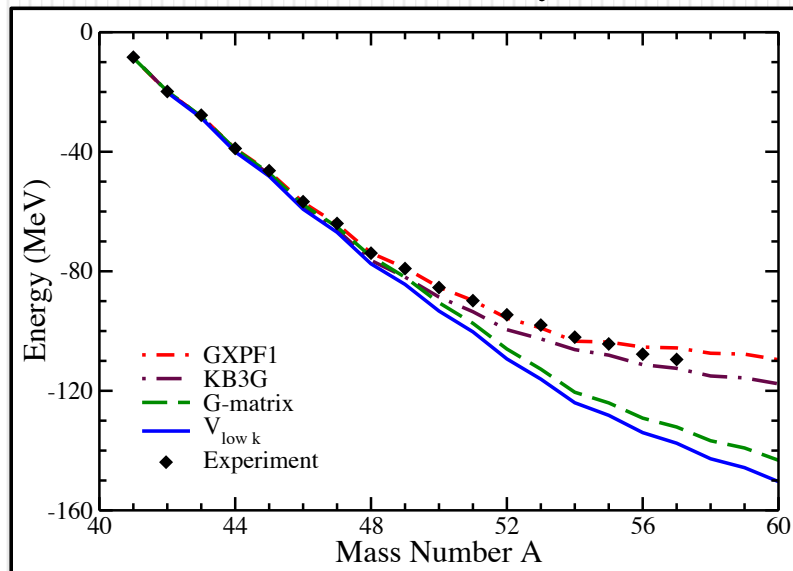
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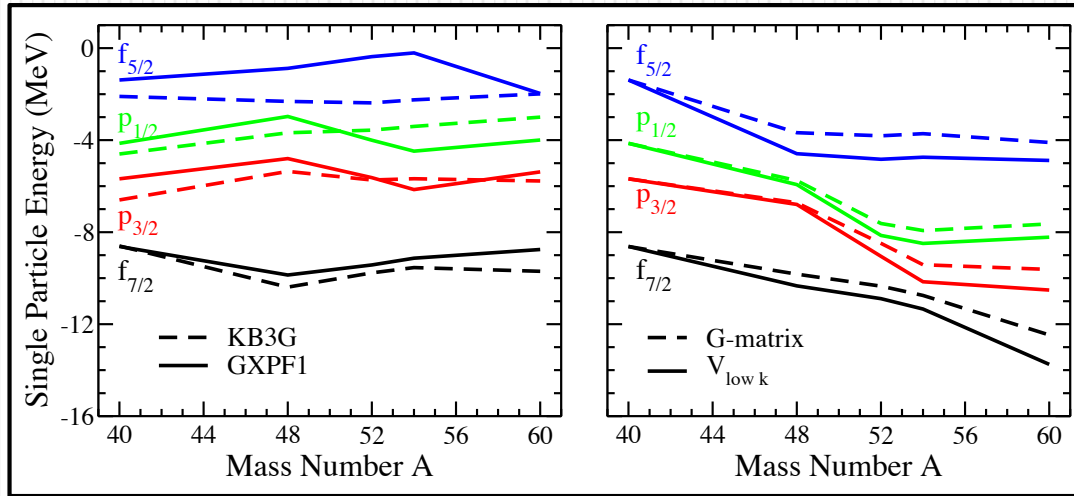
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MBPT- Overbind beyond ^{48}Ca



