Di-neutron superfluidity and the collective motions in deformed weakly-bound nuclei

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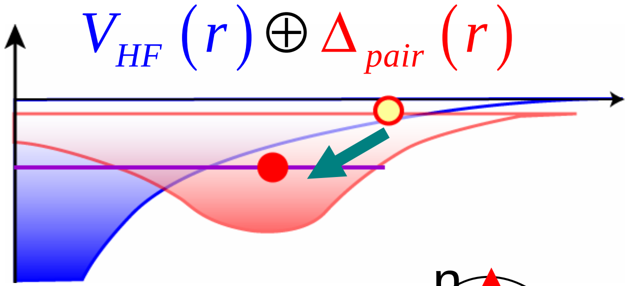
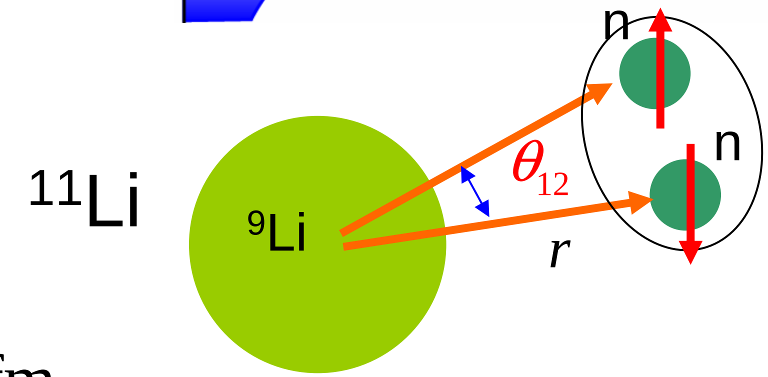
Pairing correlation in neutron rich nuclei

Crucial role in weakly-bound nuclei

$$|\lambda| \leq \Delta \quad (\lambda \to 0)$$



Light neutron rich nuclei



- Borromean: ^{11}Li, ^{6}He
- Scattering length $a_{nn}({}^1S_0) \approx -18$ fm

  $\Longrightarrow$ Di-neutron correlation suggested

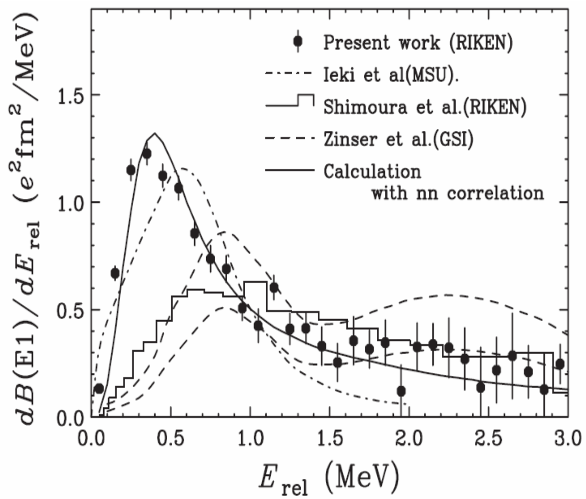
- New data of soft *E1* excitation

  T. Nakamura, *et al*., Phys. Rev. Lett. 96, 252502 (2006)

  $\Longrightarrow$ $\langle \theta_{12} \rangle = 48^{+14}_{-18}$ degree

  cf. $\langle \theta_{12} \rangle_{no-correlation} = 90$ degree

Soft E1 excitation in ^{11}Li



Discussion

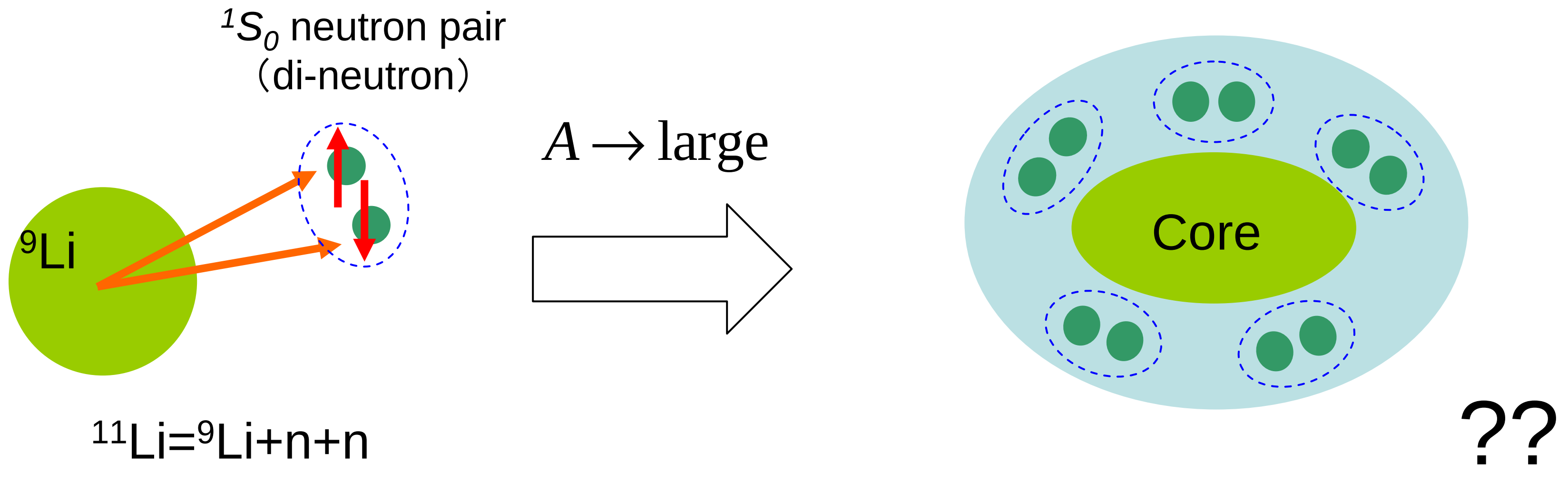
In heavy n-rich nuclei ⇨ Many weakly-bound neutrons

- Formation of several di-neutrons?

⇨ BEC condensation (di-neutron superfluidity) ?

- The rotational motion

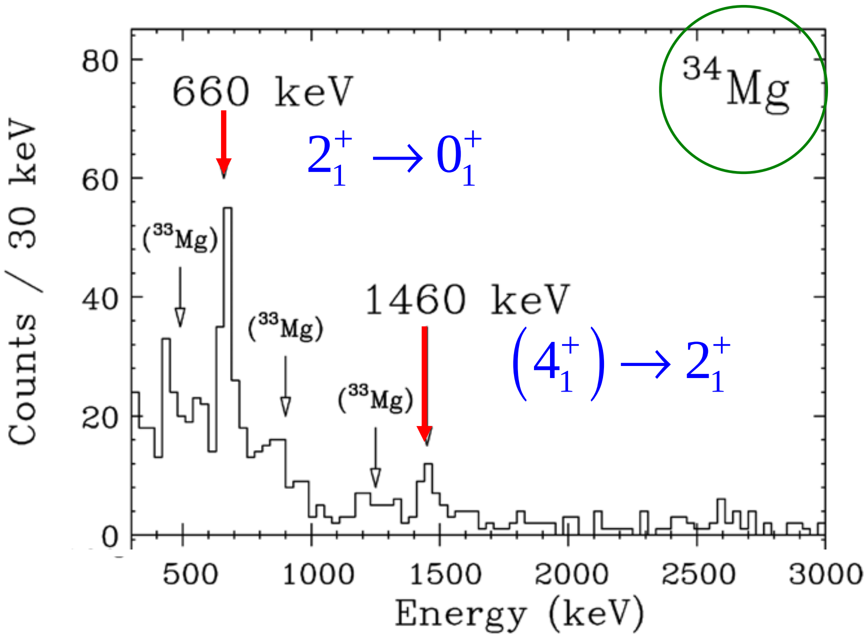
New features? Experimental indications?



Deformation of neutron-rich Mg isotopes

Experiment for ^{34}Mg

K.Yoneda *et al*., Phys. Lett. B499 (2001) 233



$$\frac{E(4_1^+)}{E(2_1^+)} = \frac{2120(\mathrm{keV})}{660(\mathrm{keV})} = 3.2$$

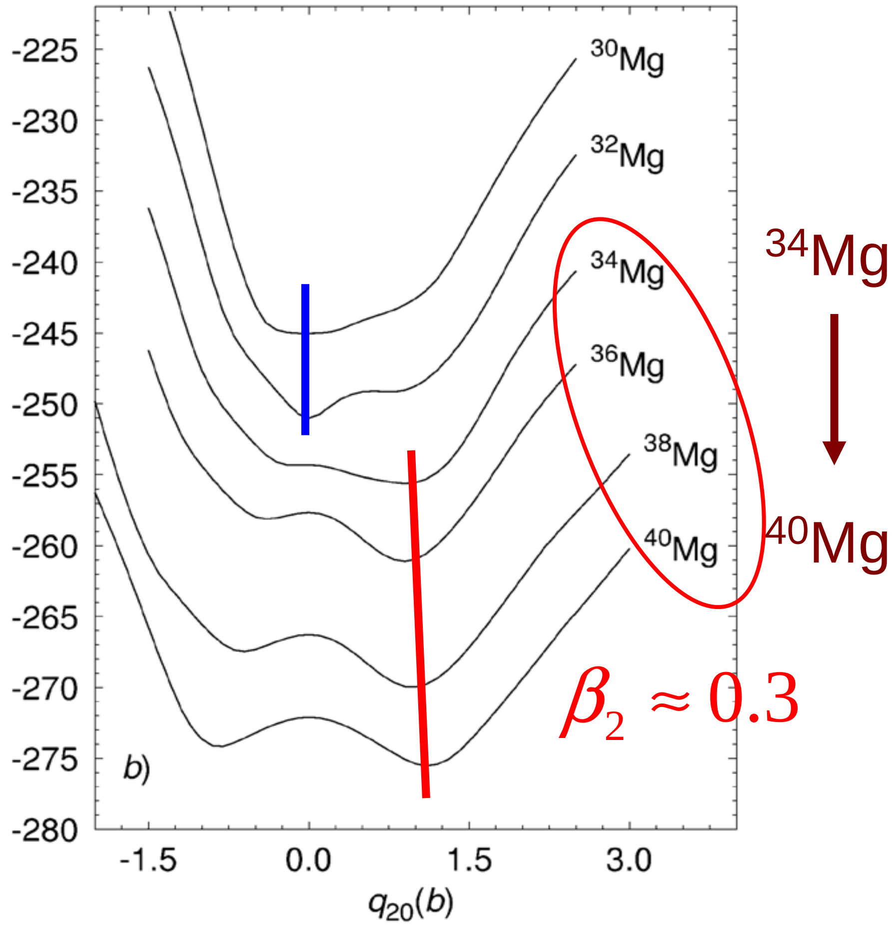
^{34}Mg is deformed

Skyrme-HFB deformed nuclear mass table

M. Stoitsov *et al.*,Phys. Rev. C68 (2003) 054312

Gogny-HFB calculation using D1S

R. Rodríguez-Guzmán *et al.,* Nucl. Phys. A709 (2002) 201



Pairing problem in weakly-bound nuclei

Coordinate space HFB theory (weak-binding, continuum)

A. Bulgac, FT-194-1980, CIP-IPNE, Bucharest Romania, 1980

J. Dobaczewski, H. Flocard, J. Treiner, Nucl. Phys. A422, 103 (1984)

$$\begin{pmatrix} T+V(\vec{r})-\lambda & \Delta(\vec{r}) \\ \Delta(\vec{r}) & -T-V(\vec{r})+\lambda \end{pmatrix} \begin{pmatrix} \varphi_1(\vec{r}) \\ \varphi_2(\vec{r}) \end{pmatrix} = E \begin{pmatrix} \varphi_1(\vec{r}) \\ \varphi_2(\vec{r}) \end{pmatrix}$$

HF potential (central + LS): $V(\vec{r}) = Y_{20}$ deformed Woods-Saxon pot.

$$V_{WS} = -51 + 30\frac{N-Z}{A} \ \text{MeV}, \quad V_{LS} = -0.44 V_{WS} \ \text{MeV}$$

$$R = 1.27 A^{1/3} \ \text{fm}, \quad a = 0.67 \ \text{fm}, \quad \beta_{\text{WS}} = 0.3$$

Self-consistent pairing potential:

$$V_{pair}(\vec{r},\vec{r}\,') = \frac{V_{pair}}{2}(1-P_\sigma)\left(1 - \frac{\rho((\vec{r}+\vec{r}\,')/2)}{\rho_0}\right)\delta(\vec{r}-\vec{r}\,')$$

$$\Rightarrow \quad \Delta(\vec{r}) = \frac{V_{pair}}{2}\left(1 - \frac{\rho(\vec{r})}{\rho_0}\right)\tilde{\rho}(\vec{r}) \qquad \rho_0 = 0.16 \ \text{fm}^{-3}, \quad V_{pair} \Rightarrow \text{parameter}$$

2D-polar grid space representation

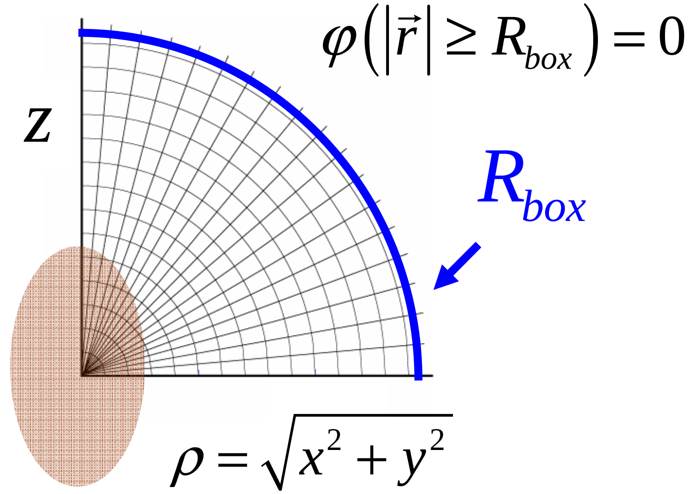
Grid space representation

⇨ *Promising for <u>weakly-bound & continuum states</u>*

$$\vec{r} \;\Rightarrow\; \vec{r}_i = \text{grid points}$$

$$\sum_j H\left(\vec{r}_i, \vec{r}_j\right)\Phi\left(\vec{r}_j\right) = \varepsilon\Phi\left(\vec{r}_i\right)$$

(NG: Expensive numerical cost)

