Study of exotic nuclei with the Gamow Shell Model

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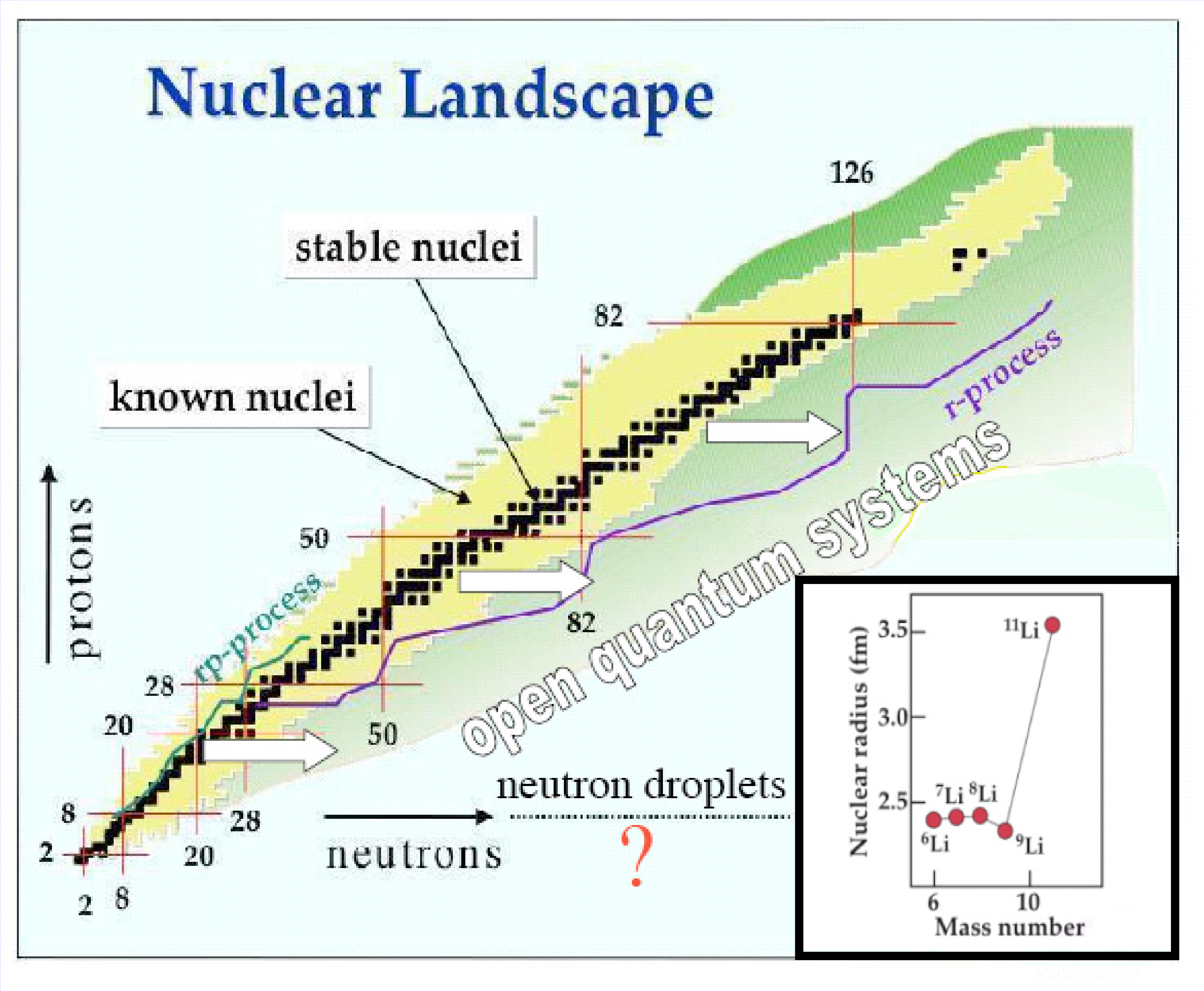
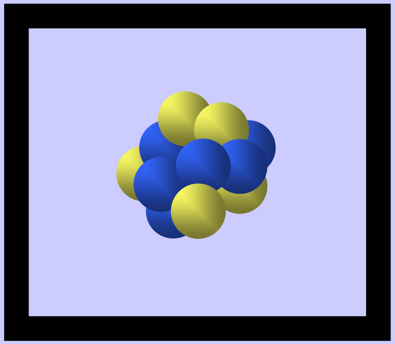
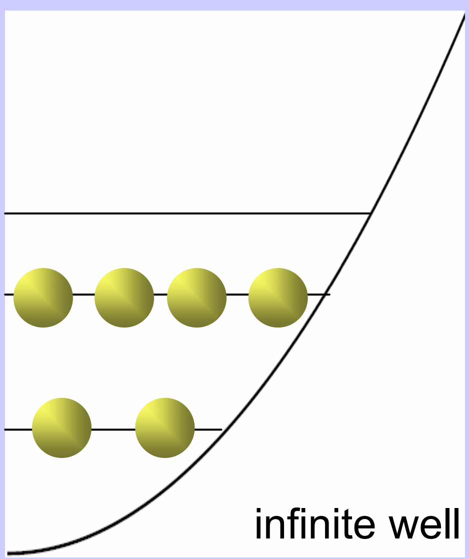
March 5-8, 2007 JUSTIPEN_LACM Meeting

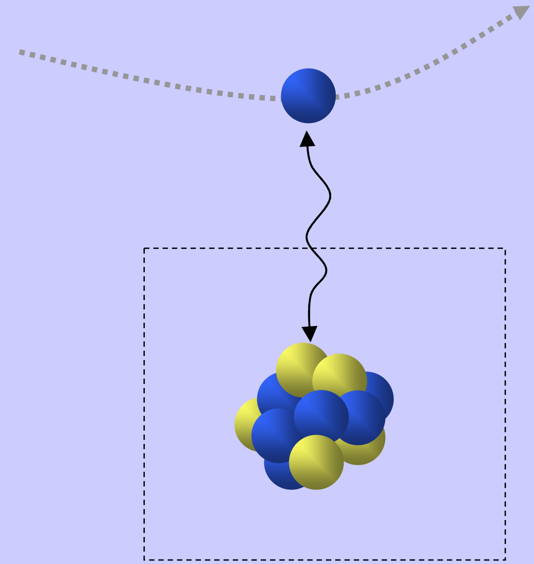

Nuclear Landscape
stable nuclei
known nuclei
126
82
50
28
20
8
2
protons
rp-process
r-process
open quantum systems
neutron droplets
?
neutrons
Nuclear radius (fm)
3.5
3.0
2.5
^{11}Li
^{7}Li
^{8}Li
^{6}Li
^{9}Li
6
10
Mass number

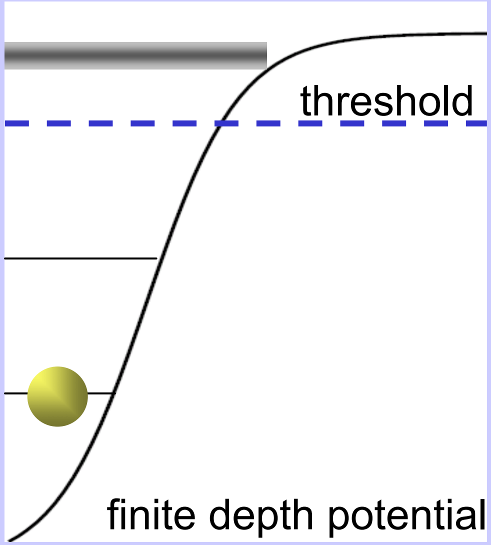
Closed quantum system





Opened quantum system





Continuum coupling in Shell Model approach

- Continuum Shell Model (1969)
  - C. Mahaux and H. Weidenmüller: "Shell Model Approach to Nuclear Reactions" 1969
  - H. W. Bartz et al., Nucl. Phys. A275, 111 (1977)