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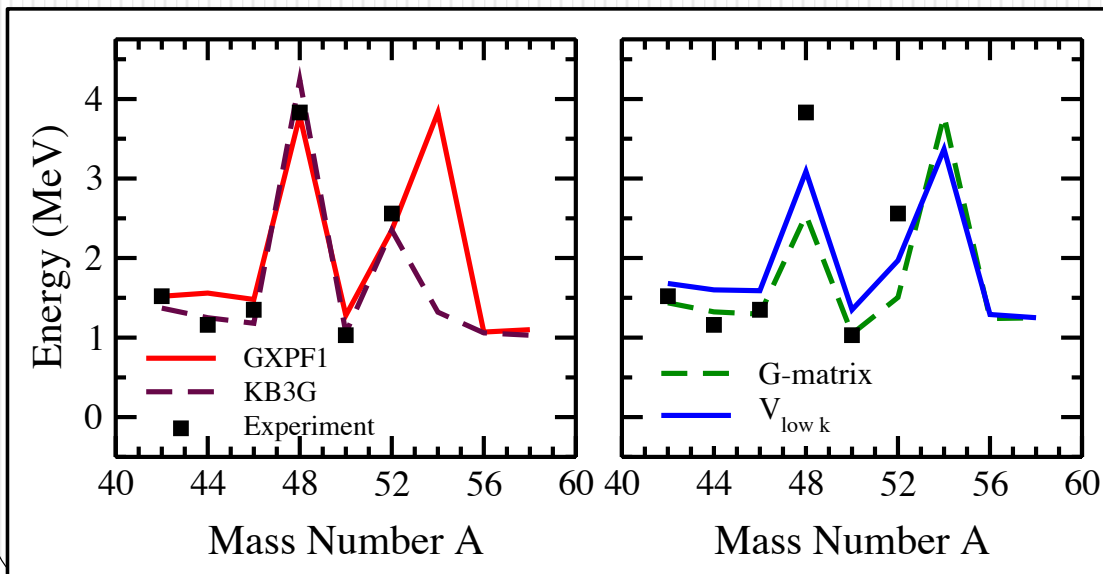
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Initial SPE important factor

If 3N forces explain monopole shifts, should also impact BE and shell structure predictions

How can we include them?

MBPT: No shell closure at ^{48}Ca – failure of NN



Chiral Effective Field Theory

	2N forces	3N forces	4N forces
LO			
NLO			
N ² LO			
N ³ LO			
	+ ...	+ ...	+ ...

Nucleons interact via π exchange and contact interactions

Explains hierarchy: $V_{NN} > V_{3N} > \dots$

Short-range couplings fit to experiment

Systematic way to include 3NF

Only two new couplings at N²LO:

c terms: already constrained by NN, π N data

$$c_1 = -0.9^{+0.2}_{-0.5}, \quad c_3 = -4.7^{+1.2}_{-1.0}$$

$$c_4 = 3.5^{+0.5}_{-0.2}$$

No new couplings at N³LO N⁴LO

Chiral 2N: large cutoffs not suitable for MBPT – need to renormalize...

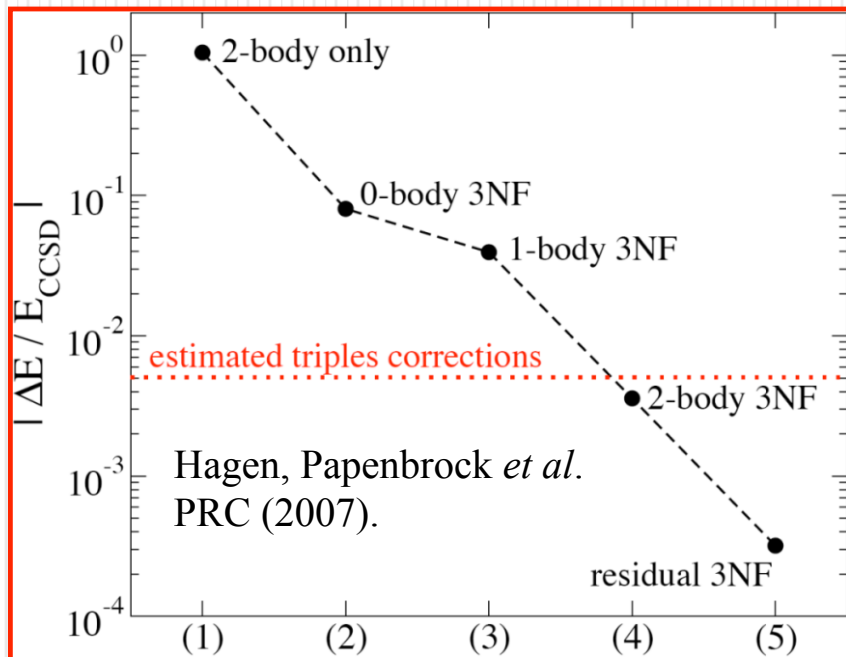
Evolve to lower cutoff using **RG methods** (smooth regulator):

