Indirect Techniques in Nuclear Astrophysics: Asymptotic Normalization Coefficents and Trojan Horse

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Plan:

- Asymptotic Normalization Coefficents (ANCs) in nuclear astrophysics.
- Trojan Horse

I. ANC method

1. The astrophysical S factors for the radiative capture $b+c \rightarrow a+\gamma$ are contributed by the tail of the projection of the bound state wave function a on the corresponding two-body channel $b+c$ (overlap function $I^{a}_{bc}(r) = < bc \,|\, a > =_{r>R} C^{a}_{bc} W_{-\eta,l+1/2}(r)/r$).

2. The amplitude of this tail is the ANC.

3. The ANC determines the absolute normalization of the astrophysical factors for peripheral reactions

$$S(E) = \lambda (C^{a}_{bc})^{2} \left| < \frac{W_{-\eta,l+1/2}(r)}{r} \,|\, r^{L} \,|\, \psi_{lk_i}(r) > \right|^{2}$$

ANC method is a powerful indirect technique in nuclear astrophysics and nuclear reaction theory

A little history of ANC

ANC determines the residue in the bound state pole of the S matrix. In the field theory appears as a renormalized coupling constant or nuclear vertex constant.

Relationship between the coupling constant and the amplitude of the tail of the bound state wave function has been shown by

H. A. Kramers, 1938, W. Heisenberg, 1946

For charged particles- A. M. Perelomov et al., 1966

General case- bound state and resonances – E. I. Dolinskii and A. M. Mukhamedzhanov, 1977

First review papers about Nuclear vertex constants – L. D. Blokhinstev et al., 1977, 1984

The role of the ANC (reduced normalization) in the sub-Coulomb reactions has been understood long ago- J. Rapaport and A. K. Kerman, 1968

First paper on the application of the ANC in nuclear astrophysics, A.M. Mukhamedzhanov and N.K. Timofeyuk, 1990.

Peripheral reactions

ANC
in nuclear astrophysics

- Astrophysical factor for radiative capture and reactions
- Interference sign of the resonant and nonresonant amplitudes
- Radiative width

Experimental methods of determination of the ANC (stable and radioactive beams):

peripheral transfer reactions at energies well above the Coulomb barrier;

sub-Coulomb transfer reactions;

elastic scattering;

breakup processes;

mirror symmetry

(N. K. Timofeyuk et al. PRL 91, 232501 (2003))

The extracted ANCs can be used to calculate different astrophysical reactions.

One of the important results in application of the ANC technique in nuclear astrophysics: measurements of the $^{14}N(^{3}He,d)^{15}O \rightarrow$ ANC $\rightarrow {}^{15}O \rightarrow {}^{14}N + p$ astrophysical factor for $^{14}N(p,\gamma)^{15}O$

P.F. Bertone, et al., Phys. Rev. C 66 (2002) 055804
A.M. Mukhamedzhanov, et al., Phys. Rev. C 67 (2003) 065804;
astrophysical factor $S(0) = 1.70 \pm 0.20$ keVb

Confirmed by LUNA direct measurements- A. Formicola et al., Phys. Lett. B 591, 61 (2004) $S(0) = 1.70 \pm 0.20$ keVb

A new low boundary of the universe age: increased by 700 mln- 1bln years and now is eastimated to be $\approx$12 bln years

Elastic scattering $\alpha + {}^3He$, ANC for $\alpha + {}^3He \rightarrow {}^7Be$ and astrophysical factor for $\alpha + {}^3He \rightarrow {}^7Be + \gamma$

$$C_{3/2}^2 = 20.25\,\mathrm{fm}^{-1} \rightarrow S_{3/2}(0) = 0.39\,\mathrm{keVb},$$

$$\mathrm{S}_{\mathrm{tot}}(0) = 0.54\,\mathrm{keVb};$$

$$\mathrm{S}(0) = 0.547 \pm 0.017\,\mathrm{keVb}$$

Latest LUNA result, activation method
arXiv:nucl-ex/ 0702003 (2007)

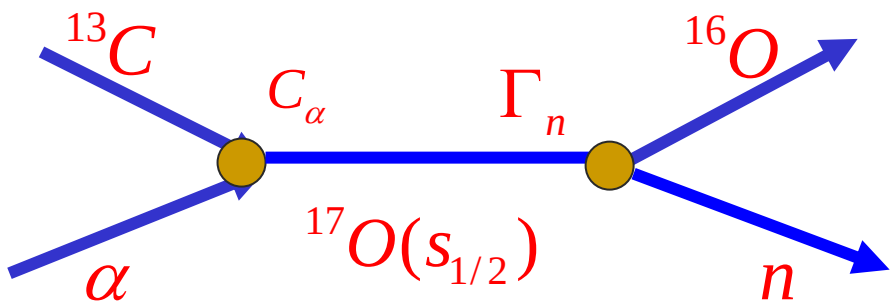
First application of the ANC technique for astrophysical nuclear reaction