- Shell Model Embedded in the continuum (1999)
  - K. Bennaceur et al., Nucl. Phys. A651, 289 (1999)
  - K. Bennaceur et al., Nucl. Phys. A671, 203 (2000)
  - N. Michel et al., Nucl. Phys. A703, 202 (2002)
  - J. Okolowicz, M. Ploszajczak, I. Rotter, Phys. Rep. 374, 271 (2003)
  - J. Rotureau et al., Phys. Rev. Lett. 95, 042503 (2005)

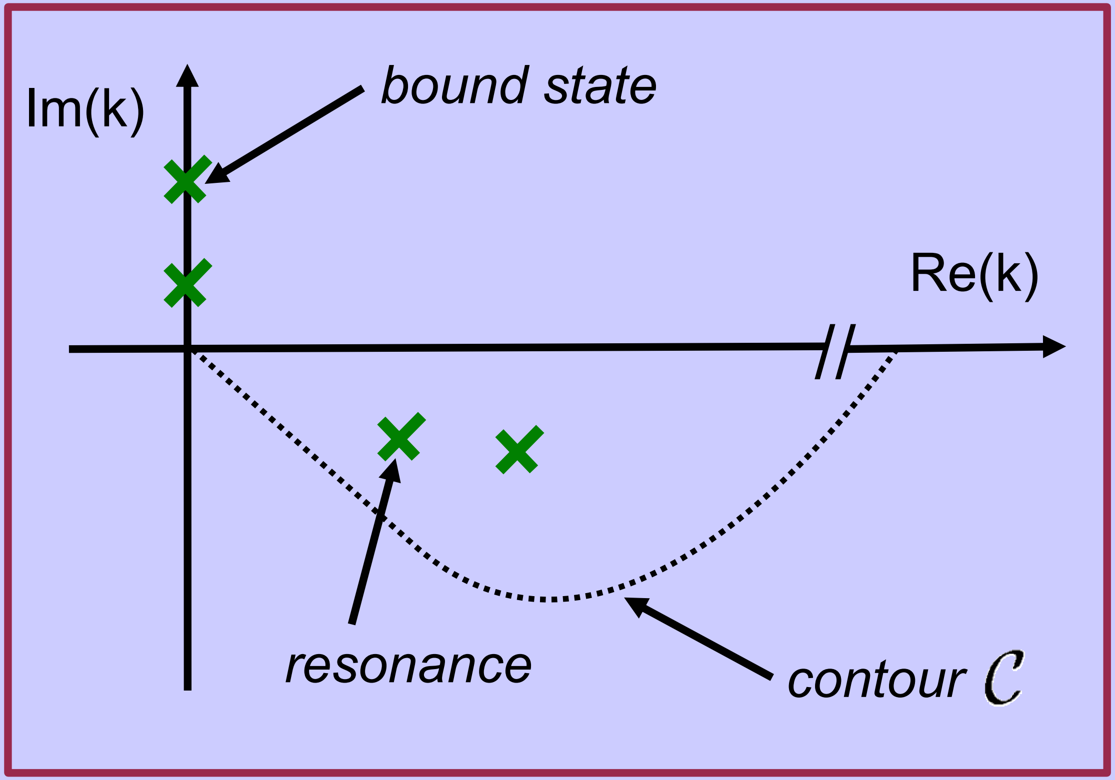
  . . . .

- Gamow Shell Model (2002)
  - N. Michel et al., Phys. Rev. Lett. 89, 042502 (2002)
  - N. Michel et al., Phys. Rev. C67, 054311 (2003)
  - N. Michel et al., Phys. Rev. C70, 064311 (2004)
  - R. Id Betan et al., Phys. Rev. Lett. 89, 042501 (2002)
  - R. Id Betan et al., Phys. Rev. C67, 014322 (2003)
  - G. Hagen et al, Phys. Rev. C71, 044314 (2005)

Outline

- Gamow Shell Model (GSM) :
  - ➔ shell model formalism with continuum states.
  - ➔ antibound (virtual) state in the structure of ^{11}Li.
- Density Matrix Renormalization Group (DMRG) :
  - ➔ reduction of the number of degrees of freedom.
  - ➔ truncation of GSM problem.
- Conclusion

Gamow Shell Model (N.Michel et al, 2002)



$$\boxed{\sum_{\text{pole}} |u_n\rangle\langle\tilde{u}_n| + \int_{\mathcal{C}} dk |u_k\rangle\langle\tilde{u}_k| = 1}$$

(Berggren completeness)

$u(r) \sim H^{+}_{l,\eta}(kr)$ (bound, resonant state)

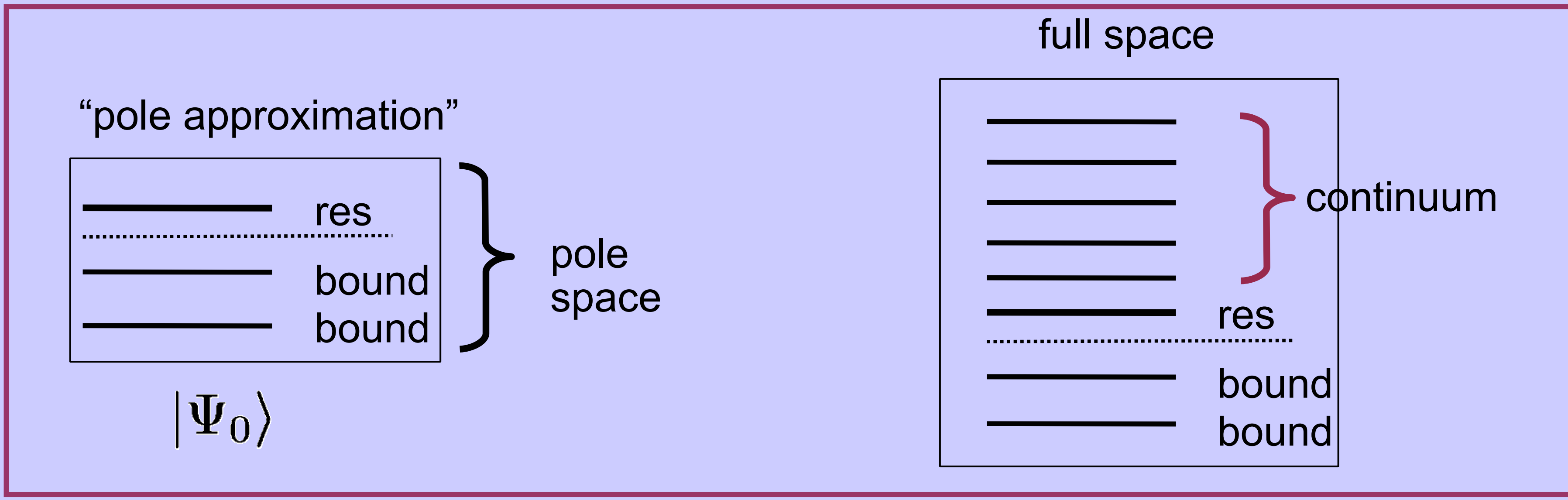
$u(r) \sim C^{+} H^{+}_{l,\eta}(kr) + C^{-} H^{-}_{l,\eta}(kr)$ (complex-continuum state)

- ✗ intrinsic hamiltonian :

