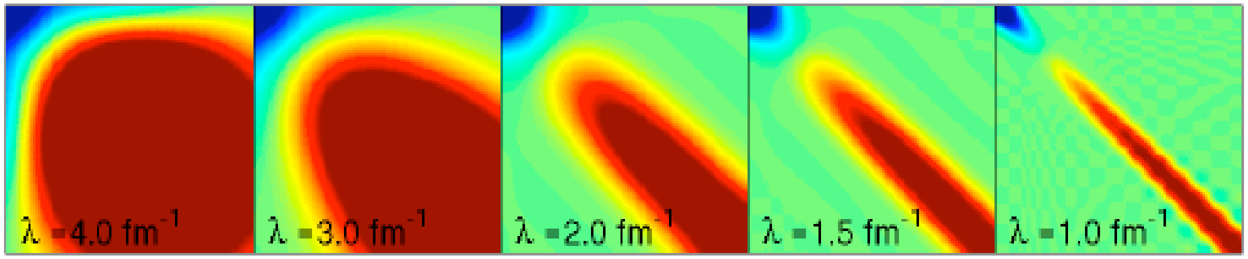
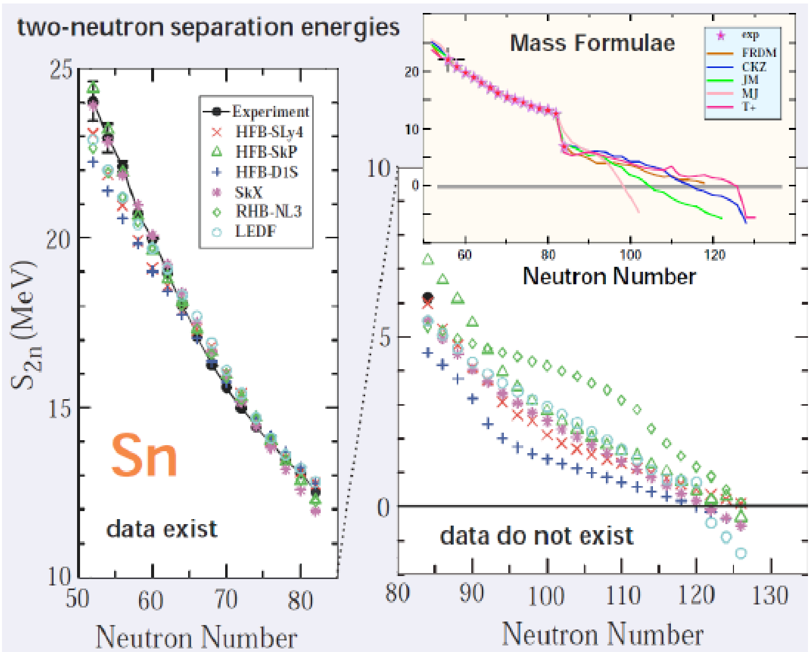
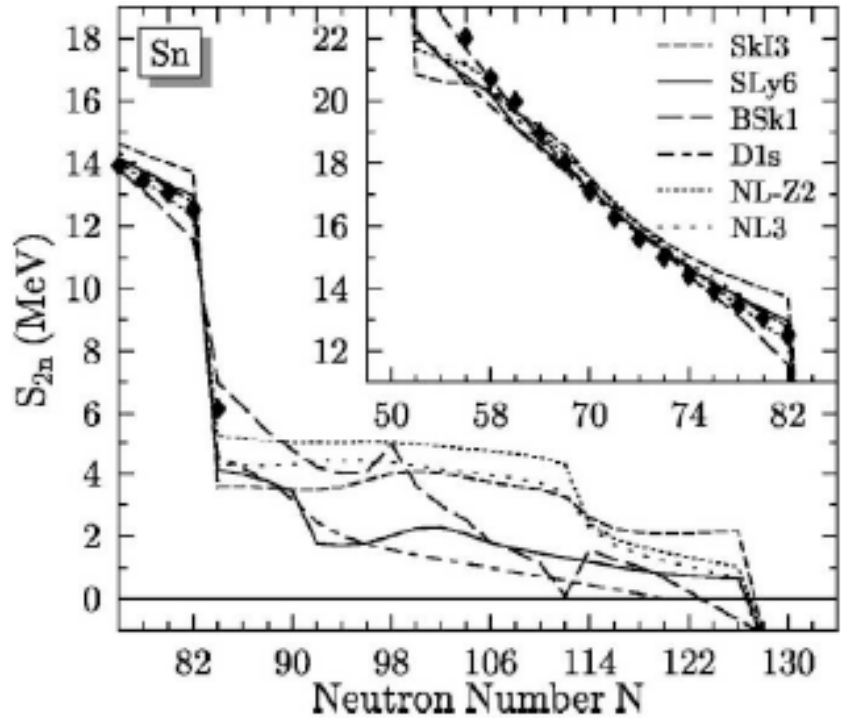


Energy functionals from chiral NN and NNN interactions

S.K. Bogner (NSCL/MSU)

Limitations of Existing Energy Functionals (Predictability)





- Uncontrolled extrapolations towards the drip-line
- Theoretical error-bars?

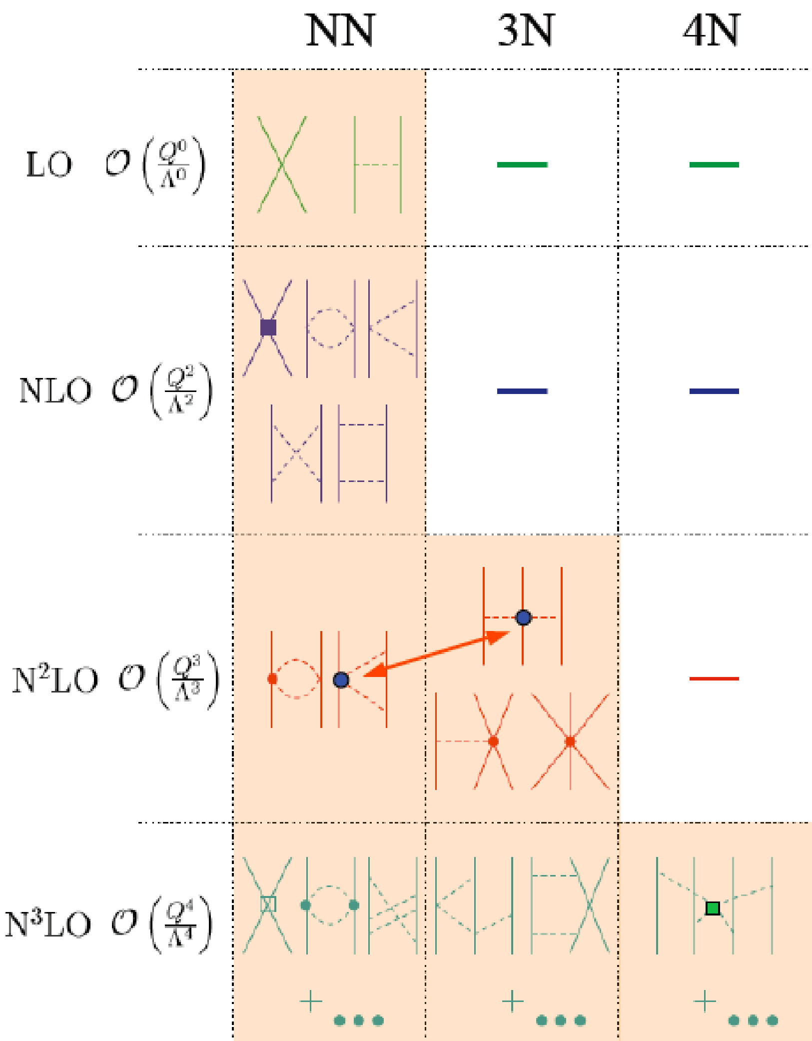
What could be missing in Skyrme?