ANC and astrophysical $^{13}C(\alpha,n)^{16}O$ process

$^{13}C(\alpha,n)^{16}O$ reaction, which is believed to be a major source of neutrons in the *s* processes in AGB with low and intermediate mass stars, competing with $^{22}Ne(\alpha,n)^{25}Mg$

Dominated by the capture through the subthreshold state 6.356 MeV, $s^{+}_{1/2}$ just 3 keV below the threshold

$$M \sim \frac{C_\alpha \Gamma_n^{1/2}}{E - E_0 + i\Gamma/2}$$



$S(0) = 2.3\times10^{7}$ MeVb NACRE

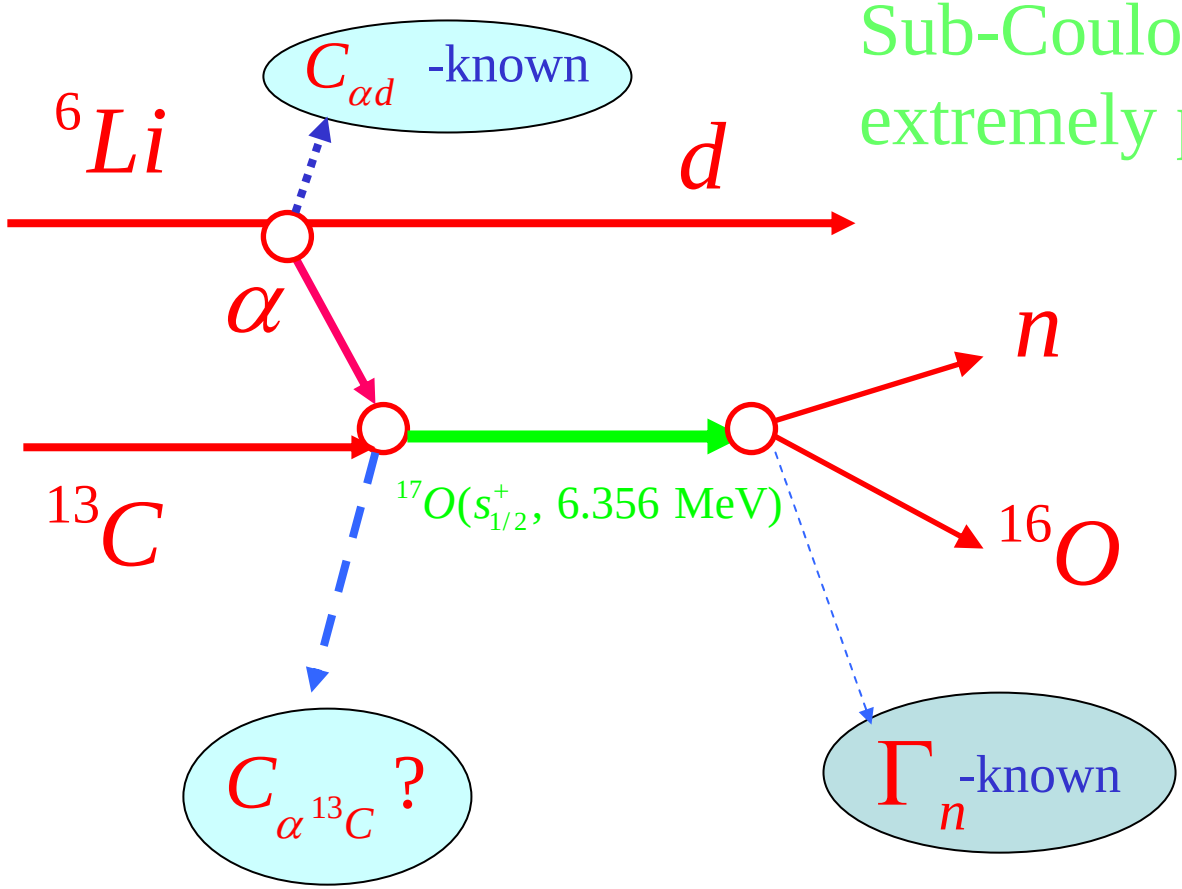
$^{13}C(^{6}Li,d)^{17}O, \quad E_{^{6}Li} = 60$ MeV

S. Kubono et al., Phys. Rev. Lett. 90, 062501 (2003)

$S(0) = 0.09\times10^{7}$ MeVb

$S(0)$ revised by N. Keeley et al., Nucl. Phys. A726, 159 (2003) gives significantly higher value