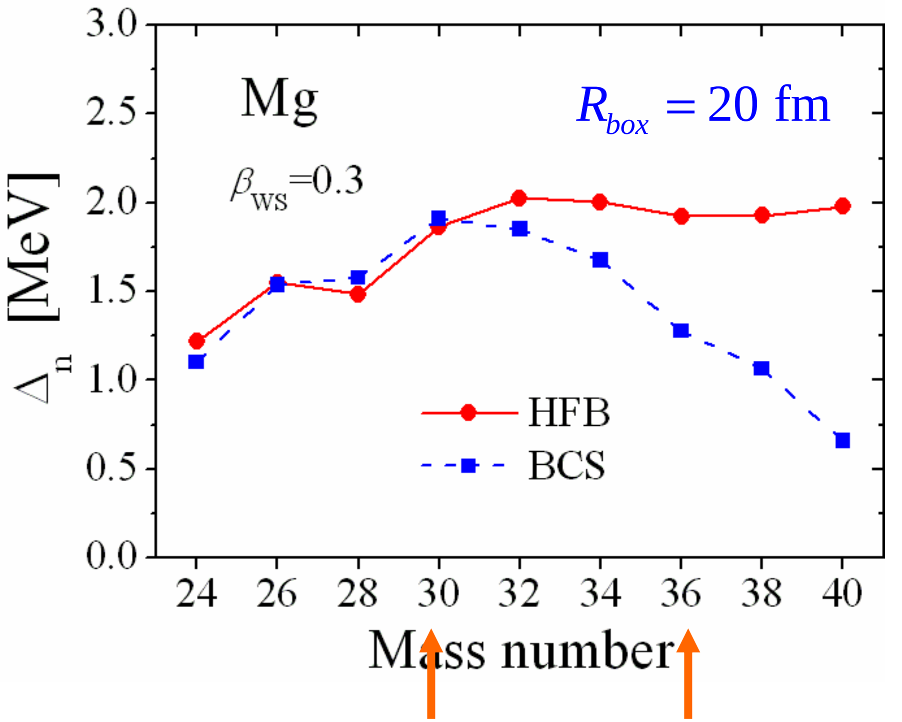


Merits of 2D-polar grid space representation

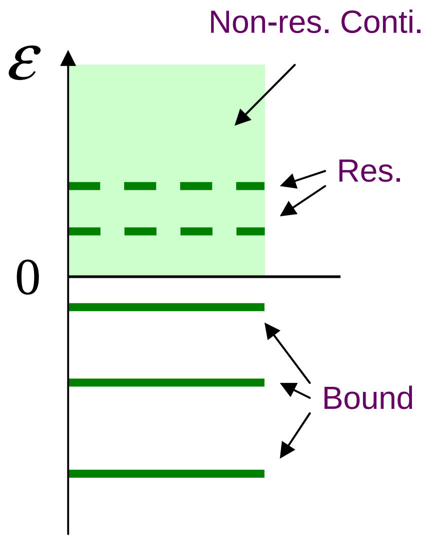
Good convergence for angular correlations
(Small number of grid points for θ-direction)

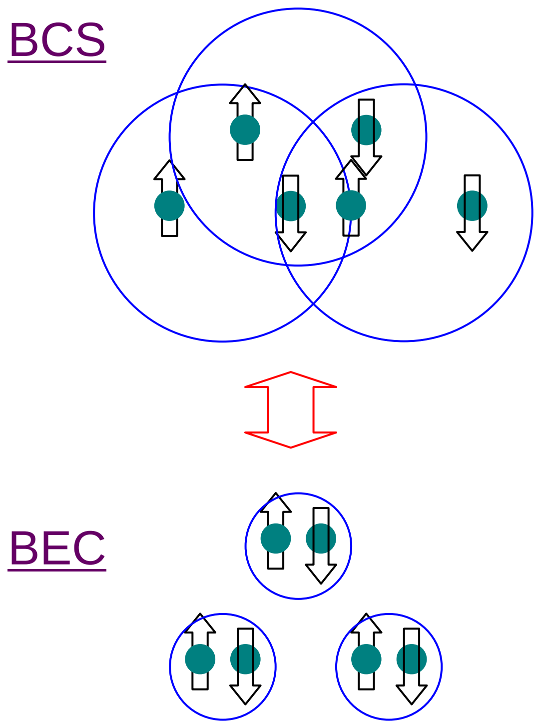
⇨ Suitable representation for di-neutron correlation

Pairing around neutron drip line: HFB vs. BCS



$\lambda_n \approx -5$ MeV $\lambda_n \approx -1.8$ MeV

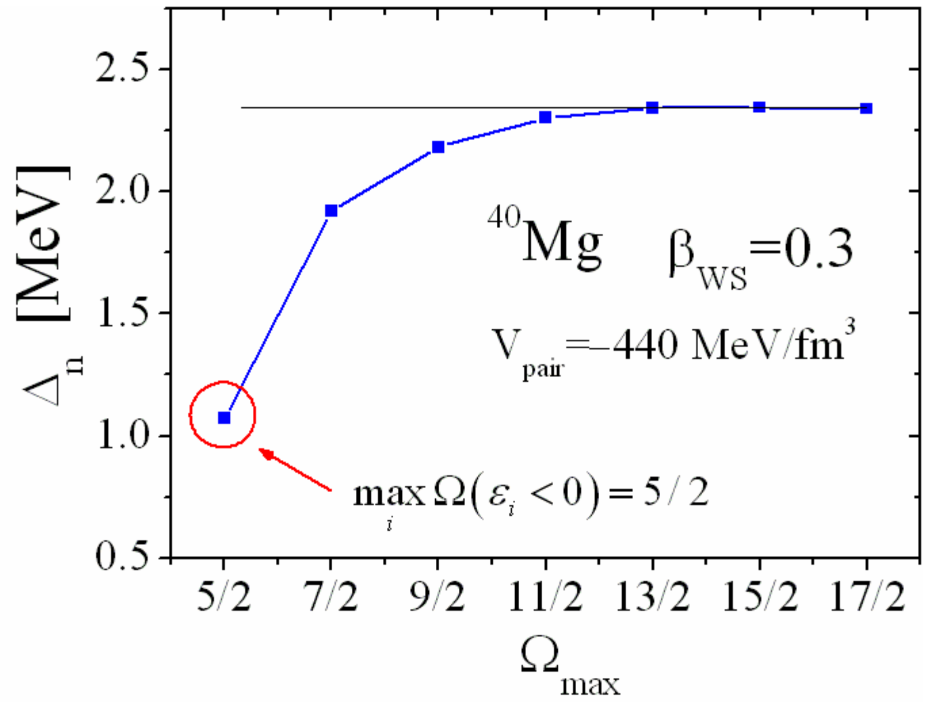




- Pairing is weak ? (BCS approximation)
  - Model space: bound & resonant states
  - Decreasing particle configurations
- Pairing is strong ? (coordinate space HFB)
  - Model space: bound & resonant states

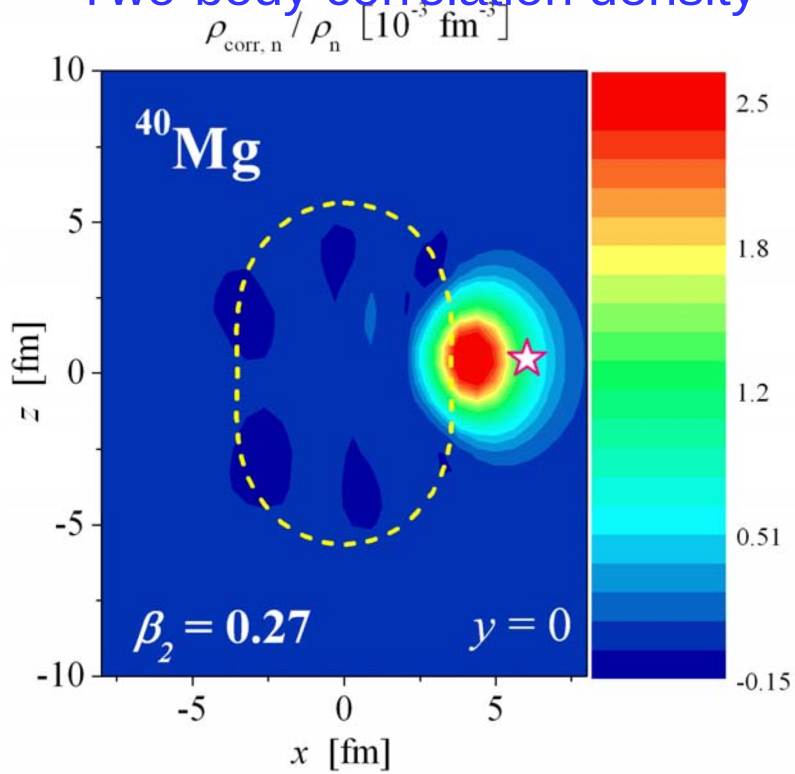
+ non-resonant continuum

Di-neutron pairing in ^{40}Mg



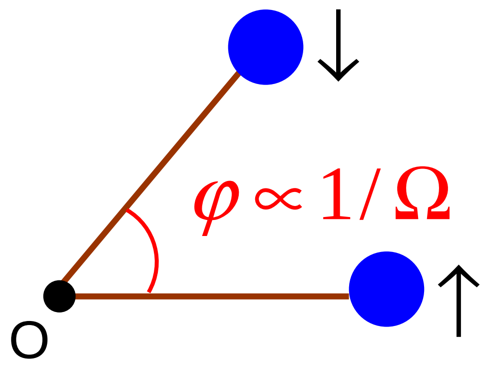
($\Omega = j_z$)

Two-body correlation density



High-Ω continuum states are required.

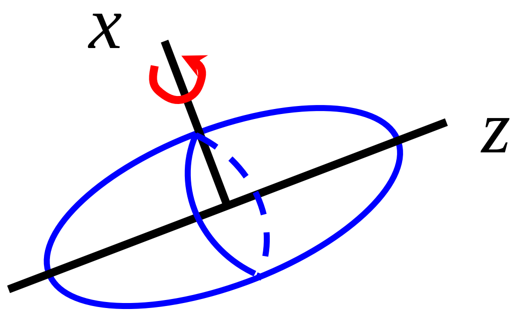
⇨ Spatial localization of the correlated pair (di-neutron correlation)



Rotational moment of inertia

- Belyaev formula

$$\mathcal{J}_{\mathrm{HFB}} = \sum_{k,k'} \frac{\left| \langle kk' | J_x | HFB \rangle \right|^2}{E_k + E_{k'}}$$

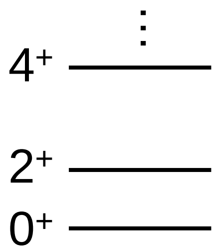


k : quasiparticle states

$\Longrightarrow$ Y_{20}-deformed coordinate space HFB

- Rotational spectrum

$$E_I = \frac{\hbar^2}{2\mathcal{J}_{\mathrm{HFB}}} I(I+1)$$



$$\frac{E(4^+)}{E(2^+)} = 3.33$$

Role of pairing correlation

$$\mathfrak{J}_{\mathrm{HFB}} = \sum_{k,k'} \frac{\left| \langle kk' | J_x | HFB \rangle \right|^2}{E_k + E_{k'}}$$

Comparison

C) Spatial extent of WF

$\mathfrak{J} \nearrow$?

$$\mathfrak{J}_{\mathrm{BCS}} = \sum_{k,k'} \frac{\left| \langle k | J_x | k' \rangle_{HF} \right|^2}{E_k + E_{k'}} \left(u_k v_{k'} - u_{k'} v_k \right)^2$$

B) Distribution of configurations

A) Quasiparticle energies

$$E_k = \sqrt{\left(\varepsilon_k^{HF} - \lambda \right)^2 + \left(\Delta_k \right)^2}$$