- Density dependencies might be too simplistic (integer powers of local densities)
- Isovector components not well constrained (no explicit pion physics)
- No systematic organization of terms in the EDF
- No way to estimate theoretical uncertainties
- What's the connection to many-body forces?
- Spin-orbit too simplistic (e.g., at microscopic level NN is short-ranged while NNN is long-ranged)

Turn to microscopic many body theory for guidance

Nuclear forces from Chiral EFT

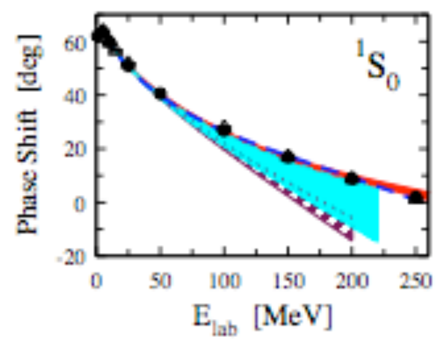
Separation of scales: low momenta Q << Λ_b breakdown scale

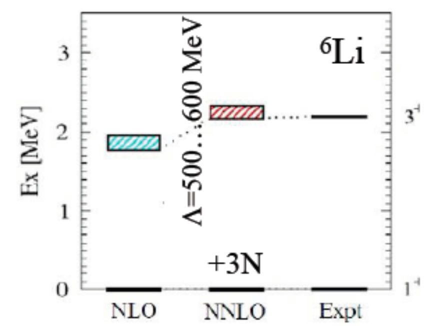


- Explains empirical hierarchy
  NN > 3N > 4N

- Formal Consistency
  NN and NNN forces
  ππ and πN, electroweak operators
  QCD, systematic expansion

- Error estimates from truncation order, lower bound from Λ variation





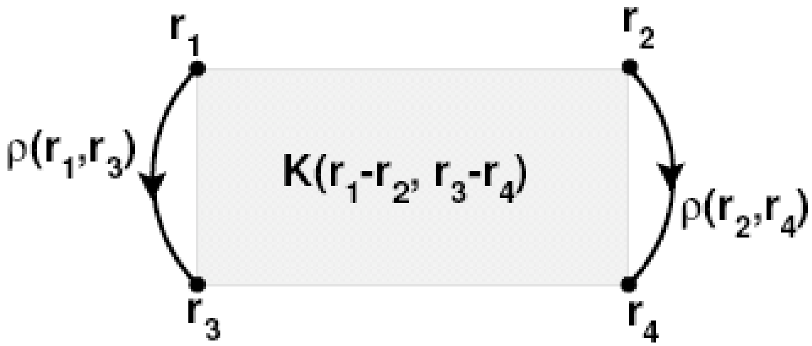
from A. Nogga

Weinberg, van Kolck, Epelbaum, Meissner, Machleidt, …

Local Functionals from Many-Body Theory

- Dominant MBPT contributions to bulk properties take the form

$$\langle V \rangle \sim \mathrm{Tr}_1 \mathrm{Tr}_2 \int d\mathbf{R}\, d\mathbf{r}_{12}\, d\mathbf{r}_{34}\, \rho(\mathbf{r_1}, \mathbf{r_3})\, K(\mathbf{r}_{12}, \mathbf{r}_{34})\, \rho(\mathbf{r_2}, \mathbf{r_4}) \; + \; \mathrm{NNN} \cdots$$



K is either free-space interaction (HF) or resummed in-medium vertex (BHF)

- Written in terms on **non-local** quantities
  - density matrices and s.p. propagators
  - finite range and non-local resummed vertices K

Connection to $E = E[\rho]$ is not obvious!

Density Matrix Expansion Revisited (Negele and Vautherin)

- Expand of DM in local operators w/factorized non-locality

$$\langle\Phi|\psi^\dagger(\mathbf{R}-\frac{1}{2}\mathbf{r})\psi(\mathbf{R}+\frac{1}{2}\mathbf{r}|\Phi\rangle = \sum_n \Pi_n(k_F r)\langle\mathcal{O}_n(\mathbf{R})\rangle$$

$$\langle\mathcal{O}_n(\mathbf{R})\rangle = [\rho(\mathbf{R}), \nabla^2\rho(\mathbf{R}), \tau(\mathbf{R}), \mathbf{J}(\mathbf{R}), \ldots]$$

- Fall off in r controlled by local k_F

N&V => expand and resum so LO term exact in uniform limit

$$\rho(\mathbf{R}+\frac{1}{2}\mathbf{r}, \mathbf{R}-\frac{1}{2}\mathbf{r}) = \frac{3j_1(k_F r)}{k_F r}\rho(R) + \frac{35 j_3(k_F r)}{2k_F^3 r}\left(\frac{1}{4}\nabla^2\rho(R) - \tau(R) + \frac{3}{5}k_F^2\rho(R)\right) + \cdots$$

- Dependence on local densities now manifest

$$\langle V\rangle \sim \sum_{n,m}\int d\mathbf{R}\,\mathcal{O}_n(\mathbf{R})\mathcal{O}_m(\mathbf{R})\int d\mathbf{r}\,\Pi_n(k_F r)\Pi_m(k_F r)V(r)$$