3N Forces in Valence-Shell Interactions

$$V_{\text{low } k}(\Lambda) + \text{N}^2\text{LO chiral } V_{3N}(\Lambda)$$

$D(\Lambda)$, $E(\Lambda)$ couplings fit to ${}^3\text{H}$ BE, ${}^4\text{He}$ radius for given Λ

Approach: inspired by **Coupled Cluster** results for ${}^4\text{He}$



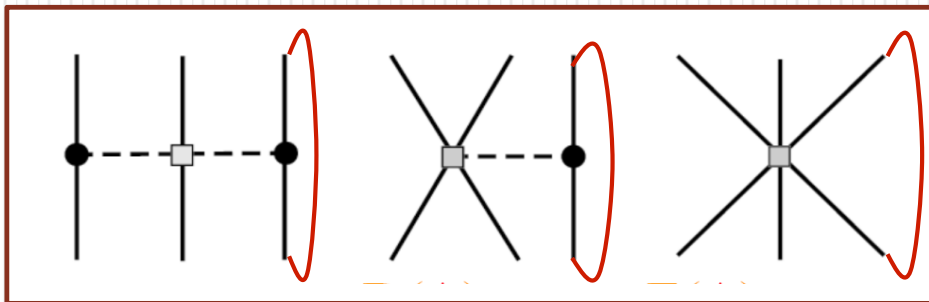
Benchmarked using low-momentum interactions and 3NF

0- 1- and 2-body parts of 3NF

dominate – **neglect residual 3NF**

Sum over occupied states (as in coupled cluster, nuclear matter):

$$\langle ab | V_{3N,\text{eff}} | a'b' \rangle = \sum_{\alpha=\text{core}} \langle ab\alpha | V_{3N} | a'b'\alpha \rangle$$