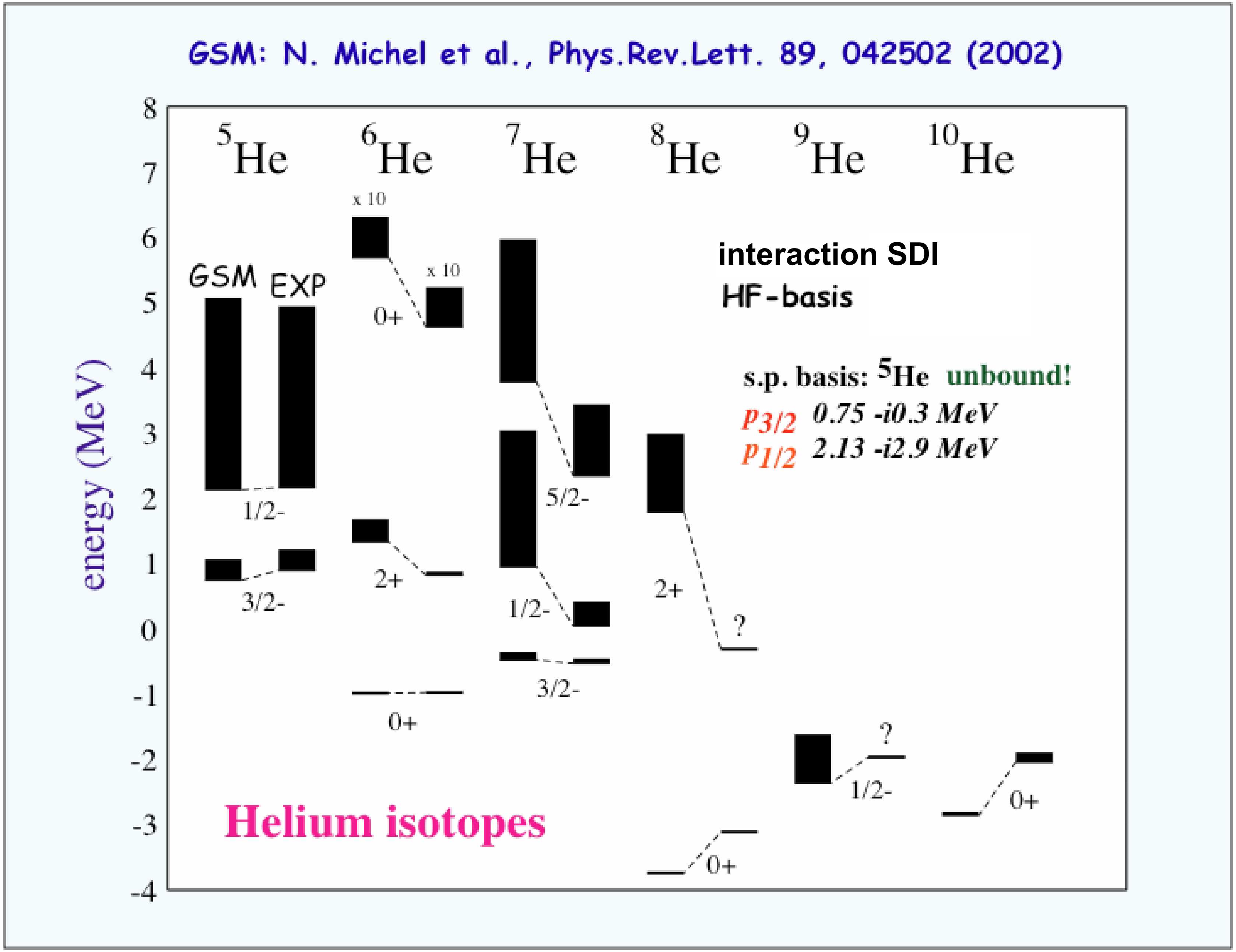
$$\boxed{H = \sum_{i=1}^{n} \left[\frac{\mathbf{p_i}^2}{2\mu} + U_i \right] + \sum_{j>i=1}^{n} \left[V_{ij} + \frac{1}{A_c}\mathbf{p_i}\mathbf{p_j} \right]}$$

recoil term

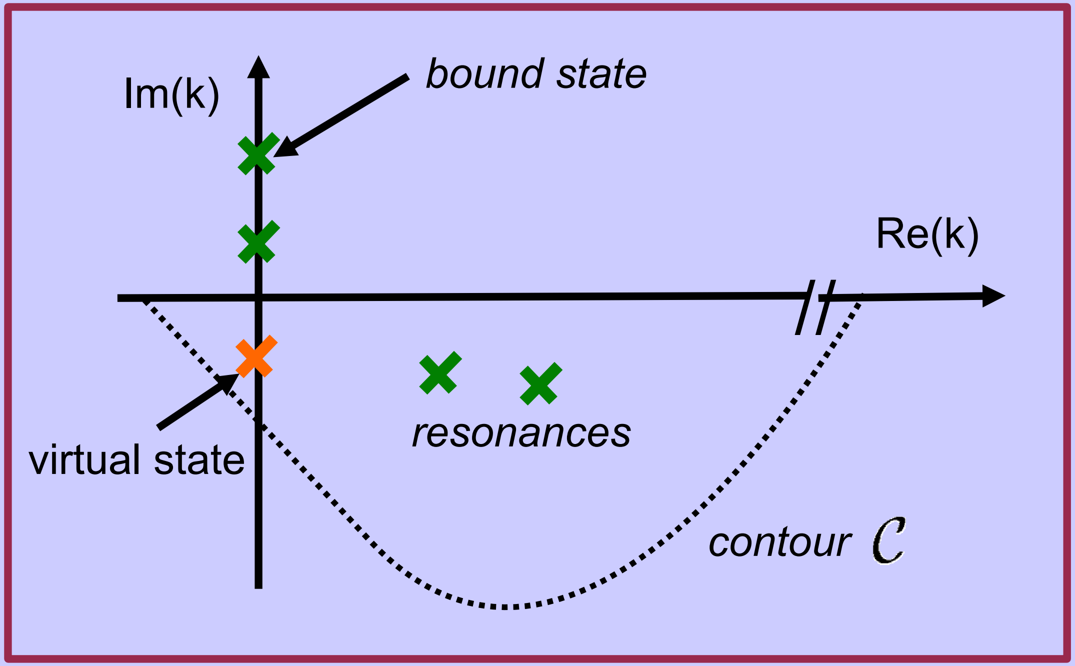
- ✗ discretization of contour $\mathcal{C}$: $\boxed{\sum_{\text{pole}} |u_n\rangle\langle\tilde{u}_n| + \sum_i |u_i\rangle\langle\tilde{u}_i| \simeq 1}$



→ Identification of N-body bound/resonant state by maximization of the overlap.


GSM: N. Michel et al., Phys.Rev.Lett. 89, 042502 (2002)
energy (MeV)
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
^{5}He
^{6}He
^{7}He
^{8}He
^{9}He
^{10}He
GSM
EXP
x 10
x 10
0+
1/2-
3/2-
2+
0+
5/2-
1/2-
3/2-
2+
?
0+
?
1/2-
0+
interaction SDI
HF-basis
s.p. basis: ^{5}He unbound!
$p_{3/2}$ 0.75 -i0.3 MeV
$p_{1/2}$ 2.13 -i2.9 MeV
Helium isotopes

Antibound state (virtual state) :



✗ real negative energy.

✗ increasing exponential behaviour.