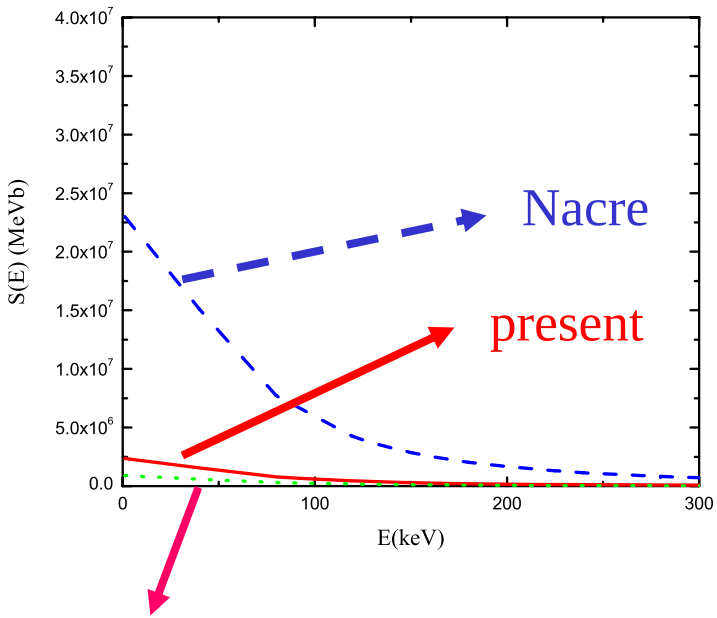
Sub-Coulomb α - transfer; extremely peripheral



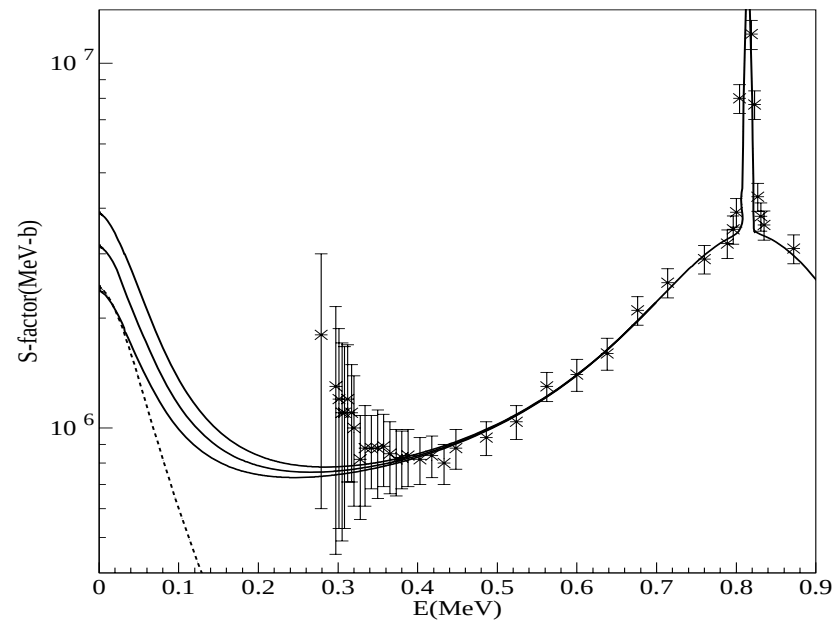
Cotribution from the subthreshold state

$$S(0) = [2.5 \pm 0.2(stat) \pm 0.2(syst) \pm 0.5(theor)] \times 10^6 \,\mathrm{MeVb}$$