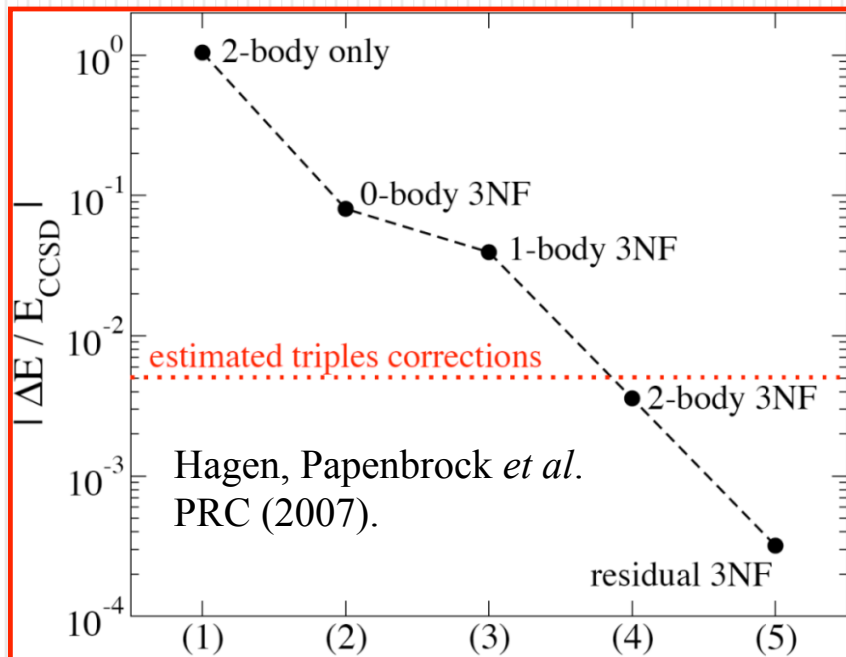


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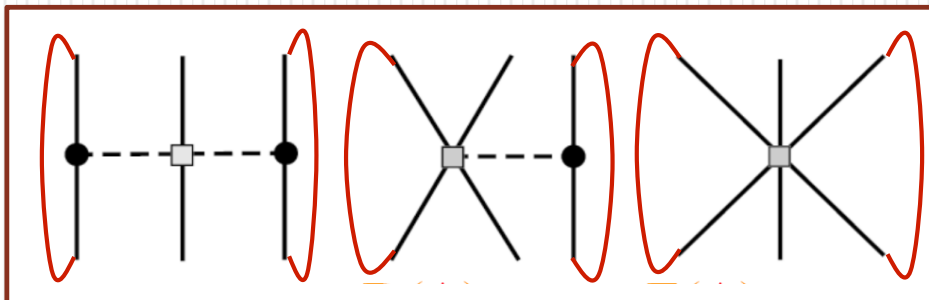


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$$\langle a | V_{3N,\text{eff}} | a' \rangle = \frac{1}{2} \sum_{\alpha\beta=\text{core}} \langle a\alpha\beta | V_{3N} | a'\alpha\beta \rangle$$

● 1- and 2-body effects – 3N forces tractable in shell model

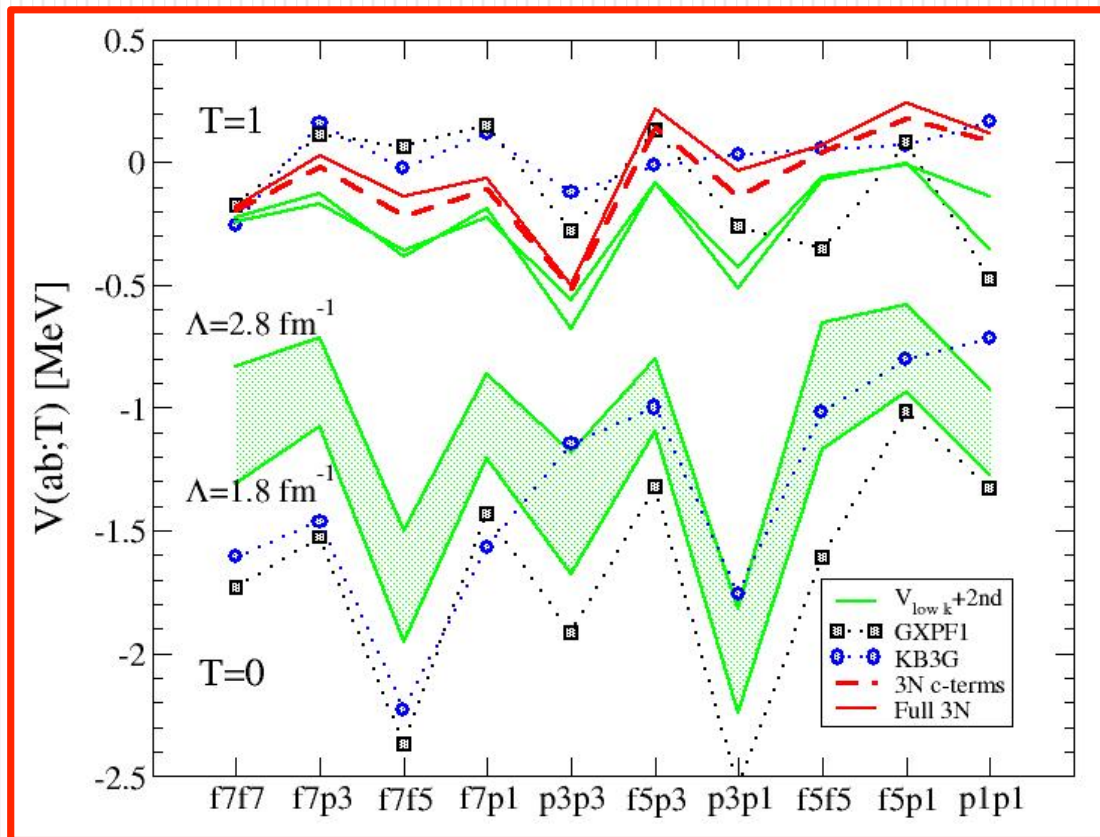
Calculation Details

Focus on $T=1$ monopoles and systems in the following details:

- 1 NN matrix elements derived from:
 - Chiral N^3LO (500MeV) using smooth-regulator $V_{low k}$ with range of cutoffs: $T=1$ results cutoff independent
 - 3rd-order in MBPT
 - $16\hbar\omega$ intermediate state configurations (converged)
- 2 3N forces: calculate complete multipole components from:
 - A) Full Chiral N^2LO
 - B) One-Delta excitation from N^2LO : specific choice of **c-terms**

Two-body 3N: Monopoles in pf -shell

- Similar picture as in sd -shell



- Calculated to first order in many-body perturbation theory

- converged with 3NF partial waves up to: $J \leq \frac{7}{2}$

--- 3N (one Δ)

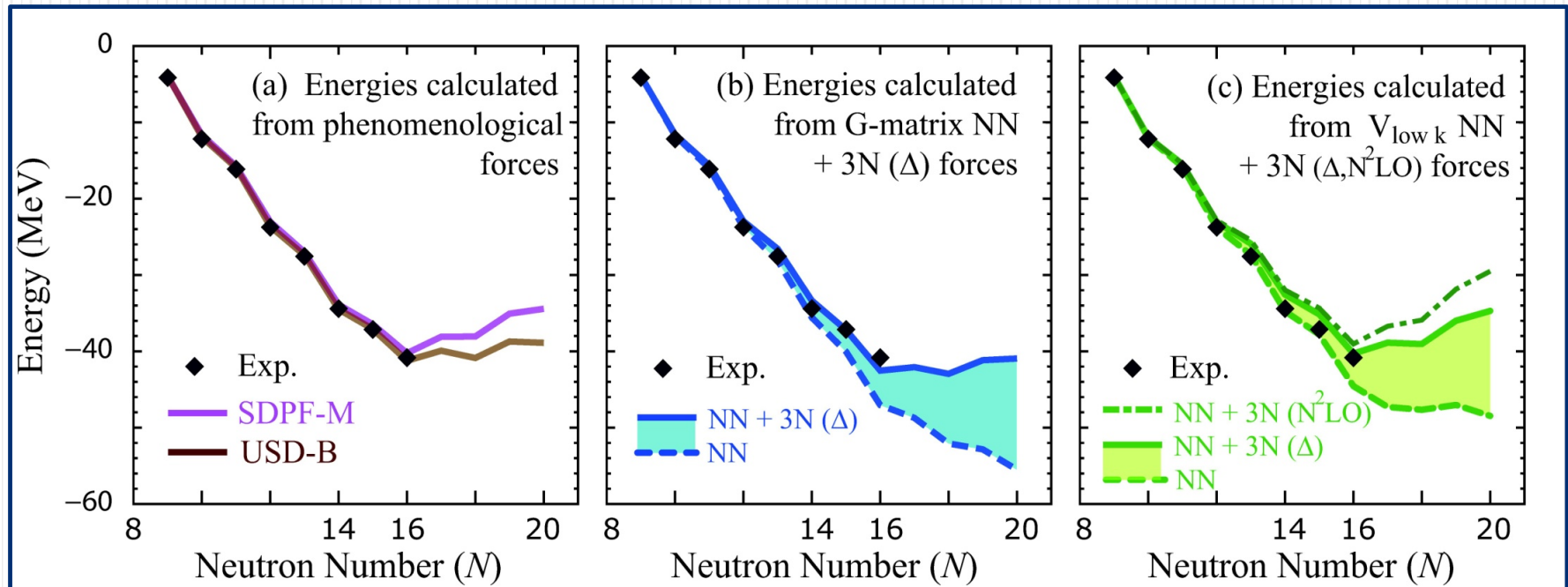
— 3N (N^2LO)