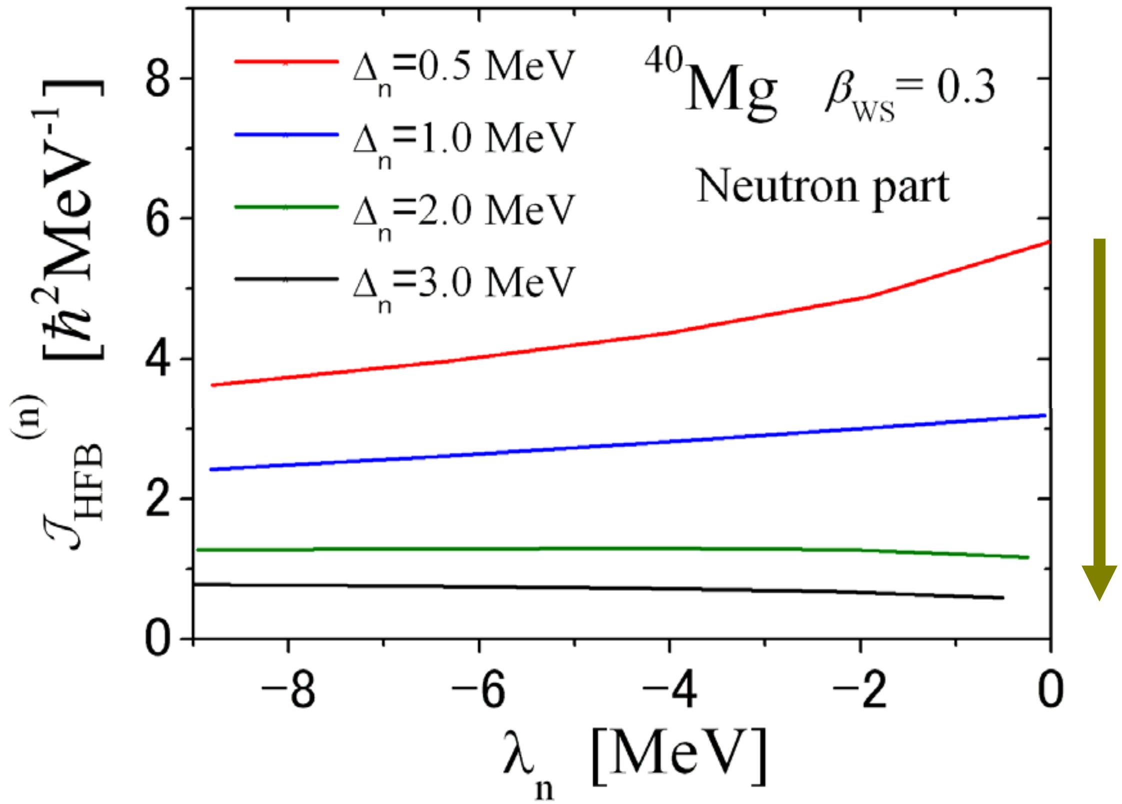
$\mathfrak{J} \searrow$

$\mathfrak{J} \searrow$

Moment of inertia in “artificial” $^{40}Mg_{28}$

$$V_{WS}\,(\text{standard}) \Rightarrow \lambda_n \approx -0.2 \text{ MeV}$$

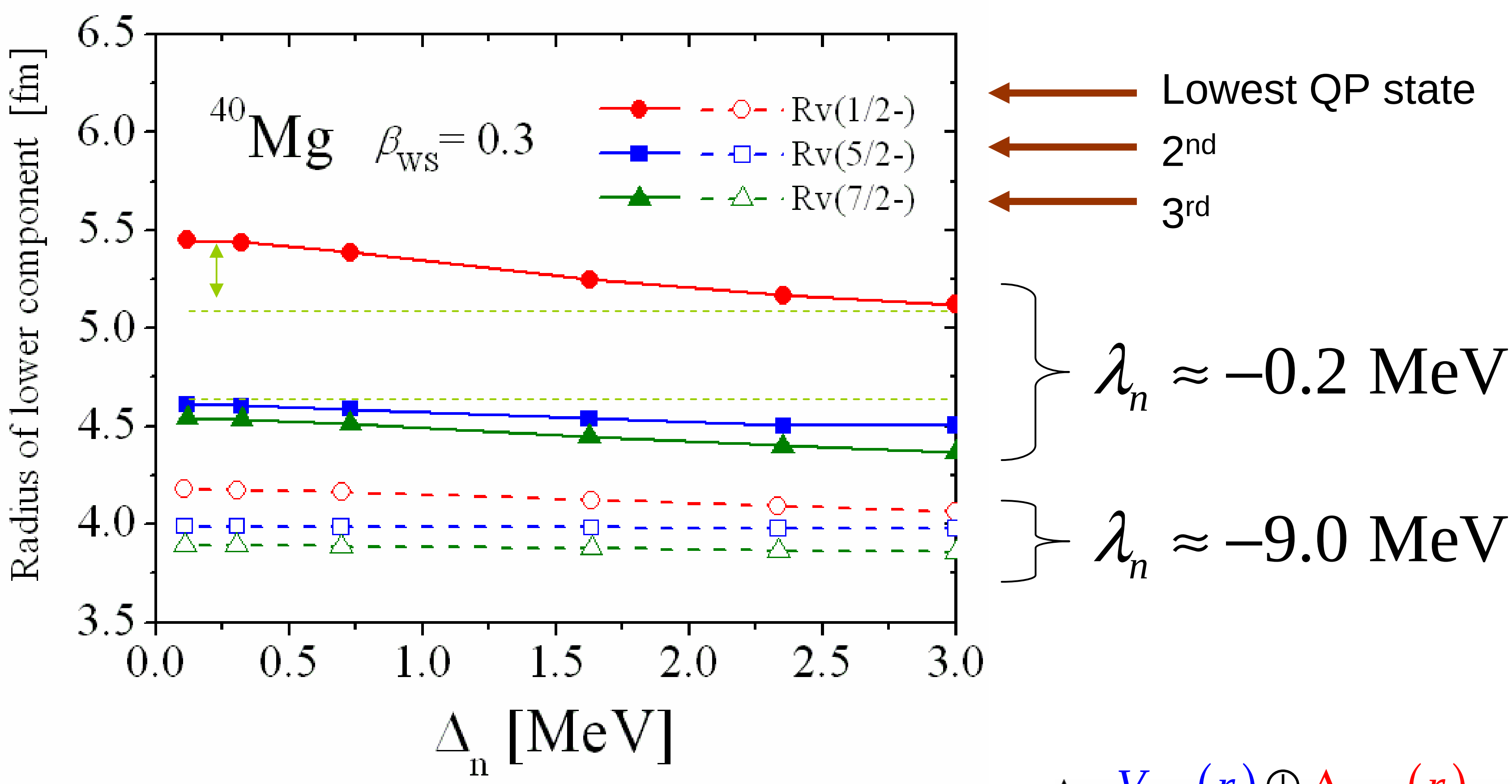
$$\text{"artificial"} \Rightarrow V_{WS}, V_{pair} \rightarrow \textit{parameters}\,(N = 28 \text{ fixed})$$

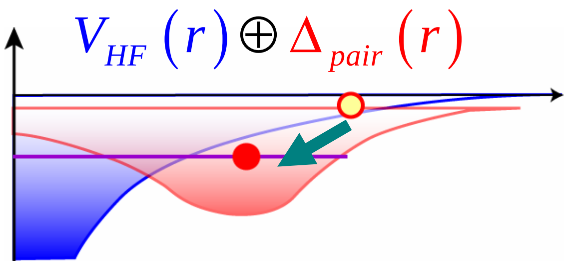


$\Delta_n \nearrow \rightarrow \mathcal{J}_{HFB} \searrow$

$$\left.\begin{array}{l} \text{Weak } \Delta_n\text{:} \;\; \lambda_n \rightarrow 0 \Rightarrow \mathcal{J}_{HFB} \nearrow \\ \text{Strong } \Delta_n\text{:} \;\; \lambda_n \rightarrow 0 \Rightarrow \mathcal{J}_{HFB} \searrow \end{array}\right\} \Rightarrow \text{Strong } \Delta_n \text{ sensitivity}$$

Spatial extent of quasiparticle states





Strong Δ_n sensitivity @ $\lambda_n \approx 0$

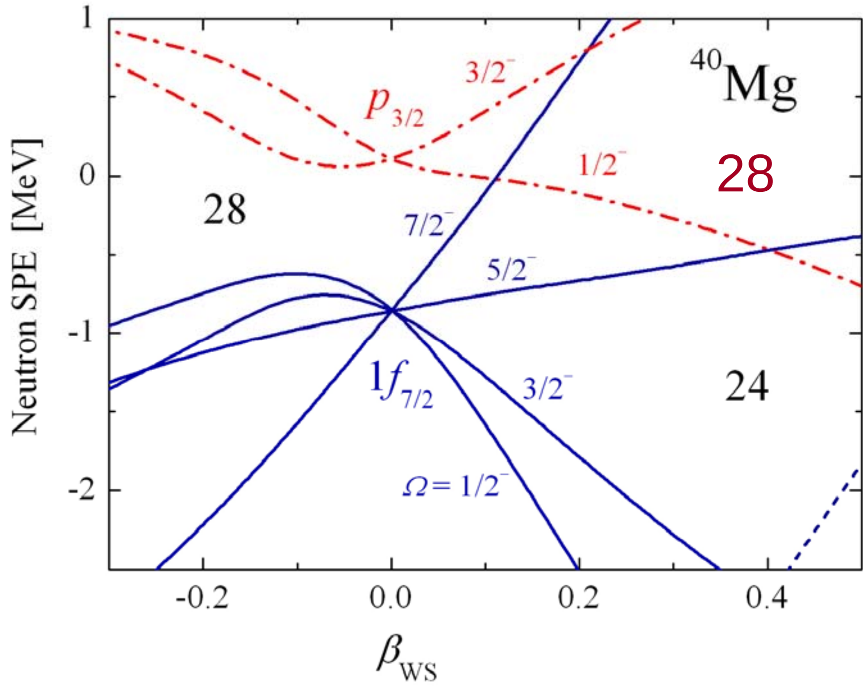
Large spatial extent $\left(\Delta_n \approx 0 \Rightarrow \mathcal{J}_{HFB} \nearrow\right)$

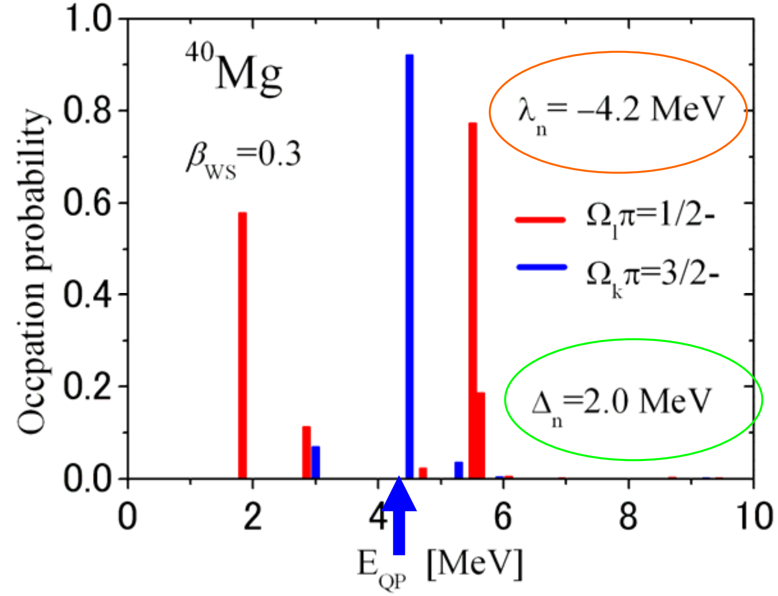
Pairing anti-halo effect $\left(\Delta_n \nearrow \Rightarrow \mathcal{J}_{HFB} \searrow\right)$

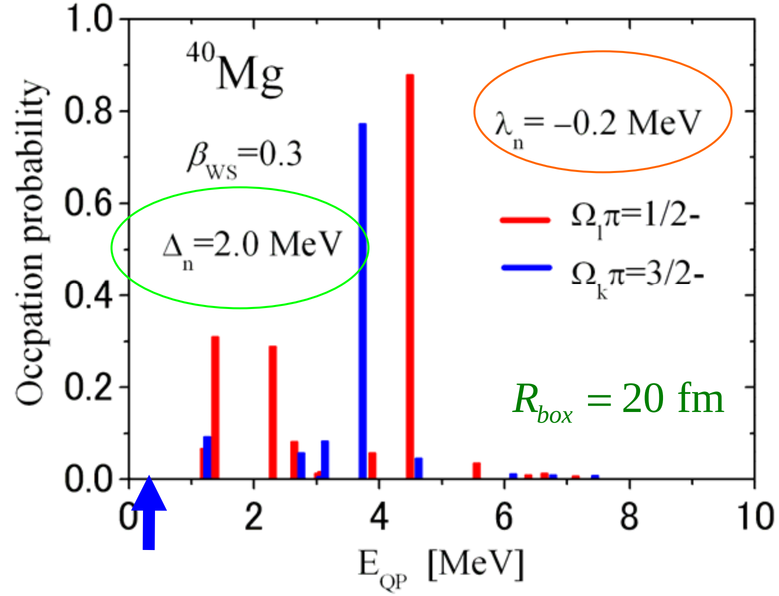
Coupling to continuum states $\left(\Delta_n \nearrow \Rightarrow \mathcal{J}_{HFB} \searrow ?\right)$ (next slide)

Quasiparticle energy spectrum

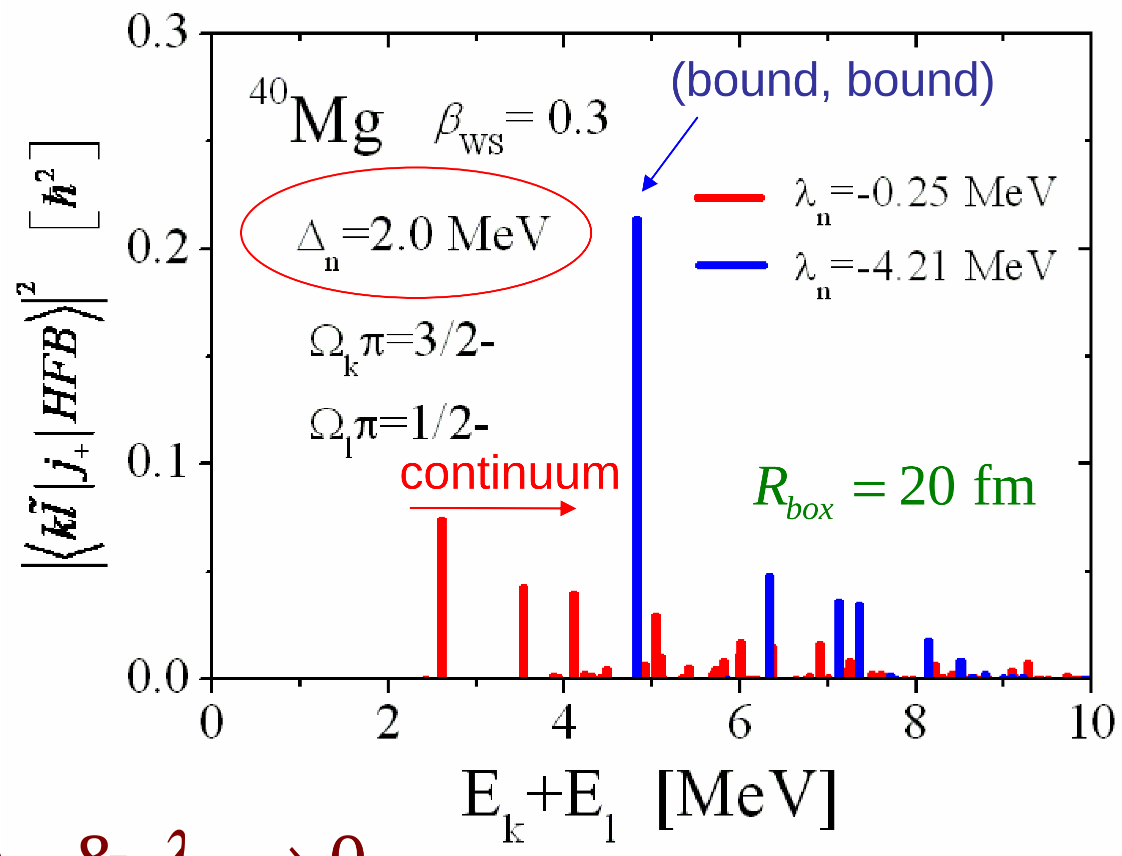
$$e.g.\quad \left\langle \left(3/2^-\right)\left(\overline{1/2^-}\right)\middle| j_+ \middle| HFB \right\rangle$$







Role of coupling to continuum states



Strong Δ_n & $\lambda_n \to 0$

$$\mathcal{J}_{HFB} = \sum_{k,l} \frac{\left|\langle kl | j_x | HFB \rangle\right|^2}{E_k + E_l}$$