$^{13}C(\alpha,n)^{16}O$



S. Kubono *et al.*, Phys. Rev. Lett. **90**, 062501 (2003).

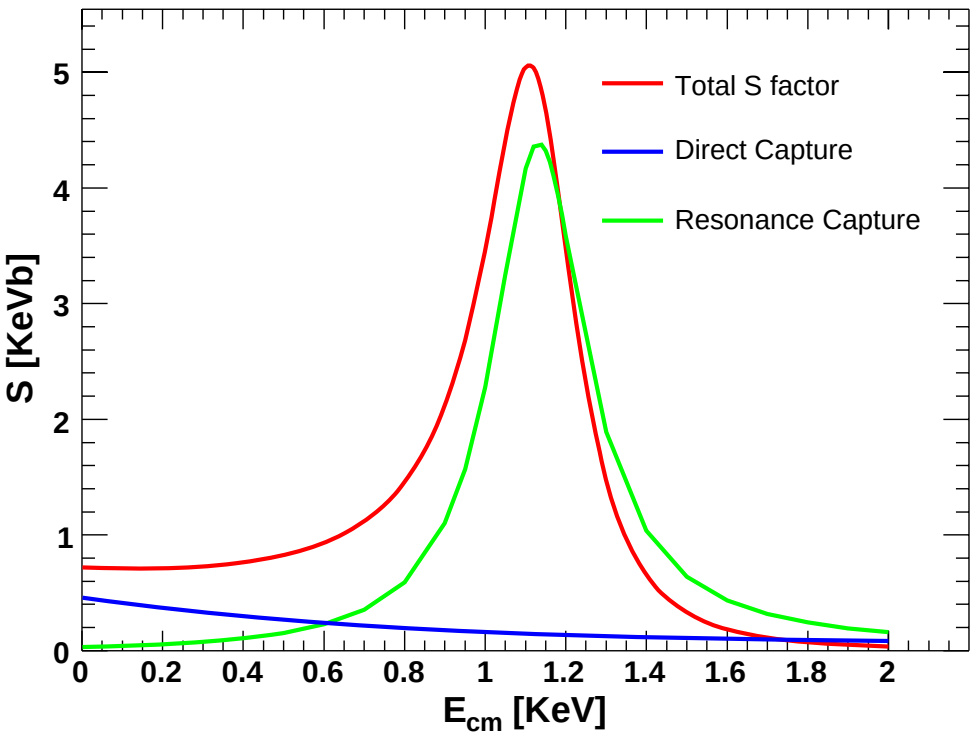


E. D. Johnson et al., Phys. Rev. Lett. 97, 192701 (2006).

S(E) factor for $^{12}N(p,\gamma)^{13}O$

- nucleosynthesis in low metalicity massive stars

B. B. Skorodumov et. al, Phys. Rev. C 75, 024607 (2007).



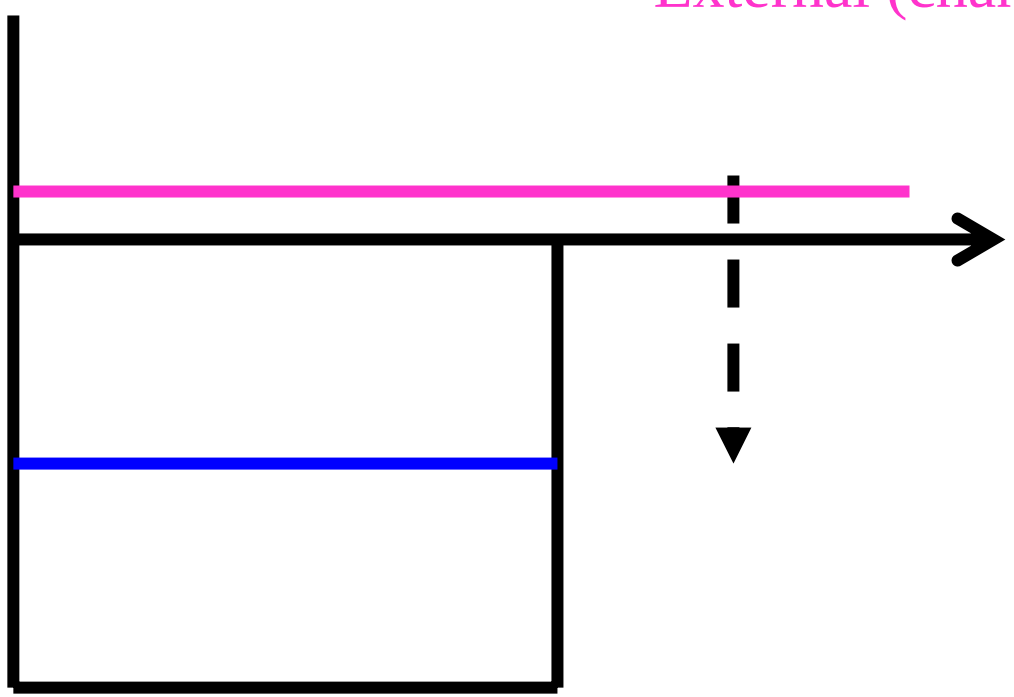
ANC method provides the nonresonant amplitude, radiative width and the interference sign of the resonant and nonresonant terms

resonant amplitude nonresonant amplitude

$$M = \frac{\Gamma_\gamma^{1/2}(E)\Gamma_p^{1/2}(E)}{E - E_R + i\dfrac{\Gamma(E)}{2}} + C < W_{-\eta,l+1/2}(r) \,|\, r^L \,|\, \psi_{kl_i}(r) >$$

$$\Gamma_\gamma^{1/2}(E) = \Gamma_{\gamma(\mathrm{int})}^{1/2}(E) + C\Gamma_p^{1/2}(E_R) < W_{-\eta,l+1/2}(r) \,|\, r^L \,|\, \psi_{(R)kl_i}(r) >$$

