Fragment mass distributions by TDHF calculations for reactions involving deformed target nuclei

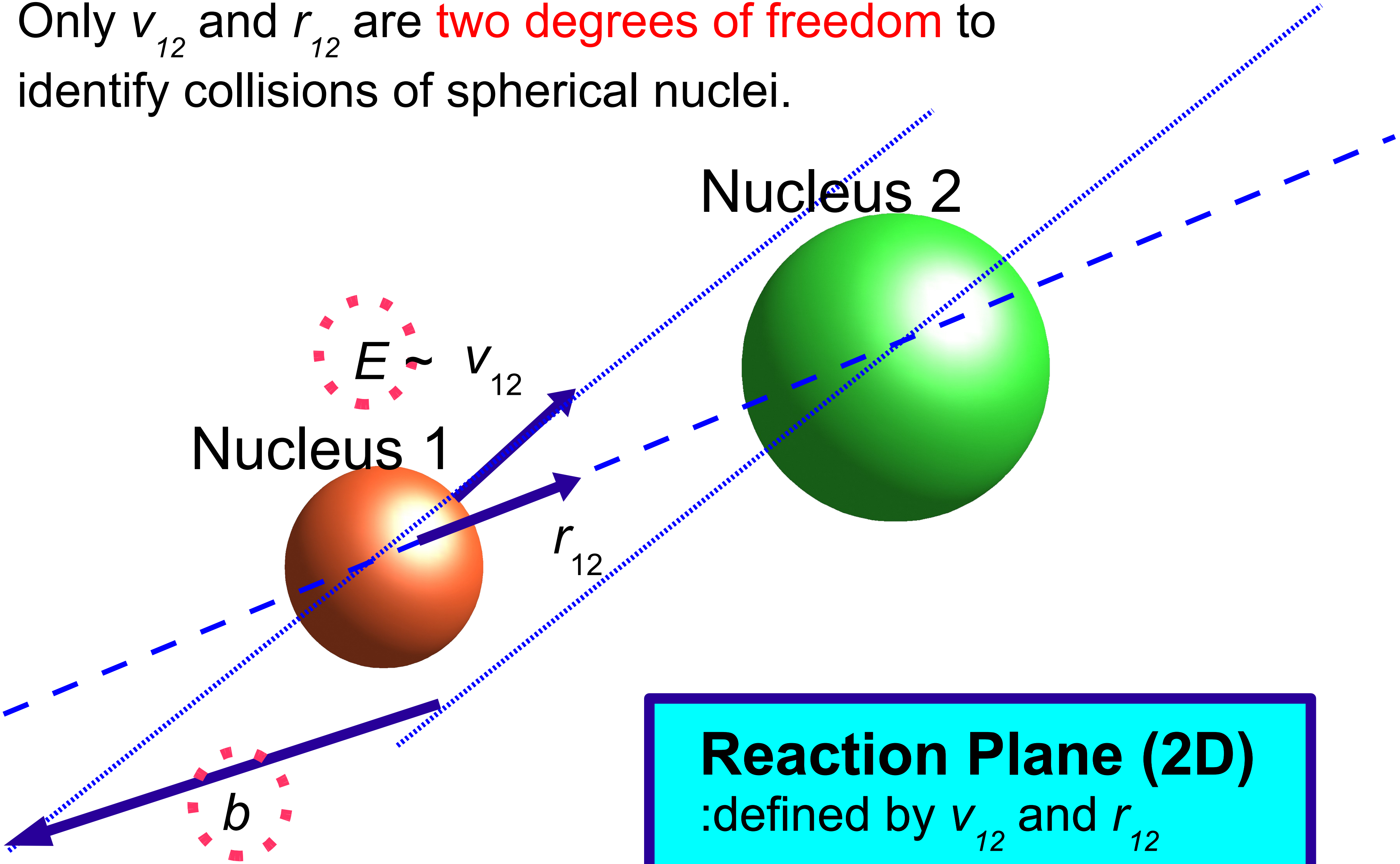
Yoritaka Iwata (University of Tokyo)
GSI (EMMI) from April 2009 -

Collaborations:
Takaharu Otsuka (Tokyo),
Naoyuki Itagaki (Tokyo),
Katsuhisa Nishio (JAEA),
Joachim A. Maruhn (Frankfurt)

Special thanks to:
H. Otsu, T. Ichikawa (Riken), C Simenel (Saclay)

Heavy-ion collisions of spherical nuclei

Only v_{12} and r_{12} are two degrees of freedom to identify collisions of spherical nuclei.



Reaction Plane (2D)
:defined by v_{12} and r_{12}

3-dimensional TDHF calculations

Systematic 3-dimensional TDHF calculations with respect to **"bombarding energy"** and **"impact parameter"** is performed for the following reactions:

Each 1MeV/A

Each 2.5fm

TDHF calculations with SLy4 interaction

^{208}Pb + ^{238}U

^{208}Pb + ^{132}Xe

^{208}Pb + ^{132}Sn

^{208}Pb + ^{24}Mg

^{208}Pb + ^{24}O

^{208}Pb + ^{4}He

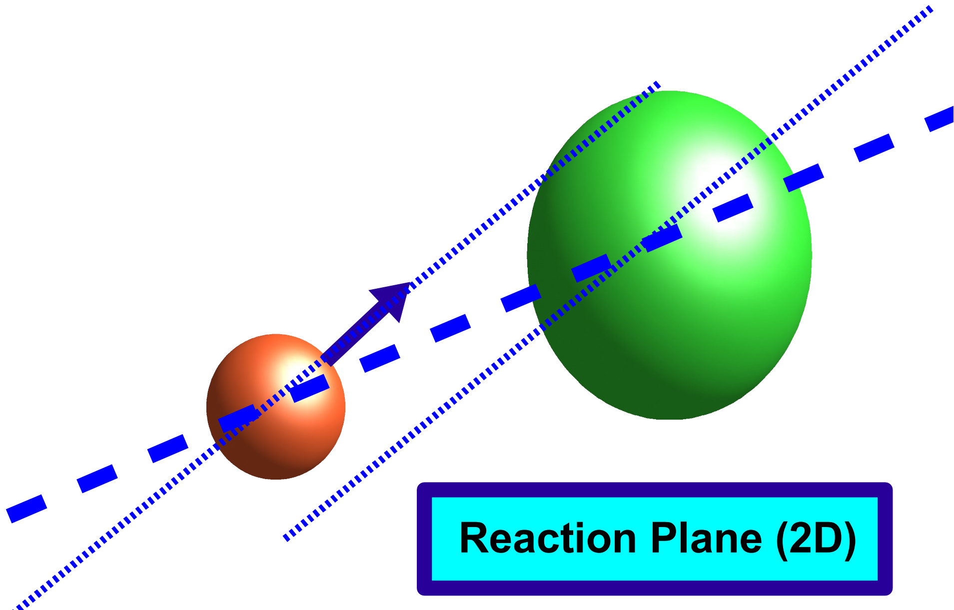
----

^{24}Mg + ^{24}O

^{208}Pb + ^{238}U

This is a large-scale calculation using ~ 30 CPU with 50GB memory (Cluster-Machine+others)

As a result, we have been calculated during 1 ~ 3 months for each collision.



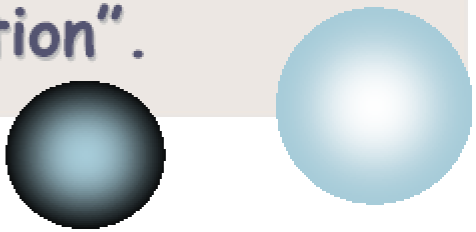
The concept: Charge equilibration

Charge equilibration is the homogenization of N/Z ratio during heavy-ion collisions.

Let us consider heavy-ion collisions of two different nuclei (more precisely, two nuclei with different N/Z ratio).

In some cases, N/Z ratio reaches the equilibrium during the collision, while it does not necessarily occurs. It depends on some conditions e.g. the bombarding energy, the impact parameter, and the sort of initial nuclei.

The equilibrium of N/Z ratio is called "**charge equilibrium**", and the process of reaching it during the collision is called "**charge equilibration**".



1) Charge equilibration upper energy-limit

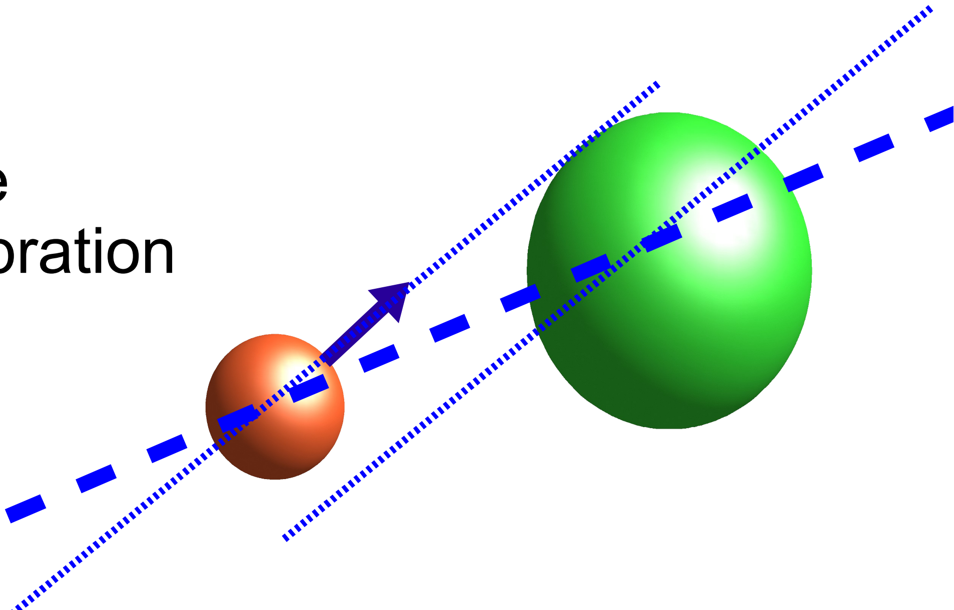
Question 1

Is there the upper energy-limit of charge equilibration ?