- Clear repulsive shift due to 3NF in $T=1$ channel

T=0: Expect attraction from 2nd order perturbation theory: **in progress**

Calculated Oxygen Binding Energies

First shell-model calculations using NN+3N monopoles: predict dripline in O
Calculate GS energies (relative to ^{16}O) with SDPF-M single particle energies



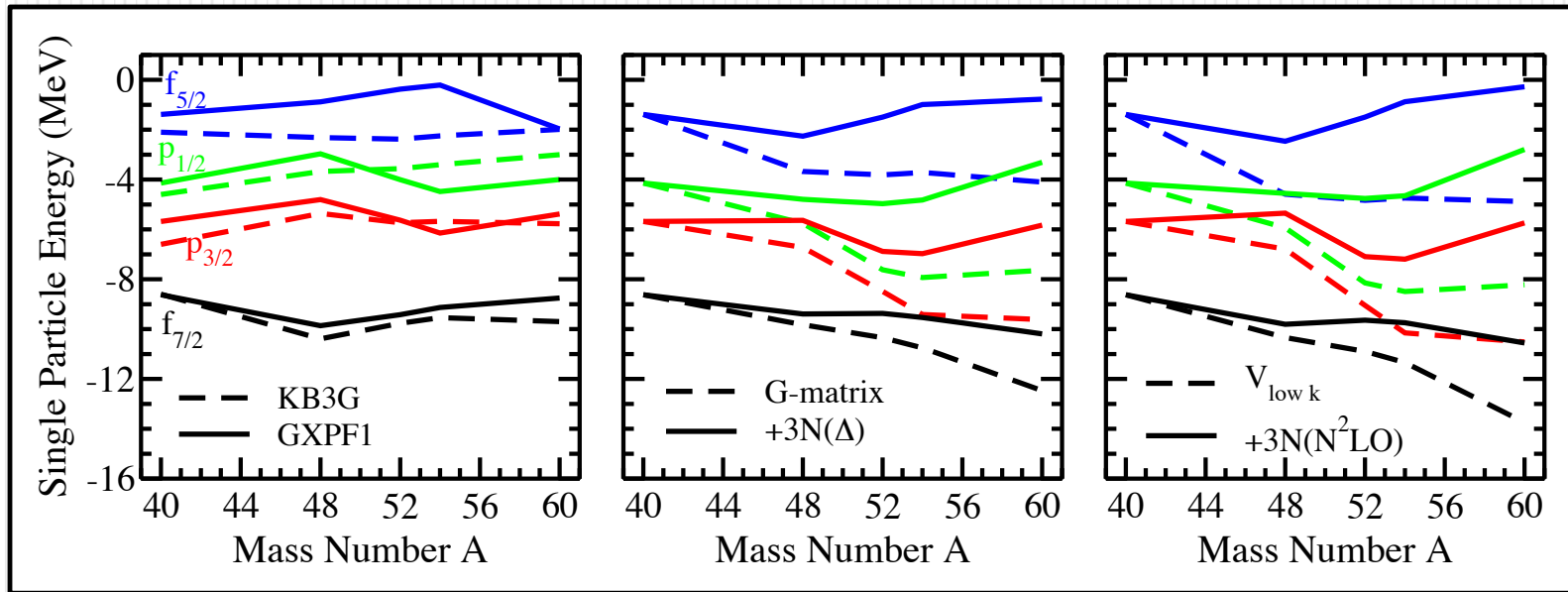
Phenomenological interactions show $^{25-28}\text{O}$ less bound than ^{24}O

BEs increase for NN-only through ^{28}O

Adding 3NF: isotopes beyond ^{24}O less bound: dripline correctly predicted at ^{24}O !