✗ not “physical’ but :

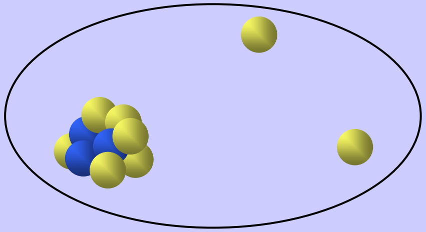
- influence on scattering cross section
- increased localization of real energy states above threshold

Should they be included in the Gamow basis ?

$$\boxed{\sum_{\text{pole}} |u_n\rangle\langle\tilde{u}_n| + \int_{\mathcal{C}} dk|u_k\rangle\langle\tilde{u}_k| = 1}$$

Ground state in ^{11}Li

✓ structure assumed as :



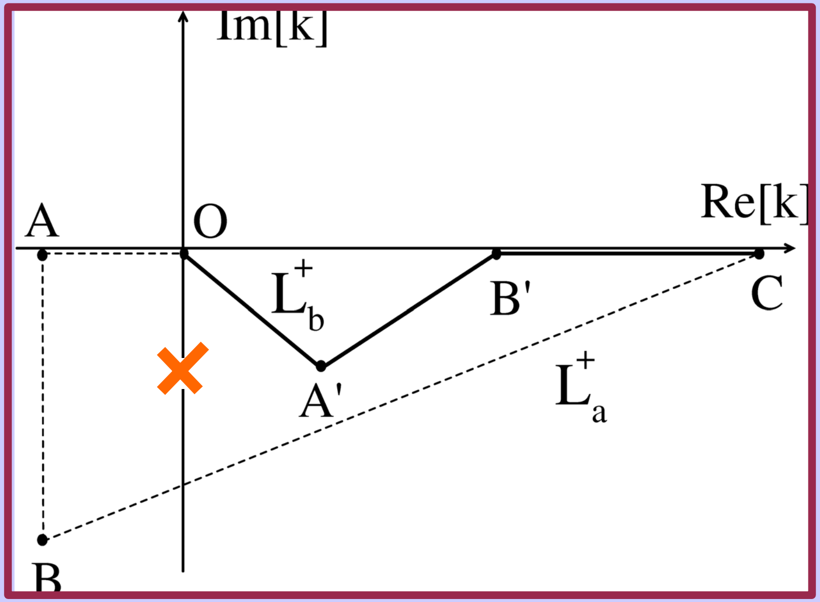
9Li core + 2n

✓ Wood Saxon + residual interaction (SGI).

✓ SGI parameters fitted to reproduce ^{11}Li ground state energy.

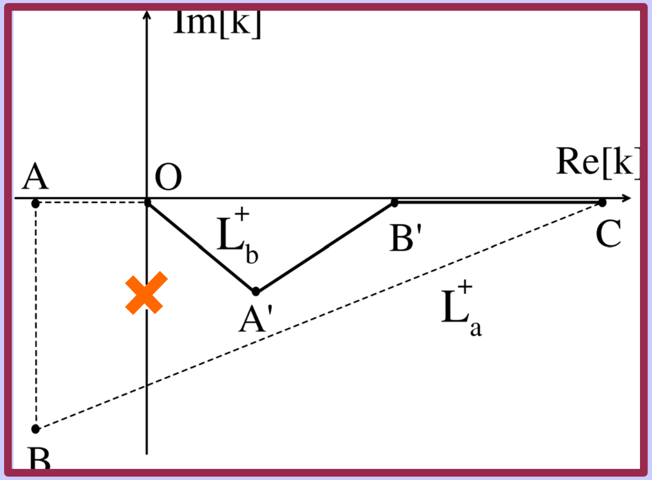
(i)
- $0p_{1/2}$ (res) + $p_{1/2}$ complex continuum
- $p_{3/2}$ real continuum
- **$1s_{1/2}$ (virtual) + $s_{1/2}$ complex continuum (L^+_a)**

(ii)
- $0p_{1/2}$ (res) + $p_{1/2}$ complex continuum
- $p_{3/2}$ real continuum
- **$s_{1/2}$ complex continuum (L^+_b)**



Energy E (MeV) and “spurious” width Γ (kev) of ^{11}Li ground state

$N_{s_{1/2}}$	$E[L_a^+]$	$E[L_b^+]$	$\Gamma[L_a^+]$	$\Gamma[L_b^+]$
10	–0.314	–0.291	65.274	–3.644
20	–0.292	–0.295	2.307	0.025
30	–0.294	–0.295	0.876	–0.003
40	–0.294	–0.295	–0.425	–0.007
50	–0.295	–0.295	0.075	–0.009
60	–0.295	–0.295	–0.005	–0.009



Components of 11Li ground state involving $s_{1/2}$ orbits

Configuration	$\mathrm{Re}[c^2]\{L_a^+\}$	$\mathrm{Re}[c^2]\{L_b^+\}$	$\mathrm{Im}[c^2]\{L_a^+\}$	$\mathrm{Im}[c^2]\{L_b^+\}$
$(1s_{1/2})^2$	0.0990	–	$-9.6033\cdot10^{-6}$	–
$(1s_{1/2}\ s_{1/2})$	–0.5887	–	$2.3369\cdot10^{-5}$	–
$(s_{1/2})^2$	1.0034	0.5137	$-8.0720\cdot10^{-6}$	$-1.4650\cdot10^{-5}$

N.Michel et al, Phys.Rev. C74 (2006) 054305

⟶ no advantage in including the antibound state

Density Matrix Renormalization Group (S.R.White, 1992)

- Iterative method to take into account *all degrees of freedom* of a physical system (quantum lattice, nucleus ....)

- increasing number of degrees of freedom (enlarged space)
- solving in this enlarged space
- shrinking number of states

  → *truncation (with density matrix)*

- calculating "effective" hamiltonian in this truncated space

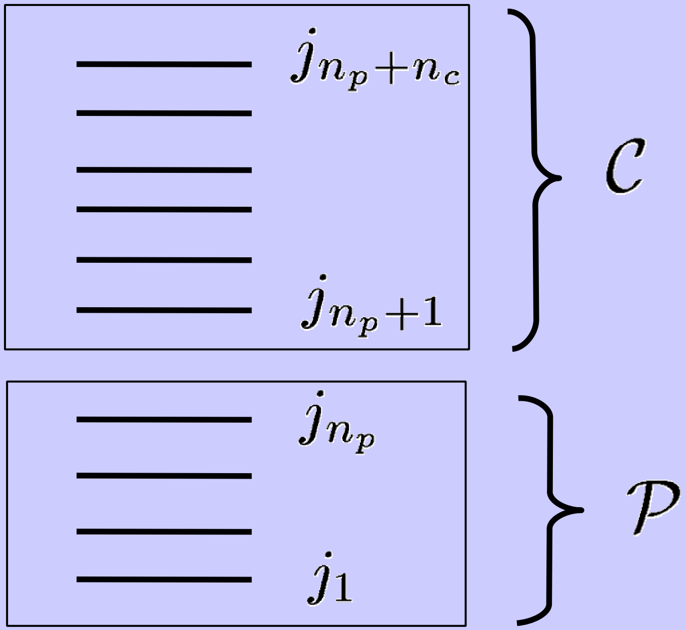
  → renormalization

. . . . .

Gamow Shell Model +DMRG

- n_c continuum shells
- n_p bound or resonant shells
- J – scheme

$$|\Psi\rangle^J = \sum_{p,c} \Psi_{pc} \left[|p\rangle^{J_p} |c\rangle^{J_c} \right]^J$$

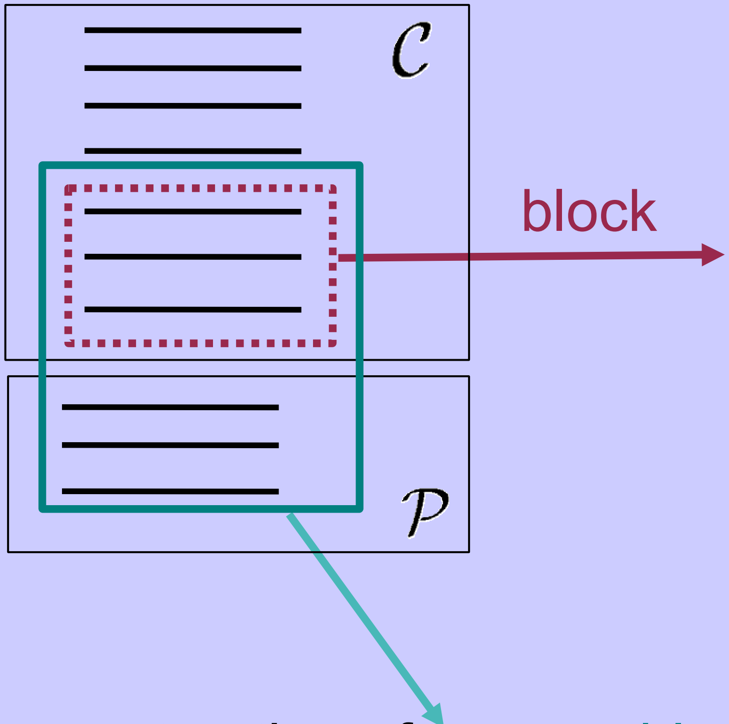


DMRG → truncation in $\mathcal{C}$

- “ construction ” of $\mathcal{P}$

$$\begin{cases} \{|p\rangle\} \equiv |(j_1)^{L_1}_{n_1} .. (j_i)^{L_i}_{n_i} .. \, ; K..K_i.. \, \rangle^{J_k} \\ a^\dagger \;\; , \;\; [a^\dagger a^\dagger]^\Gamma \; , ([a^\dagger a^\dagger]^\Gamma \tilde{a})^K \; . \; . \; . \; . \\ |\Psi_0\rangle \quad \text{(pole approximation)} \end{cases}$$