Let us verify it ...

Then, what is the decisive factor for the upper energy-limit ?

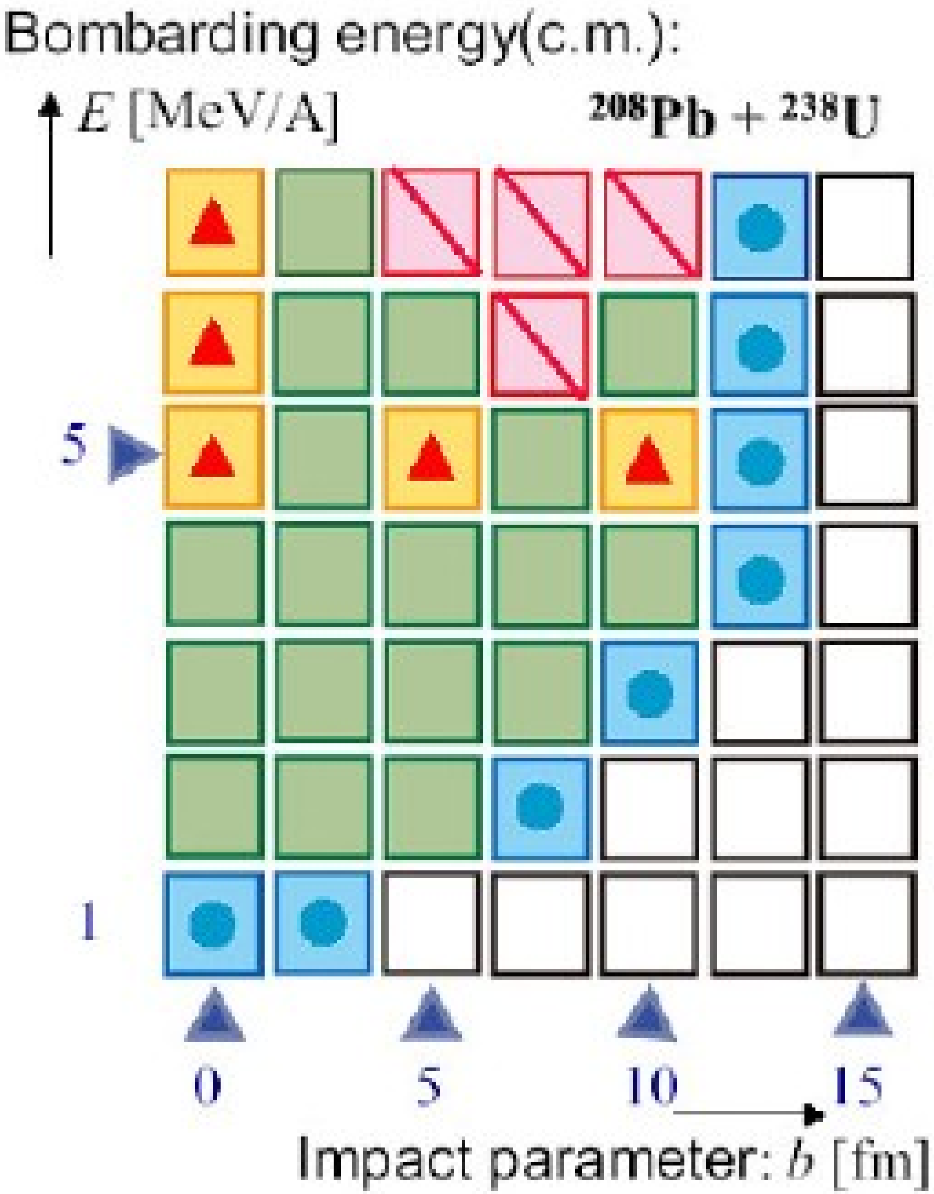
In this presentation, we discuss the decisive factor of the charge equilibration based on the 3-dimensional TDHF calculations.



Example of ^{208}Pb + ^{238}U

Systematic TDHF calculations

Sly4



☐ no contact

● contact without any nucleon transfer

🟩 fragmentation into two pieces

▲ fragmentation into three pieces

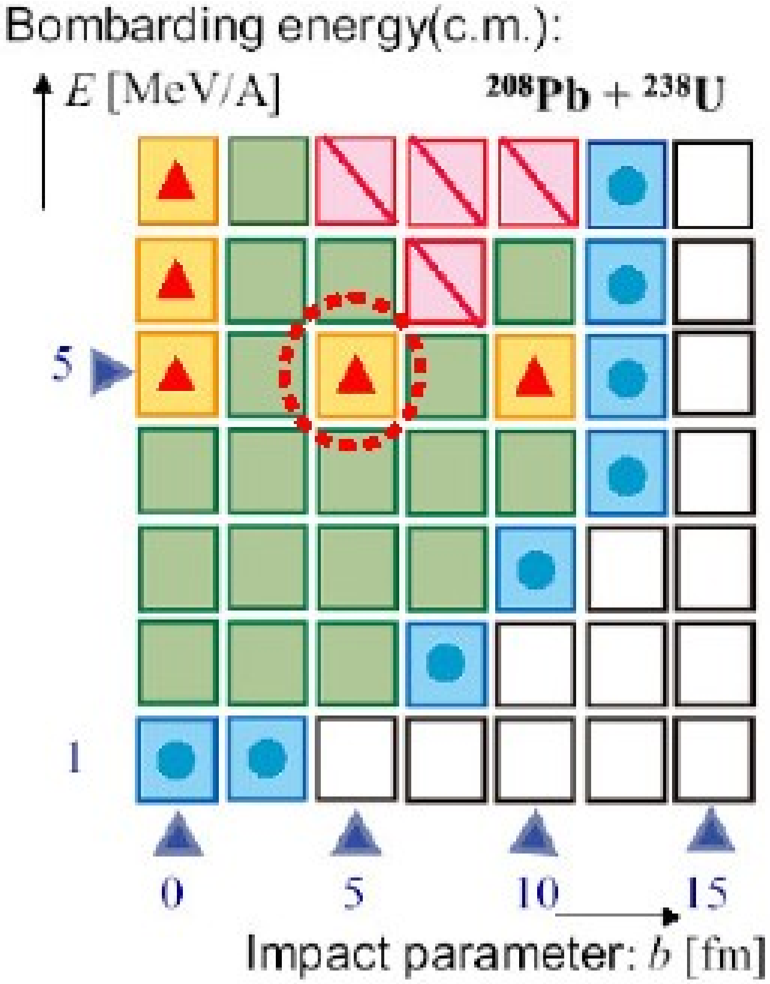
▧ fragmentation into more than three pieces

TDHF Calculations have been performed for b in steps of 2.5fm and Ec.m. in steps of 1 MeV/A.
Each box in the left figure corresponds to a single TDHF calculation, and color/markings distinguish the number of fragments.

As is shown in the figure, we have performed almost **50 different TDHF calculations** systematically.

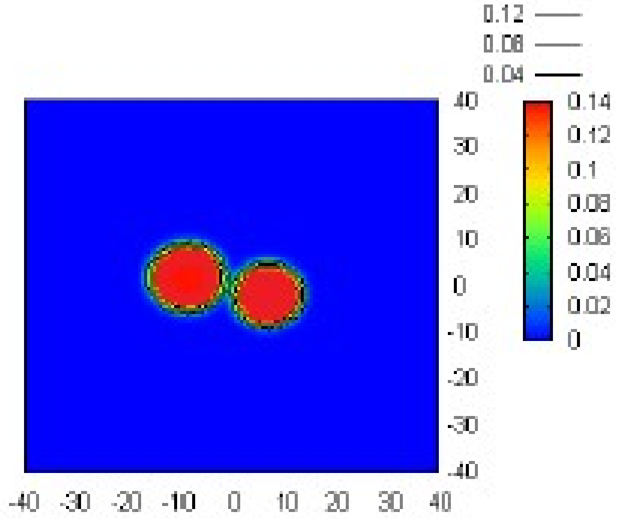
Example (deep inelastic collision)