Calcium Effective Single Particle Energies

Calculate SPE evolution with 3N forces

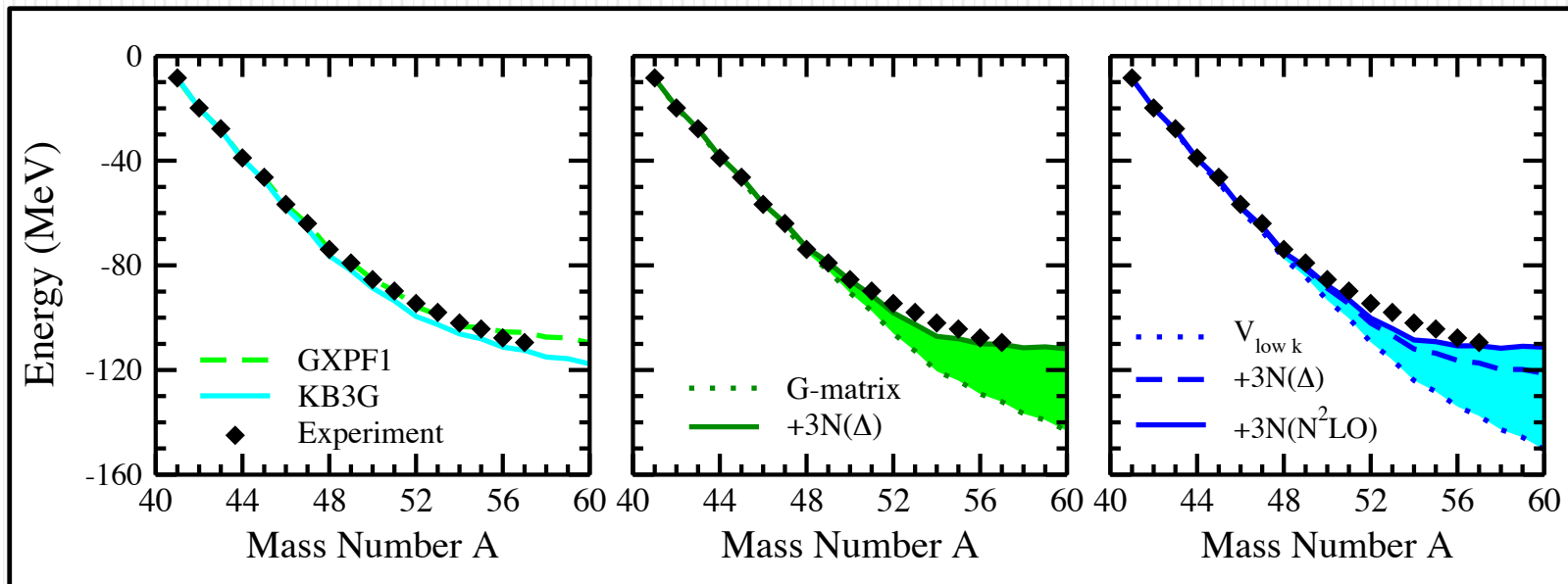


With 3N forces:

- Trend across: improve binding energies
- Increased gap at ^{48}Ca : enhancement of closed-shell features
- N=34 shell gap: robust prediction of shell gap at N=34 – supports GXPF1

Ground State Energies in Ca Isotopes

Introduce many-body correlations by diagonalization in model space
Perform shell model calculations for Ca isotopes using NN + 3N full multipoles
Calculate Binding Energies for isotope chain



