Skyrme-QRPA calculations for collective modes of excitation in deformed neutron-rich nuclei

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Magicity at N=20

J.A.Church et al.,PRC**72**(2005)054320

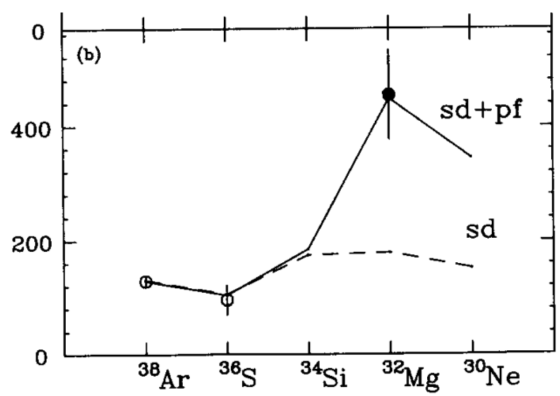
✓Low-lying 2^+ state: 885keV(^{32}Mg), 659keV(^{34}Mg)
✓Large B(E2;$0^+ \rightarrow 2^+$): 447e^2fm^4(^{32}Mg), 541e^2fm^4(^{34}Mg)

⇩

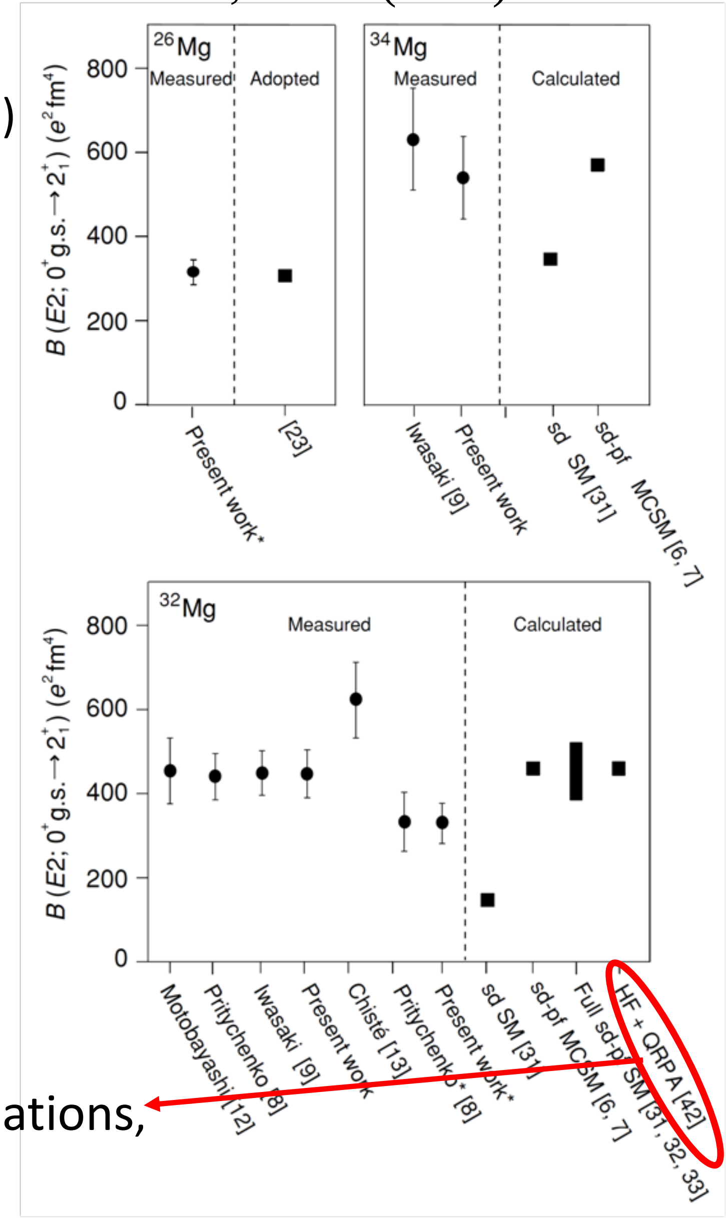
Breaking of the N=20 spherical magic number

