Pairing of a trapped unitary Fermi gas: HFB calculations

- Deformed coordinate-space HFB-AX
- Pairing of a polarized Fermi gas in an elongate trap
- Thermal effects on the superfulid and normal phase

J. Pei, W. Nazarewicz, M. Stoitsov , G.I. Fann
N. Shunck, J. Sheikh, A.K. Kerman

UTK/ORNL

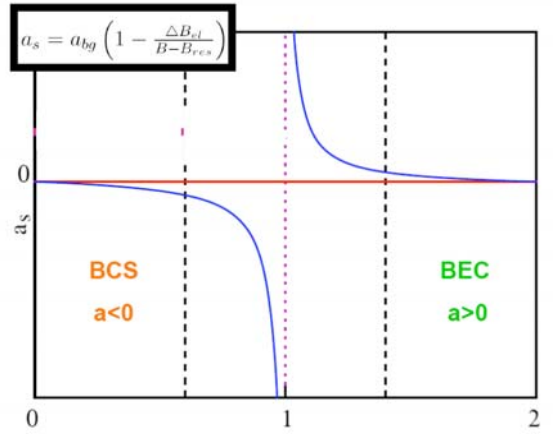
3rd LACM-EFES-JUSTIPEN Workshop, Feb.25, 2009

OAK RIDGE National Laboratory

Recent attractions in cold atoms

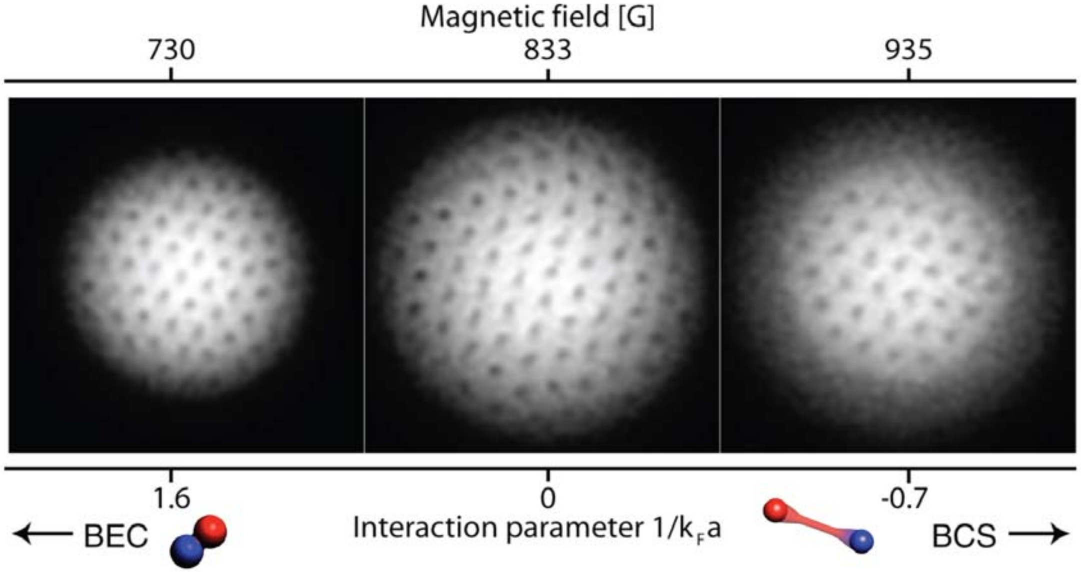
BCS-BEC crossover

Interactions are controlled by Feshbach Resonances



Unitary limit:

System is strongly correlated and its properties do not dependent on the value of scattering length a_s