NN-only comes to overbind Ca isotopes beyond $\sim^{46}\text{Ca}$

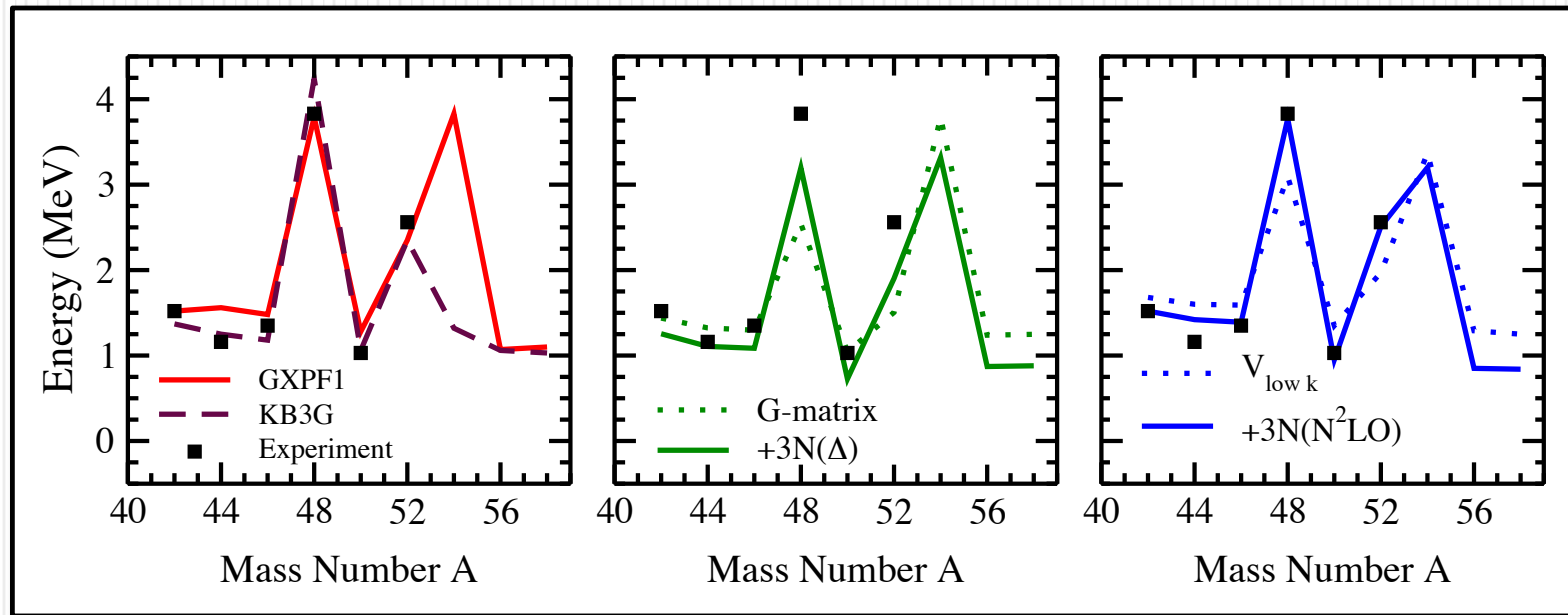
3N forces correct overbinding – good experimental agreement

Correction from 3N Δ consistent between methods

No statements about dripline – maybe including $g_{9/2}$ orbit?

Shell Gap in ^{48}Ca

Shell gap in ^{48}Ca well-reproduced with phenomenological interactions
Calculate first 2^+ excited states in calcium isotopes



3N forces: uniform improvement of ^{48}Ca 2^+ energy ~ 1 MeV

Strongly dependent on SPE chosen

- Remove ambiguity: calculate microscopically with 3N contributions

Outlook

- Exploring frontiers of nuclear structure of medium mass nuclei with 3N forces
- 3NF contribution to monopoles
 - **Repulsive shift** seen in $T=1$ monopoles due to 3N forces
- First shell model results in pf-shell using chiral 3N multipoles:
 - Leads to correct predicted binding energies and evolution of shell structure
- **In progress:**
 - One-body effects of 3N forces: single particle energies in sd, pf-shell
 - Parameter-free, fully-microscopic valence shell interactions from NN + 3N
 - Ca dripline with $g_{9/2}$ orbit?
- **Near Future:**
 - $T=0$ – need NN-3N to **2nd order**
 - Sn Isotopes
 - Continuum effects

Thanks to Collaborators: T. Otsuka , A. Schwenk, T. Suzuki