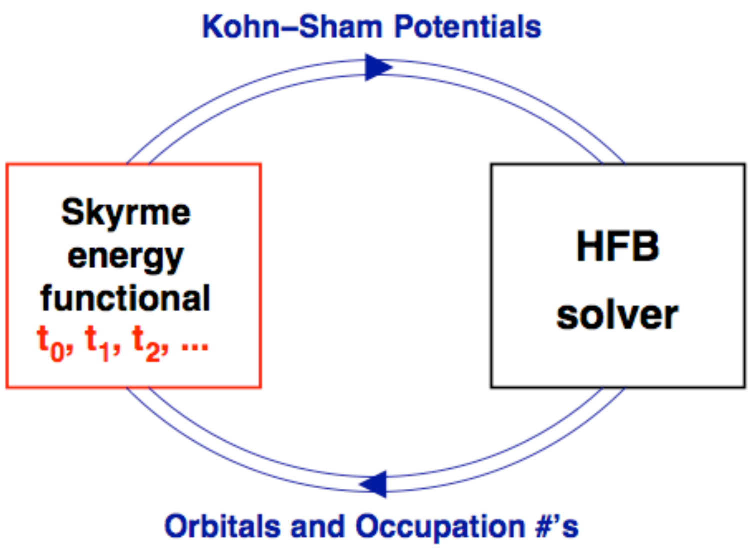
Skyrme-like EDF's from the DME

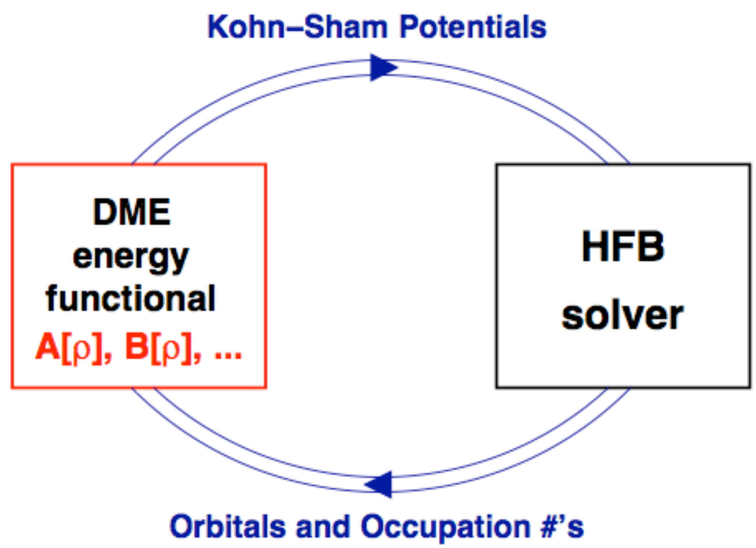
$$\mathcal{E} = \frac{\tau}{2M} + \frac{3}{8} t_0 \rho^2 + \frac{1}{16} t_3 \rho^{2+\alpha} + \frac{1}{16}(3t_1 + 5t_2)\rho\tau + \frac{1}{64}(9t_1 - 5t_2)|\nabla\rho|^2 + \cdots \quad \text{Skyrme}$$

$$\mathcal{E} = \frac{\tau}{2M} + A[\rho] + B[\rho]\tau + C[\rho]|\nabla\rho|^2 + \cdots \quad \text{DME}$$

- coupling *constants* --> coupling *functions*

  -finite range effects encoded as ρ-dependence in ABC
  - microscopic isovector, spin-orbit terms
  - well-suited for existing SkyHF codes





Including Long Range Chiral EFT in Skyrme-like EDFs

Derived the most general (N≠Z, even-even) EDF from chiral EFT thru N^2LO at HF level

$$\varepsilon_{sk}[\rho] = \sum_q \int d\vec{R} \left\{ A^{\rho\rho}\,\rho_q^2 + A^{\rho\tau}\,\rho_q\,\tau_q + A^{\rho\Delta\rho}\,\rho_q\,\Delta\,\rho_q + A^{\nabla\rho\nabla\rho}\,\vec{\nabla}\rho_q \cdot \vec{\nabla}\rho_q + A^{\nabla\rho J}\,\vec{\nabla}\rho_q \cdot \vec{J}_q + A^{J^2}\,\vec{J}_q \cdot \vec{J}_q \right\} + \ldots$$

Each coupling function splits into 2 terms

1) Λ-**dependent** Skyrme-like coupling **constants**
2) Λ-**independent** coupling **functions** from pion physics

$$A^{\rho\Delta\rho} \Rightarrow A^{\rho\Delta\rho}(\Lambda) + A^{\rho\Delta\rho}[\rho]$$

Etc…

From contact terms in EFT/RG V's

From pion exchanges

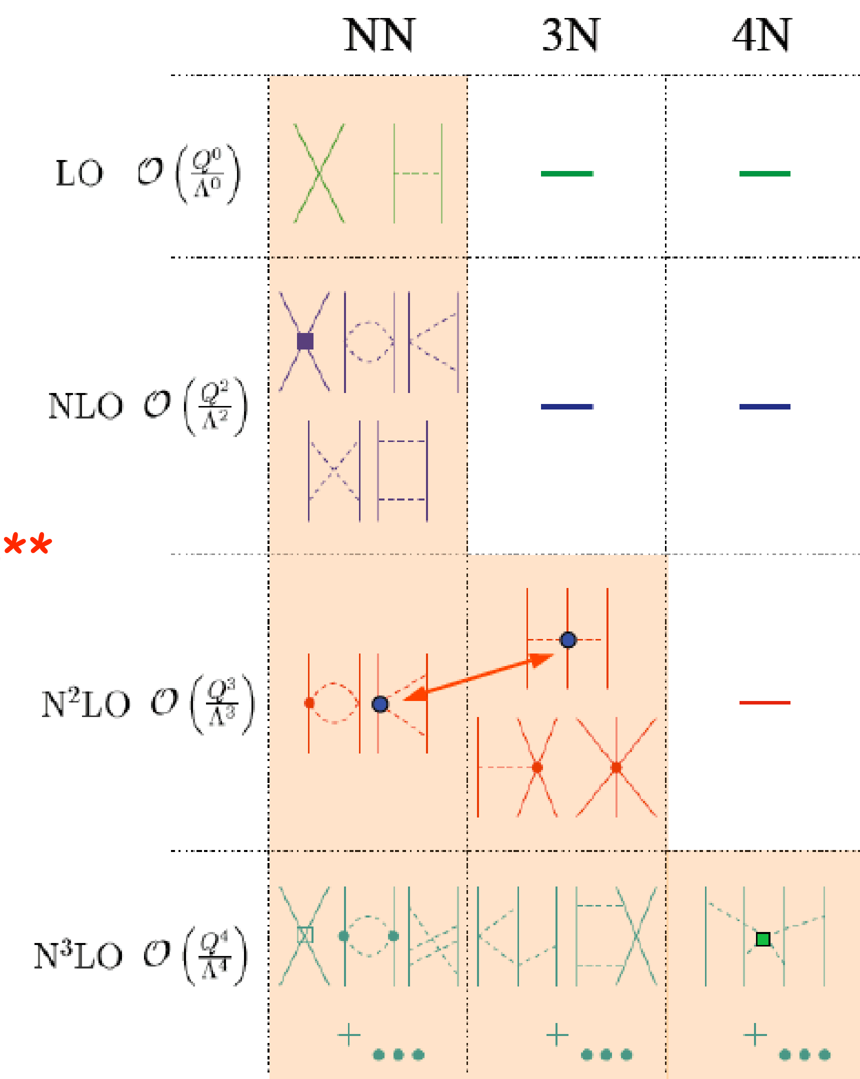
Including Long Range Chiral EFT in Skyrme-like EDFs

- DME coupling functions from **finite range** NN and NNN chiral EFT

- Refit short-distance coupling constants (EFT constraints => naturalness, Λ-dependence, etc.)**