- construction of a block in $\mathcal{C}$:



$$\{|c\rangle\} \equiv |(j_{n_p+1})^{L}_{n_p+1}..(j_i)^{L_i}_{n_i}..\,;K..K_i..\,\rangle^{J_p}$$

$$a^\dagger, \quad [a^\dagger a^\dagger]^\Gamma \;, \;. \;. \;.$$

- construction of a superblock :

$$\text{superblock} \equiv \left(|p\rangle^{J_p}\,|c\rangle^{J_c}\right)^J$$

J, number of nucleons and parity are fixed.

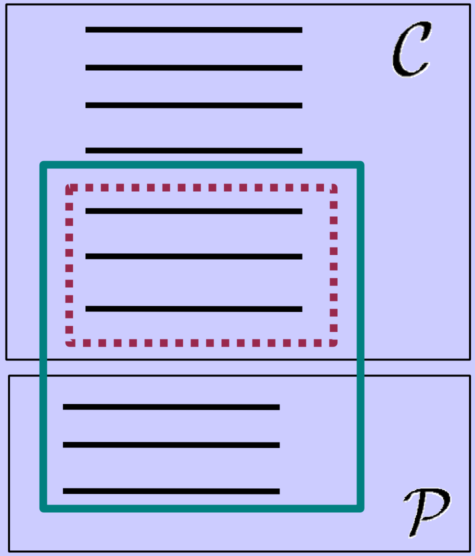
- construction of hamiltonian in the superblock :

$$\boxed{\langle (pc)^J \| H \| (p'c')^J \rangle} = \sum_{\mathcal{O}_\mathcal{P}, \mathcal{O}_\mathcal{C}} \langle p \| \mathcal{O}_\mathcal{P} \| p'' \rangle \boxed{\langle c \| \mathcal{O}_\mathcal{C} \| c' \rangle}$$

previously calculated

- diagonalization + maximization of overlap

$$\boxed{|\Psi\rangle^J = \sum_{p,c} \Psi_{pc} \left(|p\rangle^{J_p} |c\rangle^{J_c} \right)^J}$$



- truncation with the density matrix :

$$\boxed{\rho^{J_c}_{c,c'} = \sum_p \Psi_{pc} \Psi_{pc'}}$$

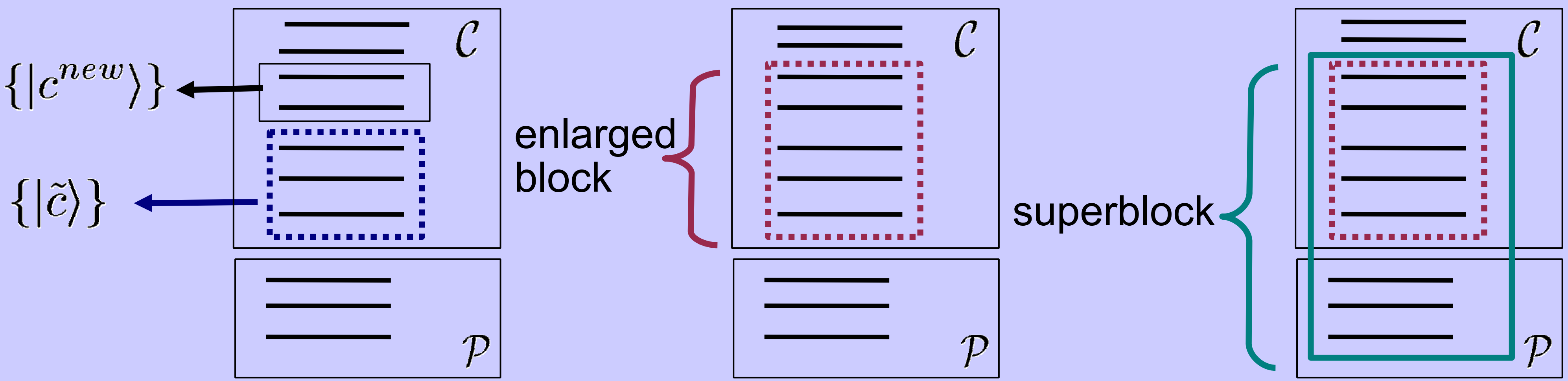
symmetric matrix, conservation of J_c

$\mathrm{N_{opt}}$ eigenstates $|\tilde{c}\rangle$ are kept.

- renormalization of sub-operators

$$\boxed{\langle \tilde{c} || a^\dagger_\alpha \;,\; (a^\dagger_\alpha a^\dagger_\beta)^\Gamma \;...|| \tilde{c}' \rangle}$$

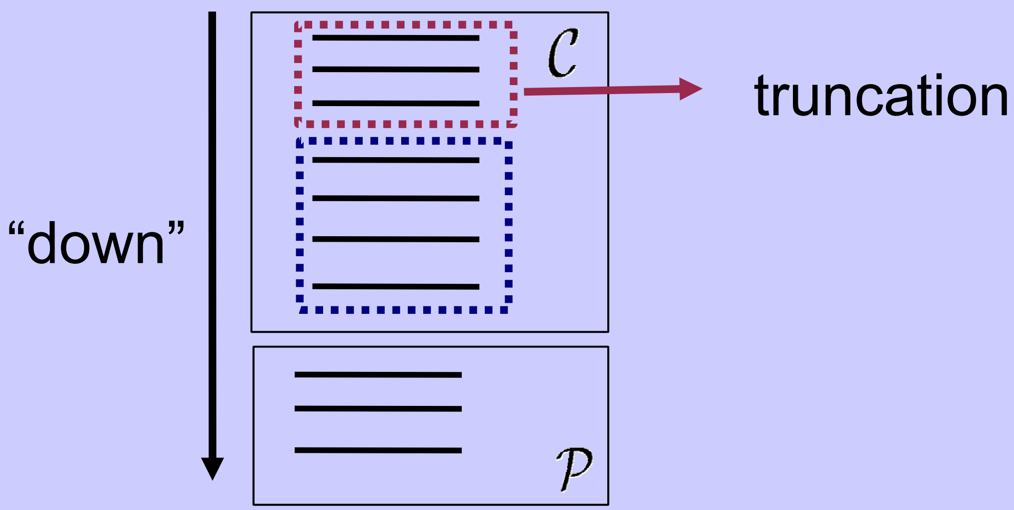
- next shells are added :