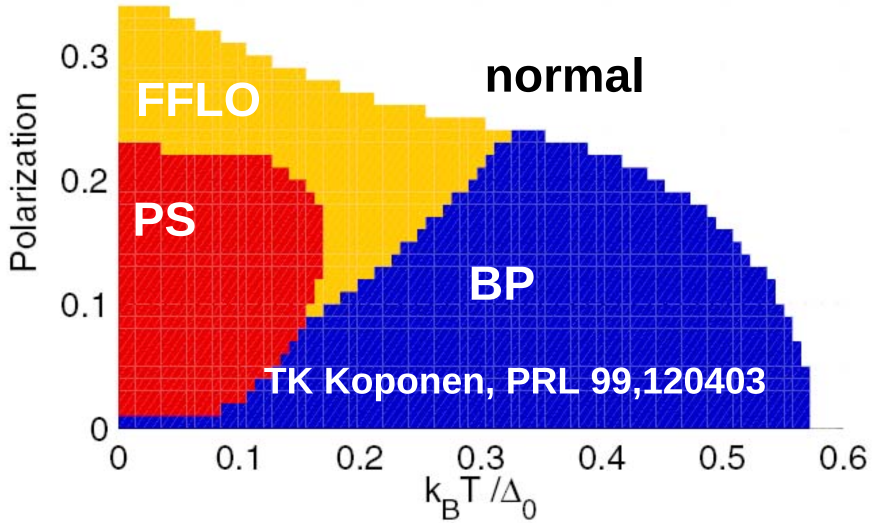
M.W. Zwierlein *et al.*, Nature 435, 1047 (2005)

Recent attractions in cold atoms

Exotic pairing states in spin polarized gas:

- Flude-Ferrell-Larkin-Ovcinnikov(FFLO) phase
- Breached pair (Sarma) phase(BP): gapless excitation
- Deformed Fermi surface phase
- Phase separation between paired and unpaired states(PS)

See review: Wenhui Li, Nucl. Phys. A 790,88c(2007)



Studies have implications for other Fermi systems:

DFT Studies

- Unitary limit: β factor

$$\mu = \frac{\hbar^2}{2m}(1+\beta)(6\pi^2 n)^{2/3}$$

which is measured to be β =-0.54±0.05. For ideal Fermi gas, it is 0. QMC calculation is 0.58

- G. Bertsch parameter (1999): study of the neutron matter

$$\frac{\mu}{\varepsilon_F} = \xi, \frac{\varepsilon}{\varepsilon_F} = \frac{3}{5}\xi$$

which is about 0.44

- The SLDA-DFT (A. Bulgac, Phys. Rev. A 76, 040502 (2007)):

$$\mathcal{E}(\boldsymbol{r}) = \alpha\frac{\tau(\boldsymbol{r})}{2} + \beta\frac{3(3\pi^2)^{2/3}\rho^{5/3}(\boldsymbol{r})}{10} + g_{\text{eff}}\frac{|\kappa(\boldsymbol{r})|^2}{\rho^{1/3}(\boldsymbol{r})},$$

HFB (BdG) of polarized Fermi systems

Two-Fermi-Level Approach (Blocking method)

$$\begin{pmatrix} H_0(\boldsymbol{r}) - \lambda_\uparrow & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -H_0(\boldsymbol{r}) + \lambda_\downarrow \end{pmatrix} \begin{pmatrix} u_i(\boldsymbol{r}) \\ v_i(\boldsymbol{r}) \end{pmatrix} = E_i \begin{pmatrix} u_i(\boldsymbol{r}) \\ v_i(\boldsymbol{r}) \end{pmatrix}$$

$$2\lambda_s = \lambda_\uparrow - \lambda_\downarrow$$

unpaired

$$m(\boldsymbol{r}) = \sum_{0 \le E_i < \lambda_s} \left(|u_i(\boldsymbol{r})|^2 + |v_i(\boldsymbol{r})|^2 \right),$$

total

$$\rho(\boldsymbol{r}) = m(\boldsymbol{r}) + \sum_{E_i > \lambda_s} 2|v_i(\boldsymbol{r})|^2,$$

$$\kappa(\boldsymbol{r}) = \sum_{E_i > \lambda_s} u_i(\boldsymbol{r}) v_i^*(\boldsymbol{r}),$$

pairing gap

$$\Delta(\boldsymbol{r}) = -g_{\mathit{eff}}(\boldsymbol{r})\kappa(\boldsymbol{r}),$$

Pairing regularization is implemented in SLDA because of local interactions:

A. Bulgac, Phys. Rev. A 76, 040502 (2007)

Deformed Coordinate-space HFB-AX

- Advantages:
  - It can precisely treat the continuums: weakly bound nuclei, QRPA
  - Precision is guaranteed for very elongated shape: two-center problem, fission
- Numerical features:

  B-spline techniques; reflection symmetry; diagonalization speedup; Broyden method for fast convergence; MPI parallel
- Challenges:

  Computation is very expensive. Currently developing a 3D Madness-HFB based on wavelets techniques. See G.I. Fann's talk.