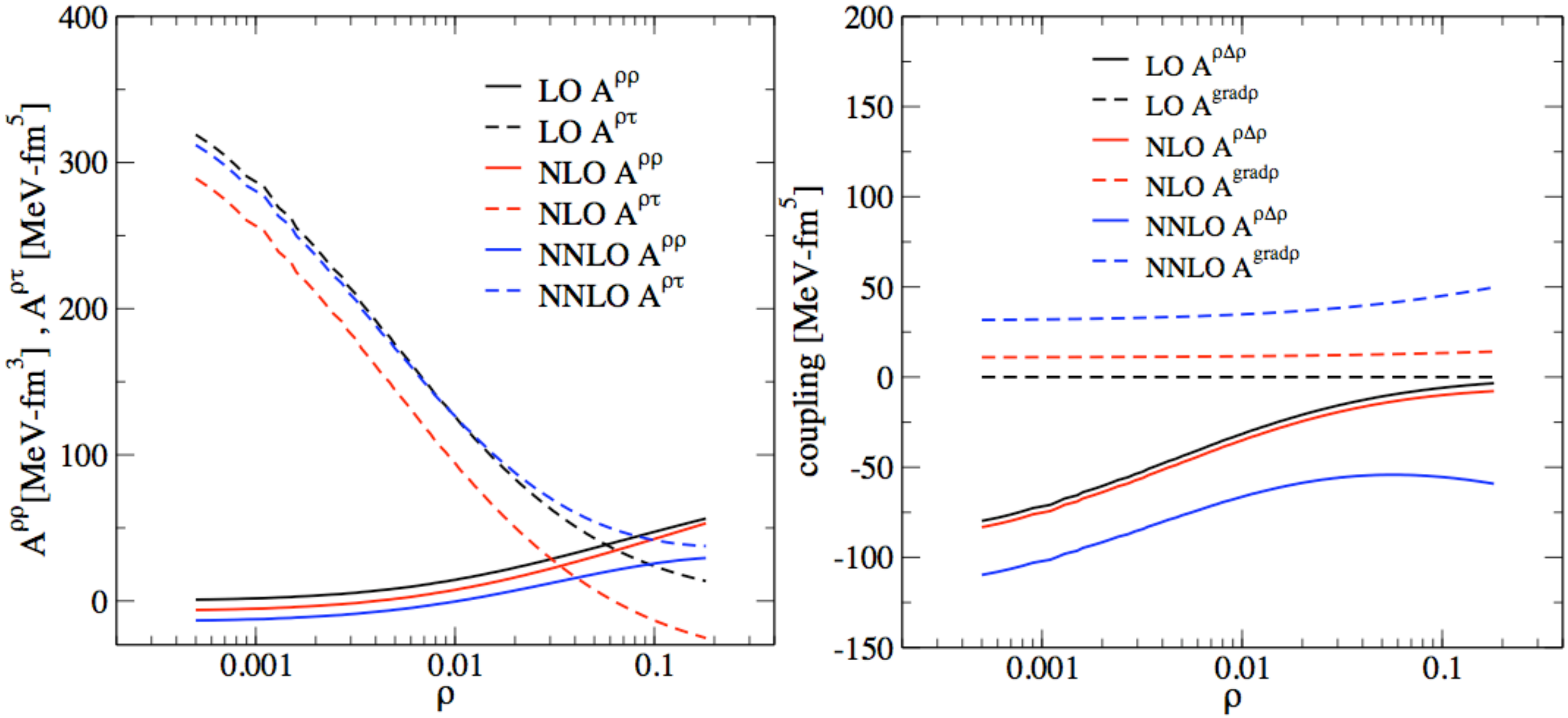
- Look for improved observables and for sensitivities

- E.g., can we "see" the pion as in NN phase shift analyses



** Bayesian methods (Schindler and Phillips)??

Long-range pion exchange contributions to the EDF



Longest range V <==> Strongest density dependence in EDF

Novel density-dependencies in EDF from 1π and 2π exchanges:

$$\rho^{7/3}, \quad \rho^{4/3}, \quad \rho^{2/3}, \quad \frac{1}{\rho^{2/3}} log(1 + c\rho^{2/3}), \quad \ldots$$

More exotic density dependencies possible

The usual LDA choice for k_F in the DME gives:

$$\rho^{7/3}, \;\; \rho^{4/3}, \;\; \rho^{2/3}, \;\; \frac{1}{\rho^{2/3}} log(1 + c\rho^{2/3}), \;\; \ldots$$

Campi-Bouyssi choice for k_F

$$k_F = \sqrt{\frac{5}{3} \frac{\tau - \frac{1}{4}\nabla^2\rho}{\rho}}$$

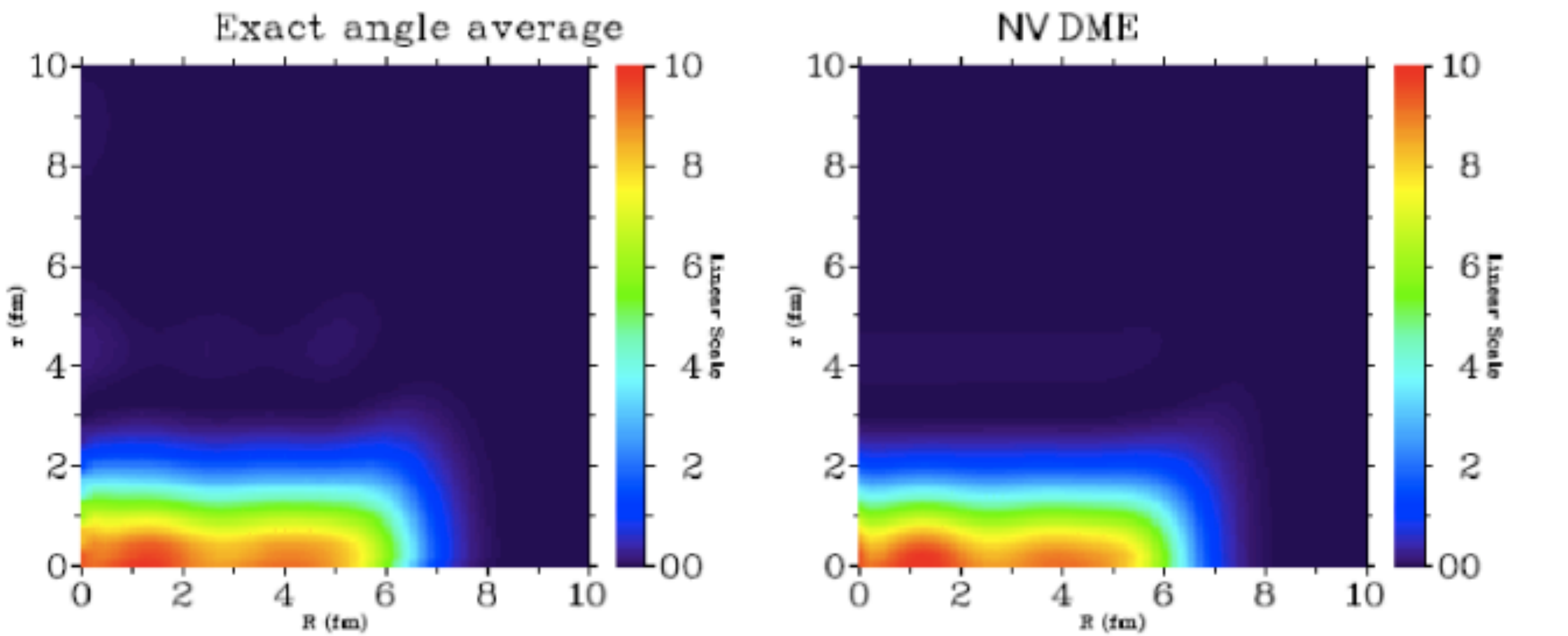
E.g., "resummed" τ and $\nabla^2 \rho$ dependence in EDF

$$\frac{1}{\rho^{2/3}} \Rightarrow \frac{\rho}{\tau - \frac{1}{4}\nabla^2\rho} \quad , \;\; \text{etc.}$$