External (channel) part



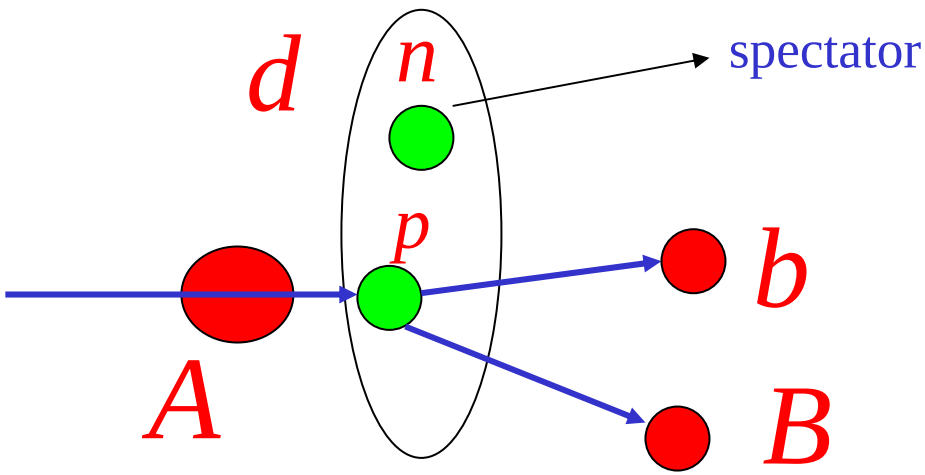
If external part dominates the relative sign of the resonant and nonresonant amplitudes is fixed

II. Trojan Horse

Trojan Horse is a unique indirect technique to measure nuclear astrophysical rearrangement reactions.
Suggested by G. Baur, Phys. Lett. B 178, 135 (1986)

Decisive contribution – Claudio Spitaleri, DMFCI, Università di Catania, Catania, Italy and INFN – Laboratori Nazionali del Sud, Catania, Italy

To overcome the Coulomb barrier, which the main obstacle to measure the astrophysical cross section, the projectile A is impinging on target $a=(xy)$ with the energy E_A higher then the Coulomb barrier . After penetration through the barrier the projectile interacts with fragment x generating the reaction of astrophysical interest $x+A\rightarrow b+B$ while the second fragment y flies away as a spectator.



sub-process

$$p+A\rightarrow b+B$$

TH reaction

$$d+A\rightarrow n+b+B$$

Advantage:

1) no barrier factor in the initial channel of the reaction $x+A\rightarrow b+B$;

2) no electron screening effects;

3) gives directly the astrophysical S factor;

4) allows one to measure the contribution from the subthreshold resonances at negative energies

Problems: particle x is off-the-energy shell; initial- and final-state interactions should be taken into account to get a correct absolute value of the astrophysical factor extracted from the TH reaction.

Practical application of the TH method:

1. Full kinematical measurement of the reaction

$$a + A \to y + b + B, \qquad a = (xy)$$

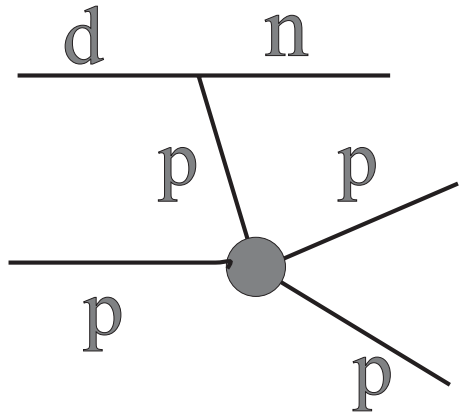
Determination of the energy dependence of the subprocess cross section $x + A \to b + B$, which is of astrophysical interest.

3. Normalization of the S factor extracted from the TH to the available astrophysical factor determined from direct measurements at higher energies