⇩

Shell inversion

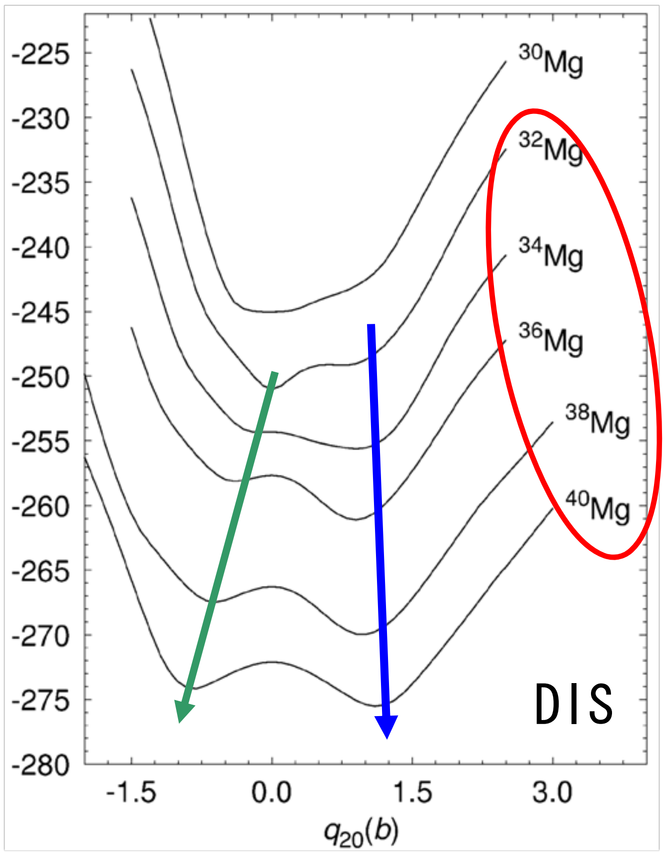


T.Motobayashi et al.,PLB**346**(1995)9



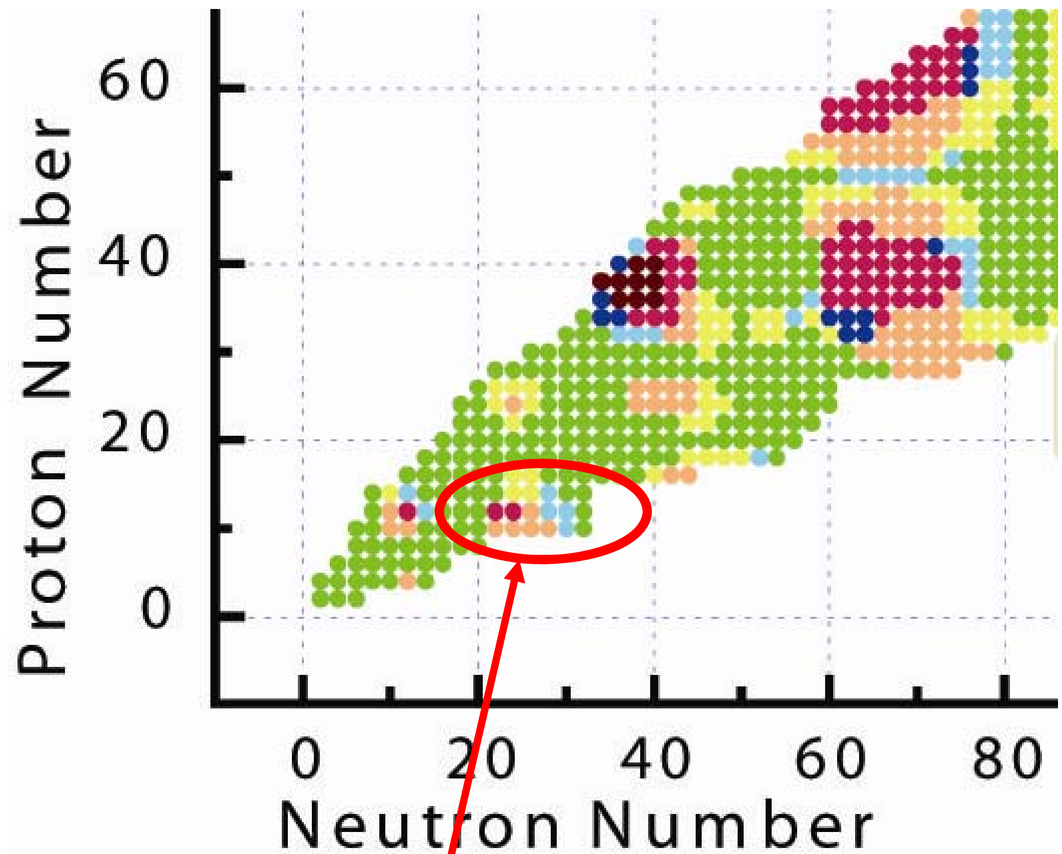
Importance of the continuum coupling and pair correlations,
M.Yamagami and N. Van Giai, PRC**69**(2004)034301