Accuracy of the NVDME - Scalar exchange

$$F_{s,\mathrm{exch}}(R,r) = \int d\Omega_r\, \rho_q(\mathbf{r}_1,\mathbf{r}_2)\rho_q(\mathbf{r}_2,\mathbf{r}_1)$$

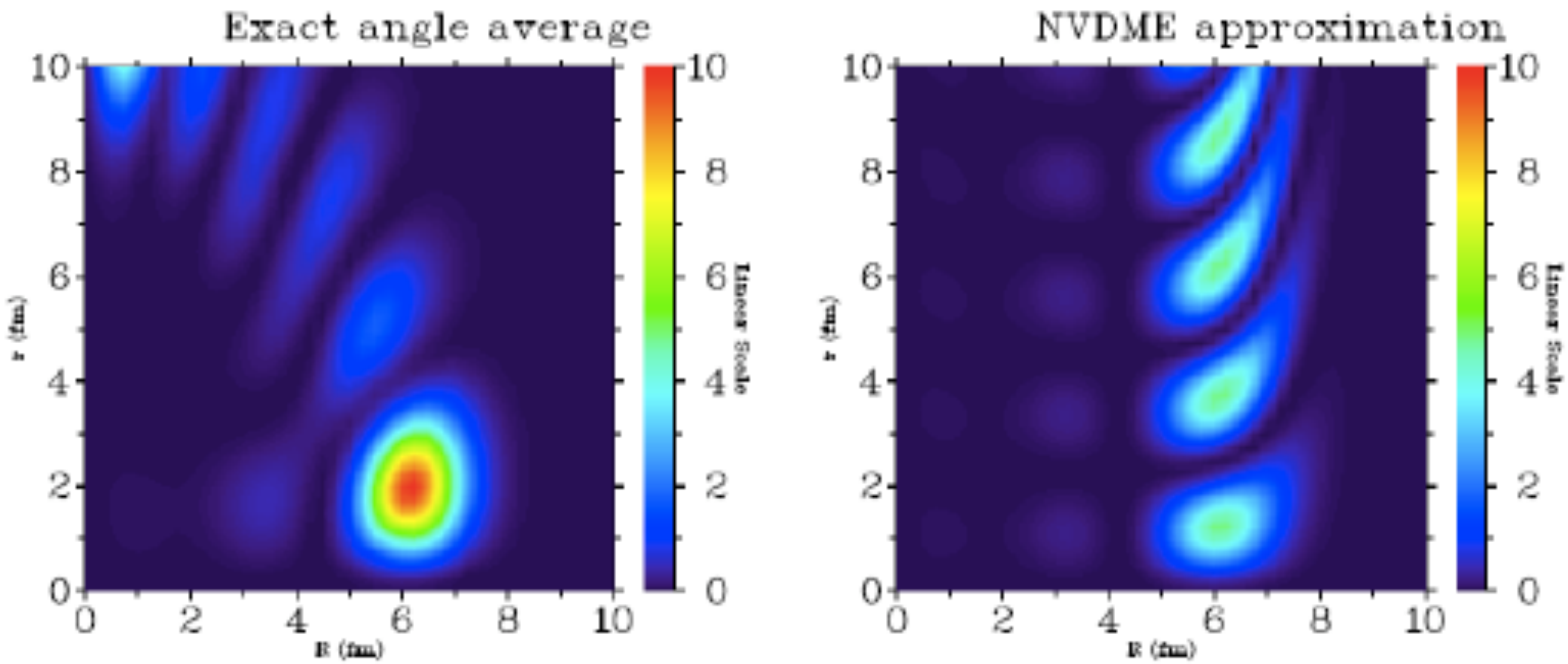
(Gebremariam et al.)



- Scalar NVDME works pretty good for exchange
- What about spin-vector piece?

Accuracy of the NVDME - Vector exchange

$$\int d\Omega_r \, \mathbf{s}_n(\mathbf{r}_1, \mathbf{r}_2) \cdot \mathbf{s}_n(\mathbf{r}_2, \mathbf{r}_1)$$

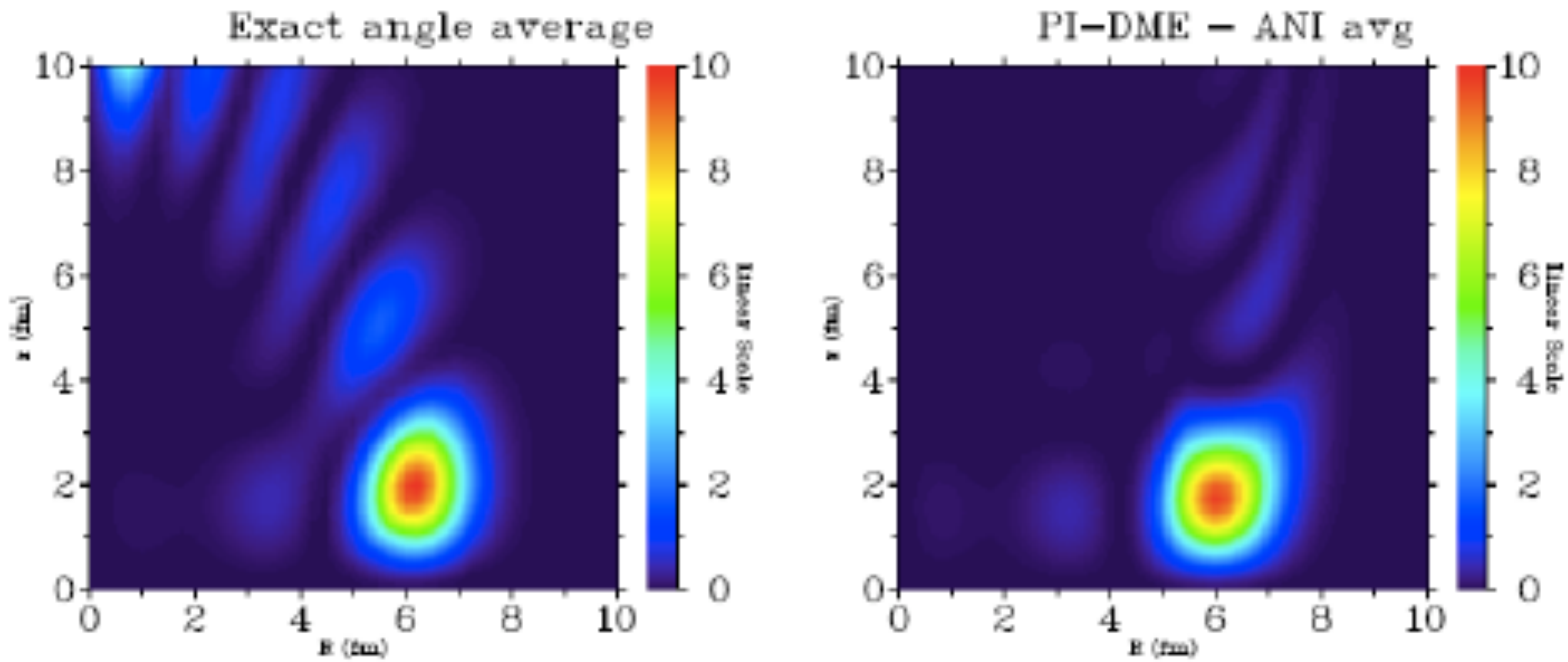


- NVDME neglects 2 key aspects of **finite** fermi systems
  - anisotropy of local momentum distribution **at the surface**
  - diffuseness of local momentum distribution **at the surface**