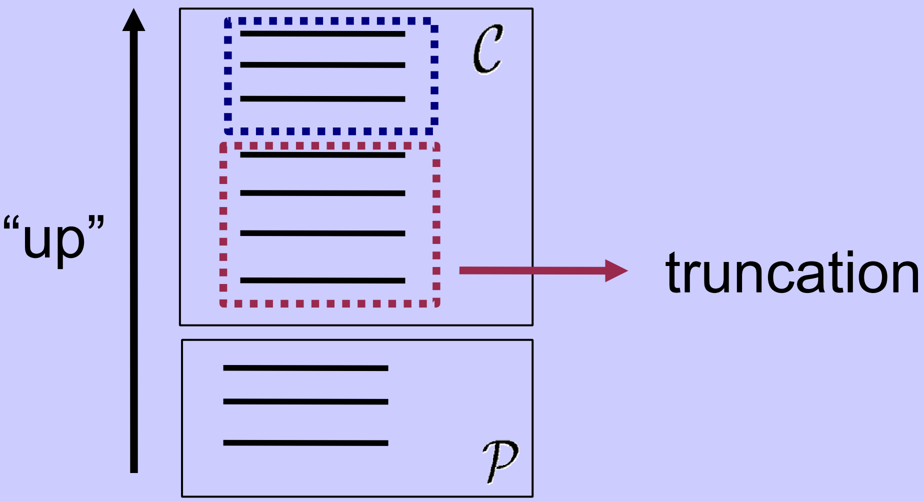


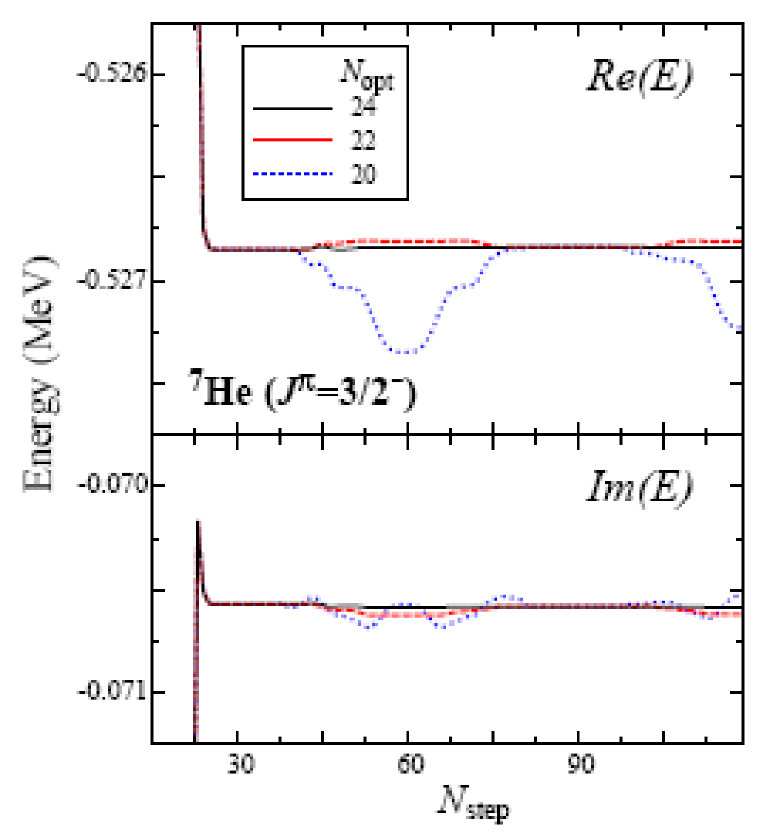
. . . until all shells in $\mathcal{C}$ are exhausted .

end of "warm up phase"

sweeping phase







$^7\mathrm{He}$, $J^\pi = 3/2^-$

- ✗ alpha core + 3 neutrons
- ✗ Woods-Saxon+SGI
- ✗ pole space : $0p_{3/2}$, $0p_{1/2}$
- ✗ continuum space :

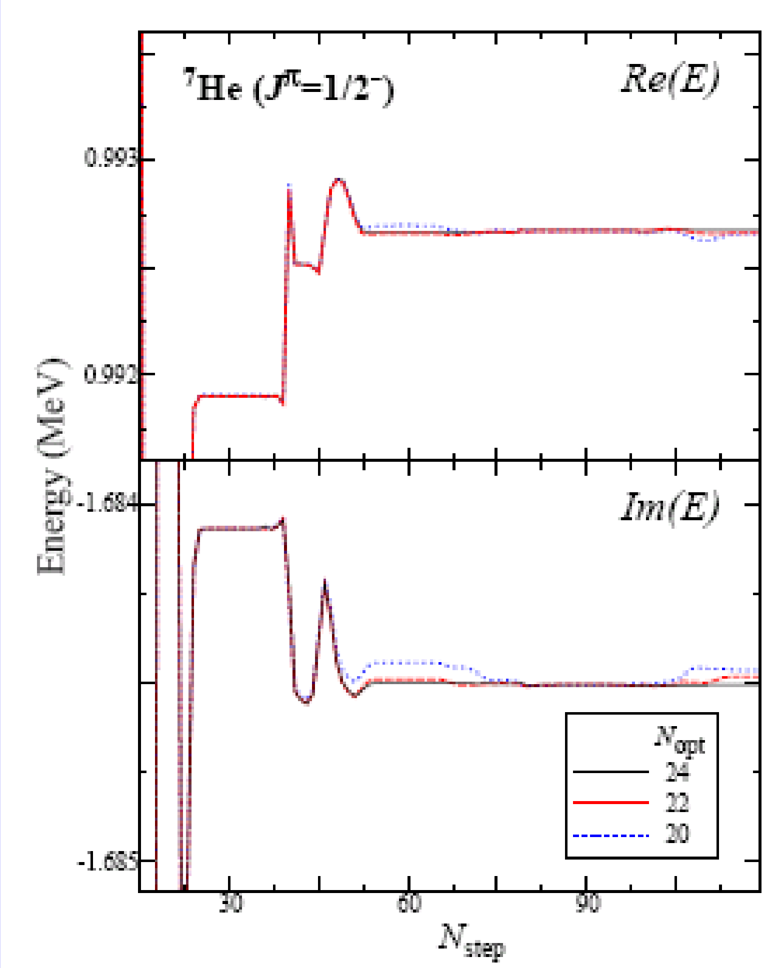
  $p_{3/2}$, $p_{1/2}$ (30 shells each)

(J.Rotureau et al., Phys.Rev.Lett. 97 (2006) 110603)

total dimension=83948
largest matrix in DMRG=1143

N_{opt}	$\Delta E/E \quad (N_{sw}=2)$	$\Delta E/E \quad (N_{sw}=4)$
20	9.4×10^{-4}	7.5×10^{-4}
22	8.0×10^{-5}	4.9×10^{-5}
24	3.0×10^{-5}	4.7×10^{-6}
26	2.1×10^{-5}	2.6×10^{-6}

Relative precision of the real part of g.s energy as a function of $\mathrm{N_{opt}}$ and N_{sw} (number of sweep).