Smaller matrix elements

$\Updownarrow$ Competition

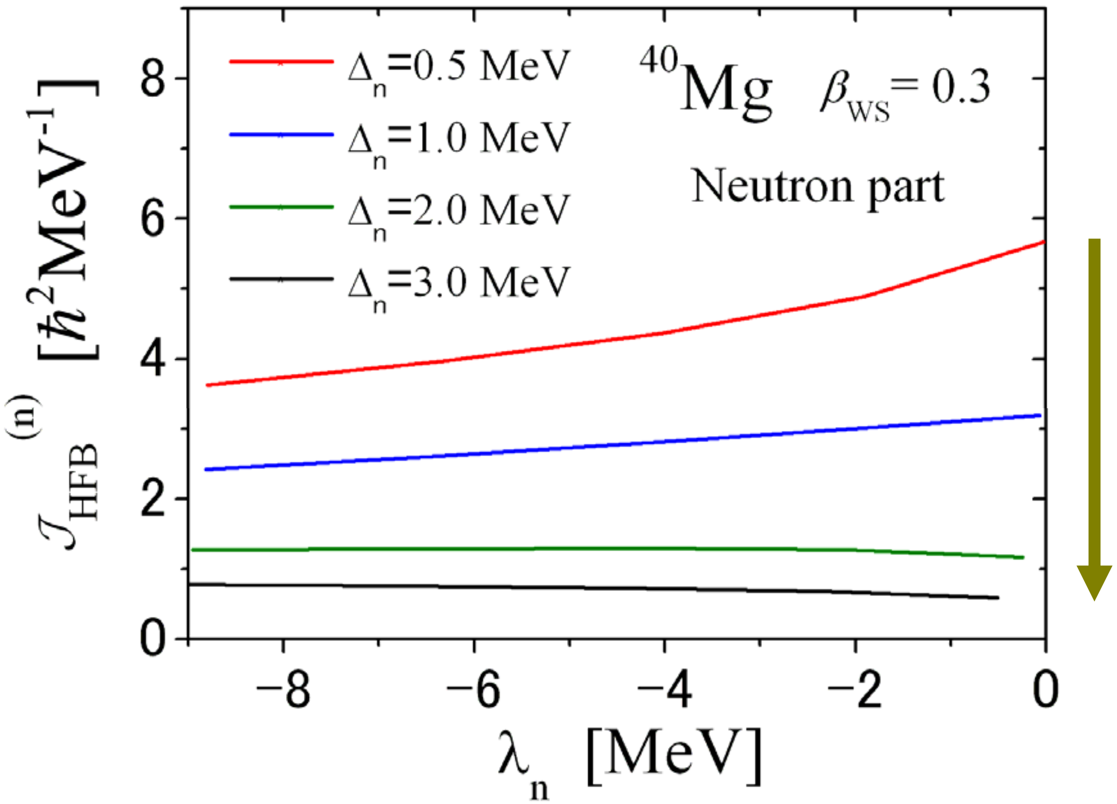
Smaller two-QP energies

$$\text{Cf.} \quad \mathcal{J}_{\text{BCS}} = \sum_{k,l} \frac{\left|\langle k | J_x | l \rangle_{HF}\right|^2}{E_k + E_l} (u_k v_l - u_l v_k)^2$$

Moment of inertia in “artificial” $^{40}Mg_{28}$

$$V_{WS}\,(\text{standard}) \Rightarrow \lambda_n \approx -0.2 \text{ MeV}$$

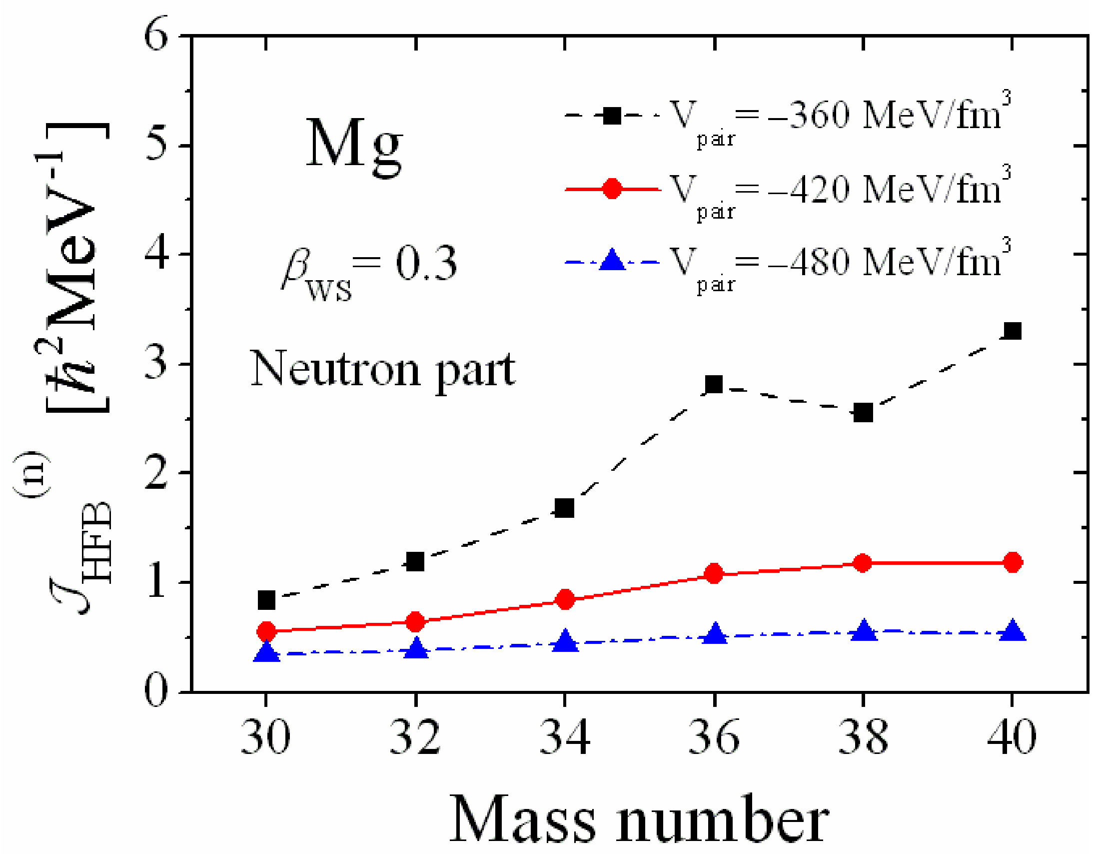
$$\text{"artificial"} \Rightarrow V_{WS}, V_{pair} \rightarrow \textit{parameters}\,(N = 28 \text{ fixed})$$



$$\Delta_n \nearrow \rightarrow \mathcal{J}_{HFB} \searrow$$

$$\left.\begin{array}{l} \text{Weak } \Delta_n:\ \lambda_n \rightarrow 0 \Rightarrow \mathcal{J}_{HFB} \nearrow \\ \text{Strong } \Delta_n:\ \lambda_n \rightarrow 0 \Rightarrow \mathcal{J}_{HFB} \searrow \end{array}\right\} \Rightarrow \text{Strong } \Delta_n \text{ sensitivity}$$

Moment of inertia in neutron rich Mg isotopes



$\Delta_n \approx 1.0$ MeV

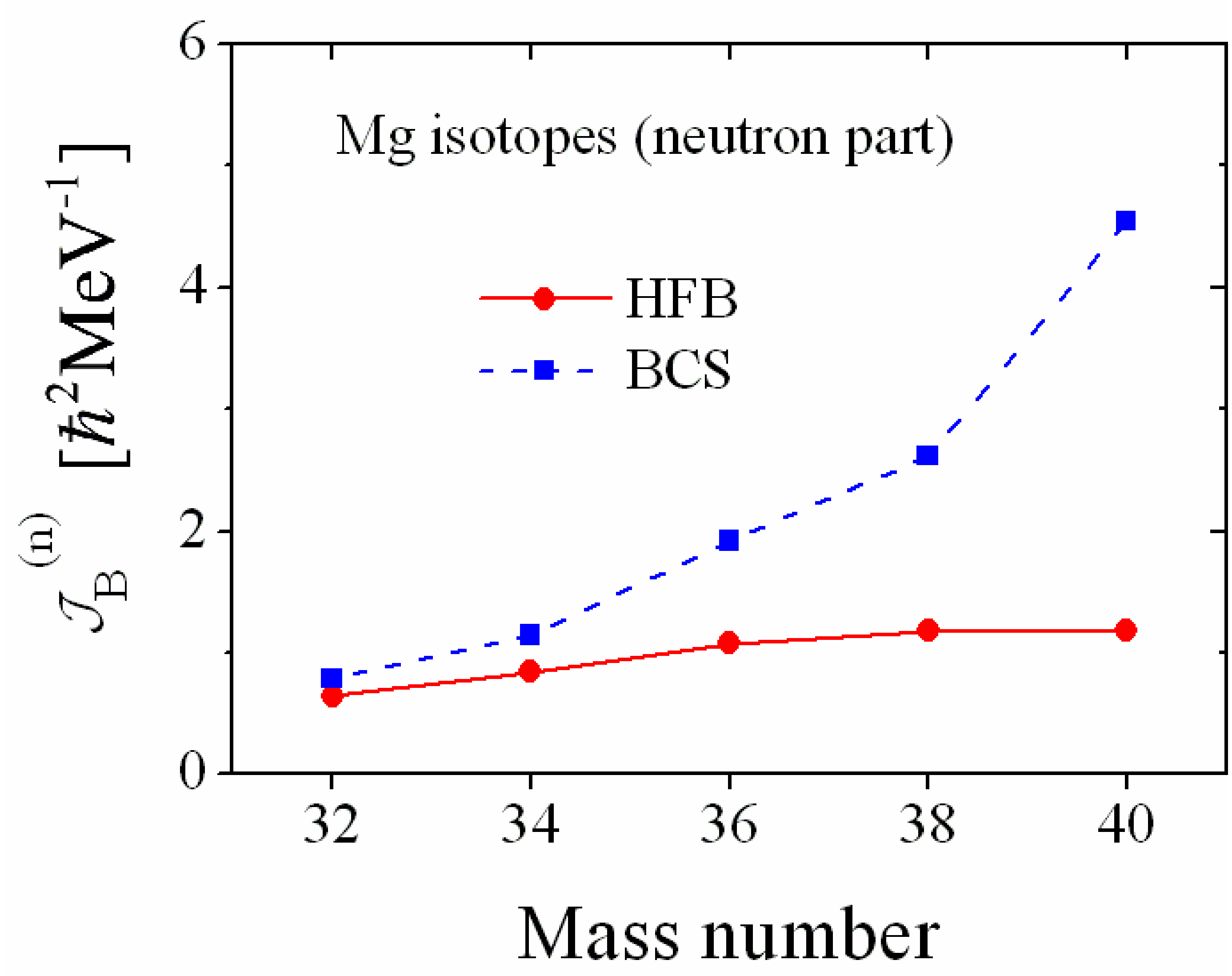
$\Delta_n \approx 2.0$ MeV $\left(\approx 12/\sqrt{40}\right)$

$\Delta_n \approx 3.0$ MeV

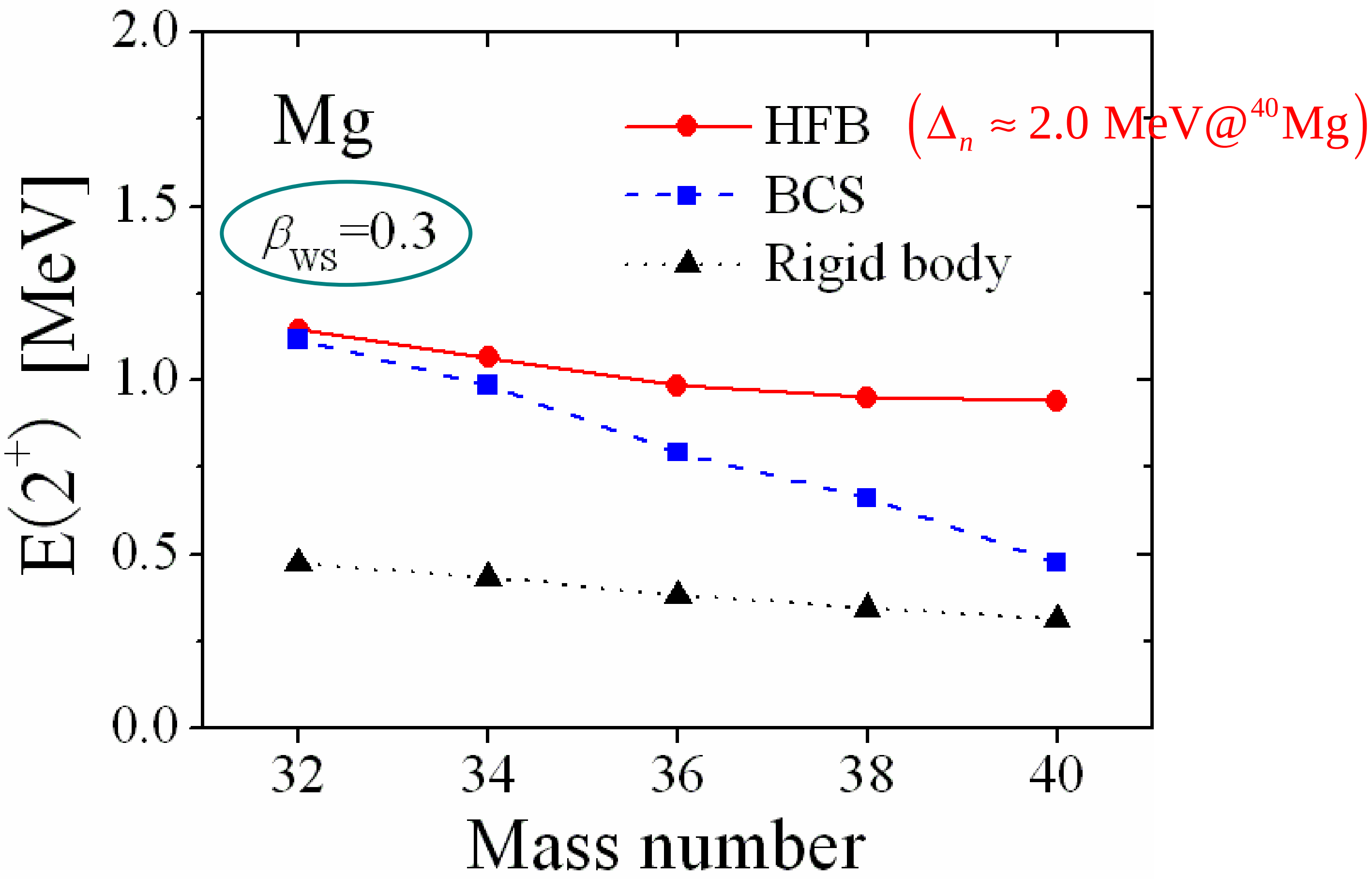
Approaching the neutron drip line ⇨ Strong pairing dependence

1) Spatial extent of WF, 2) Pairing anti-halo effect, 3) Coupling to continuum

Moment of inertia: HFB(di-neutron) vs. BCS



Excitation energies of rotational 2^+ states



Large B(E2) (β_c) & high $E(2^+)$ in HFB

Summary

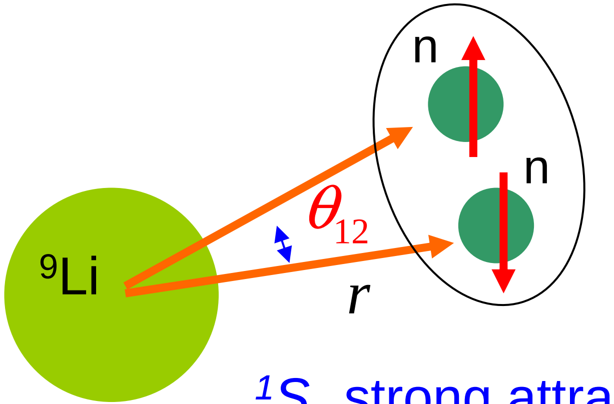
- New grid representation for deformed coordinate space HFB calculation
- Di-neutron superfluidity in neutron rich Mg isotopes
  - Role of high-Ω non-resonant continuum states
- Rotational moment of inertial based on coordinate space HFB
  - Strong pairing sensitivity (pairing strength, types (HFB or BCS))
    - ✓ Spatial extent of WF
    - ✓ Pairing anti-halo effect
    - ✓ Coupling to continuum

⇩

Experimental E(2^+) & B(E2) values in $^{34-40}$Mg can be a good indicator of pairing properties unique in neutron drip line region.