(Gebremariam et al)

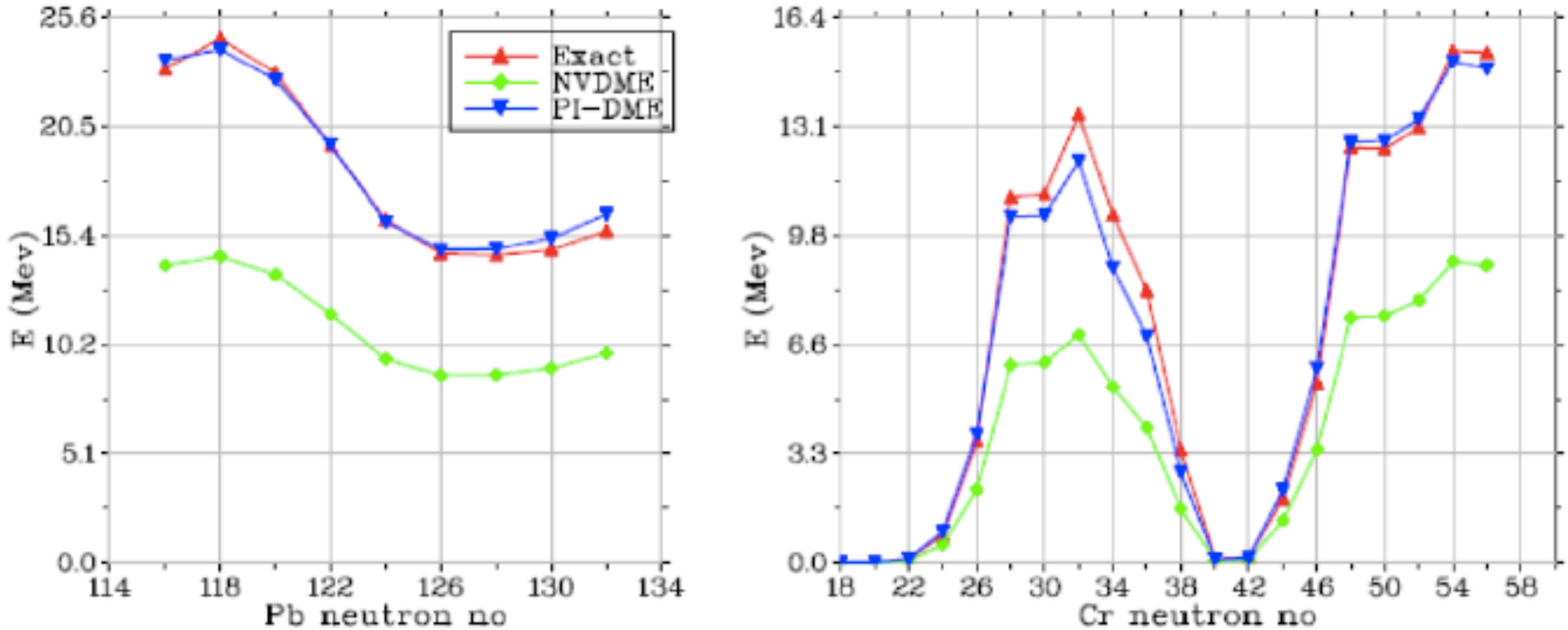
Improved Vector DME (Gebremariam et al.)

$$\int d\Omega_r\, \mathbf{s}_n(\mathbf{r}_1, \mathbf{r}_2) \cdot \mathbf{s}_n(\mathbf{r}_2, \mathbf{r}_1)$$



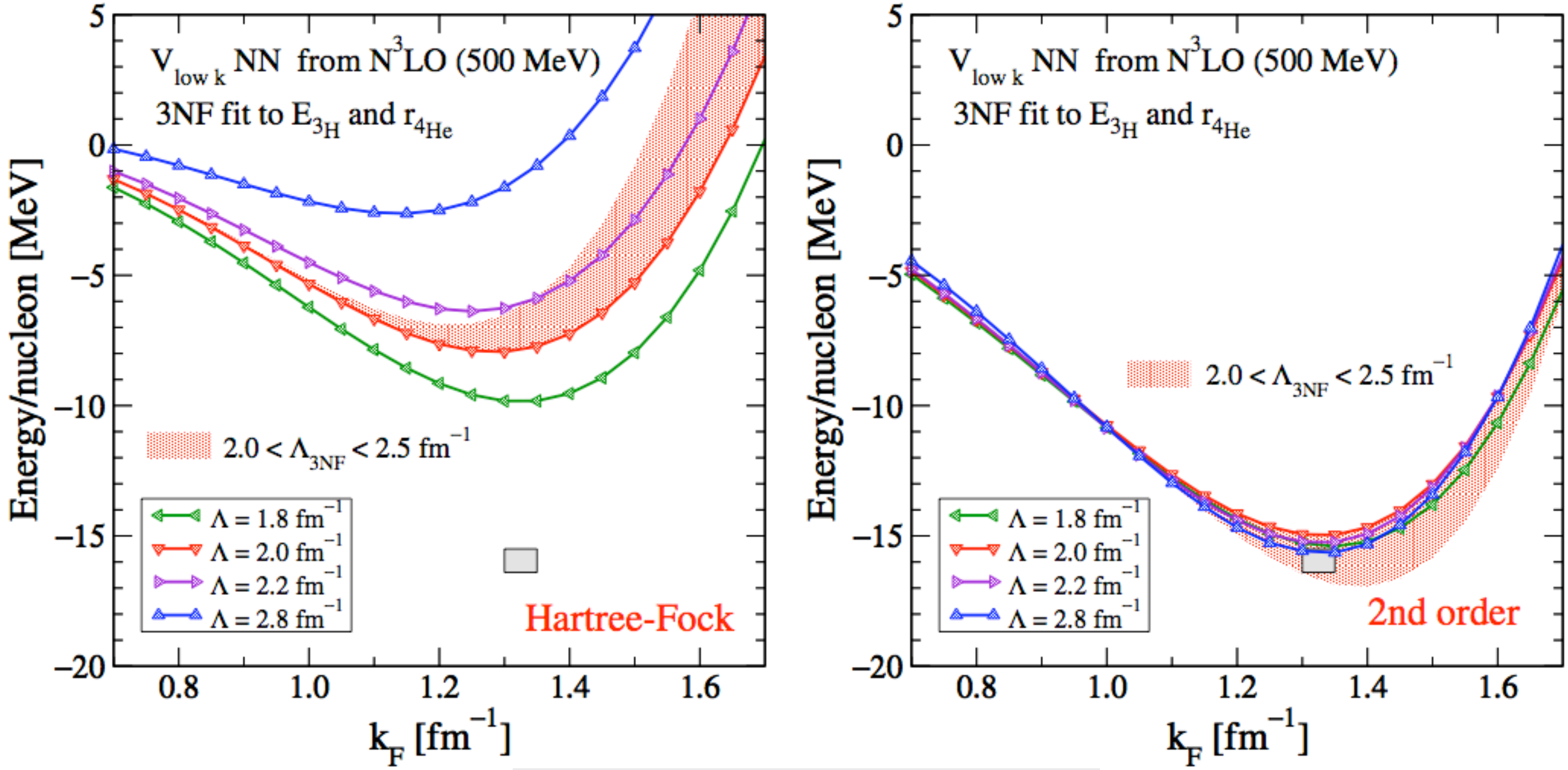
• Modified DME ("PI-DME") to include n(k,R) surface anisotropy and diffuseness effects appropriate for finite fermi systems