New shell structures – new regions of deformation



R. Rodríguez-Guzmán *et al.*, NPA709(2002)201

HFB mass table



Neutron-rich Mg region between N=20 and 28

M.V. Stoitsov *et al.*, Phys. Rev. C68(2003) 054312

Unique features in neutron-rich nuclei

Shallow Fermi level

- Spatially extended structure of the single-(quasi)particle wave functions

  Neutron skins and halos

- New shell structures

  ✓ Appearance of new magic numbers/disappearance of traditional magic numbers

  New regions of deformation

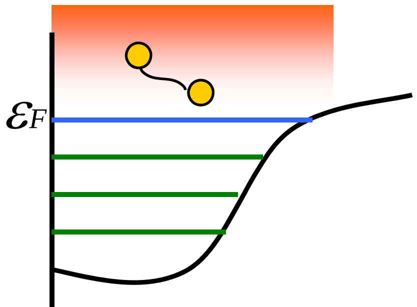
- Pairing in the continuum

  ✓ Changes the spatial structure of the quasiparticle wave functions

  M.Yamagami, PRC72(2005)064308

  ✓ Emerges the di-neutron correlation

  M.Matsuo *et al.*, PRC71(2005)064326

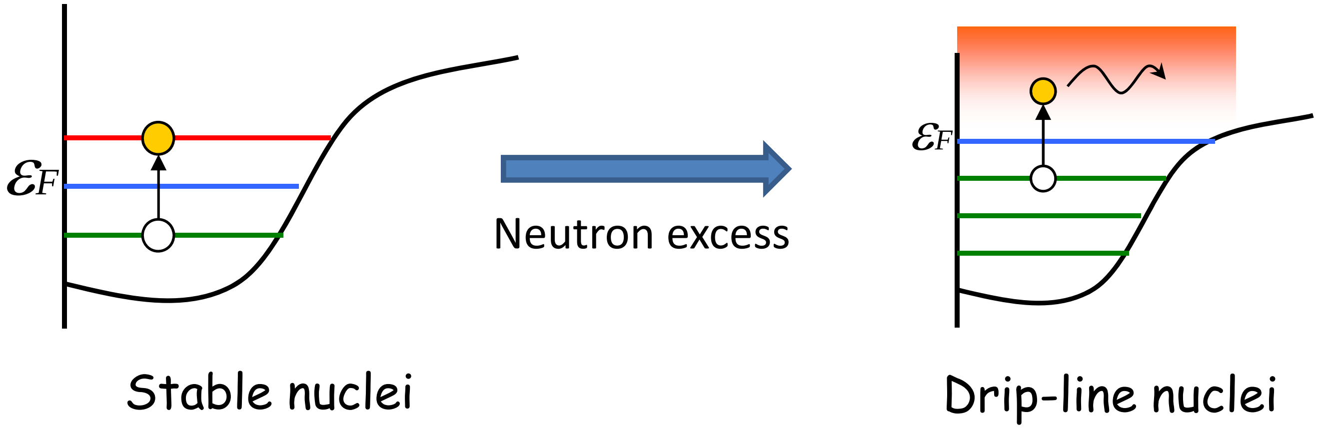


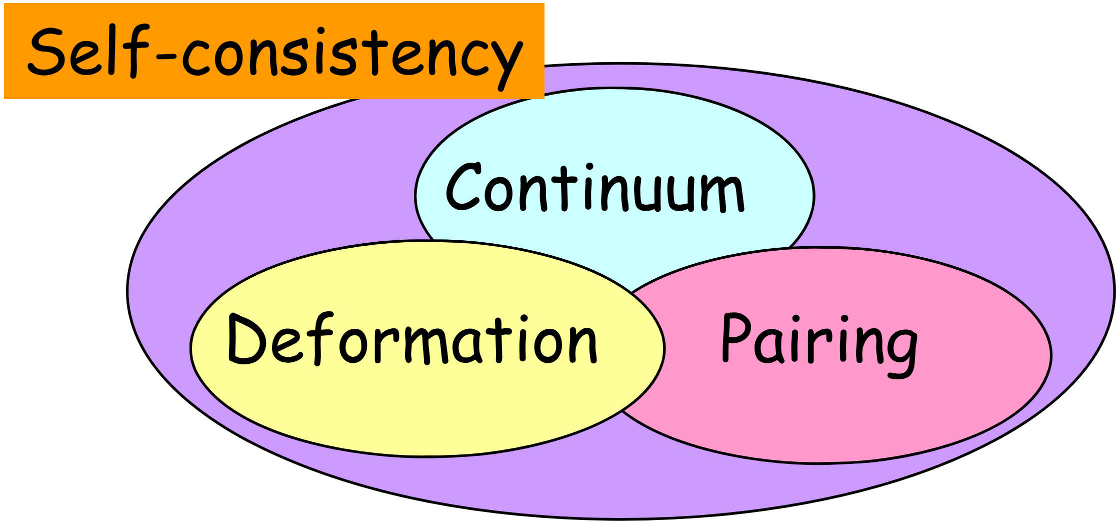
Microscopic description of collective excitations

Collective excitation modes

=coherent superposition of 2qp (1p-1h) excitations

$$|\mathrm{vib.}\rangle = \hat{B}^\dagger_\lambda |\mathrm{gr.}\rangle = \sum_{ij} \left[X^\lambda_{ij} \hat{\beta}^\dagger_i \hat{\beta}^\dagger_j - Y^\lambda_{ij} \hat{\beta}_j \hat{\beta}_i \right] |\mathrm{gr.}\rangle$$





Self-consistent calculation

The coordinate-space Hartree-Fock-Bogoliubov theory

$$\begin{pmatrix} h^q(\boldsymbol{r},\sigma)-\lambda^q & \tilde{h}^q(\boldsymbol{r},\sigma) \\ \tilde{h}^q(\boldsymbol{r},\sigma) & -(h^q(\boldsymbol{r},\sigma)-\lambda^q) \end{pmatrix} \begin{pmatrix} \varphi^q_{1,\alpha}(\boldsymbol{r},\sigma) \\ \varphi^q_{2,\alpha}(\boldsymbol{r},\sigma) \end{pmatrix} = E_\alpha \begin{pmatrix} \varphi^q_{1,\alpha}(\boldsymbol{r},\sigma) \\ \varphi^q_{2,\alpha}(\boldsymbol{r},\sigma) \end{pmatrix}$$

J.Dobaczewski, H.Flocard and J.Treiner, NPA422(1984)103
A.Bulgac, FT-194-1980 (Institute of Atomic Physics, Bucharest)

Cf. BCS → Unphysical nucleon-gas problem in drip-line nuclei

One can properly treat the pairing correlation in the continuum.

□Mean-field Hamiltonian

$$h = \frac{\delta\mathcal{E}}{\delta\varrho}$$

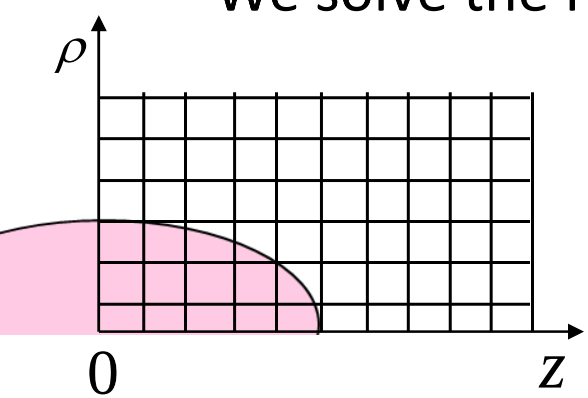
SkM* interaction

□Pairing field

$$\tilde{h} = \frac{\delta\mathcal{E}}{\delta\tilde{\varrho}}$$

mixed-type delta interaction

We solve the HFB equations directly on the 2D lattice.



11-point formula for derivative

✓Simple
✓Appropriate for describing the spatially extended structure of wavefunctions ⟷ H.O. basis

Deformed Quasiparticle RPA

KY, N.V. Giai, PRC**78**(2008)064316

HFB equations → Quasiparticle basis

$$\sum_{\gamma\delta}\begin{pmatrix} A_{\alpha\beta\gamma\delta} & B_{\alpha\beta\gamma\delta} \\ B_{\alpha\beta\gamma\delta} & A_{\alpha\beta\gamma\delta}\end{pmatrix}\begin{pmatrix} X^{\lambda}_{\gamma\delta} \\ Y^{\lambda}_{\gamma\delta}\end{pmatrix} = \hbar\omega_\lambda \begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix}\begin{pmatrix} X^{\lambda}_{\alpha\beta} \\ Y^{\lambda}_{\alpha\beta}\end{pmatrix}$$

Residual interactions

✓particle-hole channel: $\dfrac{\delta^2\mathcal{E}}{\delta^2\varrho}$

$$\begin{aligned} v_{ph}(\boldsymbol{r},\boldsymbol{r}') = & (a_0 + a_0'\tau\cdot\tau' + (b_0 + b_0'\tau\cdot\tau')\sigma\cdot\sigma')\delta(\boldsymbol{r}-\boldsymbol{r}') \\ & + (a_1 + a_1'\tau\cdot\tau' + (b_1 + b_1'\tau\cdot\tau')\sigma\cdot\sigma') \\ & \times (\boldsymbol{k}^{\dagger 2}\delta(\boldsymbol{r}-\boldsymbol{r}') + \delta(\boldsymbol{r}-\boldsymbol{r}')\boldsymbol{k}^2) \\ & + (a_2 + a_2'\tau\cdot\tau' + (b_2 + b_2'\tau\cdot\tau')\sigma\cdot\sigma') \\ & \times (\boldsymbol{k}^{\dagger}\cdot\delta(\boldsymbol{r}-\boldsymbol{r}')\boldsymbol{k}) \end{aligned}$$

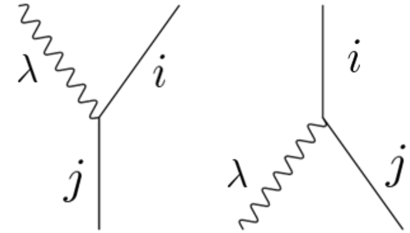
We neglect the residual spin-orbit and Coulomb interactions.