Sly4



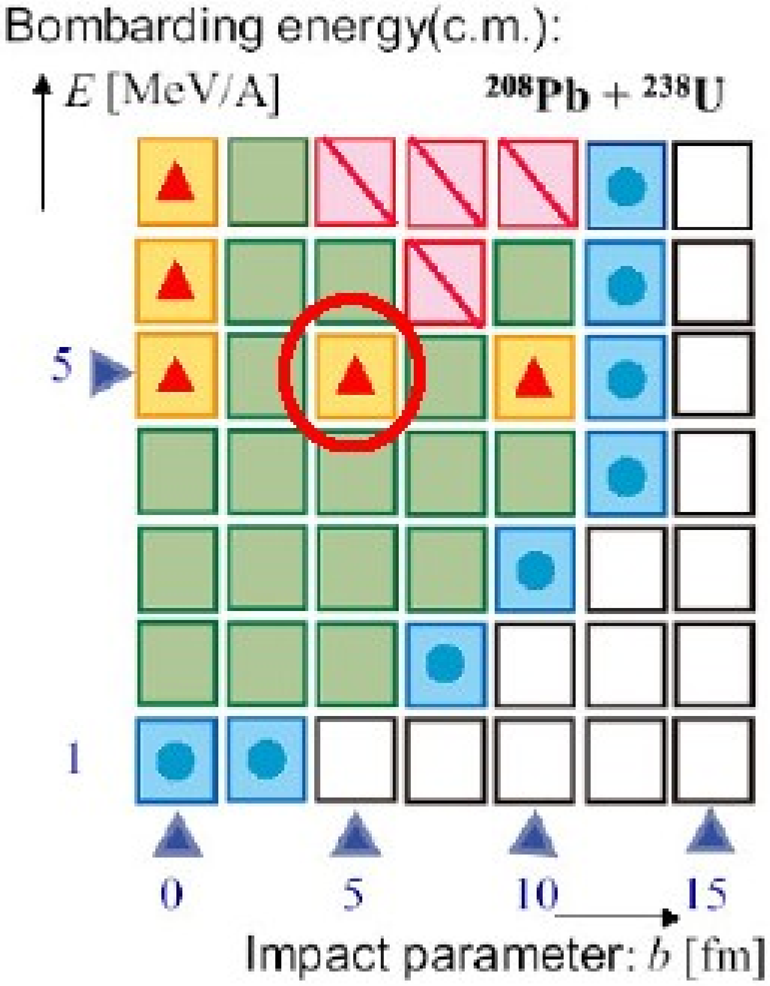
	▲	🟩	▨	▨	▨	●	☐
	▲	🟩	🟩	▨	🟩	●	☐
5	▲	🟩	▲	🟩	▲	●	☐
	🟩	🟩	🟩	🟩	🟩	●	☐
	🟩	🟩	🟩	🟩	●	☐	☐
	🟩	🟩	🟩	●	☐	☐	☐
1	●	●	☐	☐	☐	☐	☐
	0		5		10		15

- ☐ no contact
- ● contact without any nucleon transfer
- 🟩 fragmentation into two pieces
- ▲ fragmentation into three pieces
- ▨ fragmentation into more than three pieces

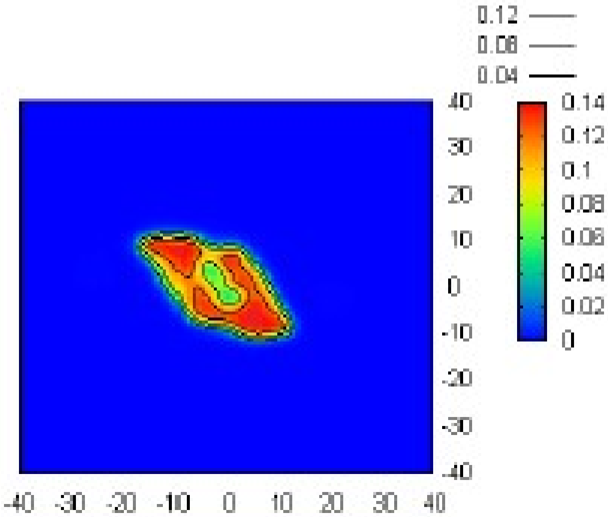


Example (deep inelastic collision)

Sly4

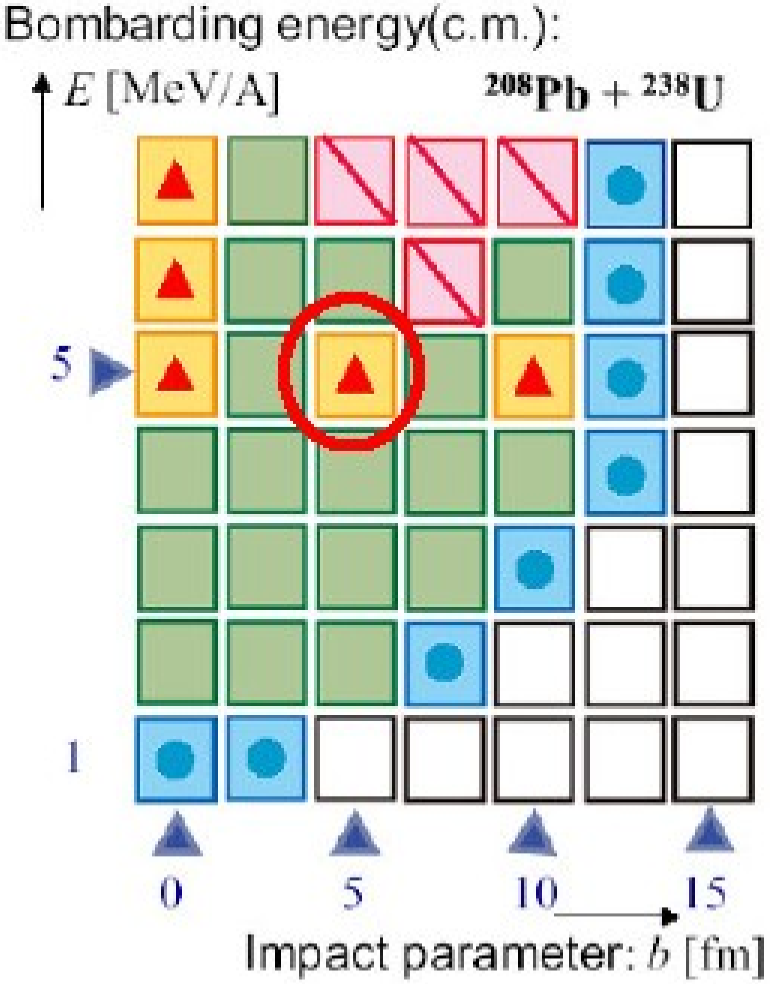


- ☐ no contact
- ● contact without any nucleon transfer
- 🟩 fragmentation into two pieces
- ▲ fragmentation into three pieces
- ▨ fragmentation into more than three pieces



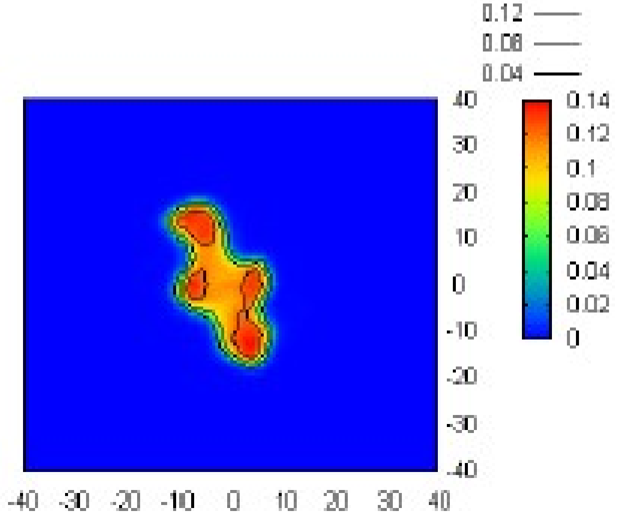
Example (deep inelastic collision)

Sly4



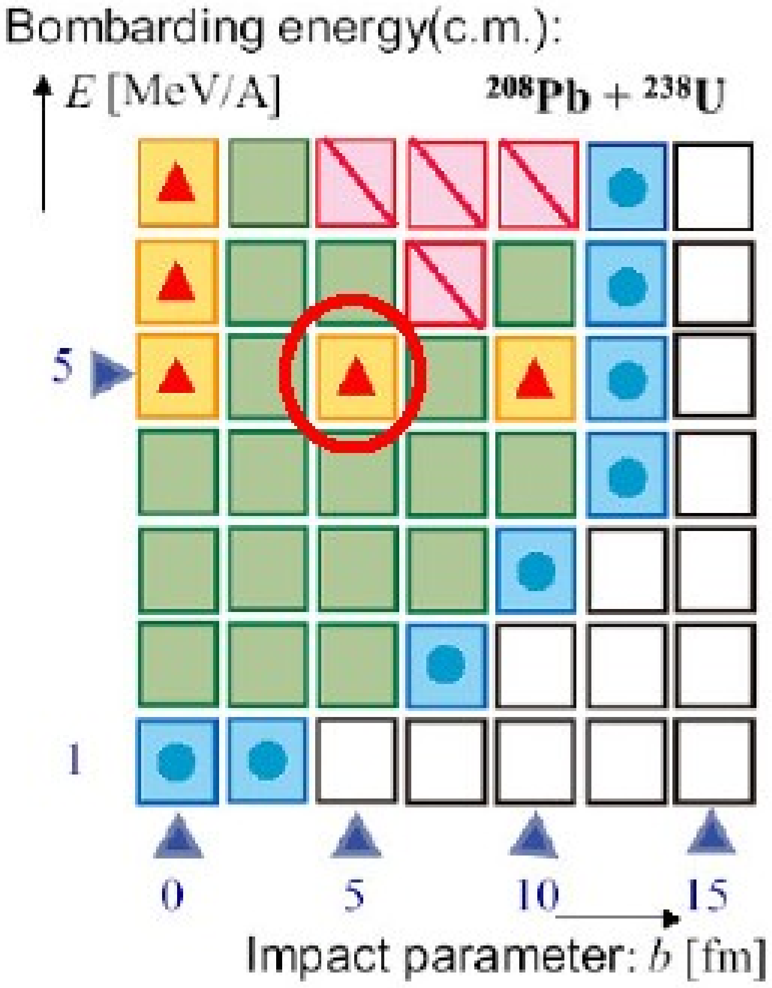
	▲	🟩	▨	▨	▨	●	☐
	▲	🟩	🟩	▨	🟩	●	☐
5	▲	🟩	▲	🟩	▲	●	☐
	🟩	🟩	🟩	🟩	🟩	●	☐
	🟩	🟩	🟩	🟩	●	☐	☐
	🟩	🟩	🟩	●	☐	☐	☐
1	●	●	☐	☐	☐	☐	☐
	0		5		10		15