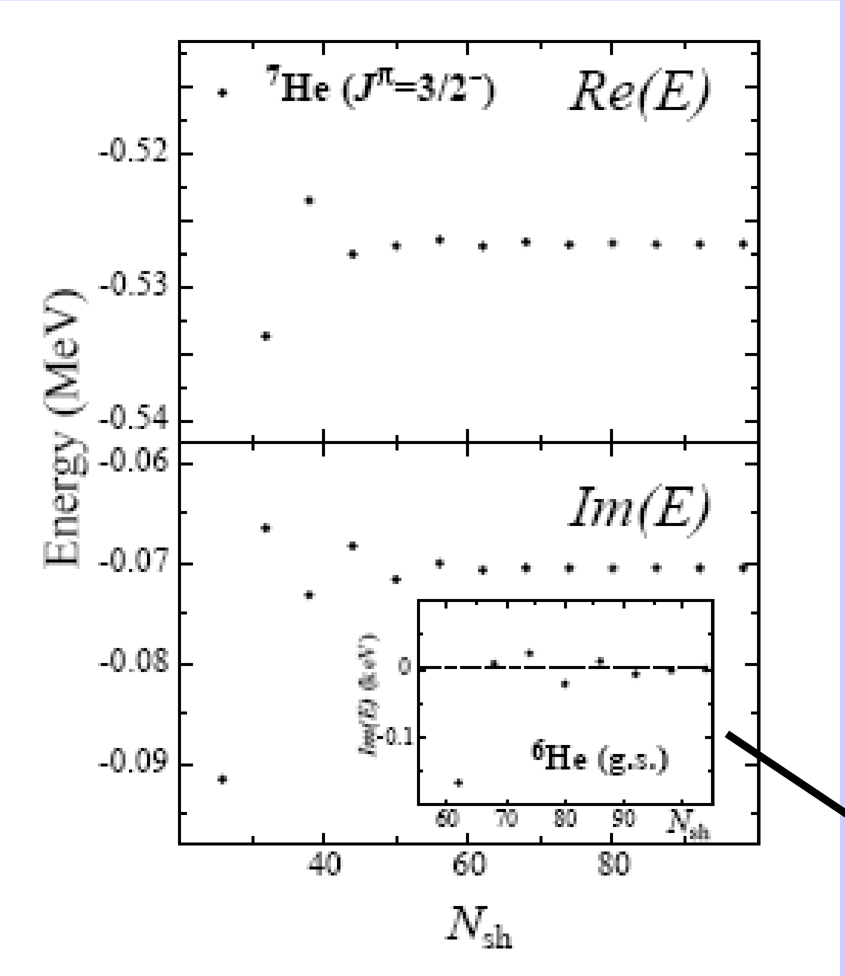
$^{7}\mathrm{He}$, $J^\pi = 1/2^-$

- ✗ alpha core + 3 neutrons
- ✗ Woods-Saxon+SGI
- ✗ pole space : $0p_{3/2}$, $0p_{1/2}$
- ✗ continuum space :

  $p_{3/2}$, $p_{1/2}$ (30 shells each)

total dimension=64046
largest matrix in DMRG=1492

$^{7}\mathrm{He}$, $J^{\pi} = 3/2^{-}$



(N_{opt} = 22)

Convergence of the real (top) and imaginary part (bottom) of the g.s. energy as a function of the total number of shells N_{sh} .

total dimension:

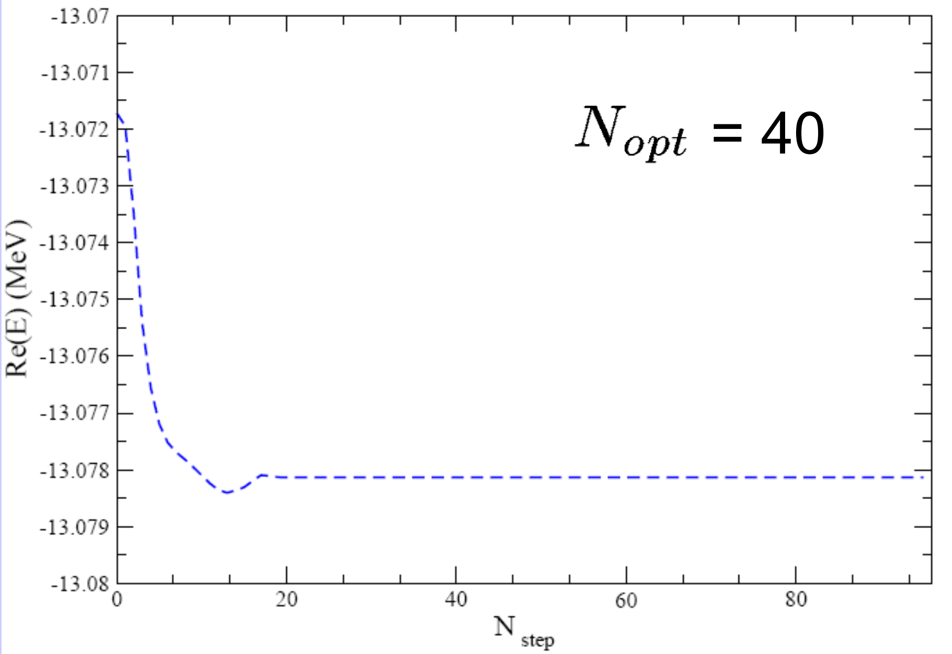
6149 ⟷ 332171

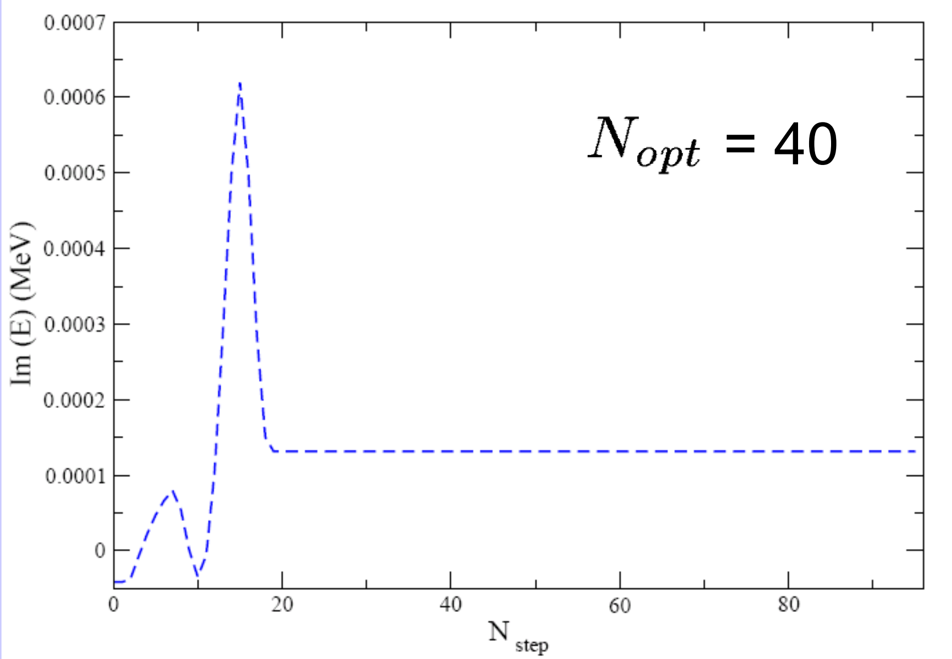
DMRG dimension:

941 ⟷ 1001

Convergence of the imaginary part of the g.s. energy of $^{6}\mathrm{He}$ as a function of the total number of shells N_{sh} .