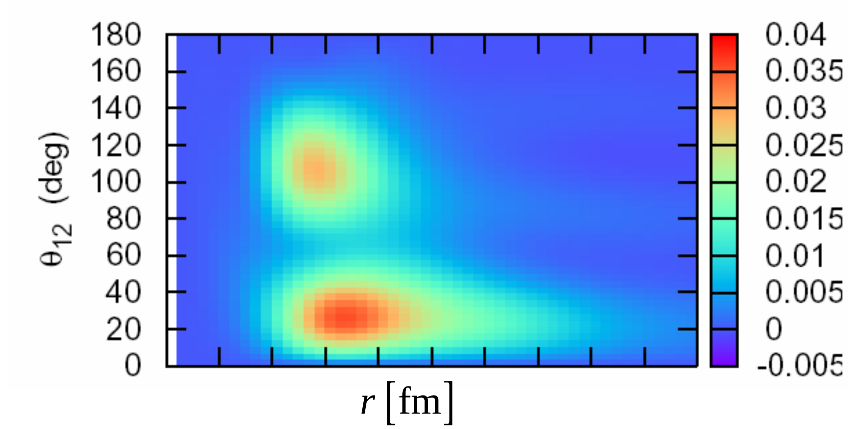
- Future work for more quantitative discussion
  - Skyrme-HFB
  - Correlations (Migdal term) $\longrightarrow$ Deformed QRPA

Pairing correlation in borromean nucleus ^{11}Li



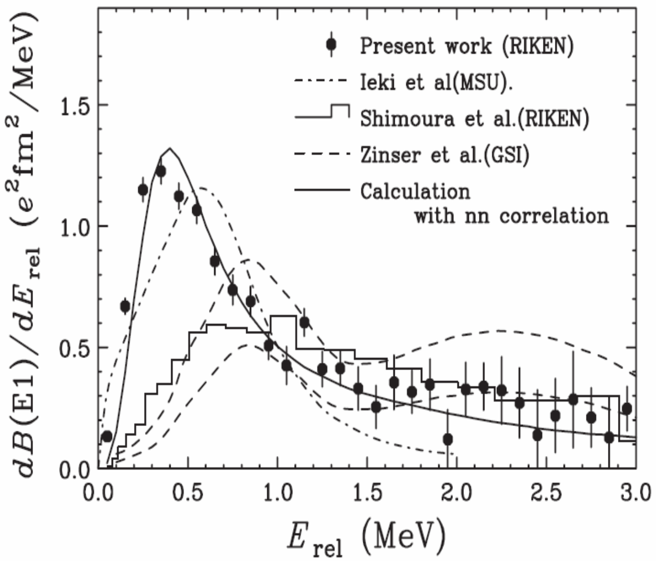
$^{1}S_{0}$ strong attraction in low-r
>> di-neutron correlation?

Two-particle density in ^{11}Li



K.Hagino, H.Sagawa, Phys.Rev. C 72, 044321 (2005)

Soft E1 excitation



MSU@ 28MeV/nucleon
PRL 70 (1993) 730.
PRC 48(1993) 118.

RIKEN @ 43MeV/nucleon
PLB348 (1995) 29.

GSI @280MeV/nucleon
NPA 619 (1997) 151.

New data: T. Nakamura, *et al.*, Phys. Rev. Lett. 96, 252502 (2006)

$$B(E1) = 1.42(18)\ \mathrm{e^2 fm^2},\ E_{\mathrm{rel}} < 3\ \mathrm{MeV}$$

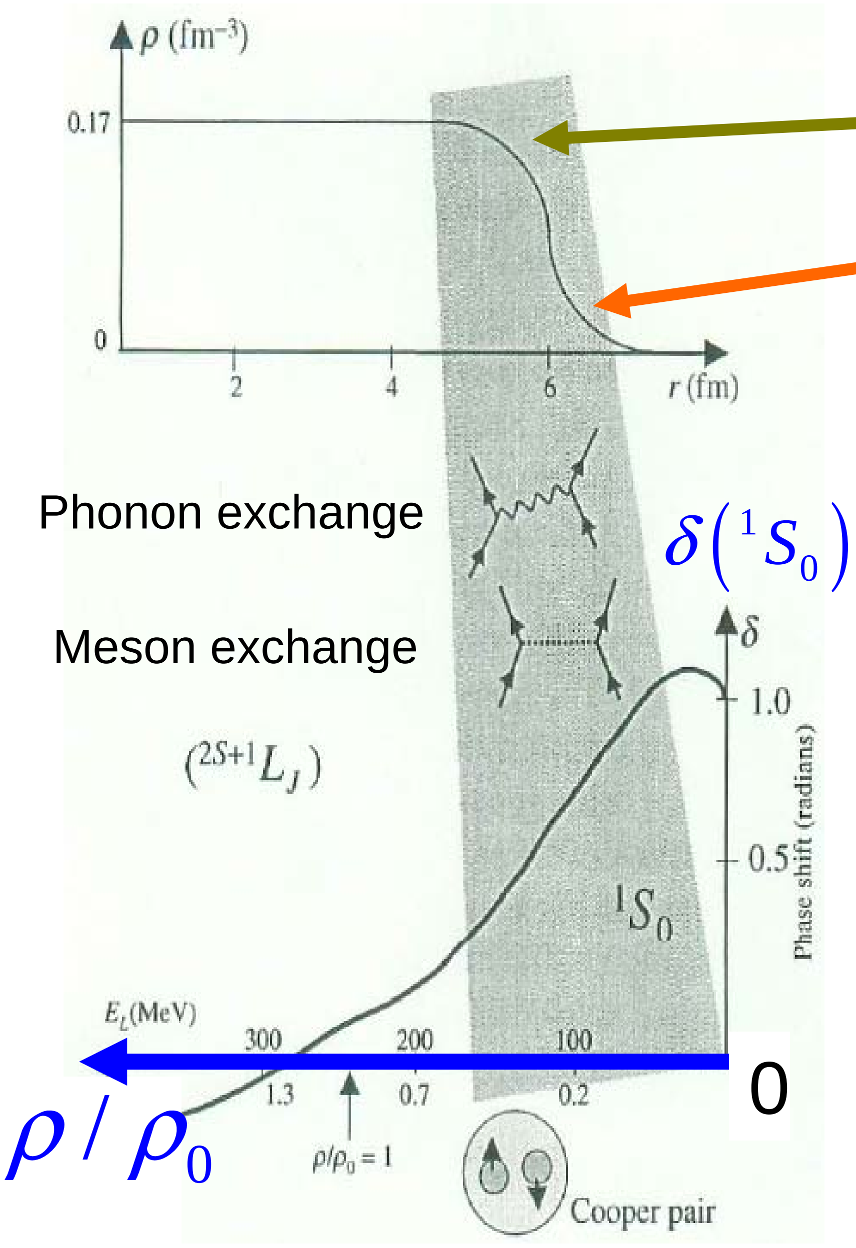
$$\Rightarrow \langle \theta_{12} \rangle = 48^{+14}_{-18}\ \text{degree}$$

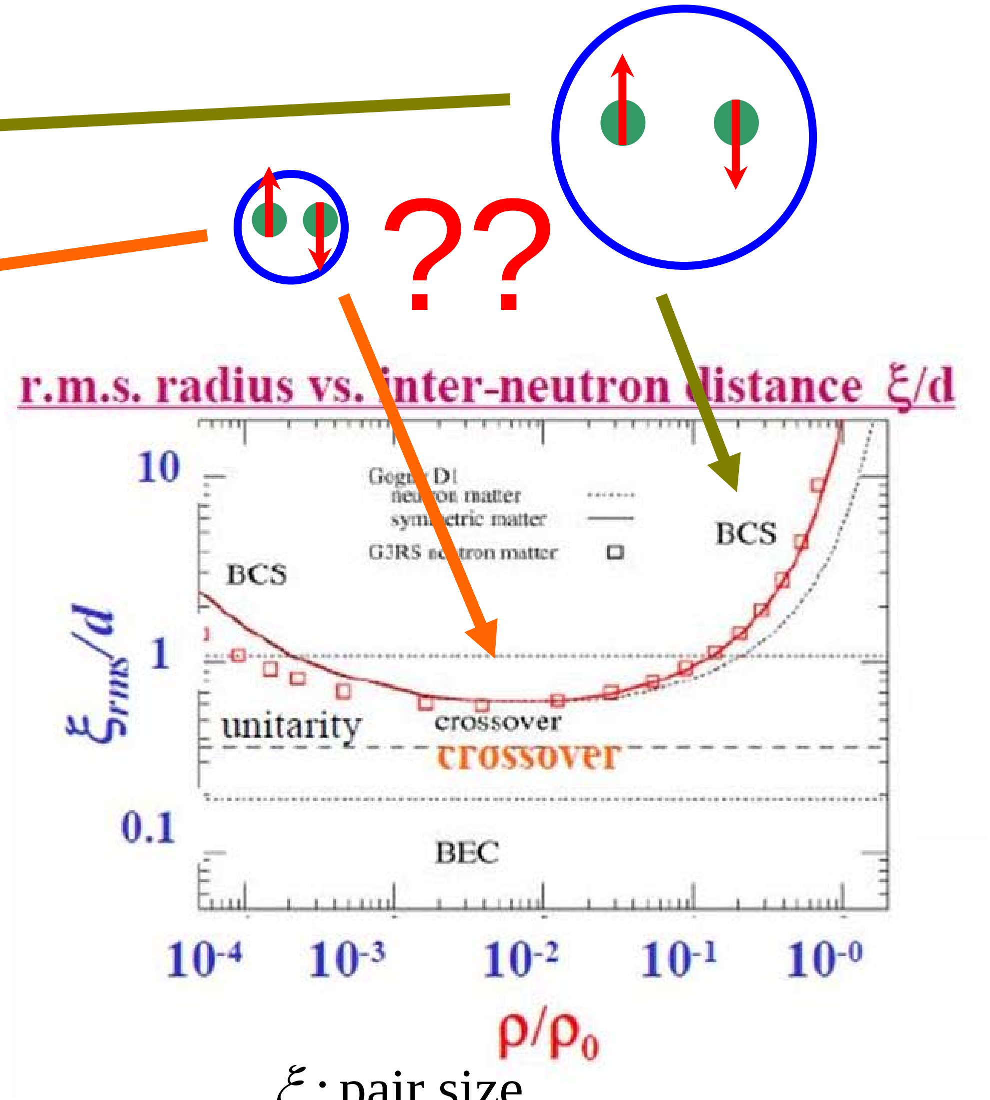
$$\text{cf. } \langle \theta_{12} \rangle_{no-correlation} = 90\ \text{degree}$$

Appreciable two neutron spatial correlation

→ Di-neutron correlation implied

Density dependence of pairing correlation





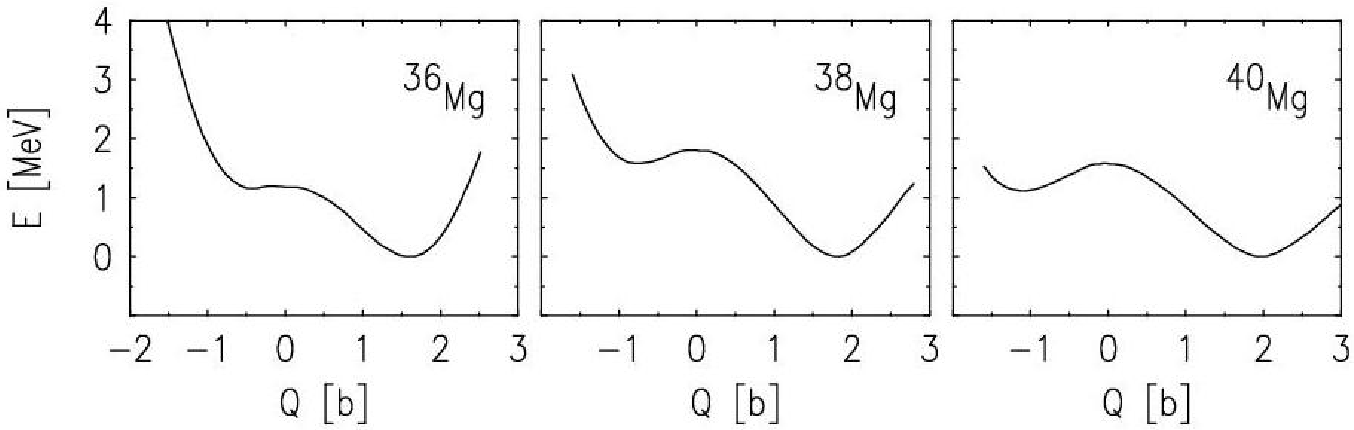
ξ : pair size

d : inter-neutron distance

D.M.Brink, R.A.Broglia, *Nuclear Superfluidity*, Cambridge University Press, 2005

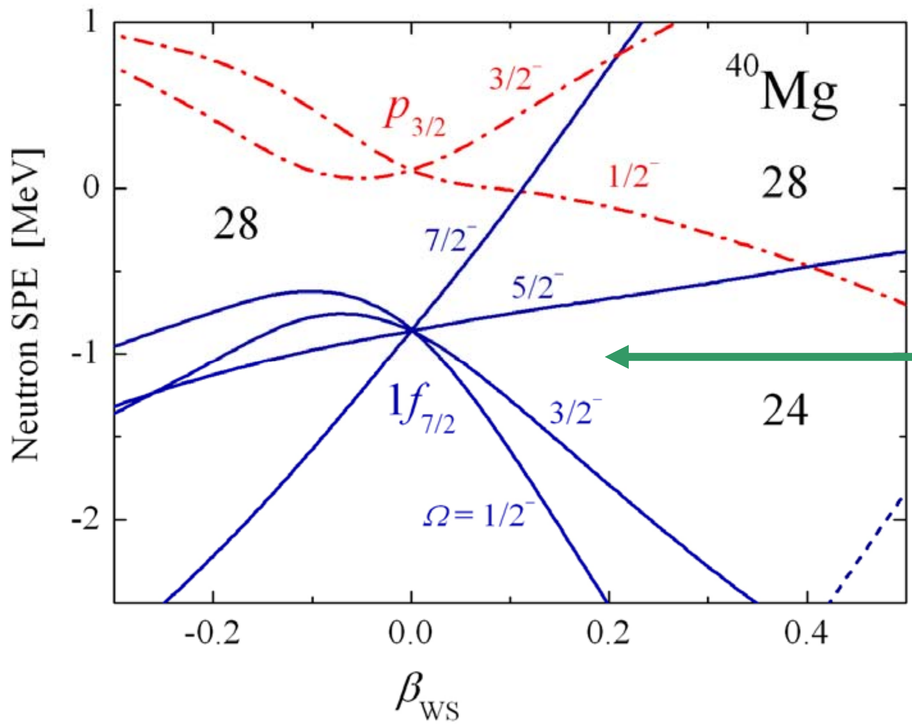
M.Matsuo, Phys.Rev. C 73, 044309 (2006)