☐ no contact
● contact without any nucleon transfer
🟩 fragmentation into two pieces
▲ fragmentation into three pieces
▨ fragmentation into more than three pieces



Example (deep inelastic collision)

Sly4



	▲	🟩	▨	▨	▨	●	☐
	▲	🟩	🟩	▨	🟩	●	☐
5	▲	🟩	▲	🟩	▲	●	☐
	🟩	🟩	🟩	🟩	🟩	●	☐
	🟩	🟩	🟩	🟩	●	☐	☐
	🟩	🟩	🟩	●	☐	☐	☐
1	●	●	☐	☐	☐	☐	☐
	0		5		10		15

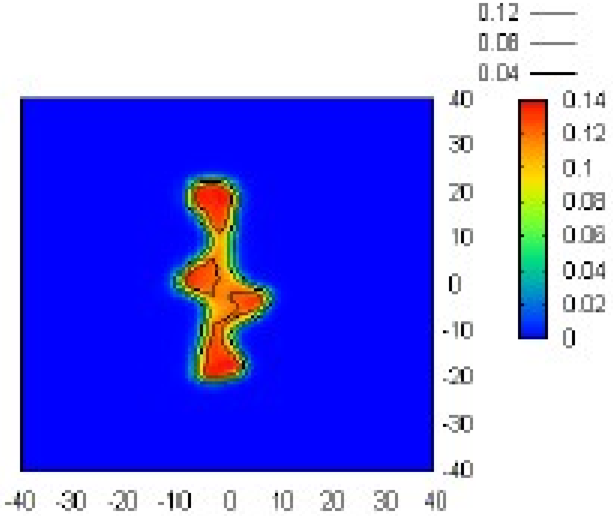
☐ no contact

● contact without any nucleon transfer

🟩 fragmentation into two pieces

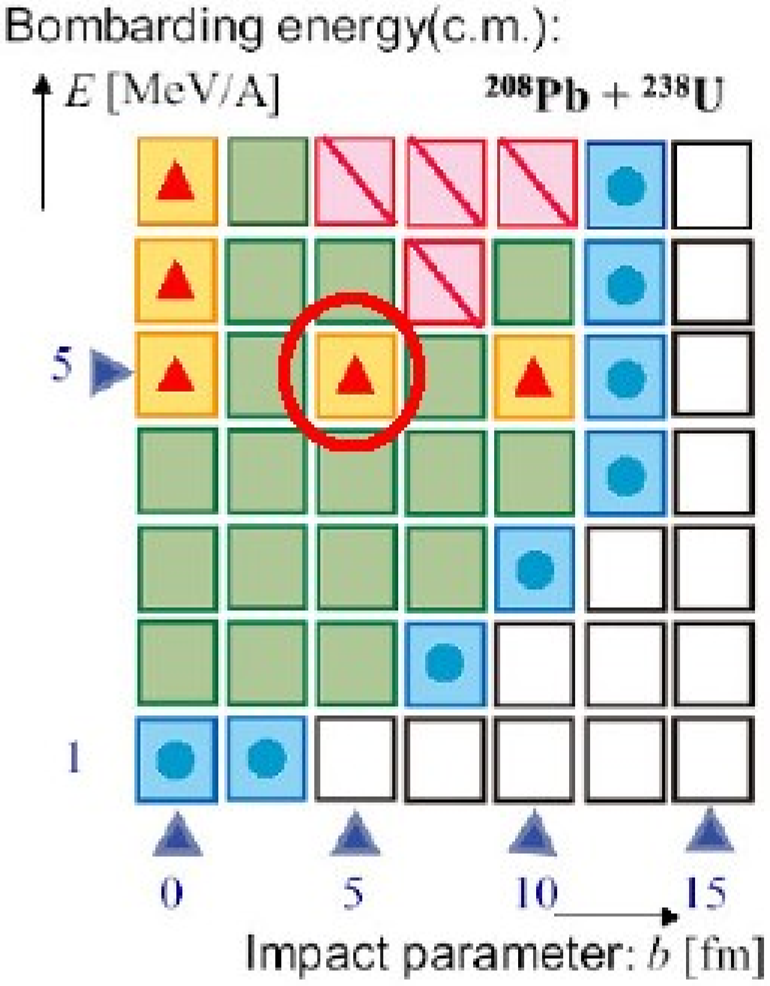
▲ fragmentation into three pieces

▨ fragmentation into more than three pieces



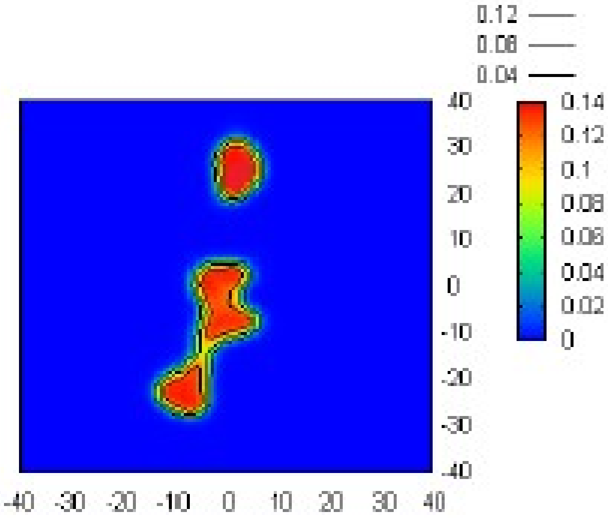
Example (deep inelastic collision)

Sly4

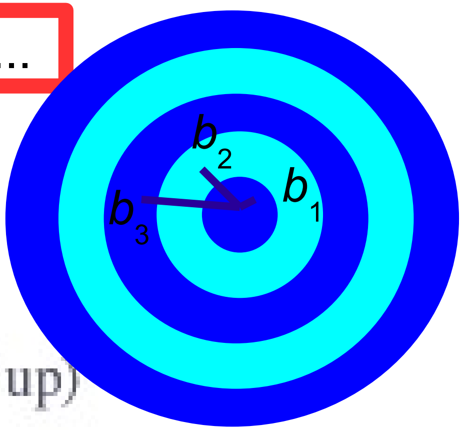


E [MeV/A]	0		5		10		15
	▲	🟩	▨	▨	▨	●	□
	▲	🟩	🟩	▨	🟩	●	□
5	▲	🟩	▲	🟩	▲	●	□
	🟩	🟩	🟩	🟩	🟩	●	□
	🟩	🟩	🟩	🟩	●	□	□
	🟩	🟩	🟩	●	□	□	□
1	●	●	□	□	□	□	□

- □ no contact
- ● contact without any nucleon transfer
- 🟩 fragmentation into two pieces
- ▲ fragmentation into three pieces
- ▨ fragmentation into more than three pieces



Weight ~ πb_1^2, $\pi(b_2^2 - b_1^2)$, $\pi(b_3^2 - b_2^2)$, ...



Statistics of Final Products

Histograms of final products (impact parameters are summed up)

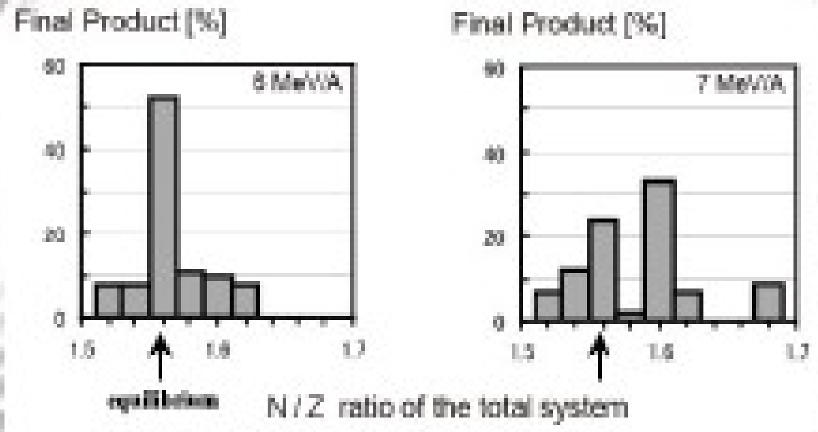


Fig.5-1 Distribution of N/Z ratio

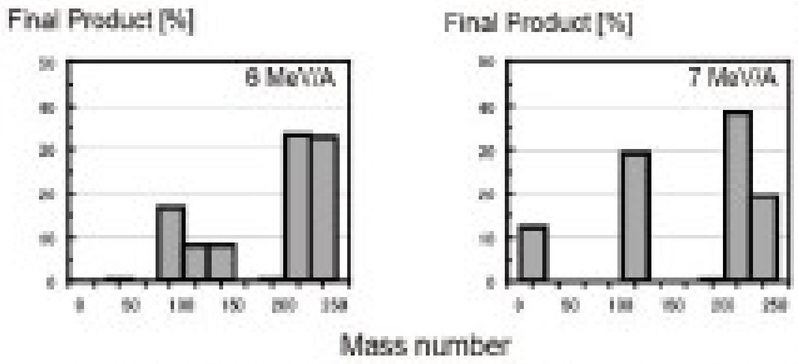


Fig.5-2 Distribution of Mass

The TDHF results shown in figures 5-1 and 5-2 are the summed up in terms of the impact parameters.