J. Pei et al. PRC 78:064306, 2008
E. Teran, V.E. Oberacker, A.S. Umar, PRC 67 (2003) 064314

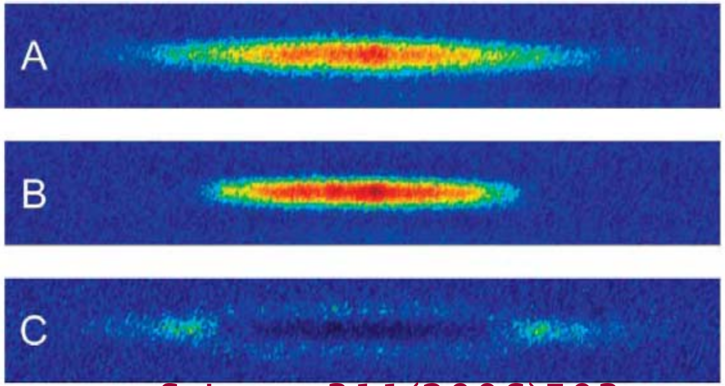
Calculation settlement

- Experiment observation dependents on the atoms and elongation.

$$V_{ext}(r,z)=\frac{m\omega^2}{2}(r^2+z^2/\eta^2)$$

MIT: N=10^7, η=5
Rice: N=10^5, η=50 (phase separation)

significant finite size effects



Science 311(2006)503

- Calculation challenges:

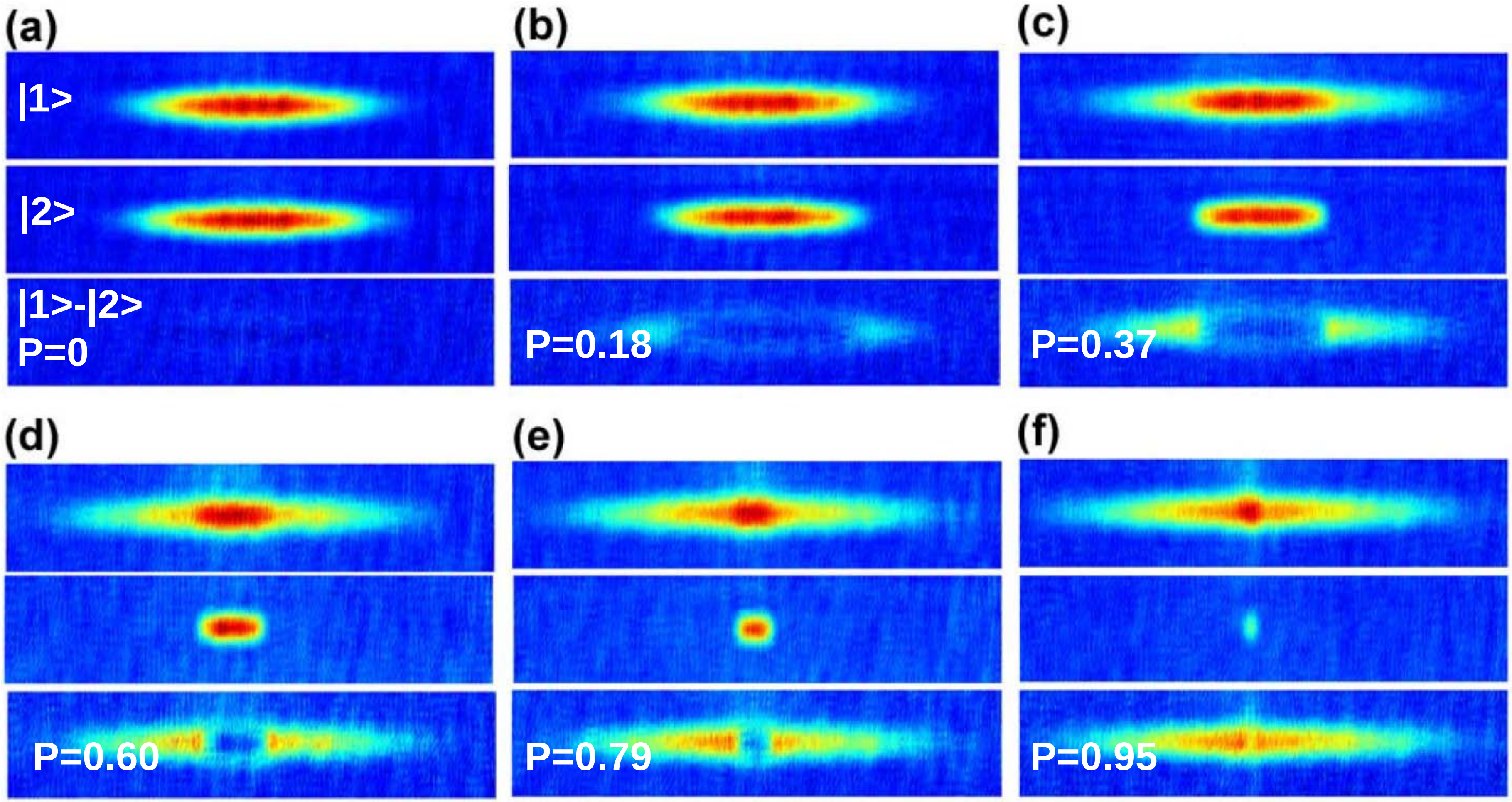
1000 atoms employed.
dimensionless 2D-box: R=25×0.3, Z=125×0.3 (very dense spectrum)
the elongation is set to be η=10.