✓particle-particle channel: $\dfrac{\delta^2\mathcal{E}}{\delta^2\tilde{\varrho}}$

$$v_{pp}(\boldsymbol{r},\boldsymbol{r}') = \frac{1-P_\sigma}{2}\left[t_0' + \frac{t_3'}{6}\varrho(\boldsymbol{r})\right]\delta(\boldsymbol{r}-\boldsymbol{r}')$$

Quadrupole moment of odd-A nuclear systems

The nuclear Hamiltonian



$$\hat{H} = \sum_i E_i \hat{\beta}_i^\dagger \hat{\beta}_i + \sum_\lambda \hbar\omega_\lambda \hat{B}_\lambda^\dagger \hat{B}_\lambda + \hat{H}_{\mathrm{couple}}$$

$$f_{ij}^\lambda = \int d\boldsymbol{r} v^{\mathrm{pv}}(\boldsymbol{r}) \delta\varrho_\lambda(\boldsymbol{r}) \left[\varphi_{1,i}(\boldsymbol{r})\varphi_{1,j}^*(\boldsymbol{r}) - \varphi_{2,i}(\boldsymbol{r})\varphi_{2,j}^*(\boldsymbol{r})\right]$$

is diagonalized within the subspace $\{\beta_i^\dagger|0\rangle, B_\lambda^\dagger \beta_j^\dagger |0\rangle\}$

The eigenstate of the odd-A systems:

$$|\phi\rangle = \sum_i c_i^0 \beta_i^\dagger |0\rangle + \sum_{\lambda j} c_{\lambda j}^1 B_\lambda^\dagger \beta_j^\dagger |0\rangle$$

The electric quadrupole moment: $\langle\phi|e\hat{Q}|\phi\rangle$

$$\hat{Q} = \langle\hat{Q}\rangle + \sum_{ij\in\pi} Q_{ij} \hat{\beta}_i^\dagger \hat{\beta}_j + \sum_\lambda (Q_\lambda \hat{B}_\lambda^\dagger + Q_\lambda^* \hat{B}_\lambda)$$

$$\begin{aligned} Q_{ij} &= \langle 0|\hat{\beta}_i \hat{Q} \hat{\beta}_j^\dagger|0\rangle \\ &= \int d\boldsymbol{r}(3z^2 - r^2)[\varphi_{1,i}(\boldsymbol{r})\varphi_{1,j}^*(\boldsymbol{r}) - \varphi_{2,i}(\boldsymbol{r})\varphi_{2,j}^*(\boldsymbol{r})], \end{aligned}$$

$$Q_\lambda = \langle 0|[\hat{B}_\lambda, \hat{Q}]|0\rangle = \int d\boldsymbol{r}(3z^2 - r^2)\delta\varrho_\lambda^\pi(\boldsymbol{r})$$

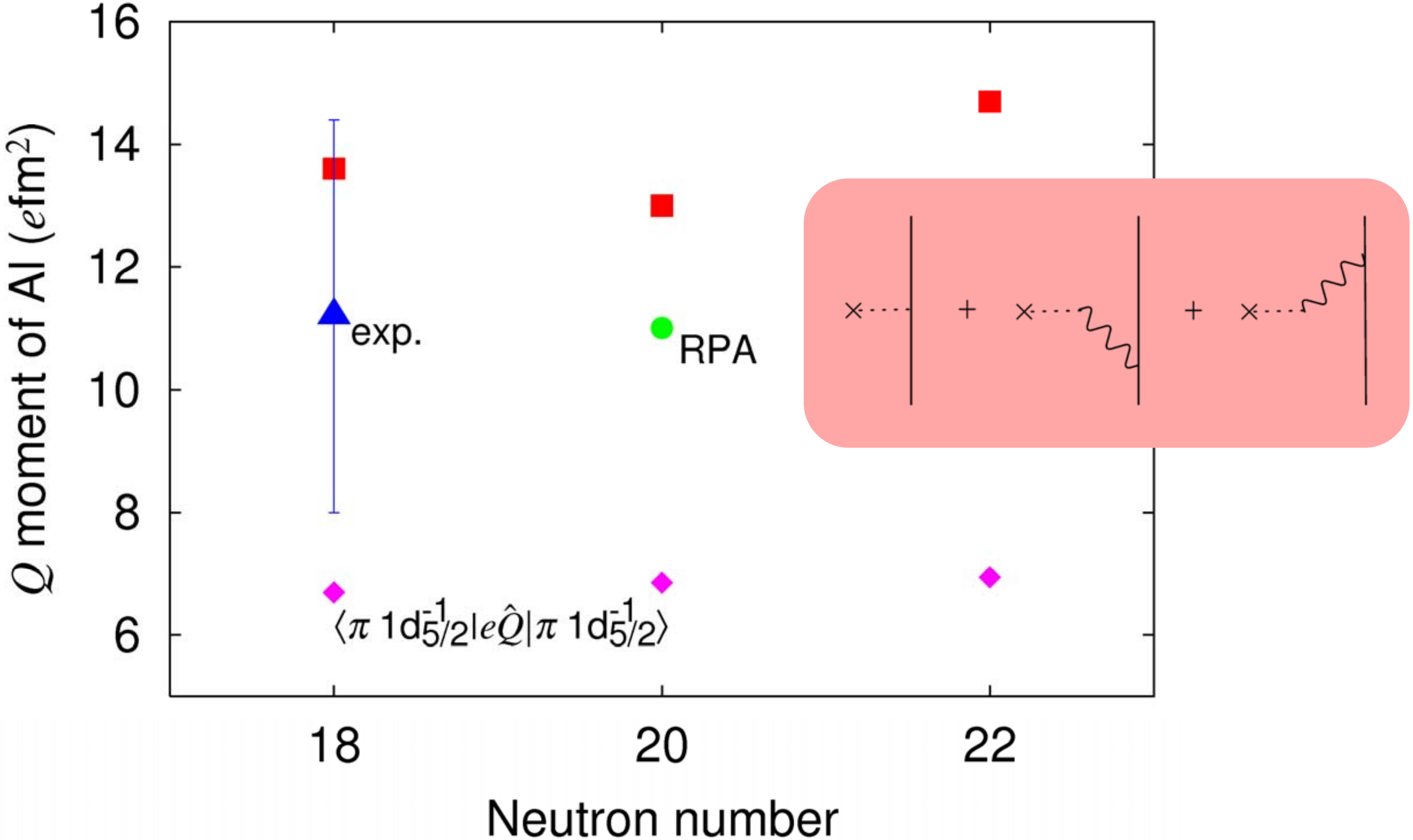
Quadrupole moment of Al isotopes

SkM*+mixed-type pairing (V_0=-295 MeV fm^3)