From Fig.5-1, TDHF calculations show that 52% of the final products are equilibrated at 6MeV/A, but 23% at 7MeV/A. The shape of the N/Z distribution changes drastically.

From Fig.5-2, TDHF calculations show that the masses of final fragments are located around 110 and 230 for 6MeV/A, but 25, 100 and 230 for 7MeV/A. In the former case, the reaction is dominated by fragmentation into two or three pieces. In the latter case, fragmentation into four pieces appears, which produces lighter-mass fragments.

Pb:1.54 U:1.59

Table 1. Summary of TDHF calculations

Reactions	Upper energy-limit [Mev/A]
$^{208}Pb + {}^{238}U$	$6.0 < E_{CE} < 7.0$
$^{208}Pb + {}^{132}Xe$	$6.0 < E_{CE} < 7.0$
$^{208}Pb + {}^{132}Sn$	$6.0 < E_{CE} < 7.0$
$^{208}Pb + {}^{24}Mg$	$2.0 < E_{CE} < 3.0$
$^{208}Pb + {}^{24}O$	$2.0 < E_{CE} < 3.0$
$^{208}Pb + {}^{16}O$	$1.0 < E_{CE} < 2.0$
$^{208}Pb + {}^{4}He$	$E_{CE} < 1.0$

<u>Mass-dependence</u> can be found in this table.
How can we understand it ?

“Charge Equilibration” upper energy-limit Formula

By comparing __the Fermi velocity__ with __the relative velocity__ …

If __the touching time__ is large enough for a single nucleon with Fermi velocity to propagate throughout the two colliding nuclei, the charge equilibration can take place.

Based on the Fermi-gas with the droplet model, we propose

$$E_{\mathrm{CE}} = \frac{\hbar^2 \left(3\pi^2 \rho_{min}\right)^{2/3}}{2m} \frac{A_1 A_2}{A_1 + A_2} + \frac{e^2}{4\pi\epsilon_0 r_0} \frac{Z_1 Z_2}{A_1^{1/3} + A_2^{1/3}} \qquad (1)$$

AIP conference proceedings “Fusion 08”

$$|\mathbf{v}_F| = \hbar|\mathbf{k}_F|/m = \hbar(3\pi^2\rho_{\min})^{1/3}/m,$$

$$|\mathbf{v}_r| = \sqrt{\frac{2}{\mu}\left(E_{\mathrm{cm}} - \frac{1}{4\pi\epsilon}\frac{Z_1 Z_2 e^2}{r_0(A_1^{1/3} + A_2^{1/3})}\right)},$$

$$\rho_{\min} = \min_i \left(\rho_{ni}, \rho_{zi}\right) \quad = \min_i \left(\frac{N_i\left(\frac{4\pi r_0}{3}A_i^{1/3}\right)^{-1}}{(1-3\varepsilon)(1+\delta)}, \frac{Z_i\left(\frac{4\pi r_0}{3}A_i^{1/3}\right)^{-1}}{(1-3\varepsilon)(1-\delta)}\right),$$

By solving $|v_r| = |v_F|$, we obtain the upper enrgy-limit (1).

“Charge Equilibration” upper energy-limit Formula

By comparing <u>the Fermi velocity</u> with <u>the relative velocity</u> ...

If <u>the touching time</u> is large enough for a single nucleon with Fermi velocity to propagate throughout the two colliding nuclei, the charge equilibration can take place.

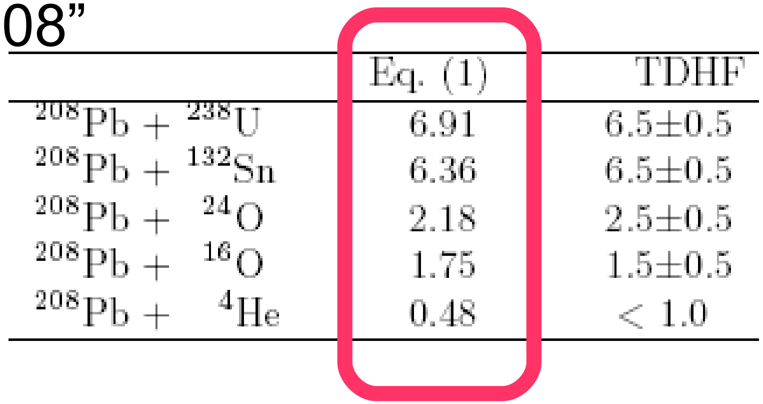
Based on the Fermi-gas with the droplet model, we propose

$$E_{\mathrm{CE}} = \frac{\hbar^2\left(3\pi^2\rho_{min}\right)^{2/3}}{2m}\frac{A_1A_2}{A_1+A_2} + \frac{e^2}{4\pi\epsilon_0 r_0}\frac{Z_1Z_2}{A_1^{1/3}+A_2^{1/3}} \qquad (1)$$

$$\rho_{\min} = \min_i\left(\rho_{ni}, \rho_{zi}\right) = \min_i\left(\frac{N_i\left(\frac{4\pi r_0}{3}A_i^{1/3}\right)^{-1}}{(1-3\varepsilon)(1+\delta)}, \frac{Z_i\left(\frac{4\pi r_0}{3}A_i^{1/3}\right)^{-1}}{(1-3\varepsilon)(1-\delta)}\right),$$

The derivation and the meaning of this formula should be referred to AIP conference proceedings “Fusion 08”

This formula <u>agrees quite well</u> with TDHF3d calculations



	Eq. (1)	TDHF
$^{208}\mathrm{Pb} + {}^{238}\mathrm{U}$	6.91	6.5±0.5
$^{208}\mathrm{Pb} + {}^{132}\mathrm{Sn}$	6.36	6.5±0.5
$^{208}\mathrm{Pb} + {}^{24}\mathrm{O}$	2.18	2.5±0.5
$^{208}\mathrm{Pb} + {}^{16}\mathrm{O}$	1.75	1.5±0.5
$^{208}\mathrm{Pb} + {}^{4}\mathrm{He}$	0.48	< 1.0

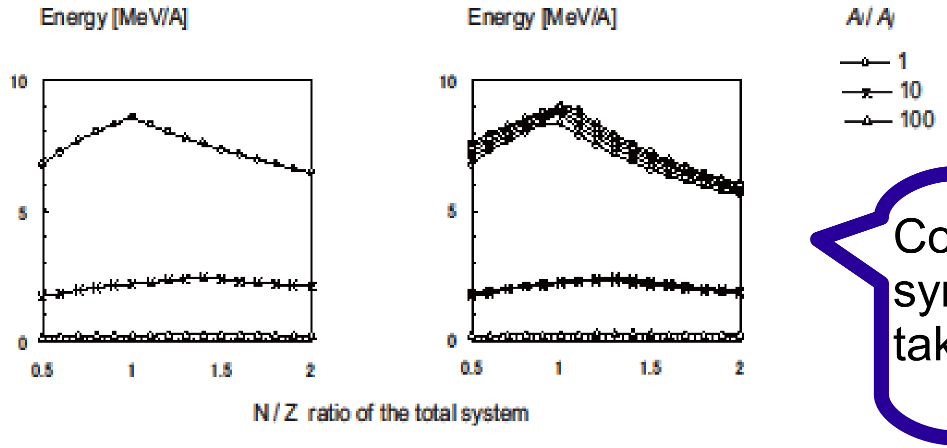
“Charge Equilibration” upper energy-limit Formula

We proposed ...

Based on the Fermi-gas with the droplet model, we propose

$$E_{\mathrm{CE}} = \frac{\hbar^2 \left(3\pi^2 \rho_{min}\right)^{2/3}}{2m} \frac{A_1 A_2}{A_1 + A_2} + \frac{e^2}{4\pi\epsilon_0 r_0} \frac{Z_1 Z_2}{A_1^{1/3} + A_2^{1/3}} \quad (1)$$

Graphical representation



Coulomb and symmetry energy is taken into account

FIG. 1: N/Z dependence of the charge equilibration upper energy limit in the center-of-mass frame based on Eq. (1), assuming that $N_1/Z_1 = N_2/Z_2$ (compare [Iwata-FUSION08]). The total N and Z values are used without any modification for the left, and the modified values by the droplet model are taken for the right in which the lines with different total masses 100, 200, 300, 400, and 500 are plotted (from lower to upper). A_i/A_j denotes the maximum of A_1/A_2 and A_2/A_1.

Question 1

Is there the upper energy-limit of charge equilibration ?

- yes !

Then, what is the decisive factor for the upper energy-limit ?

The Fermi energy is essential for determining **the upper-energy limit(presence or absence)**, while the symmetry and the Coulomb energies are the valid secondary factor for this limit.