Existing studies with HO basis are limited to small deformations.

R. Sensarma, arXiv.0706,1741,2007

J.Kinnunen, PRL 96, 110403, 2006

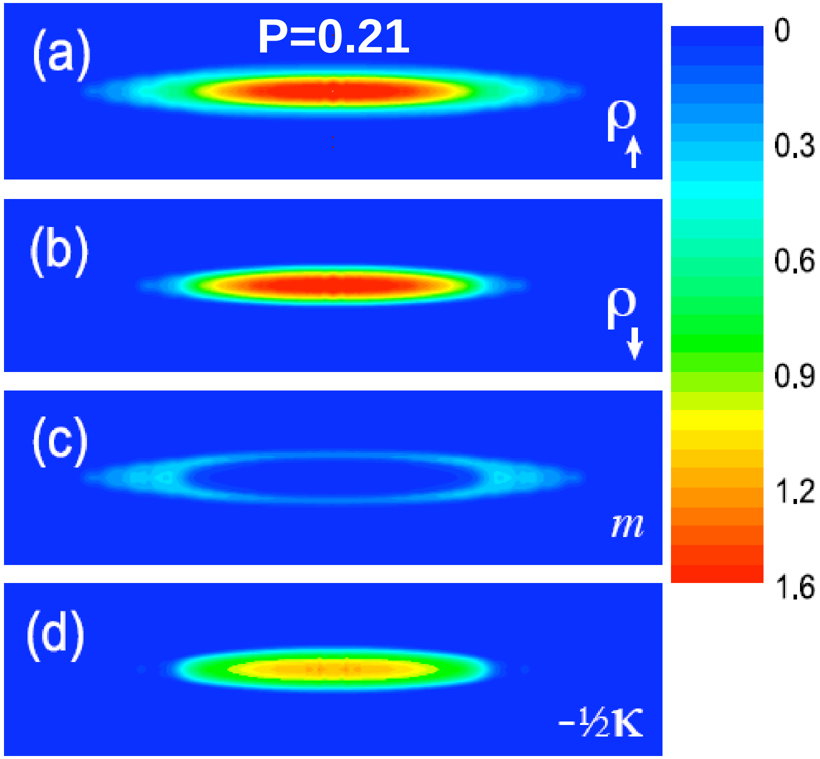
Trapped ^{6}Li gas (Rice)