KY, arXiv:0902.3054

$$|\mathrm{Al}\rangle = |\pi 1d_{5/2}^{-1} \otimes \mathrm{Si}\rangle$$

spherical



Experiment

^{31}Al at RIKEN: D. Nagae et al., PRC79(2009)027301

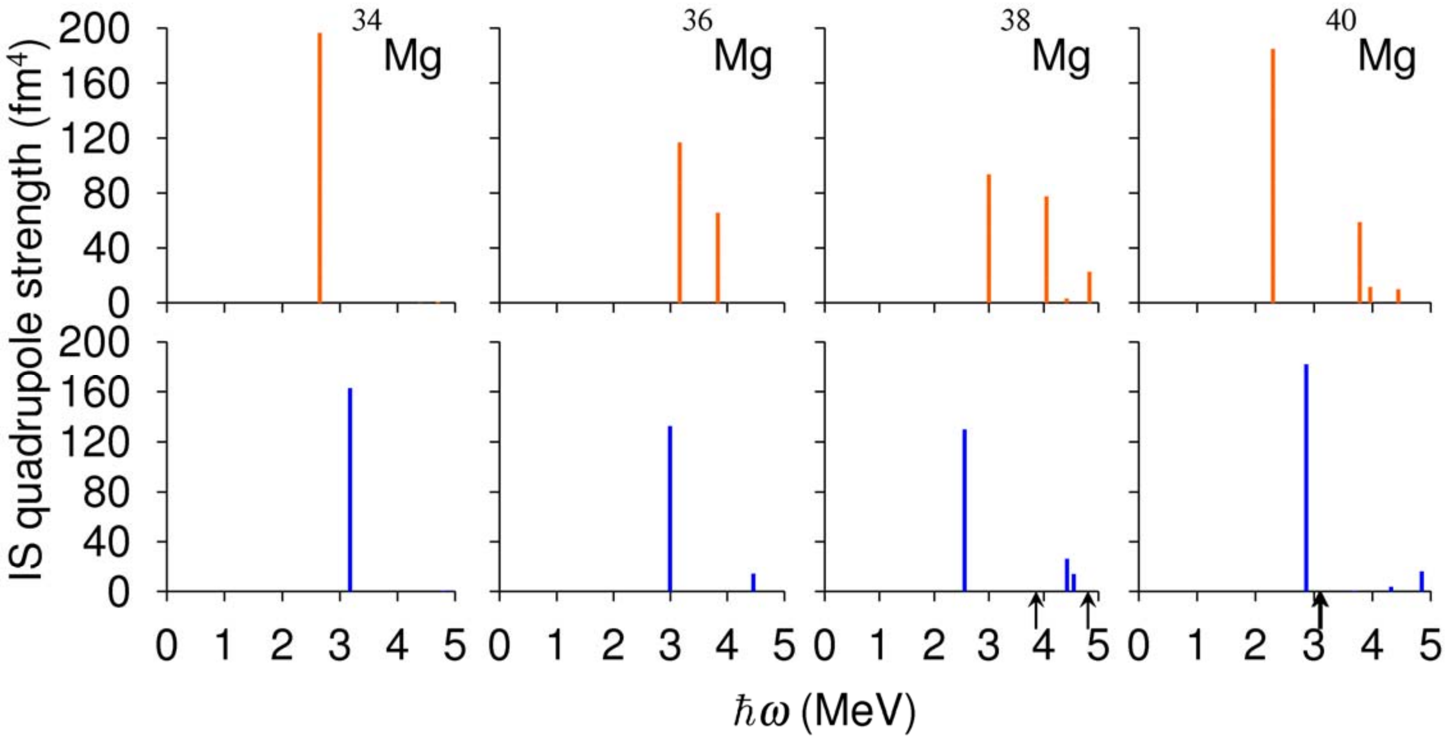
Quadrupole excitation modes in neutron-rich Mg isotopes

SkM*+mixed-type pairing (V_0=-295 MeV fm^3)

KY, arXiv:0902.3053

	^{34}Mg	^{36}Mg	^{38}Mg	^{40}Mg
$\beta_{2,n}$	0.35	0.31	0.29	0.28
$\beta_{2,p}$	0.41	0.39	0.38	0.36