Consistence

- The charge equilibration is a rapid process ~ 10^{-22} sec propagating with Fermi velocity (quantum waves in the fermionic system)
- The charge equilibration similarly occurs in the collisions involving very heavy nuclei, although there exists a large Coulomb repulsion leading to the localization of charges.

Application of C. E. formula (exotic production)

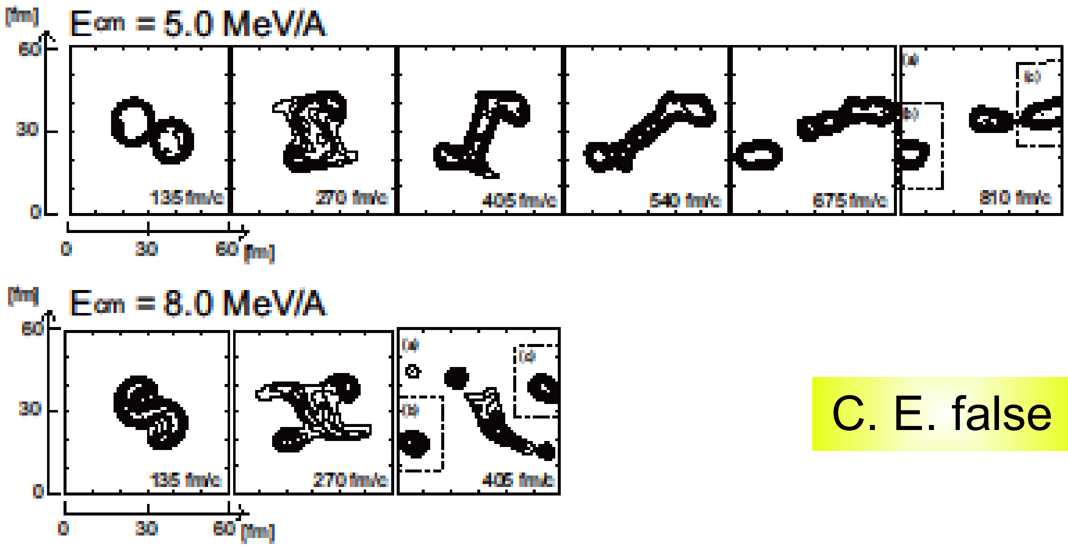


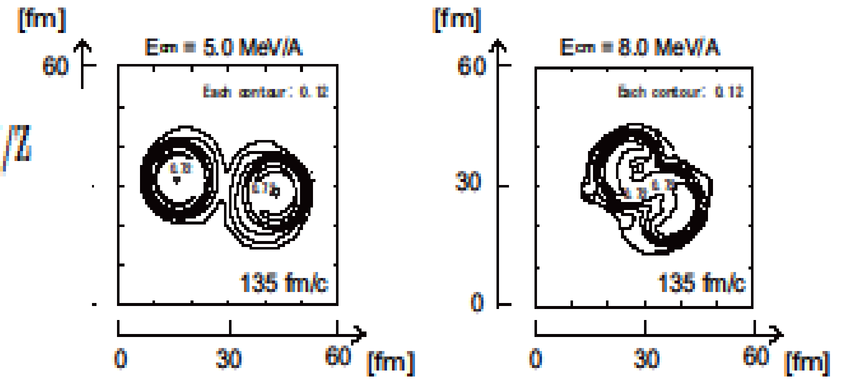
Fig. 8. Time evolution of the total density. The impact parameter is fixed to be 9.2 fm. We have separated into three parts as is shown in 810 fm/c for the case of E_{cm} = 5.0 MeV/A, and in 405 fm/c for the case of E_{cm} = 8.0 MeV/A.

C. E. true

C. E. false

Table 2. N/Z ratio of parts (a), (b) and (c) in Fig. 8 are shown, where we take the average N/Z value for the part (a) of E_{cm} = 8.0 MeV/A.

	(a)	(b)	(c)
E_{cm} = 5.0 MeV/A	1.54	1.56	1.58
E_{cm} = 8.0 MeV/A	1.64	1.42	1.47



ots of TDHF calculations at 135 fm/c, where the proton-to-neutron density ratio is shown. Each contour is set to be 0.12, while thick contours to 0.24. These two figures precisely correspond to the cases shown in Fig. 8.

2) Charge and mass distribution

Question 2

How can we understand the mass distribution in the early stage of heavy-ion collision ?

In particular, can we find the relation with charge equilibration ?

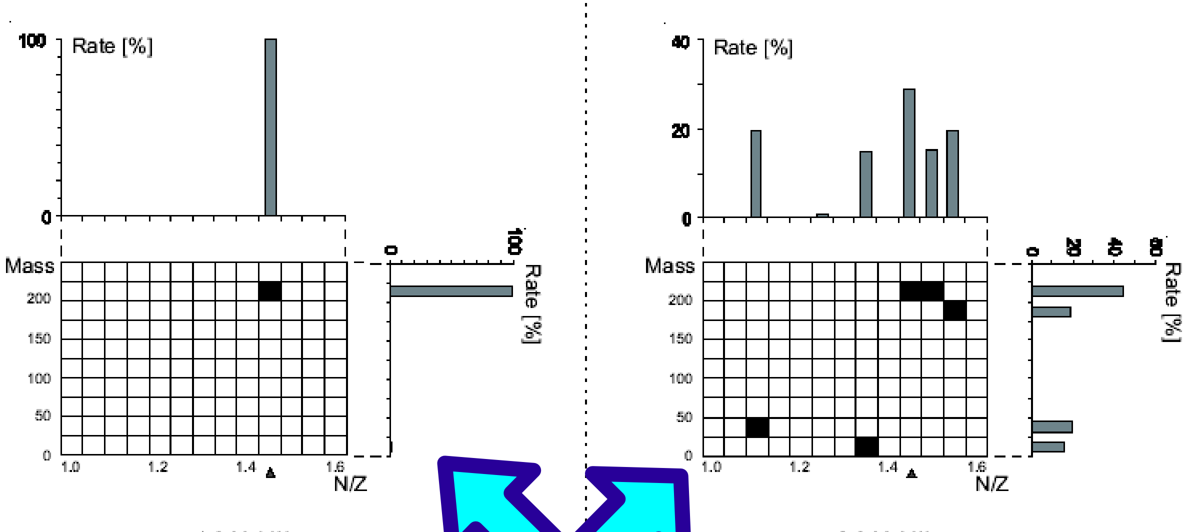
Motivation:
Is mass equilibration (fragmentation into similar masses in the case of “deep inelastic collisions”) similar to the charge equilibration.

To clarify the similarity and difference between two things

Difference in Charge and Mass Equilibration

Lighter case

^{208}Pb + ^{16}O



1.0 MeV/A

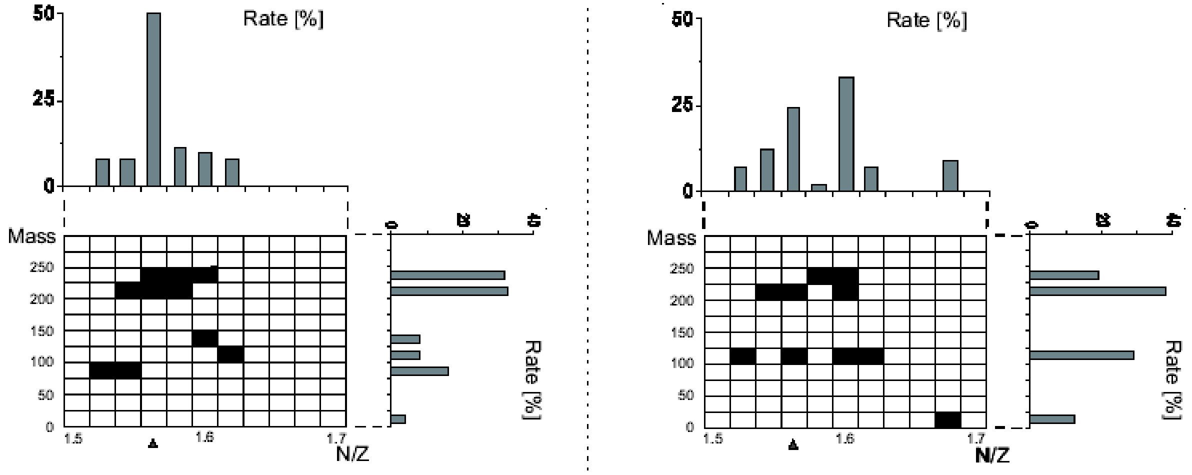
2.0 MeV/A

C. E. true

C. E. false

Fig. 5. Histogram of final fragments (weighted [illegible]pact parameter) in the central collisions between ^{208}Pb and ^{16}O, where the horizontal and vertic[illegible]xes mean N/Z ratio and the mass number, respectively. N/Z ratio is incremented by 0.05 and the mass number by 25. Black or White square shows the existence or non-existence of final products (larger than 5 % of the total products are produced or not). Triangles designate the charge equilibrium. Cases of E_{cm} = 1.0 and 2.0 MeV/A are shown.