Look at $\int d\mathbf{r}\, d\mathbf{R}\, V_{1\pi}(r)\, \mathbf{s}_n(\mathbf{r}_1, \mathbf{r}_2) \cdot \mathbf{s}_n(\mathbf{r}_2, \mathbf{r}_1)$:



- Inclusion of finite fermi phase space effects crucial for **quantitative** agreement
- NVDME still has OK systematics => fixable by scale factor?

New low-momentum NNN fits and Nuclear Matter

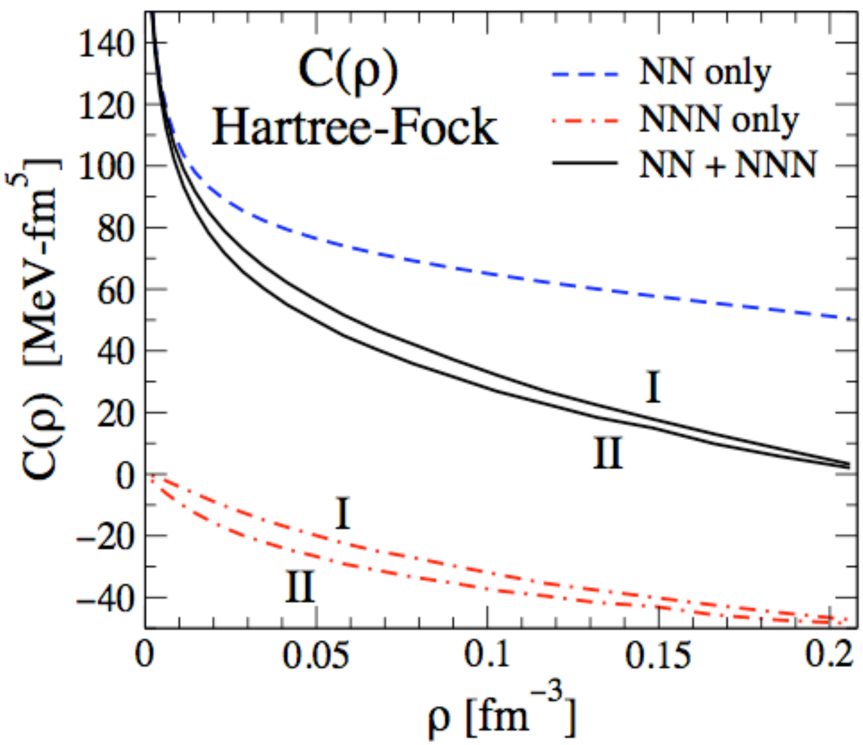


Perturbative expansion
about HF becomes sensible

OK to use DME at HF level to constrain EDF

Effects of NNN on Couplings

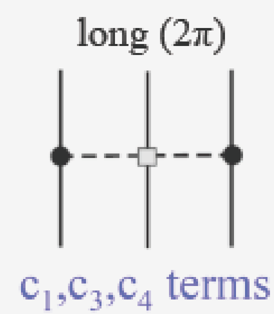
Gradient term $(\nabla\rho)^2$



SKB et. al., arXiv:0811.4198

- scalar-isoscalar terms worked out so far
- Consistent with Kaiser et al. results with explicit Δ's

Near Term:



Spin-orbit couplings from 2π 3NF

Should find interesting **density dependencies** compared to NN spin-orbit

Work in the near-term

- Generate DME couplings from N^2LO 3NF
  - 90% complete
  - Automated via Mathematica -> "easy" to generate N^3LO 3NF and 4NF contributions

- Higher order EFT contacts => higher gradients a-la Carlsson et al.

- Release f95 modules (DME couplings + s.p. fields)

- Refits of Skyrme + Long-range coupling functions (ORNL group)

- Gauging accuracy of PI-DME in self-consistent HF (B. Gebremariam)

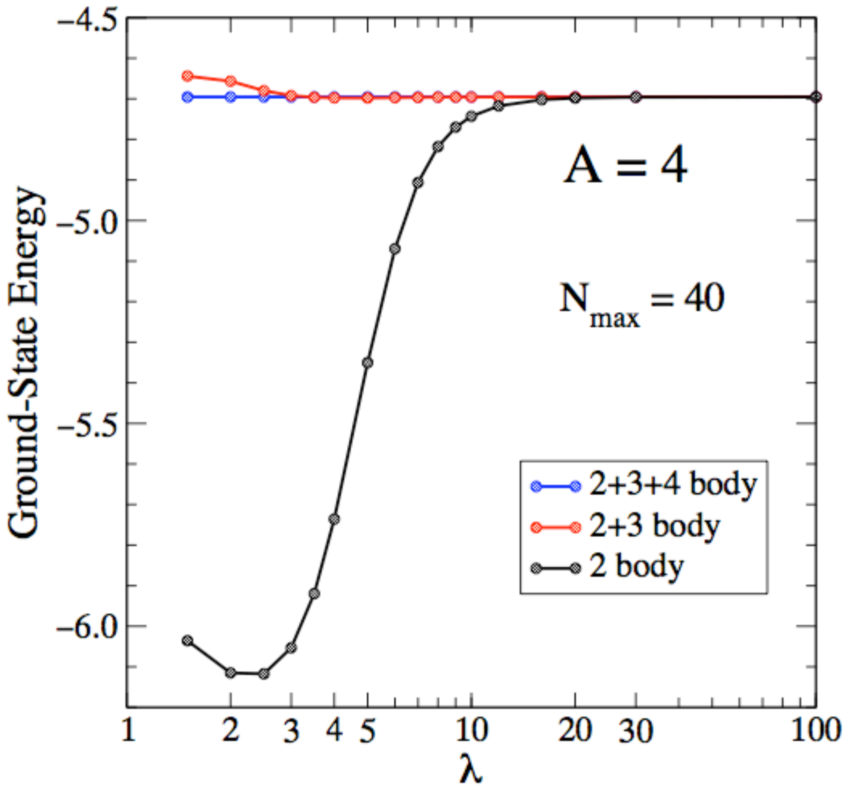
- Generalization of DME to handle non-localities in time (I.e., energy-dependence from beyond HF)