Deformed neutron rich Mg isotopes



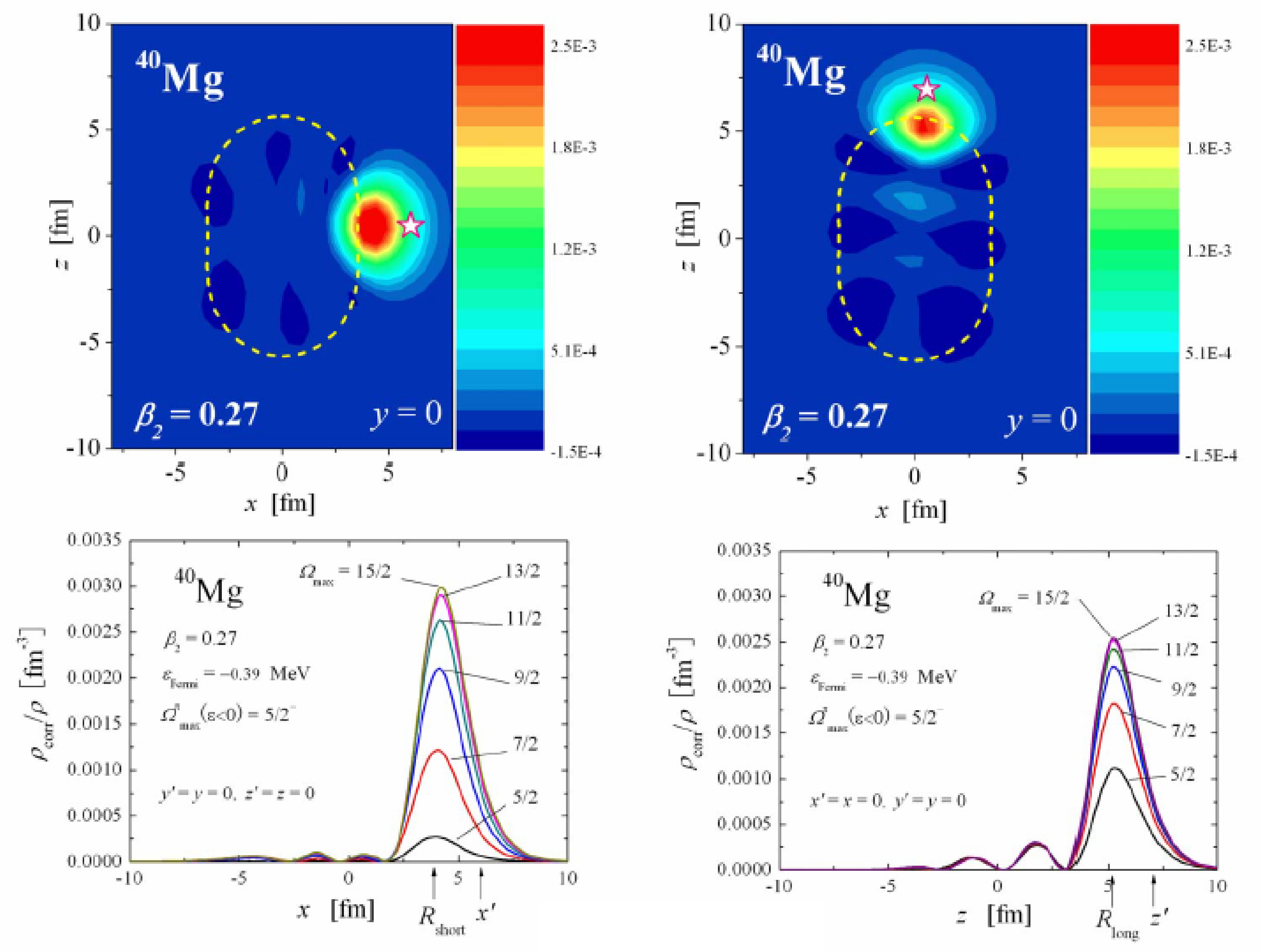
HFB (Skyrme SIII):

J.Terasaki, H.Flocard, P.-H.Heenen, P.Bonche, Nucl.Phys. A621, 706 (1997)

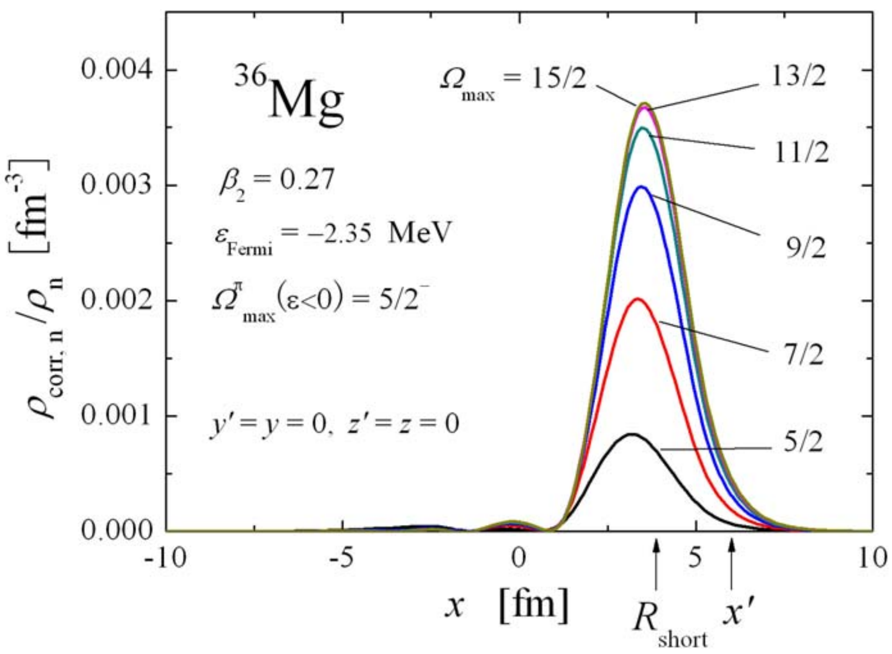


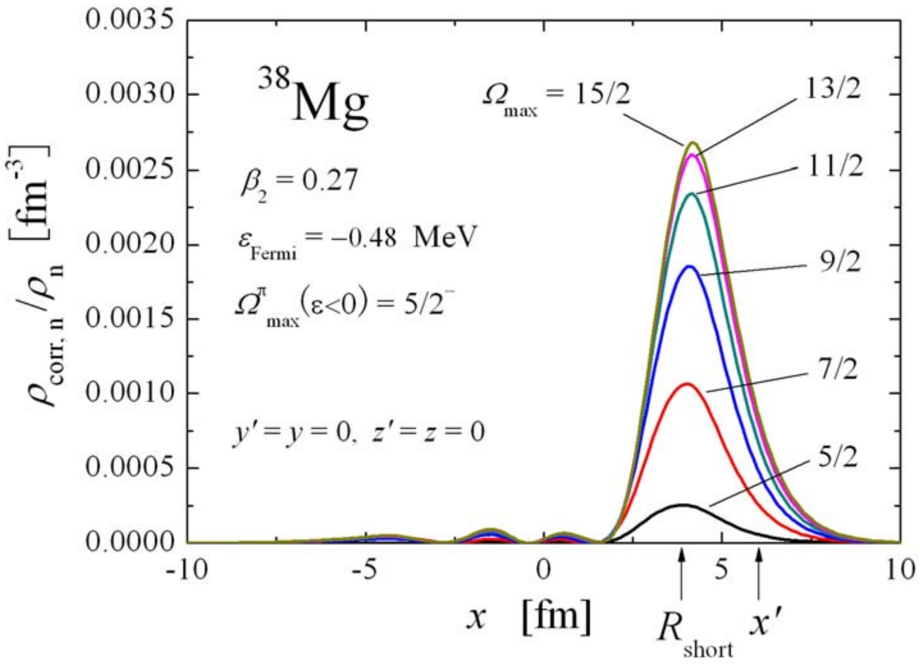
$$\varepsilon_{Fermi}\left({}^{36}\mathrm{Mg}\right)$$

Two-neutron correlation density in ^{40}Mg

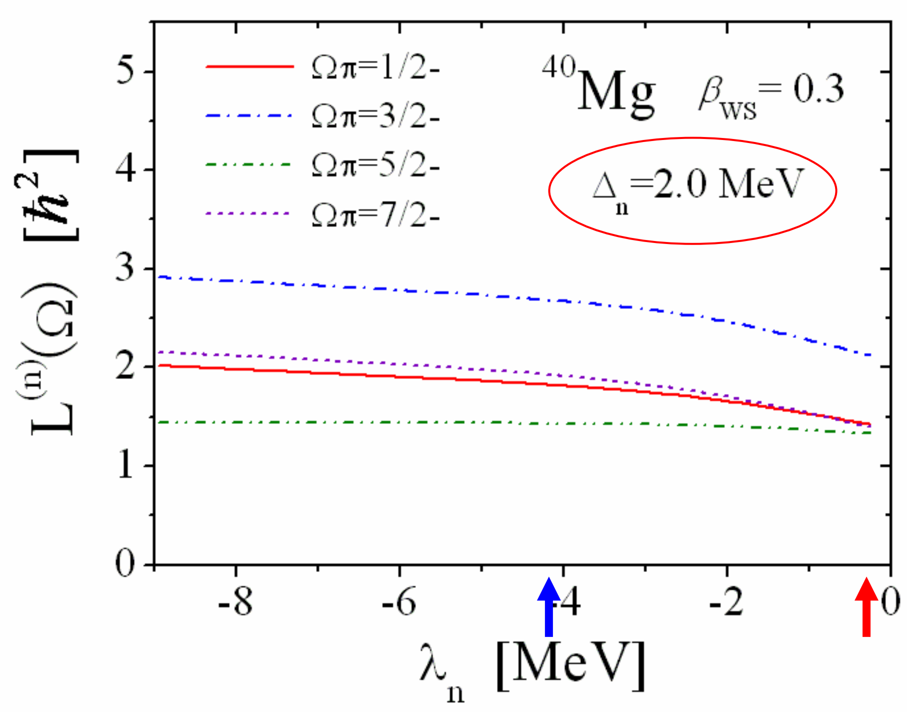


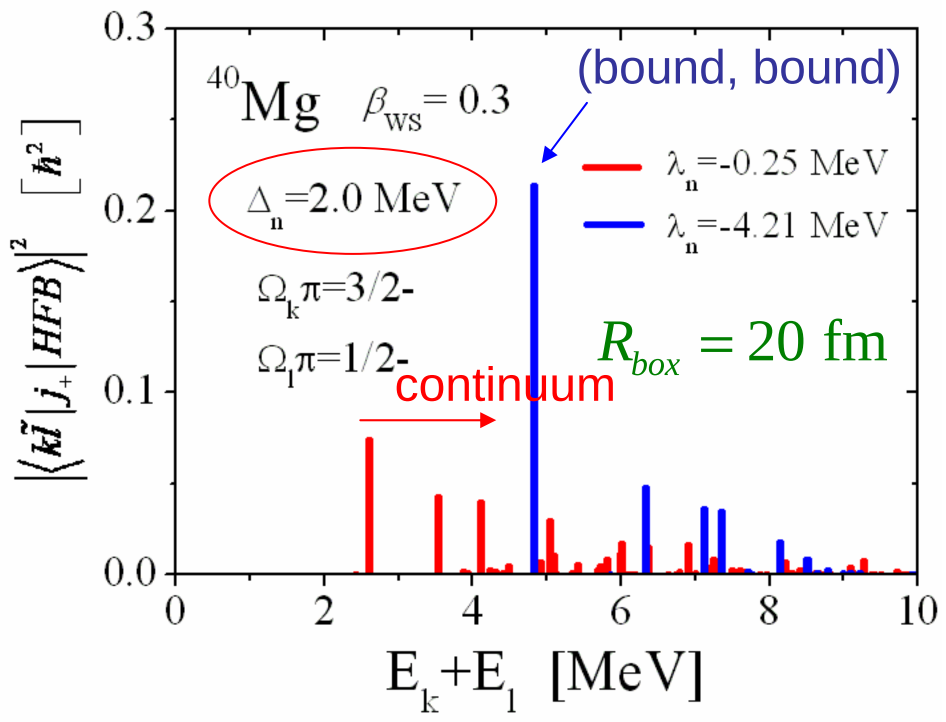
Di-neutron correlation in $^{36,38}Mg$





Role of coupling to continuum



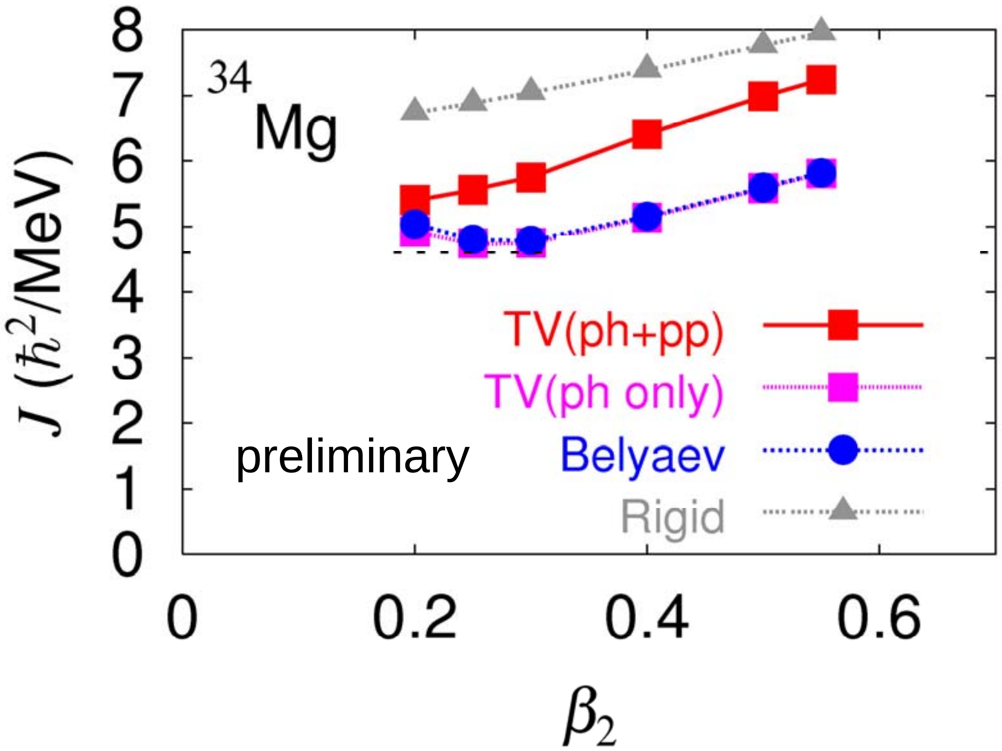


$$L = \sum_{k,l} \left| \left\langle kl \right| j_x \left| HFB \right\rangle \right|^2 = \sum_{\Omega>0,\pi=\pm} L\left(\Omega^\pi\right) \quad \left(\Omega = \max\left(\Omega_k, \Omega_l\right)\right)$$

$$\text{Cf.} \quad \mathcal{J}_{HFB} = \sum_{k,l} \frac{\left| \left\langle kl \right| j_x \left| HFB \right\rangle \right|^2}{E_k + E_l} \Leftrightarrow \mathcal{J}_{\mathrm{BCS}} = \sum_{k,k'} \frac{\left| \left\langle k \right| J_x \left| k' \right\rangle_{HF} \right|^2}{E_k + E_{k'}} \left(u_k v_{k'} - u_{k'} v_k\right)^2$$