$x + A \to b + B$ can be direct or resonant

Trojan Horse in the 3-body case

$$p + d \rightarrow p + p + n$$

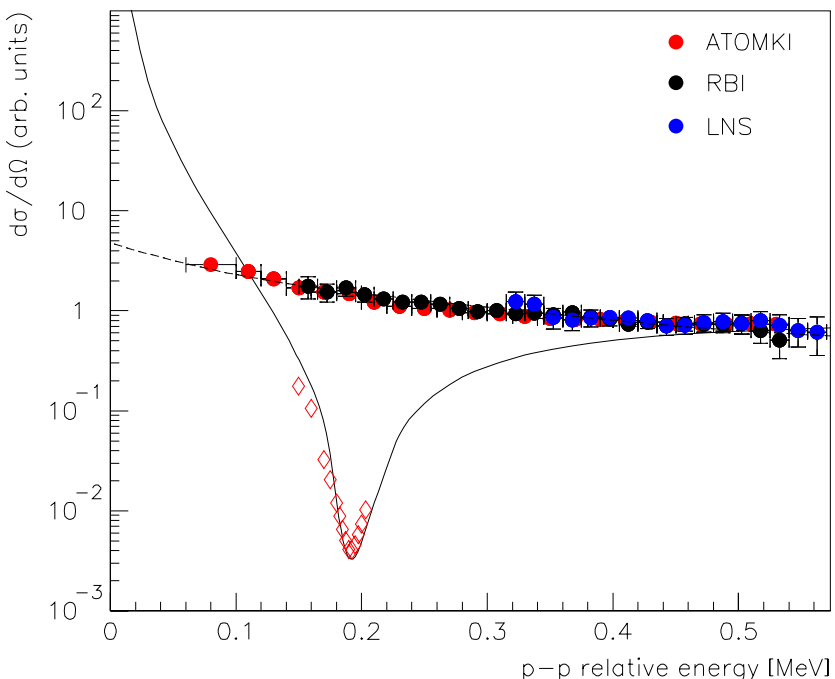


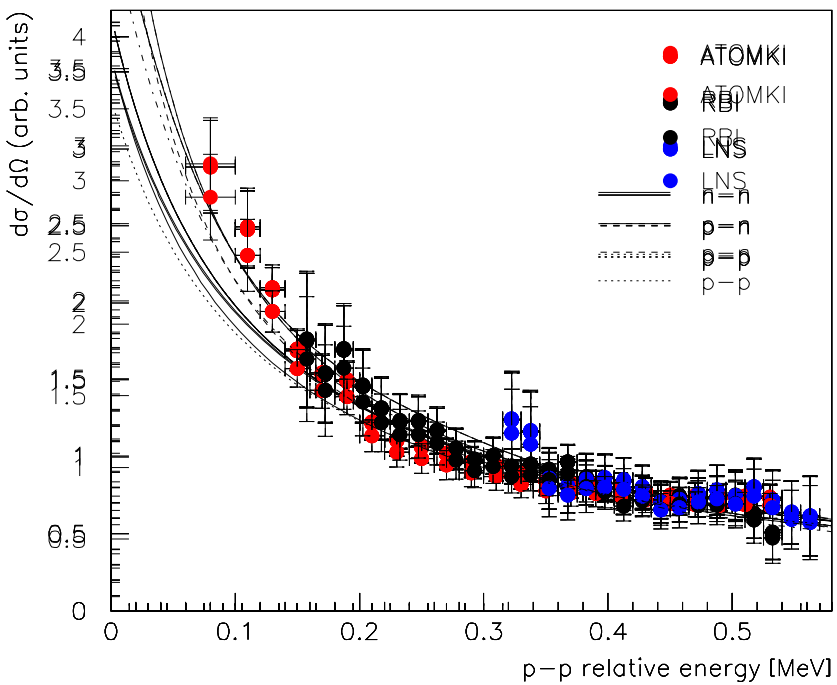
Coulomb $p - p$ scattering is suppressed due to the off-shell effects.

Only nuclear interaction contributes.

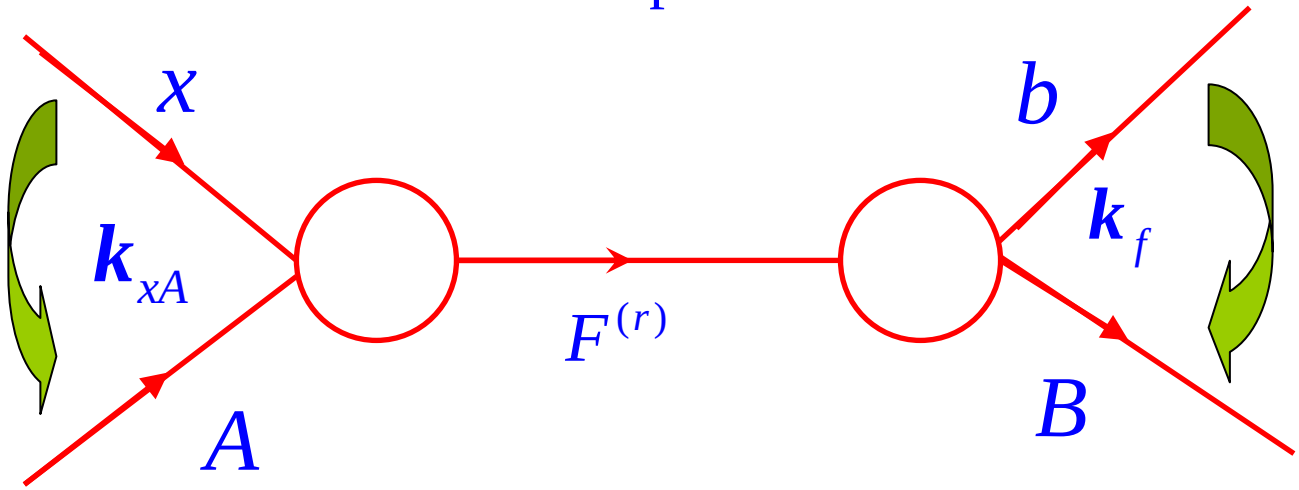
Half-off-shell $p - p$ scattering cross section follows $p - n$ and $n - n$

Due to the off-shell effects the Coulomb amplitude is suppressed!!!