Collaborators

- MSU/NSCL: B. Gebremariam
- Ohio State: R. Furnstahl, L. Platter
- ORNL: N. Schunck, M. Stoitsov
- Orsay/France: T. Duguet

Progress on “honest” 3NF RG evolutions

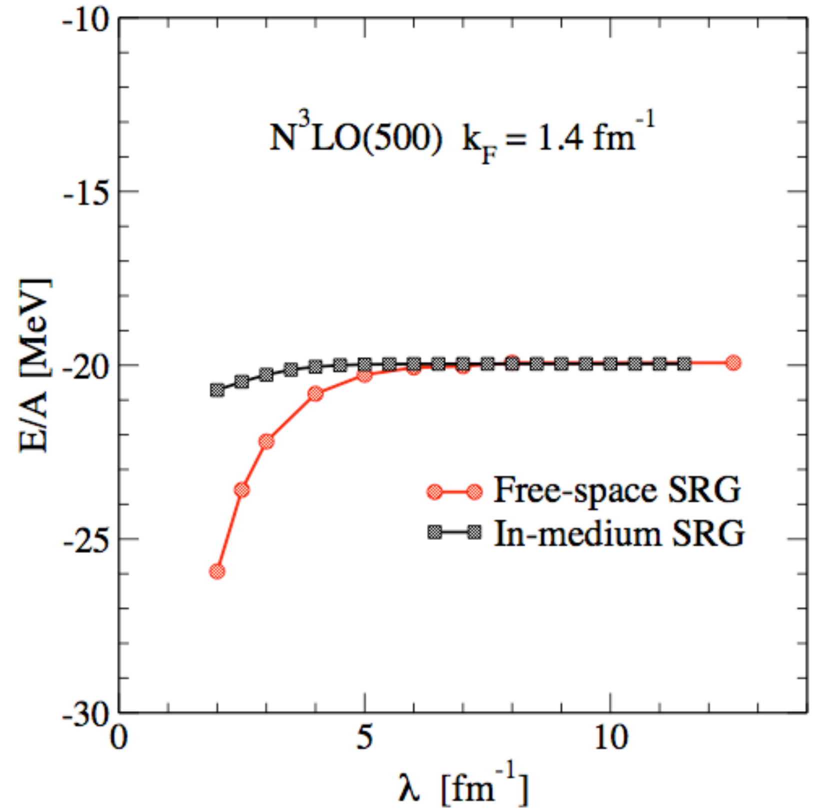
4-boson model problem



Anderson and Furnstahl 2008

SRG in H.O. basis

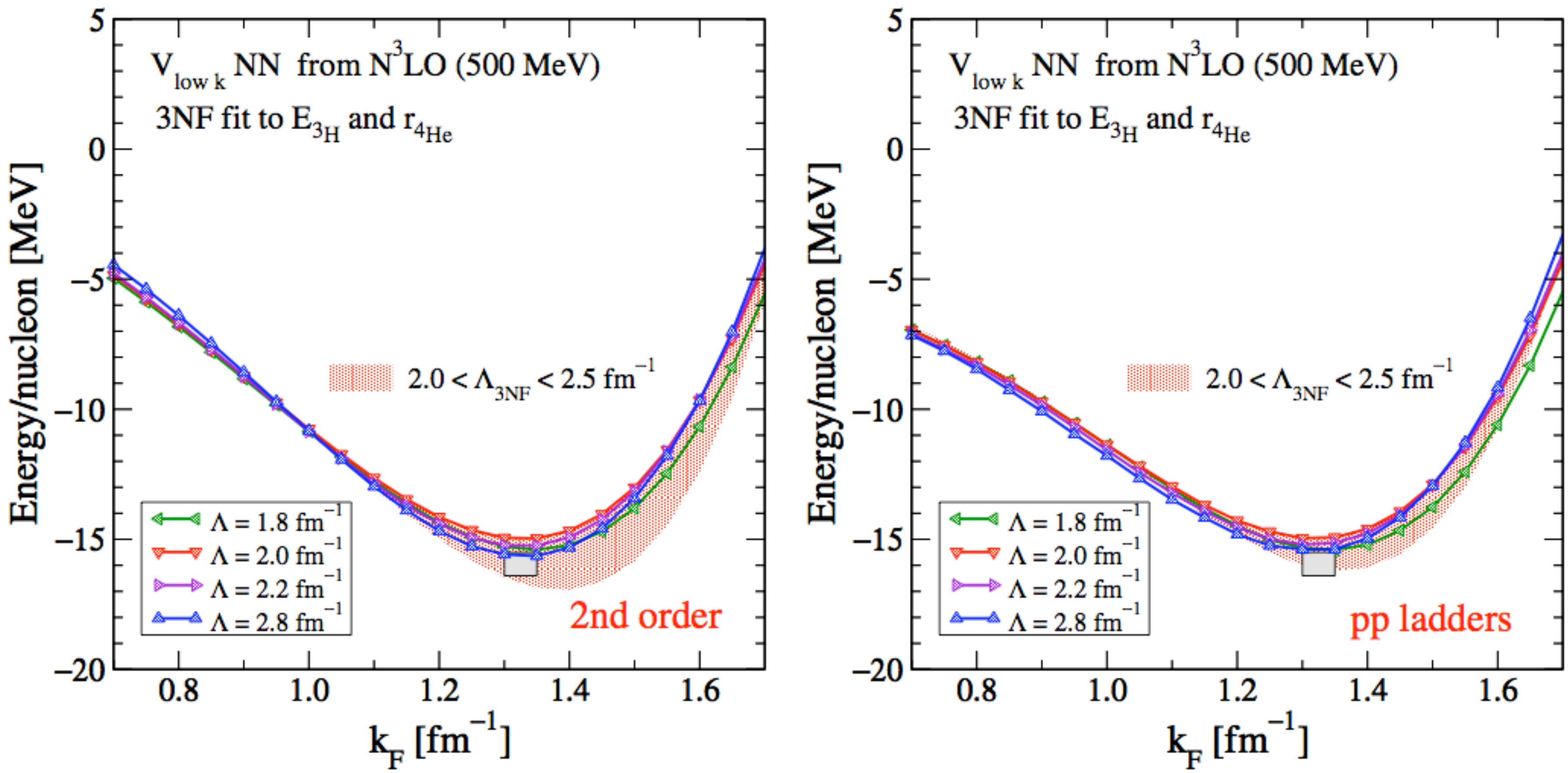
NM using in-medium SRG



S.K. Bogner, in prep

In-medium SRG w/normal-ordering

New low-momentum NNN fits and Nuclear Matter



Perturbative expansion
about HF becomes sensible