Wenhui Li et al., Nucl. Phys. A 790,88c(2007)

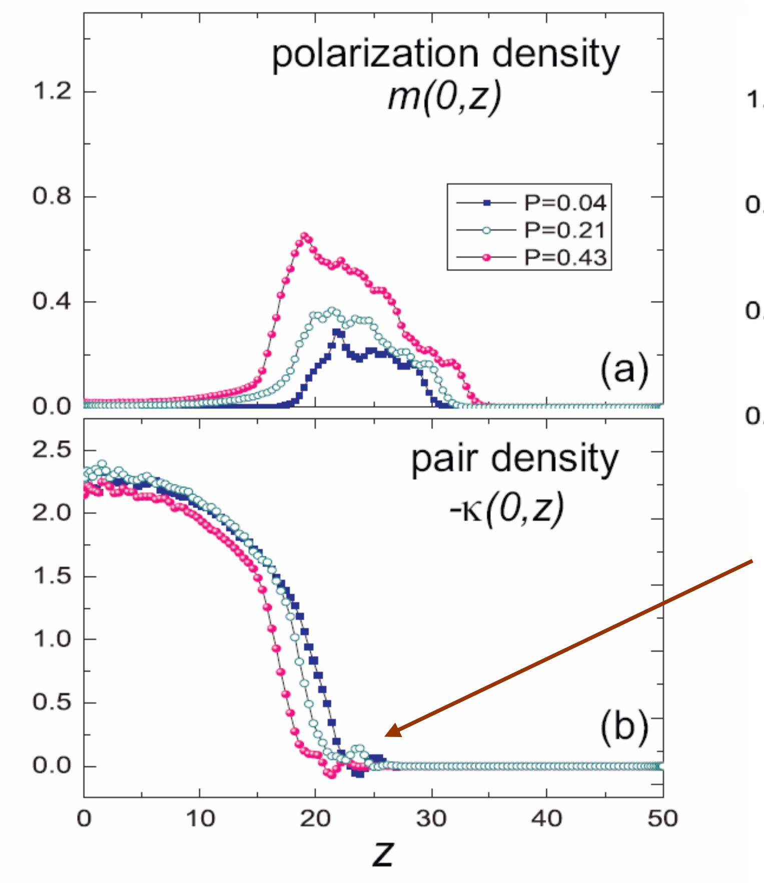
Phase separation

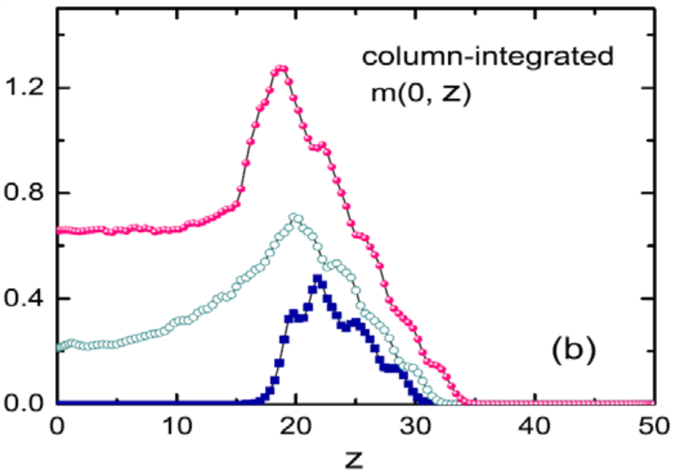
$P=(N_1-N_2)/(N_1+N_2)$



J. Pei, W. Nazarewicz, M. Stoitsov, arXiv:0901.0545, 2009

Pairing density





FFLO-type phase
is difficult to measure

Superfluid core reduced with increasing polarization

Phase separation at arbitrary small polarizations. No sharp transition to the **PS** regime.