Effect of QRPA correlation (preliminary)

Thouless-Valatin, **Belyaev**, and **rigid body** moment of inertia

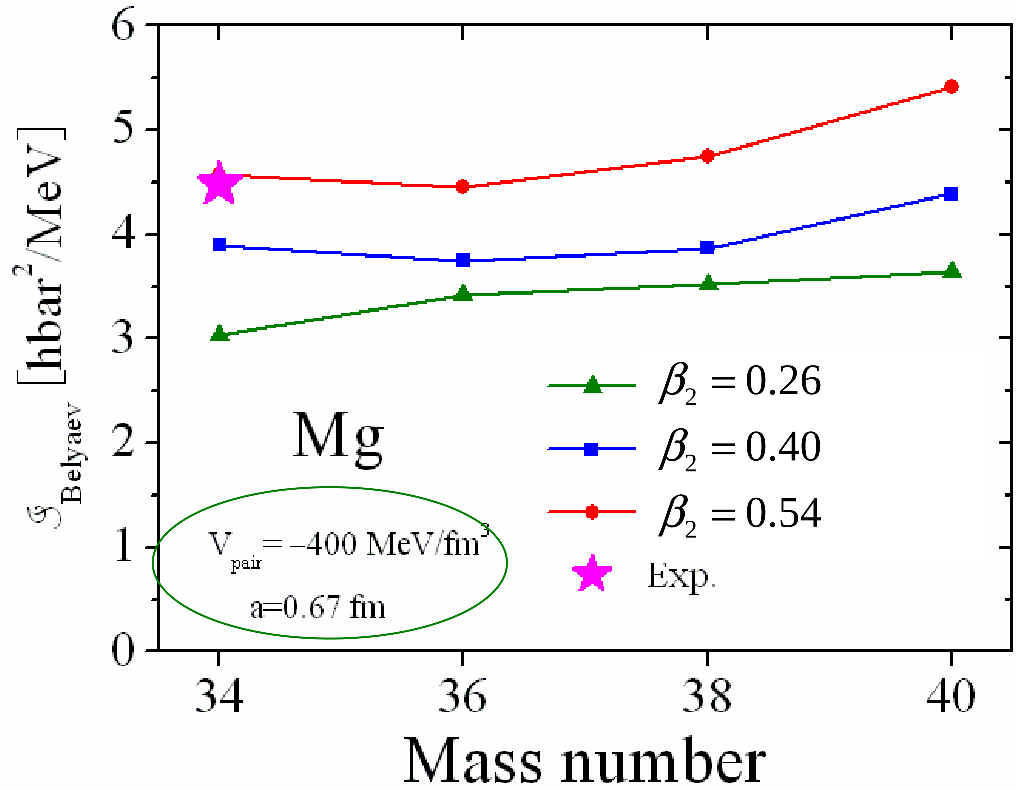


QRPA correlation enhances 20-30% (Migdal term)

For more quantitative discussion, extensive deformed QRPA calculation *in progress*.

Collaboration with Y.R.Shimizu (Kyushu), K.Matsuyanagi, K.Yoshida (Kyoto)

Comparison with experimental data



$\beta_2 = 0.54$

$\beta_2 = 0.26$

Experimental data for ^{34}Mg

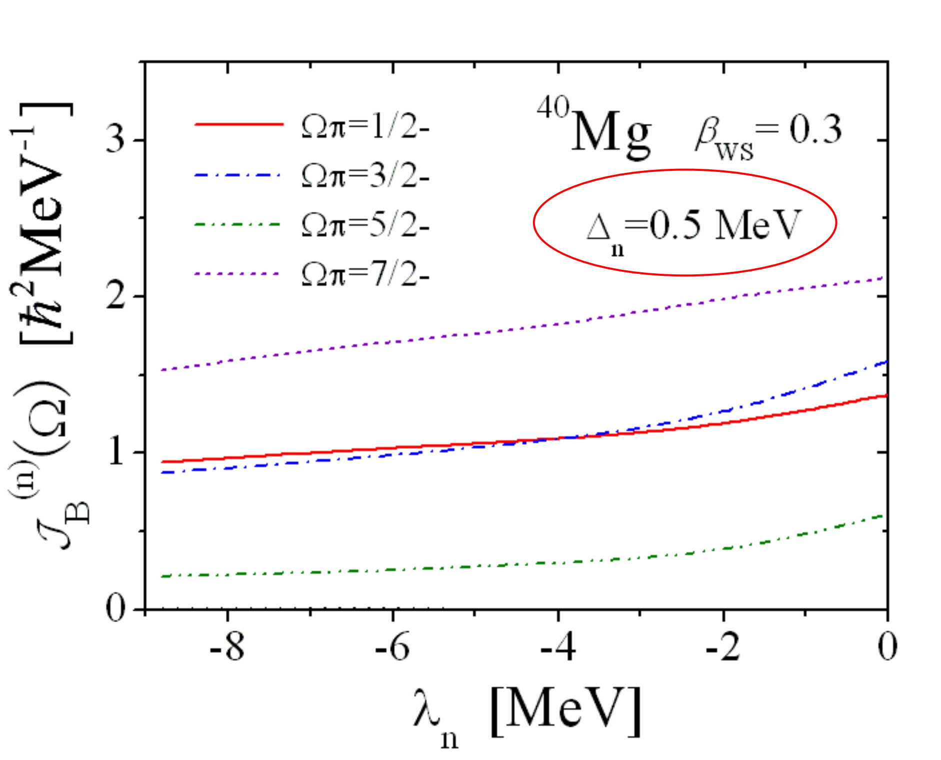
$E\left(2^+\right) = 660$ keV, $E(4_1^+)/E(2_1^+) = 3.2$: K.Yoneda *et al*., Phys. Lett. 499B, 233 (2001)

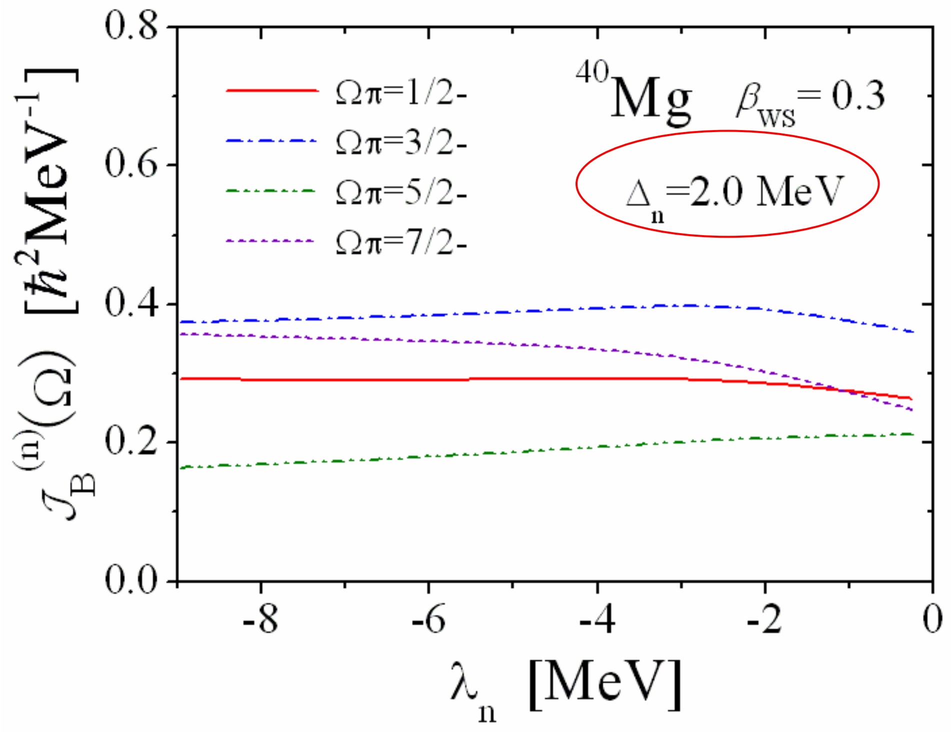
$\beta_c = 0.58 \pm 0.06$: H.Iwasaki, *et al*., Phys. Lett. 522B, 227 (2001)

$\beta_M = 0.68 \pm 0.16$, $\beta_n = 0.70 \pm 0.13$: Z.Elekes, *et al*., Phys. Rev. C 73, 044314 (2006)

Note: $\beta_2 = \sqrt{\pi/5}\left\langle r^2 Y_{20}\right\rangle / A\left\langle r^2\right\rangle$

Moment of inertia: Ω-decomposition





$$\mathcal{J}_{HFB} = \sum_{k,l} \frac{\left|\left\langle kl \middle| j_x \middle| HFB \right\rangle\right|^2}{E_k + E_l} = \sum_{\Omega_k > 0} \mathcal{J}_{HFB}\left(\Omega_k\right)$$

(Axial and Time-reversal symmetries)

$$\left\langle kl \middle| j_+ \middle| HFB \right\rangle = \hbar \delta_{\Omega_k + \Omega_l, 1} M^+_{kl},$$

$$\left\langle k\tilde{l} \middle| j_+ \middle| HFB \right\rangle = \hbar \delta_{\Omega_k, \Omega_l + 1} M^+_{k\tilde{l}},$$

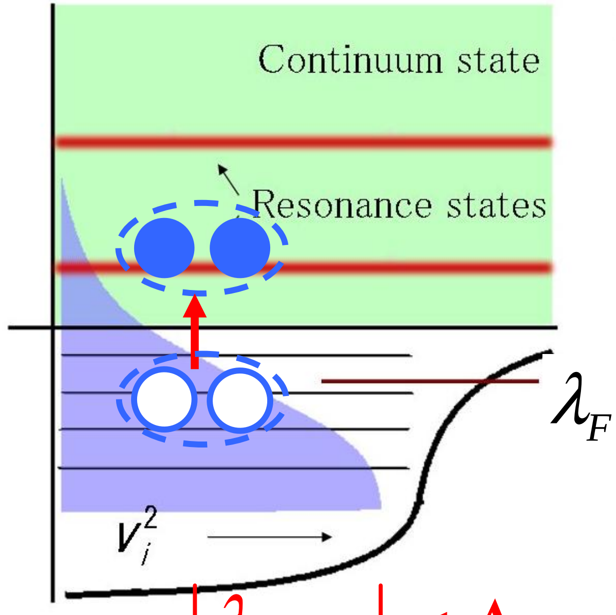
…

Pairing correlation in weakly-bound nuclei

Pair scattering into continuum states

⇩

Break down of BCS approximation



$$|\lambda_{Fermi}| < \Delta_{pair}$$

$$\Psi_k(\vec{r}) = \begin{pmatrix} u_k(E,\vec{r}) \\ v_k(E,\vec{r}) \end{pmatrix} \overset{BCS}{\approx} \begin{pmatrix} u_k \varphi_k^{HF}(\vec{r}) \\ v_k \varphi_k^{HF}(\vec{r}) \end{pmatrix}$$

- Pairing correlation dose NOT change the spatial structure.
- Neutron gas problem

$$\rho(\vec{r}) = \sum_{\varepsilon_k<0} \left\{ v_k \varphi^{HF}(\varepsilon_k,\vec{r}) \right\}^2 + \int_{\varepsilon>0} dn(\varepsilon) \left\{ v_\varepsilon \varphi^{HF}(\varepsilon,\vec{r}) \right\}^2 \to div.$$

$$\varphi_k^{HF} \to \exp(-\alpha_k r)/r \qquad \varphi_\varepsilon^{HF} \to \sin(kr+\delta_{lj})/r$$

J.Dobaczewski, H.Flocard, J.Treiner, Nucl. Phys. A422, 103 (1984)

Coordinate space Hartree-Fock-Bogoliubov theory

$$\begin{pmatrix} \hat{T} + V_{HF}(\vec{r}) - \lambda & \Delta(\vec{r}) \\ \Delta(\vec{r}) & -\hat{T} - V_{HF}(\vec{r}) + \lambda \end{pmatrix} \begin{pmatrix} u_k(E,\vec{r}) \\ v_k(E,\vec{r}) \end{pmatrix} = E \begin{pmatrix} u_k(E,\vec{r}) \\ v_k(E,\vec{r}) \end{pmatrix}$$