Results of the Skyrme-QRPA calculation

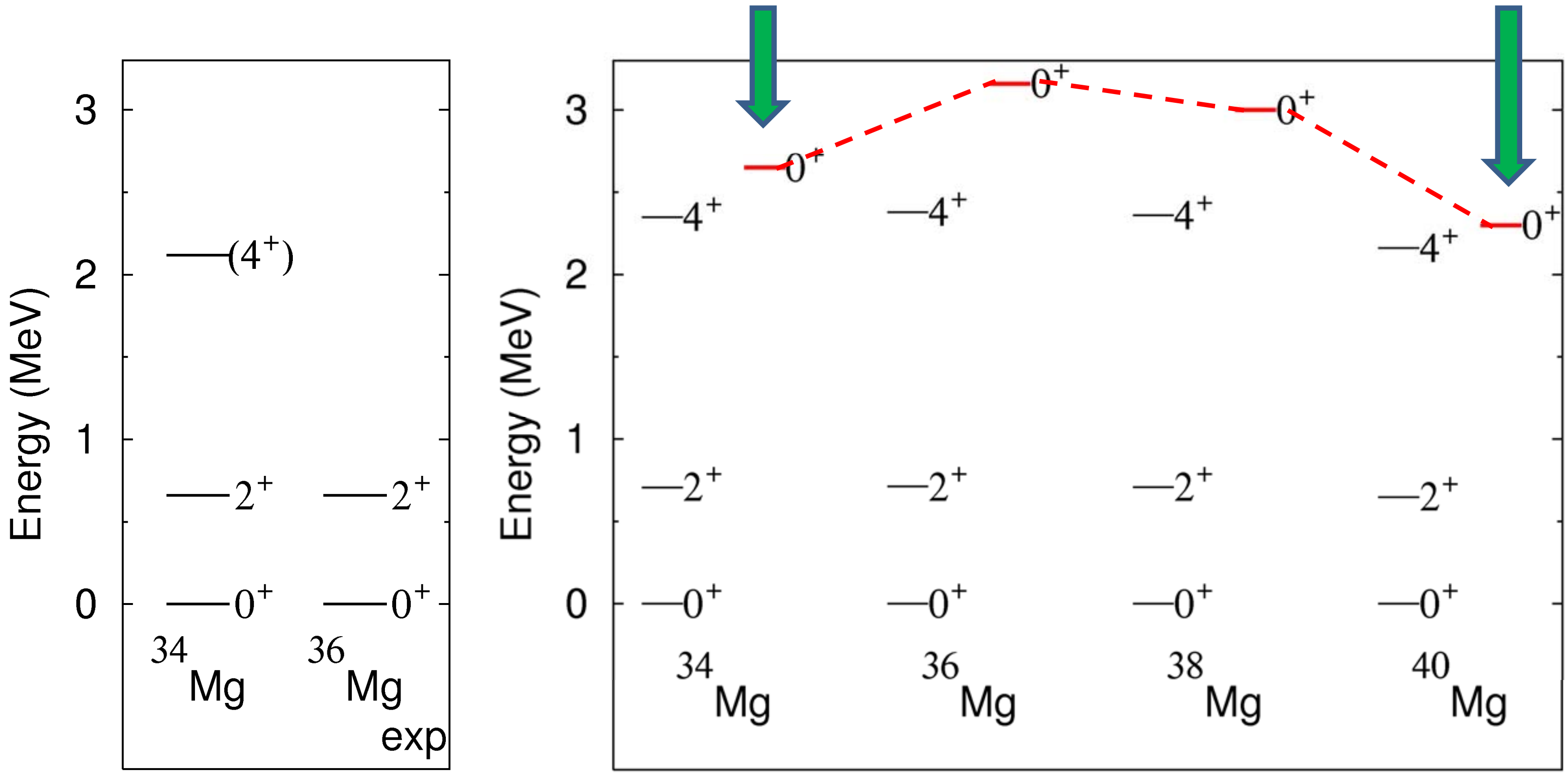


$K^\pi = 0^+$

$K^\pi = 2^+$

Low-lying levels in neutron-rich Mg isotopes

Sensitive to the shell structure



$$E(I,K) = \hbar\omega_{\mathrm{RPA}} + \frac{\hbar^2}{2\mathcal{J}_{\mathrm{TV}}}\left[I(I+1) - K^2\right]$$

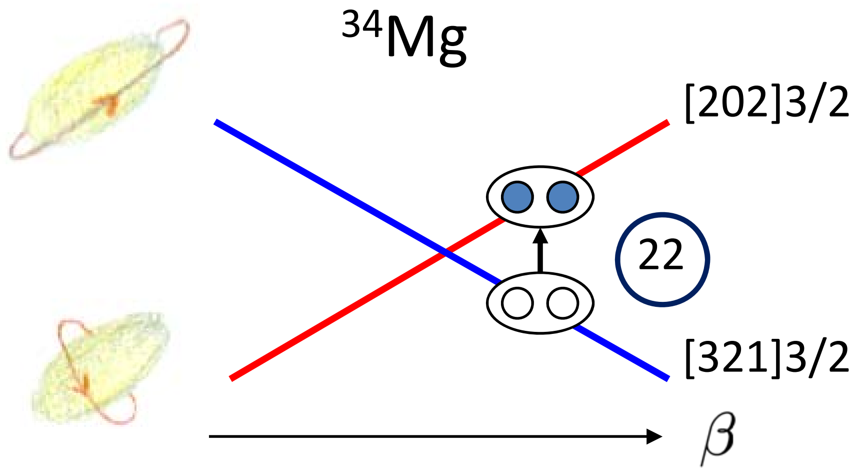
Microscopically calculated

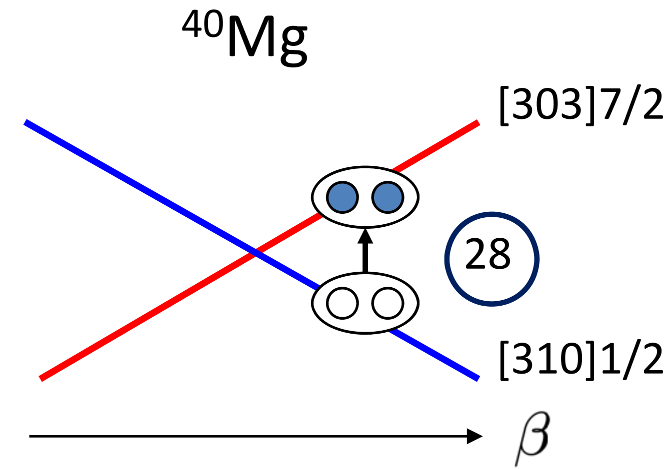
Experiments

^{34}Mg at RIKEN: K.Yoneda et al., PLB**499**(2001)233

^{36}Mg at NSCL: A.Gade et al., PRL**99**(2007)072502

Microscopic structure of the soft K=0$^+$ mode





Two level model (Bohr-Mottelson)

Ground state

$$|0_1^+\rangle = \frac{1}{\sqrt{a^2+b^2}}\left(a|\nu_1\bar{\nu}_1\rangle + b|\nu_2\bar{\nu}_2\rangle\right)$$

Excited state

$$|0_2^+\rangle = \frac{1}{\sqrt{a^2+b^2}}\left(-b|\nu_1\bar{\nu}_1\rangle + a|\nu_2\bar{\nu}_2\rangle\right)$$

Opposite sign → **Enhancement**

Transition matrix element

$$\Longrightarrow \langle K^\pi = 0_1^+|r^2Y_{20}|K^\pi = 0_2^+\rangle = \frac{2ab}{a^2+b^2}\left[\langle\nu_2|r^2Y_{20}|\nu_2\rangle - \langle\nu_1|r^2Y_{20}|\nu_1\rangle\right]$$