Finite-Temperature HFB

- Generally,The FT-HFB:

A.L. Goodman, Nucl. Phys. A 352(1981)30

$$\rho = \tilde{U} f U^* + V^+ (1 - f) V$$

$$\kappa = \tilde{U} f V^* + V^+ (1 - f) U$$

$$f_i = \frac{1}{1 + e^{E_i / k_B T}}$$

Thermal factors

- Thus for the heated polarized gas:

$$\rho_{\text{paired}}(T) = 2 \times \sum_{E > \lambda} [|U_i|^2 f_i + |V_i|^2 (1 - f_i)]$$

$$\rho_{\text{blocked}}(T) = \sum_{E < \lambda} [|U_i|^2 + |V_i|^2]$$

No explicit thermal effects!

$$\kappa_{\text{paired}}(T) = 2 \times \sum_{E > \lambda} [U_i V_i (1 - 2 f_i)]$$

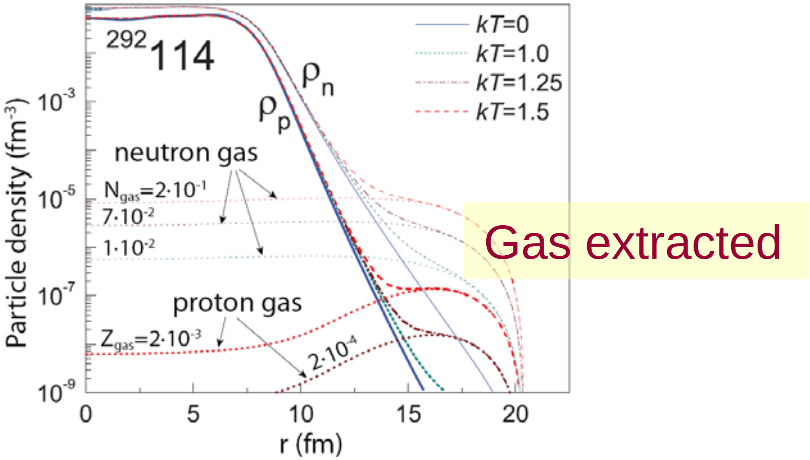
Pairing decrease with increasing T

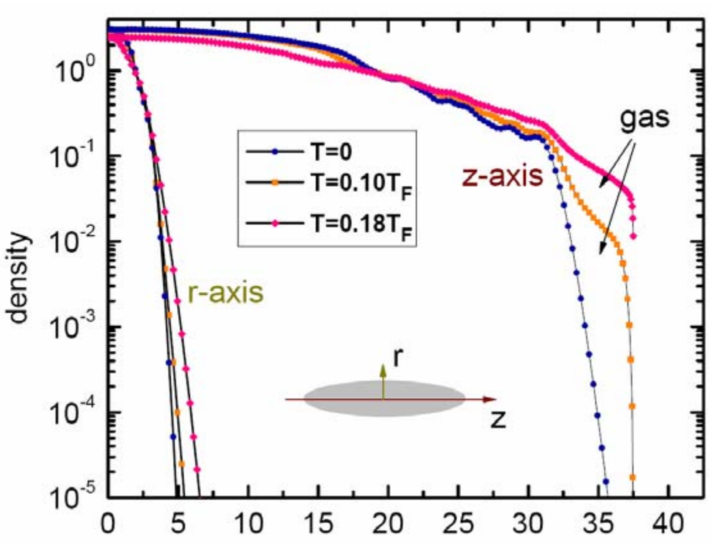
$$\kappa_{\text{blocked}}(T) = 0$$

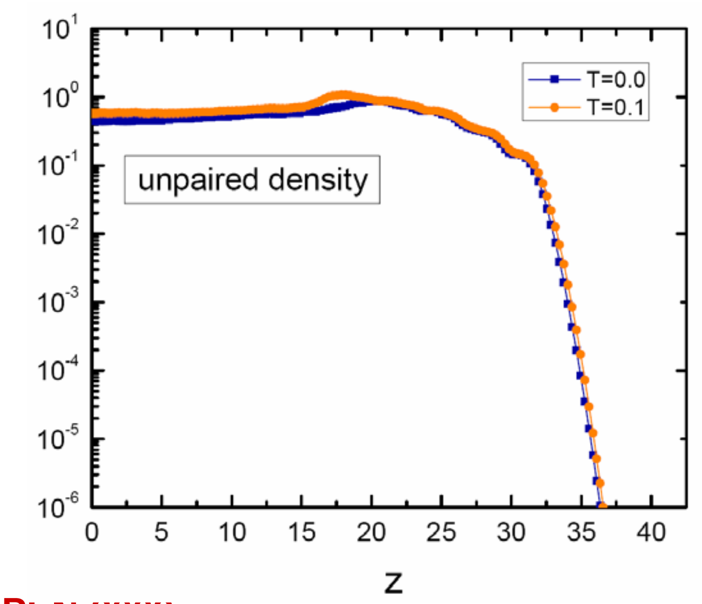
Finite-Temperature HFB

- Thermal *gas* effects:

  In FT-HFB, external gas produces the pressure necessary to obtain equilibrium with the particle-decaying hot nucleus.







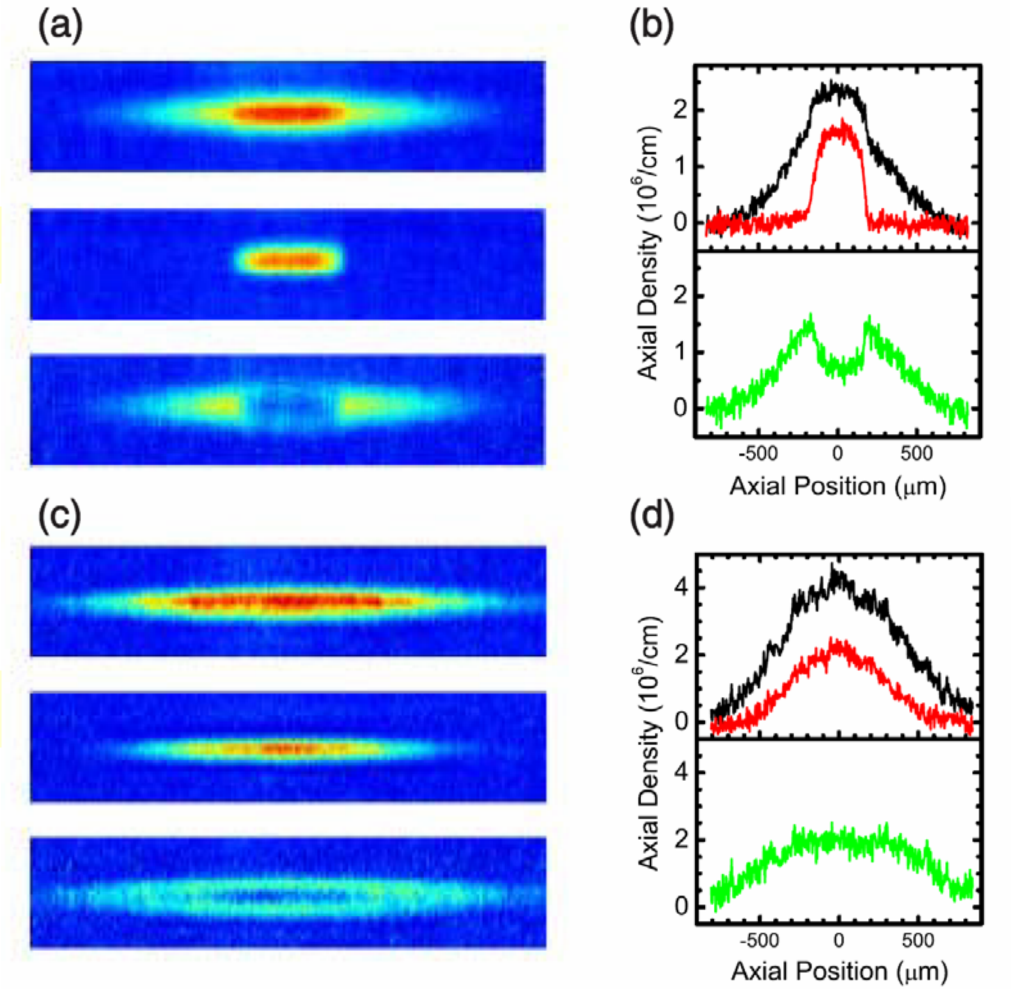
Thermal effects

- Experiments:

$T<0.05T_F$

- The density profiles are less deformed.
- The sharp phase boundary doesn't exist

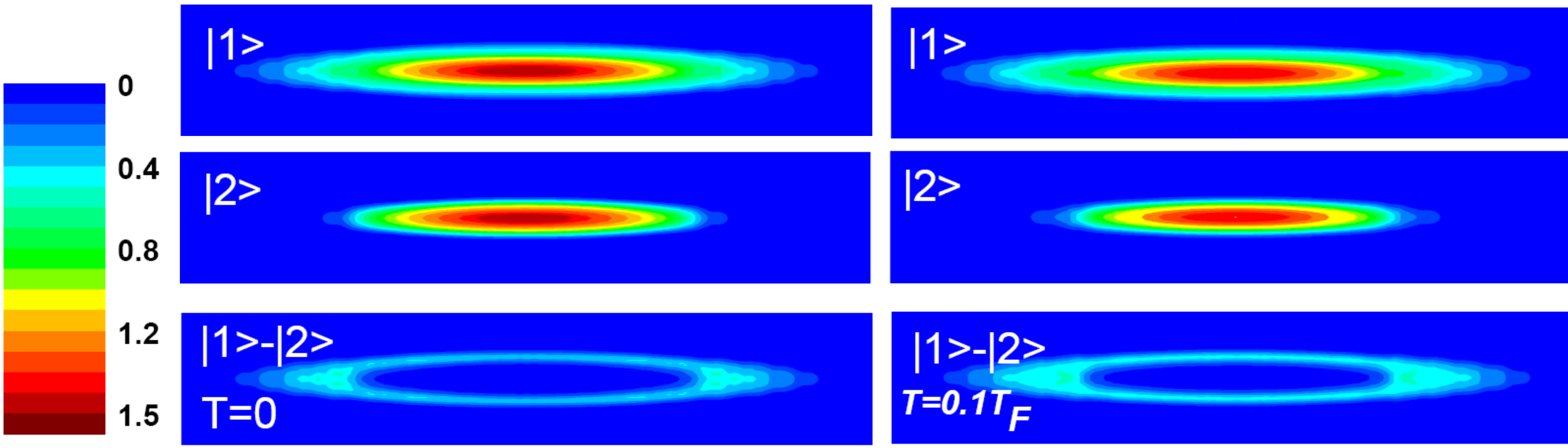
$T\approx 0.2T_F$

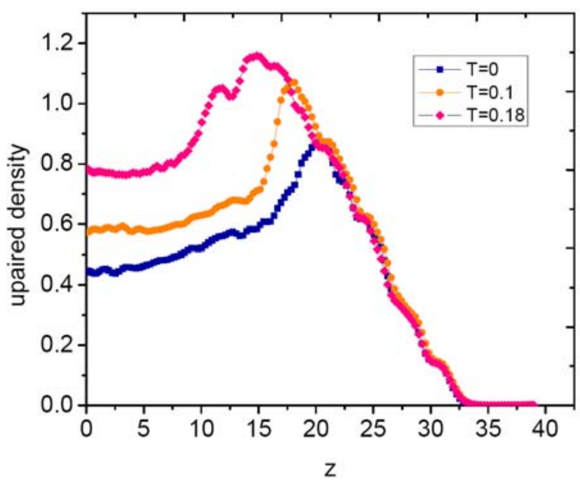


Wenhui Li et al., Nucl. Phys. A 790,88c(2007)

Thermal effects

- Calculations:





- Superfluid core decrease
- Surface density of unpaired states doesn't change

Conclusions

- Coordinate-space HFB-AX is very useful for the description of Fermi-gas in elongated trap.
- The phase separation, FFLO have been demonstrated.
- Remarkable thermal gas is predicted in the elongated trap.
- Next step, to analyze the entropy; also improved DFT parameters for unbalanced gas have to be studied; the self-consistent remove of gas in heat nuclei and atom condenses.