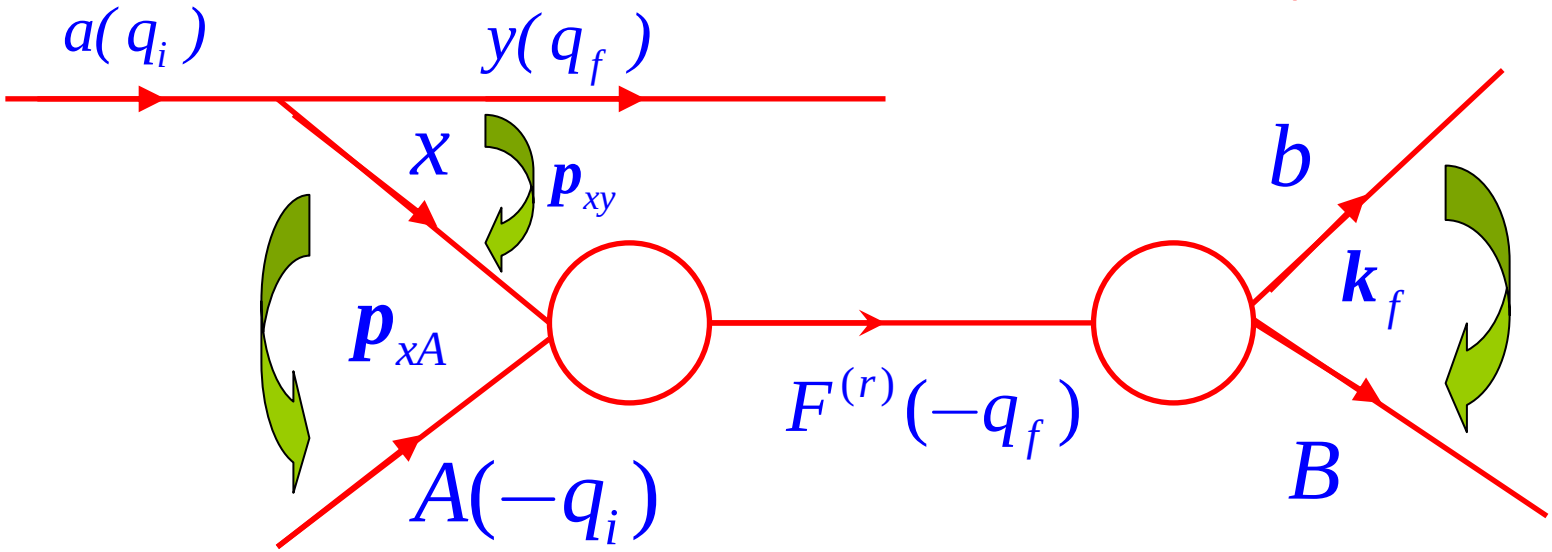
On-shell resonant reaction amplitude



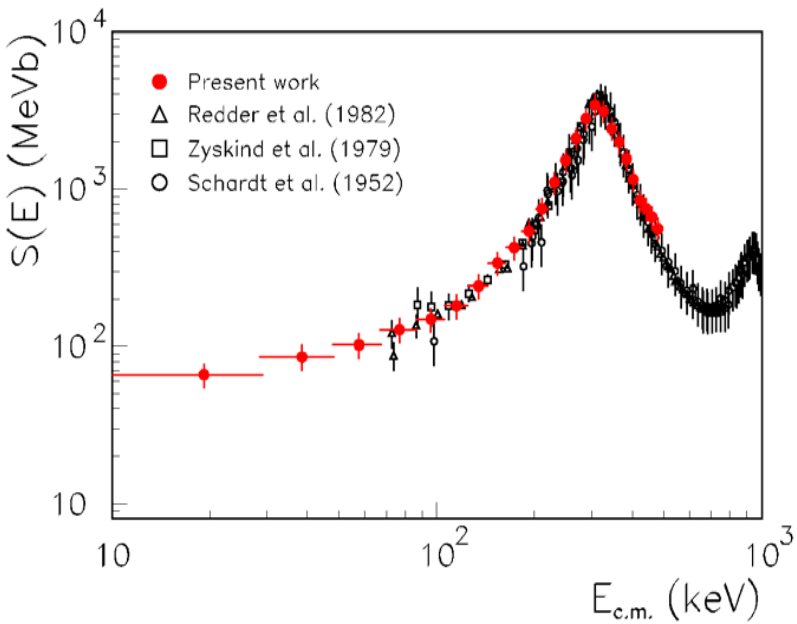
Trojan Horse for resonant sub-processes

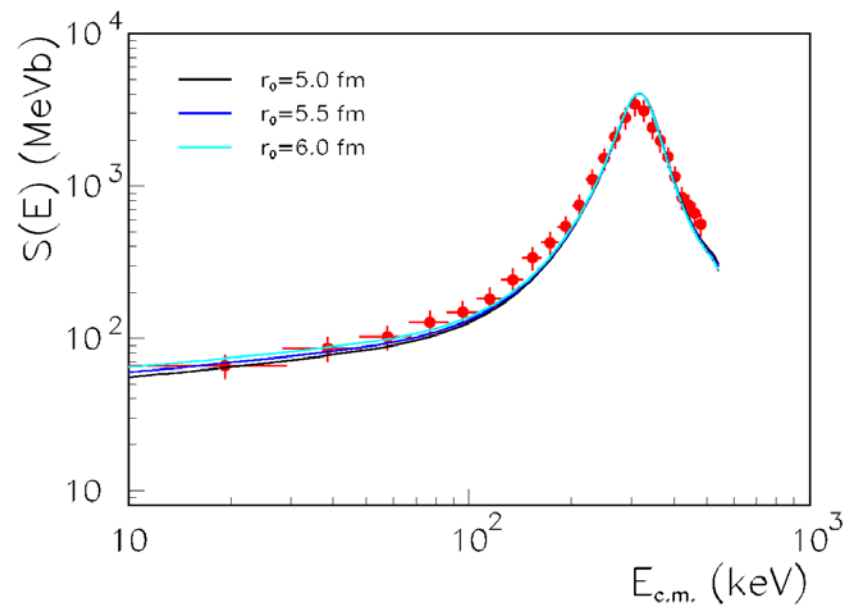
Half-off-shell resonant reaction amplitude

(ingoing state is off-shell, final state is on-shell) $(p_{xA}^2/2\mu \neq E_{xA},\ k_f{}^2/2\mu = E_{bB})$



$^{15}N(p,\alpha)^{12}C$ reaction from the Trojan Horse reaction $^{15}N(d,n\alpha)^{12}C$





Half-off-shell R matrix

Two resonances interfere: 312 keV and 960 keV

Direct measurements:

$$M = \frac{\Gamma^{1/2}_{\alpha(1)}(E)\Gamma^{1/2}_{p(1)}(E)}{E - E_{R(1)}} + \frac{\Gamma^{1/2}_{\alpha(2)}(E)\Gamma^{1/2}_{p(2)}(E)}{E - E_{R(2)}} = \Gamma^{1/2}_{p(1)}(E)\left[\frac{\Gamma^{1/2}_{\alpha(1)}(E)}{E - E_{R(1)}} + \frac{\gamma_{p(2)}}{\gamma_{p(1)}}\frac{\Gamma^{1/2}_{\alpha(2)}(E)}{E - E_{R(2)}}\right]$$

Direct measurements: on-shell R matrix

$$M = \frac{\Gamma^{1/2}_{\alpha(1)}(E)\Gamma^{1/2}_{p(1)}(E)}{E - E_{R(1)}} + \frac{\Gamma^{1/2}_{\alpha(2)}(E)\Gamma^{1/2}_{p(2)}(E)}{E - E_{R(2)}} = \Gamma^{1/2}_{p(1)}(E)\left[\frac{\Gamma^{1/2}_{\alpha(1)}(E)}{E - E_{R(1)}} + \frac{\gamma_{p(2)}}{\gamma_{p(1)}}\frac{\Gamma^{1/2}_{\alpha(2)}(E)}{E - E_{R(2)}}\right]$$