KY, M.Yamagami, PRC77(2008)044312

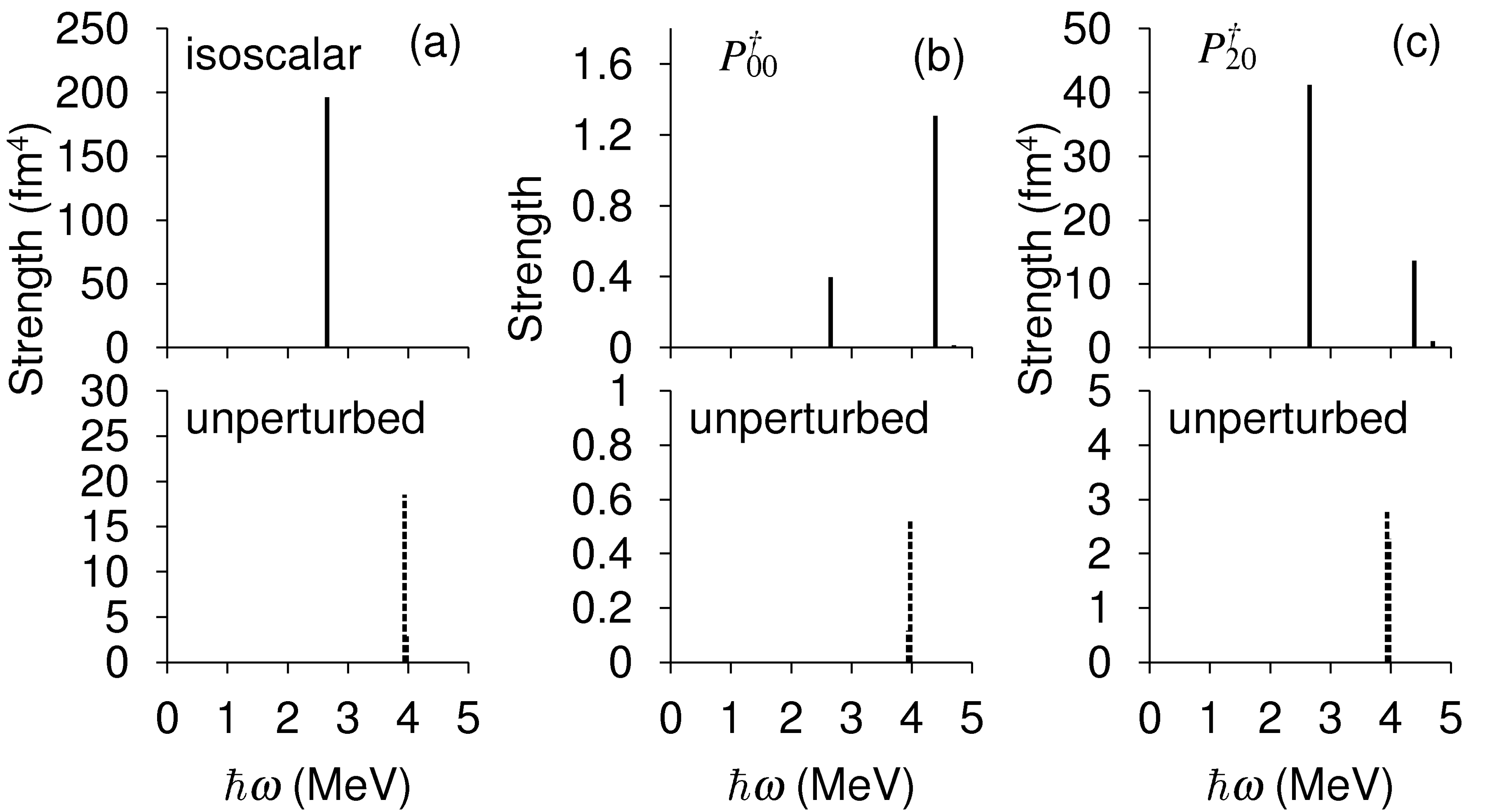
Pairing collectivity – neutron pair transition in ^{34}Mg

Monopole pairing

$$\hat{P}^{\dagger}_{00} = \int d\boldsymbol{r} \hat{\psi}^{\dagger}_{\nu}(\boldsymbol{r},\uparrow)\hat{\psi}^{\dagger}_{\nu}(\boldsymbol{r},\downarrow)$$

Quadrupole pairing

$$\hat{P}^{\dagger}_{20} = \int d\boldsymbol{r} r^2 Y_{20} \hat{\psi}^{\dagger}_{\nu}(\boldsymbol{r},\uparrow)\hat{\psi}^{\dagger}_{\nu}(\boldsymbol{r},\downarrow)$$



Separation of the center of mass motion

Fully self-consistent RPA

→ free from the spurious modes

☹ We have to take care of them.

"physical" states $|\tilde{n}\rangle$

$$\hat{X}_{\tilde{n}} = \hat{X}_n - \chi_P^n \hat{P} - \chi_R^n \hat{R}$$

$$[\hat{R}, \hat{P}] = i\hbar$$

The physical states satisfy the following conditions:

(i) The vacuum condition

$$\hat{X}_{\tilde{n}}|0\rangle = 0$$

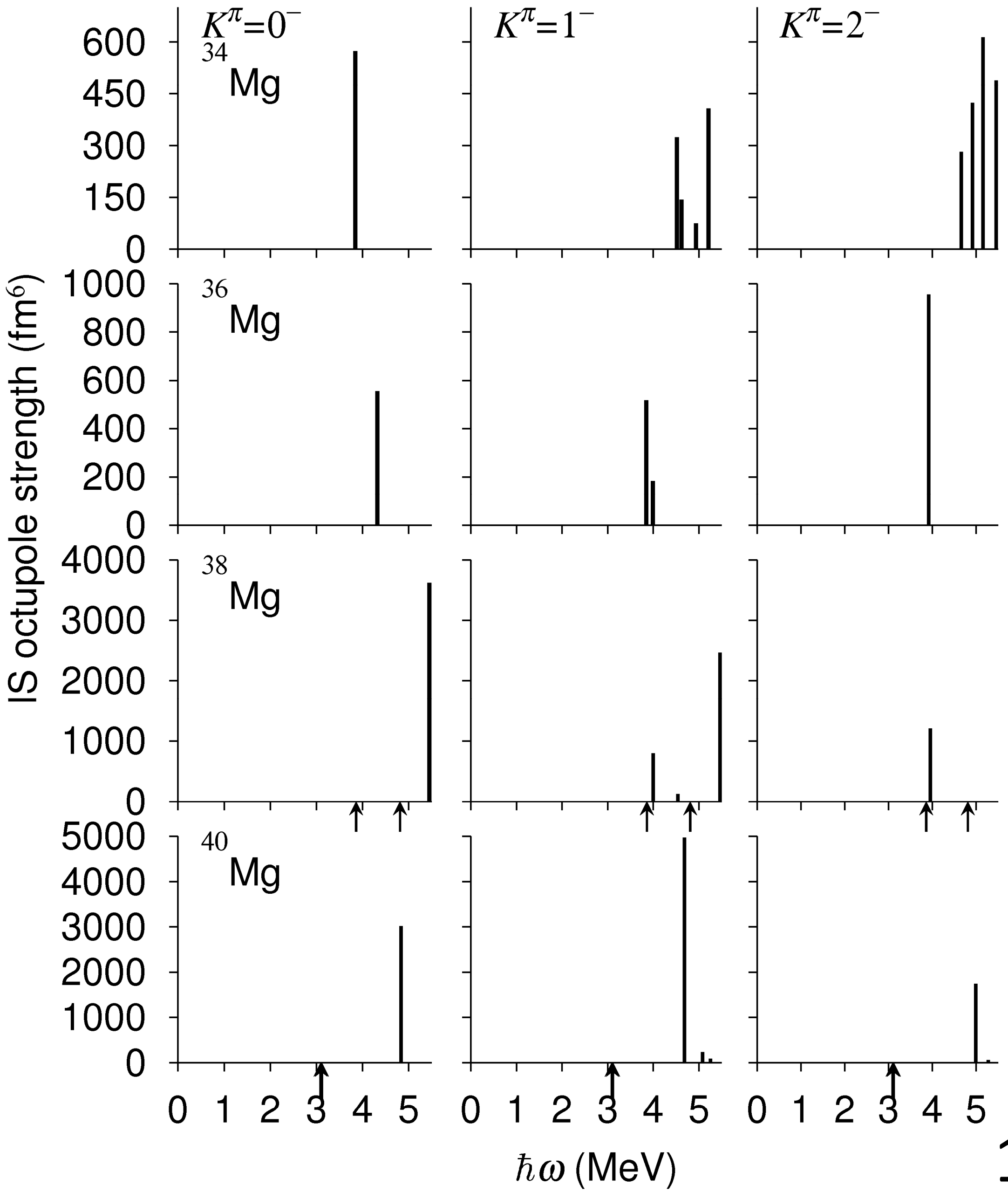
(ii) The decoupling condition

$$\langle 0|[\hat{X}_{\tilde{n}}, \hat{P}]|0\rangle = 0, \langle 0|[\hat{X}_{\tilde{n}}, \hat{R}]|0\rangle = 0$$

→ Determine the coefficients χ_P^n, χ_R^n;

$$\chi_P^n = i\langle 0|[\hat{X}_n, \hat{R}]|0\rangle/\hbar, \chi_R^n = -i\langle 0|[\hat{X}_n, \hat{P}]|0\rangle/\hbar$$

Low-lying octupole excitations

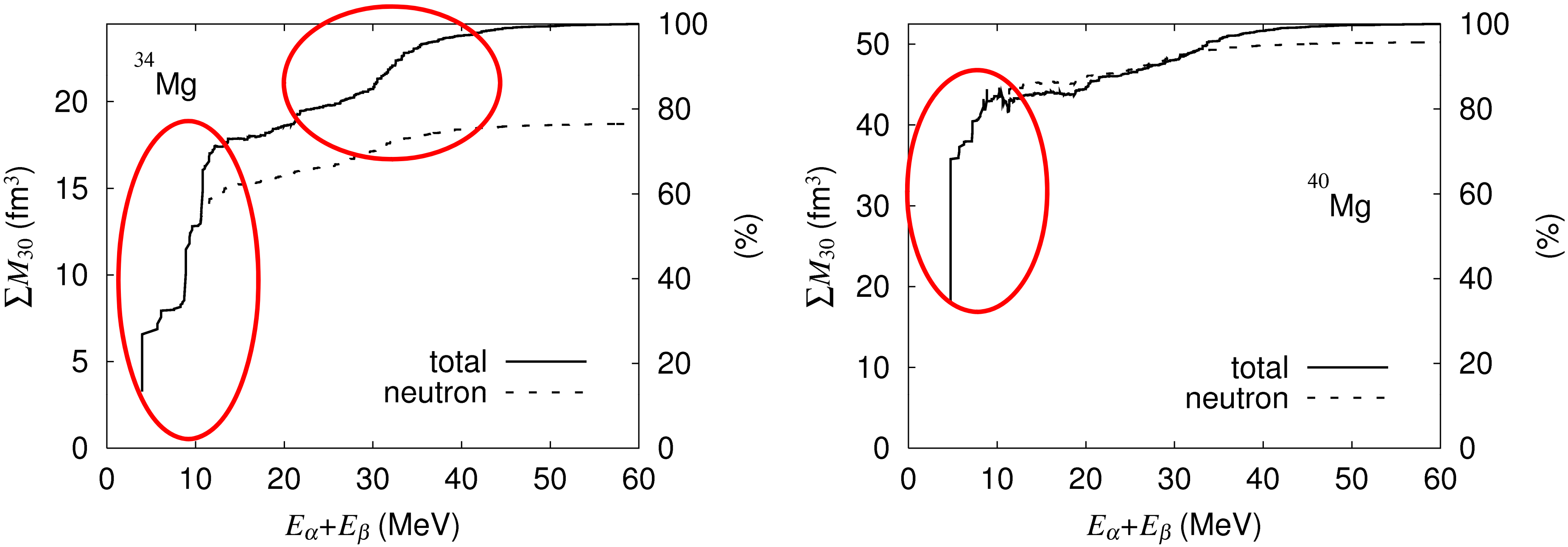


1W.u. ≈70-90 fm^6

Microscopic structure of the low-lying octupole modes

$$K^\pi = 0^- \text{ mode}$$

$$\langle \tilde{n} | \hat{O}_{30} | 0 \rangle = \sum_{\alpha\beta} M_{30,\alpha\beta}$$



coupling to the giant resonance

$$B(E3; 0^+ \to 3^-_{K=0^-}) = 33e^2\text{fm}^6 \quad \Longleftrightarrow \quad 5.0e^2\text{fm}^6$$

Summary

Collectivity at N=20

Neutron pairing correlations are crucial for the anomalous B(E2) in ^{32}Mg and enhancement of the quadrupole moment of ^{33}Al.

Beyond N=20

Deformed ground state in $^{34,36,38,\mathbf{40}}$Mg

Soft K=0$^+$ mode especially in $^{34,\mathbf{40}}$Mg

Sensitive to the neutron number (shell structure around the Fermi level)

In the deformation region, where orbitals both of up-sloping and of down-sloping exist.

The coherent coupling between **the pairing vibration** and **the beta vibration of neutrons**