A. Bulgac, FT-194-1980, CIP-IPNE, Bucharest Romania, 1980 (nucl-th/9907088)

J. Dobaczewski, H. Flocard, J. Treiner, Nucl. Phys. A422, 103 (1984)

Asymptotic behavior at infinity

$$\hat{T} u_k(E,\vec{r}) = (E + \lambda) u_k(E,\vec{r})$$

$$\hat{T} v_k(E,\vec{r}) = -(E - \lambda) v_k(E,\vec{r})$$

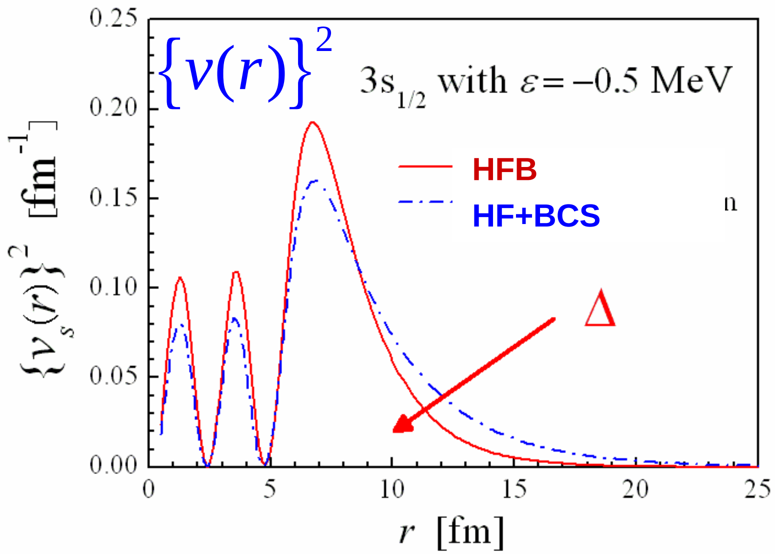
- Determined by $E \Rightarrow$ Pairing correlation changes the spatial structure
- $u_k(E,\vec{r}), v_k(E,\vec{r}) \Rightarrow$ Different asymptotic behavior

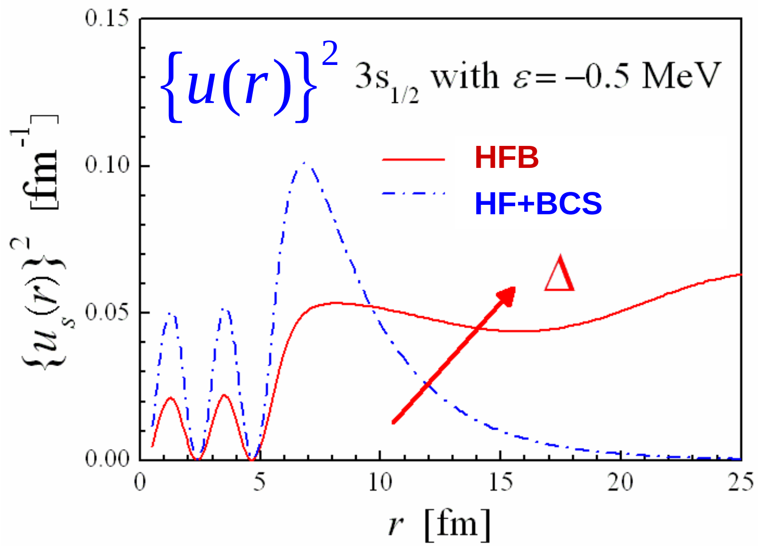
Quasiparticle states in weakly-bound nuclei

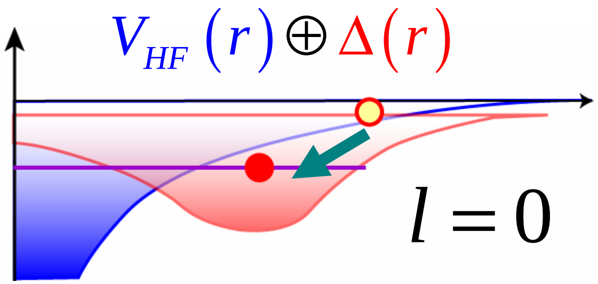
$$\left|\lambda_{Fermi}\right| \le E_k \quad \Rightarrow \quad \begin{aligned} u(E_k,\vec{r}) &\to \sin(\alpha_k r + \delta_k)/r \\ v(E_k,\vec{r}) &\to \exp(-\beta_k r)/r \end{aligned}$$

No neutron gas

e.g., $3s_{1/2}$ state at $\varepsilon^{HF} = -0.5$ MeV







Pairing anti-halo effect

K. Bennaceur, et al., Phys. Lett. 496B, 154 (2000)

Quasiparticle states in weakly-bound nuclei

Coordinate space Hartree-Fock-Bogoliubov theory (Dobaczewski (1984))

$$\begin{pmatrix} \hat{T}+V_{HF}(\vec{r})-\lambda_{Fermi} & \Delta(\vec{r}) \\ \Delta(\vec{r}) & -\hat{T}-V_{HF}(\vec{r})+\lambda_{Fermi} \end{pmatrix}\begin{pmatrix} u(E_k,\vec{r}) \\ v(E_k,\vec{r}) \end{pmatrix} = E_k \begin{pmatrix} u(E_k,\vec{r}) \\ v(E_k,\vec{r}) \end{pmatrix}$$

Quasiparticle motions ⇒ Self-consistency between $V_{HF}(\vec{r})$ and $\Delta(\vec{r})$

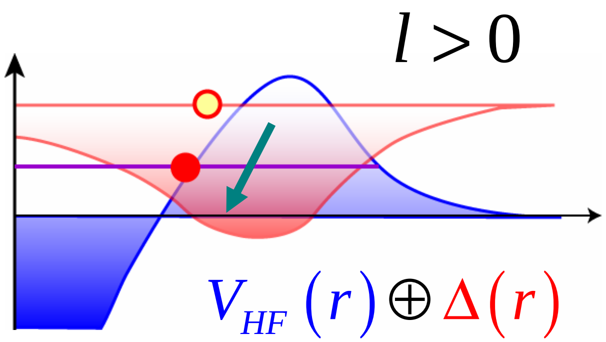
Weakly-bound system $|\lambda_{Fermi}| \leq about\ 2\ \mathrm{MeV}$ $\left(E_k \geq |\lambda_{Fermi}|\right)$

$$\boxed{\begin{aligned} v(E_k,\vec{r}) &\to \exp(-\beta_k r)/r \\ u(E_k,\vec{r}) &\to \sin(\alpha_k r+\delta_k)/r \end{aligned}}$$

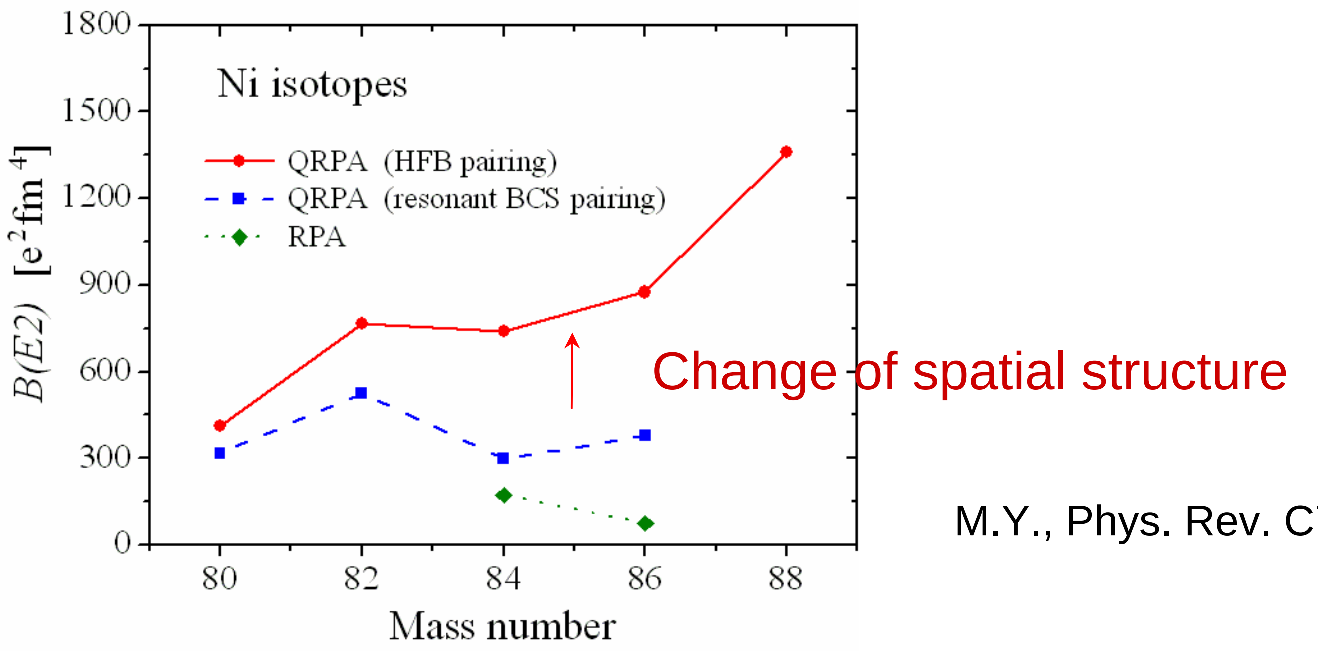
Independent of ε^{HF}

Localization of particle density

$$\rho(\vec{r}) = \sum_{E_k>0} \{v(E_k,\vec{r})\}^2 \to \exp(-2\beta_{\min} r)/r^2$$

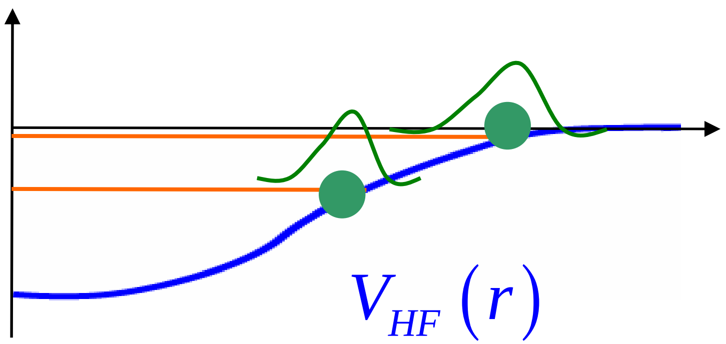


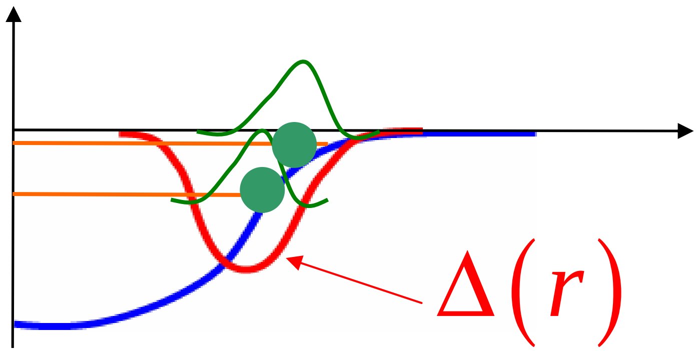
Collective motions: pairing and spatial structure



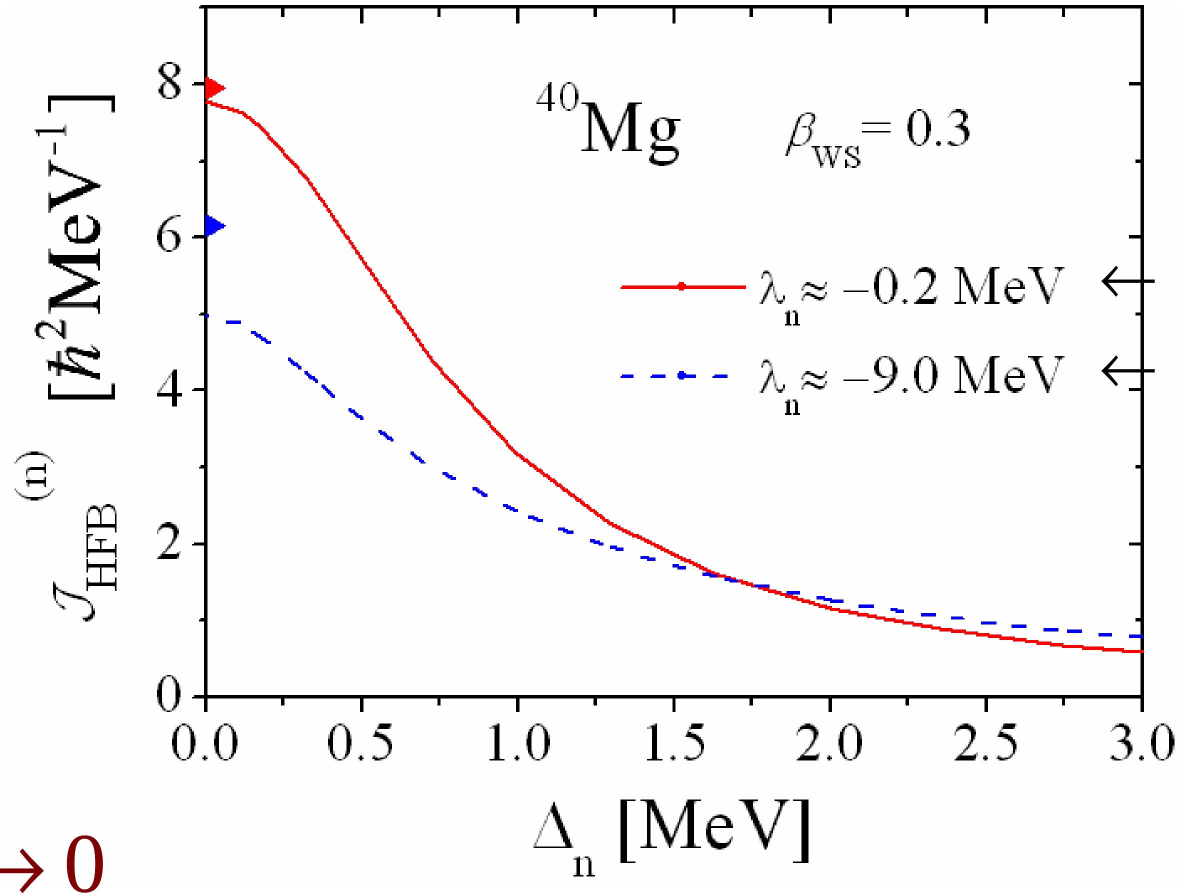
M.Y., Phys. Rev. C72 (2005) 064308

- Pairing NO : Spatial decoupling →Suppression of coherence
- Pairing YES : Spatial localization by pairing potential
  →new mechanism for generating coherence





Moment of inertia in weakly-bound nuclei



$\leftarrow \quad V_{WS} = -39.0$ MeV

$\leftarrow \quad V_{WS}' = V_{WS} \times 1.4$

$\lambda_n \to 0$

$\Delta_n \approx 0 \;\Rightarrow\; \mathcal{J}_{HFB}\left(\lambda_n \approx -0.2 \text{ MeV}\right) \gg \mathcal{J}_{HFB}\left(\lambda_n \approx -9.0 \text{ MeV}\right)$

$\Delta_n \nearrow \;\Rightarrow$ Strong Δ_n sensitivity

$\Delta_n > 1.5 \text{ MeV} \;\Rightarrow\; \mathcal{J}_{HFB}\left(\lambda_n \approx -0.2 \text{ MeV}\right) < \mathcal{J}_{HFB}\left(\lambda_n \approx -9.0 \text{ MeV}\right)$!!