Explicit difference exist between lighter and heavier reactions



6.0 MeV/A

7.0 MeV/A

C. E. true

C. E. false

Fig. 6. Histogram of final fragments [illegible] parameter) in the central collisions between ^{208}Pb and ^{238}U, where the horizontal [illegible]an N/Z ratio and the mass number, respectively. N/Z ratio is incremented by 0.05 a[illegible]ass number by 25. Black or White square shows the existence or non-existence of final products [illegible]rger than 5 % of the total products are produced or not). Triangles designate the charge equilibrium. Cases of E_{cm} = 4.0 and 8.0 MeV/A are shown.

Heavier case

^{208}Pb + ^{238}U

Question 2

How can we understand the mass distribution in the early stage of heavy-ion collision ?

In particular, can we find the relation with charge equilibration ?

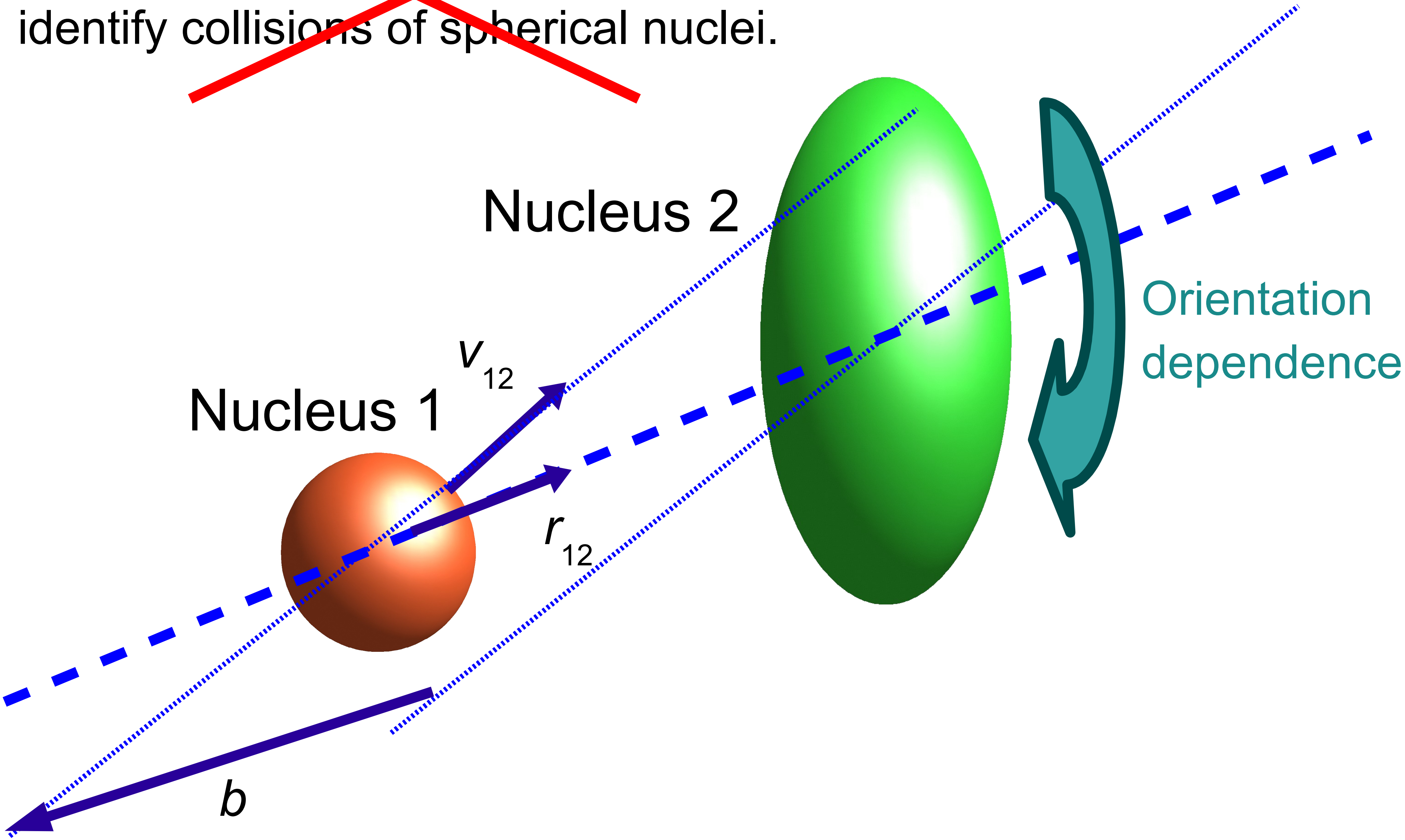
Results at Present:

The mass equilibration is similarly occurs to the charge equilibration in the **lighter nuclear reactions,** while it is not in the **heavier nuclear reactions.**

We point out the importance of the formation of merged system to understand the mass distribution.

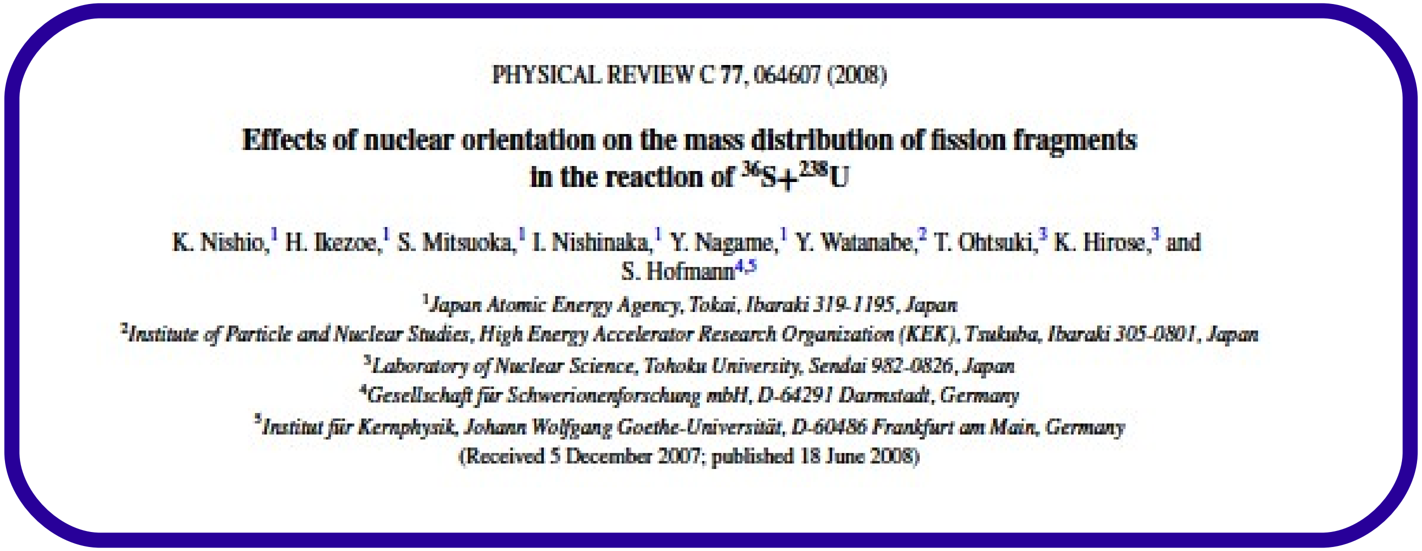
3) Orientation effect (preliminary)

~~Only v_{12} and r_{12} are two degrees of freedom to identify collisions of spherical nuclei.~~



The purpose of studying orientation effect ?

The importance of orientation effect has been already discussed.



PHYSICAL REVIEW C 77, 064607 (2008)

Effects of nuclear orientation on the mass distribution of fission fragments in the reaction of ^{36}S+^{238}U

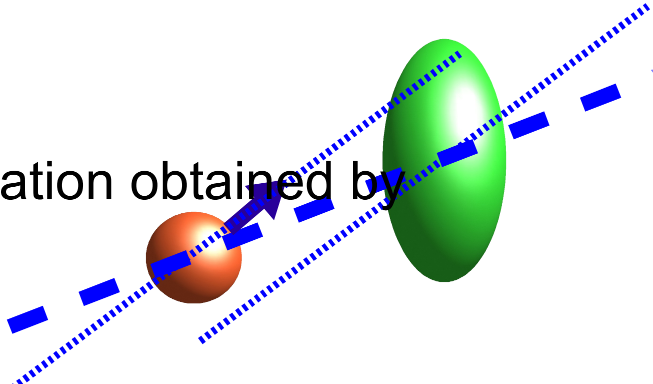
K. Nishio,[1] H. Ikezoe,[1] S. Mitsuoka,[1] I. Nishinaka,[1] Y. Nagame,[1] Y. Watanabe,[2] T. Ohtsuki,[3] K. Hirose,[3] and S. Hofmann[4,5]

[1]*Japan Atomic Energy Agency, Tokai, Ibaraki 319-1195, Japan*
[2]*Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan*
[3]*Laboratory of Nuclear Science, Tohoku University, Sendai 982-0826, Japan*
[4]*Gesellschaft für Schwerionenforschung mbH, D-64291 Darmstadt, Germany*
[5]*Institut für Kernphysik, Johann Wolfgang Goethe-Universität, D-60486 Frankfurt am Main, Germany*
(Received 5 December 2007; published 18 June 2008)

There exists another degree of freedom (orientation) to fix the initial condition compared to the spherical cases.

The relation between charge equilibration and the orientation effect should be interesting.

(Private reason) First time comparison of charge equilibration obtained by TDHF3d to experimental results.