The Trojan Horse amplitude contains the factor can be expressed in terms of the wave function

$$\phi^{(-)}_{bB}\Delta V_{bB}G_F = \Psi^{(-)}_{bB} - \phi^{(-)}_{bB}$$

For $\Psi^{(-)}_{bB}$ we can use the spectral decomposition (shell-model R matrix, Mahaux and Weidenmuller, 1969). It leads to the half-off-shell R matrix

Trojan Horse: half-off-shell R matrix

$$M = W^{1/2}_{p(1)}(k)\left[\frac{\Gamma^{1/2}_{\alpha(1)}(E)}{E - E_{R(1)}} + \frac{W_{p(2)}(k)}{W_{p(1)}(k)}\frac{\Gamma^{1/2}_{\alpha(2)}(E)}{E - E_{R(2)}}\right]$$

$$M_{ren} = \Gamma^{1/2}_{p(1)}(E)\left[\frac{\Gamma^{1/2}_{\alpha(1)}(E)}{E - E_{R(1)}} + \frac{W_{p(2)}(k)}{W_{p(1)}(k)}\frac{\Gamma^{1/2}_{\alpha(2)}(E)}{E - E_{R(2)}}\right]$$

1. Trojan Horse is a powerful indirect method in nuclear astrophysics: allows one to determine the astrophysical factor between bare nuclei down to zero energy.

2. Has been used to determine the astrophysical factors for astrophysical rearrangement reactions.

3. For the first time we can probe the subthreshold state contribution at negative energies.

4. Allows one to determine the electron screening potential.

5. Future applications: radoiactive beams and radiative capture reactions.

6. Collaboration with Prof. S. Kubono, CNS, Tokyo University