238 U + 30 Si
(Experiment with theoretical model)

Single peak

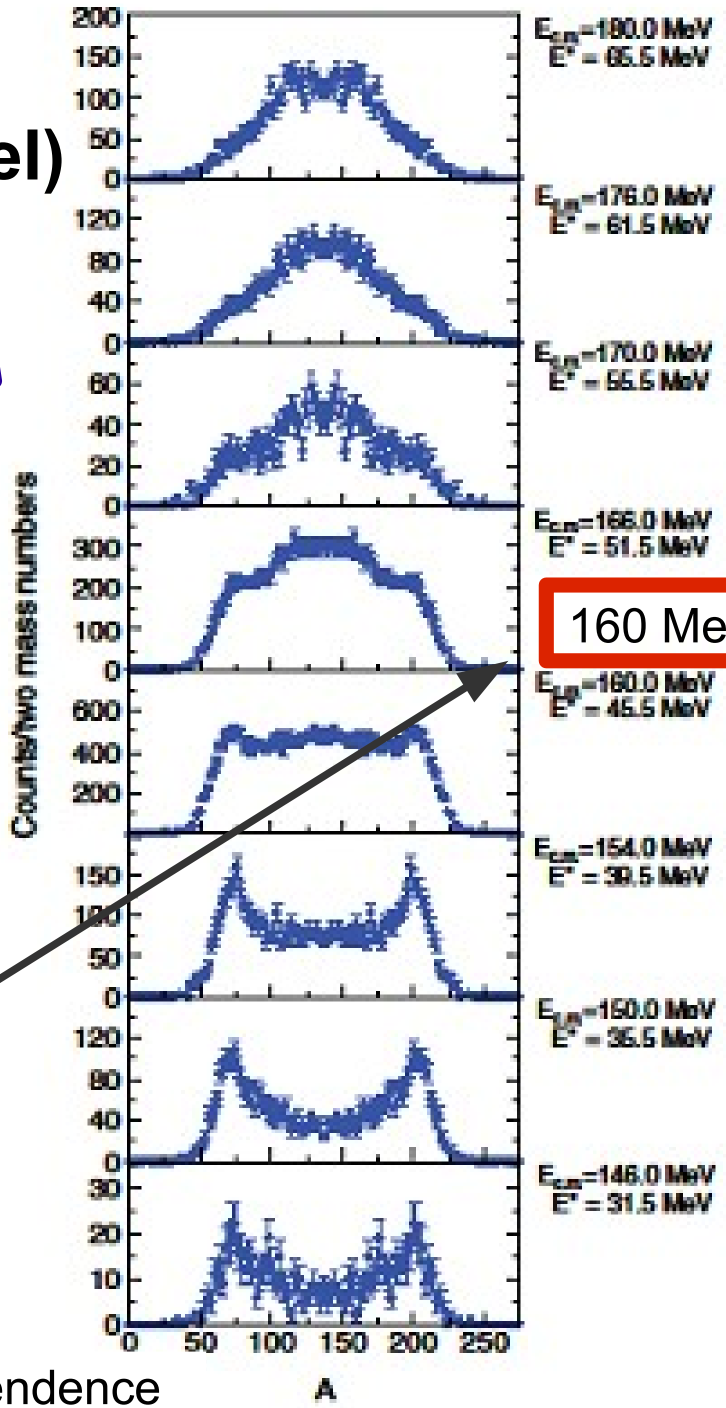
Energy dependence is very interesting, but it is so low compared to the upper energy-limit of charge equilibration

Twin peak

C.E. upper-limit

2.6 MeV/A + (238+30)
= 697 MeV

Energy dependence



160 MeV

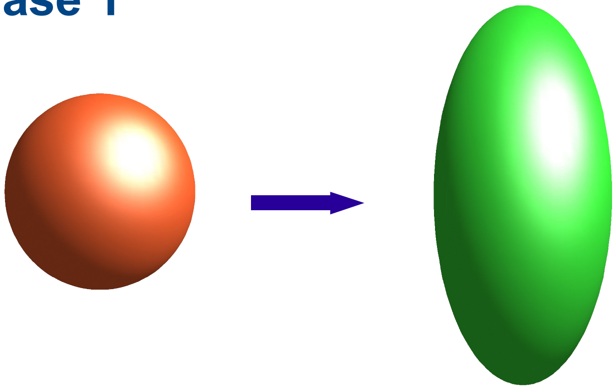
Nishio *et. al.* PRC (2008)

Orientation Effect

238 U + 30 Si

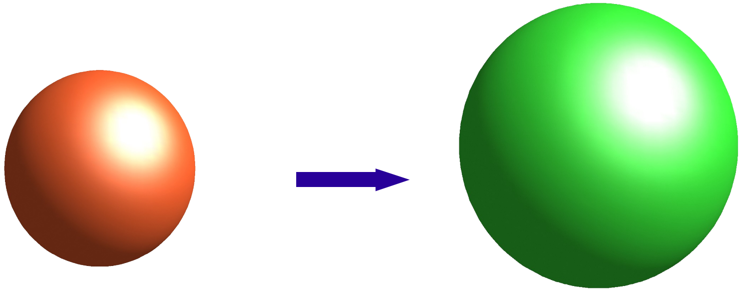
We have made a calculations of two types

Case 1



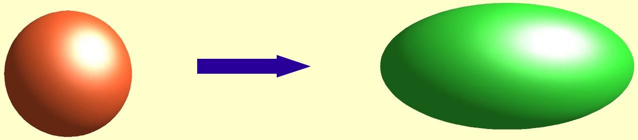
Deformed (perpendicular)

Case 2



Spherical

Now under calculation



Deformed (horizontal)

As the 1st stage of our research,

Although there should be many interesting Physics included in the orientation effect, we keep our eyes on **the charge equilibration upper energy-limit**.

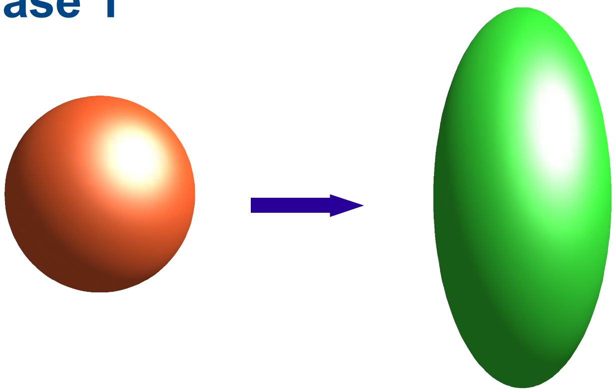
How can it be different owing to **the orientation** ?

Orientation Effect

238 U + 30 Si

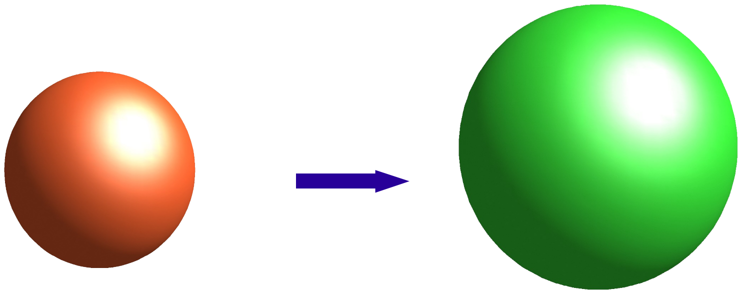
We have made a calculations of two types

Case 1



Deformed (perpendicular)

Case 2



Spherical

The charge equilibration upper energy-limit.

TDHF3d: 2.0 – 3.0 MeV/A

TDHF3d: 2.0 – 3.0 MeV/A

There is no difference between them

2.61 MeV/A (c.m.) according to the upper-limt formula

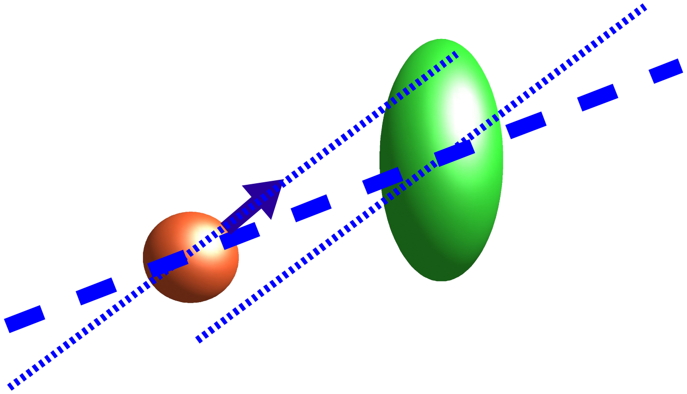
700MeV (c.m.)

The purpose of studying orientation effect ?

Our results:

•The upper energy-limit of charge equilibration is not so different owing t to the orientation.

→ Because it is so high energy compared with the experimental border energy for the single-twin mass-peak transition (most energetically orientation-affected energy).



Summary

1) Charge equilibration upper energy-limit formula

>>> **Fermi energy** is the first essential factor, **symmetry and Coulomb energies** are secondary factor.

2) Charge and mass equilibration

>>> Mass distribution (mass equilibration) has large discrepancy to the charge equilibration in heavy nuclear reactions. Indeed, the wide mass distribution in the charge equilibrium exists, where we pointed out that the formation of merged system.

3) Orientation effect (still studying)

>>> We have a possibility that **the upper-limit of charge equilibration is not so affected by the orientation**.
It will be because the upper-limit energy is not so low as affected by the orientation.