$^{8}\mathrm{He}\ ,\quad J^{\pi} = 0^{+}$

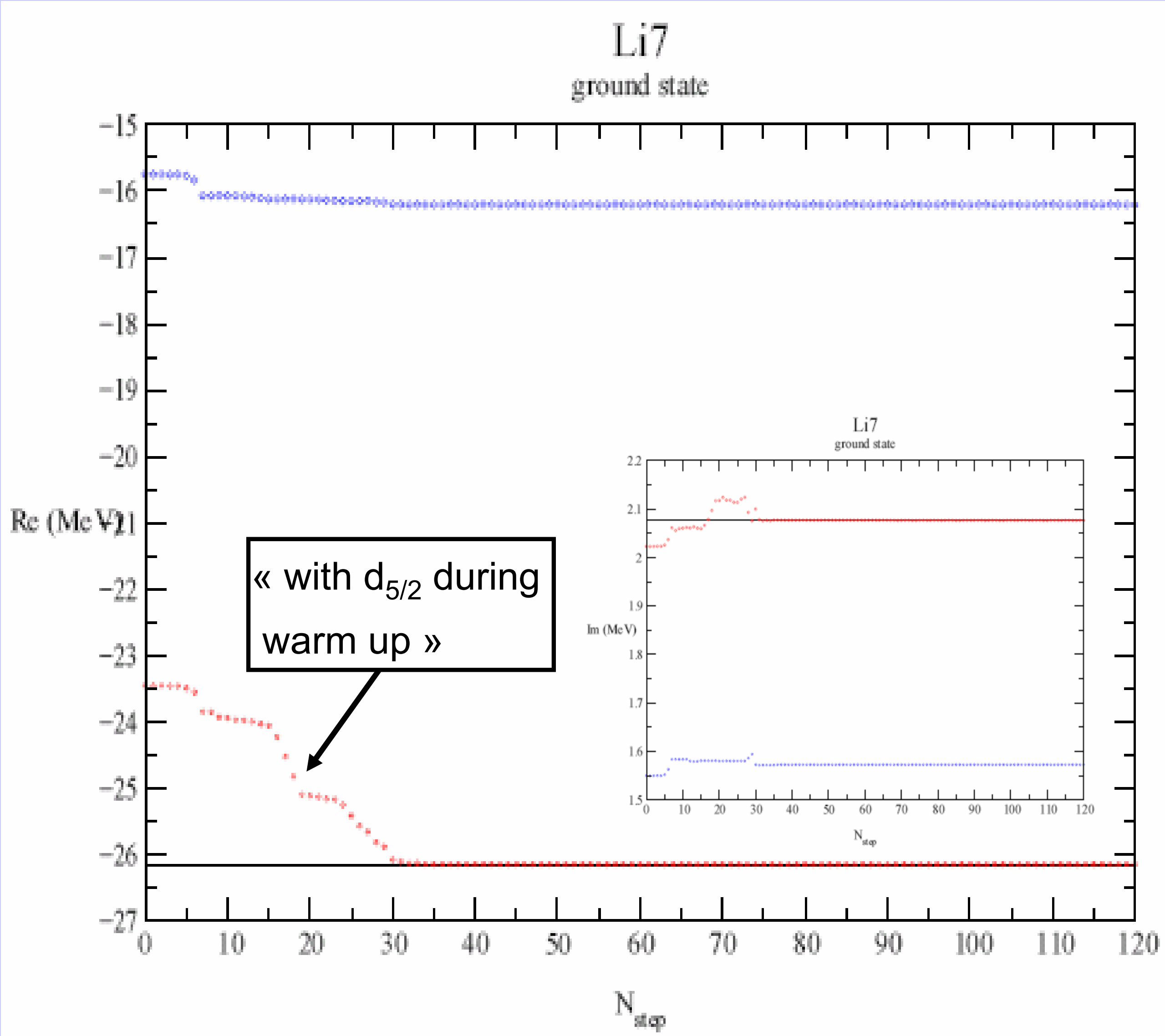




- alpha core + 4 neutrons
- Woods-Saxon+SGI
- ***Hartree Fock basis***
- pole space : $0p_{3/2}\ ,\ 0p_{1/2}$
- continuum space :

  $p_{3/2}\ ,\ p_{1/2}$ (25 shells each)

total dimension=481250
largest matrix in DMRG=2061

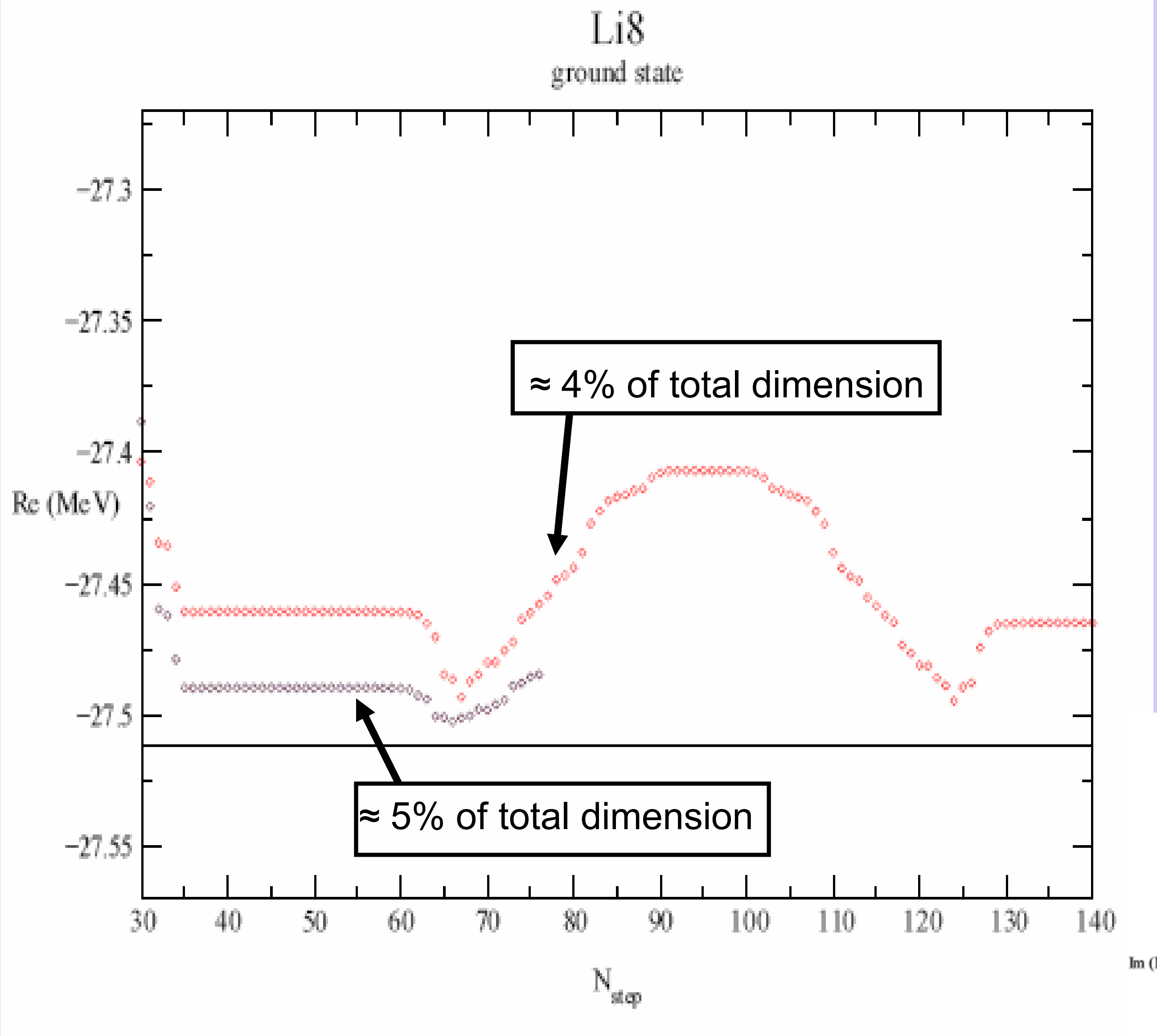


- ✗ 2 neutrons +1 proton
- ✗ ***Hartree Fock basis***
- ✗ pole space : $0p_{3/2}$, $0p_{1/2}$ (p/n)
- ✗ continuum space :

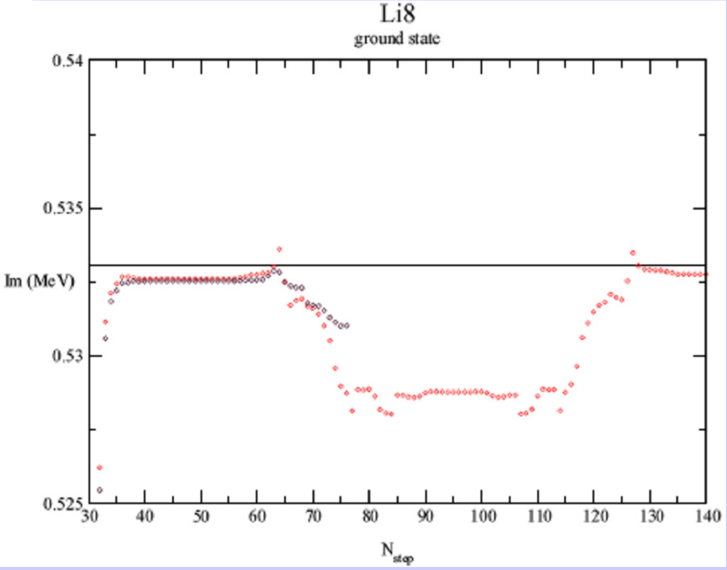
$$\begin{cases} p_{3/2},\ p_{1/2} \text{ complex continuum} \\ s_{1/2},\ d_{5/2} \text{ real continuum} \end{cases}$$

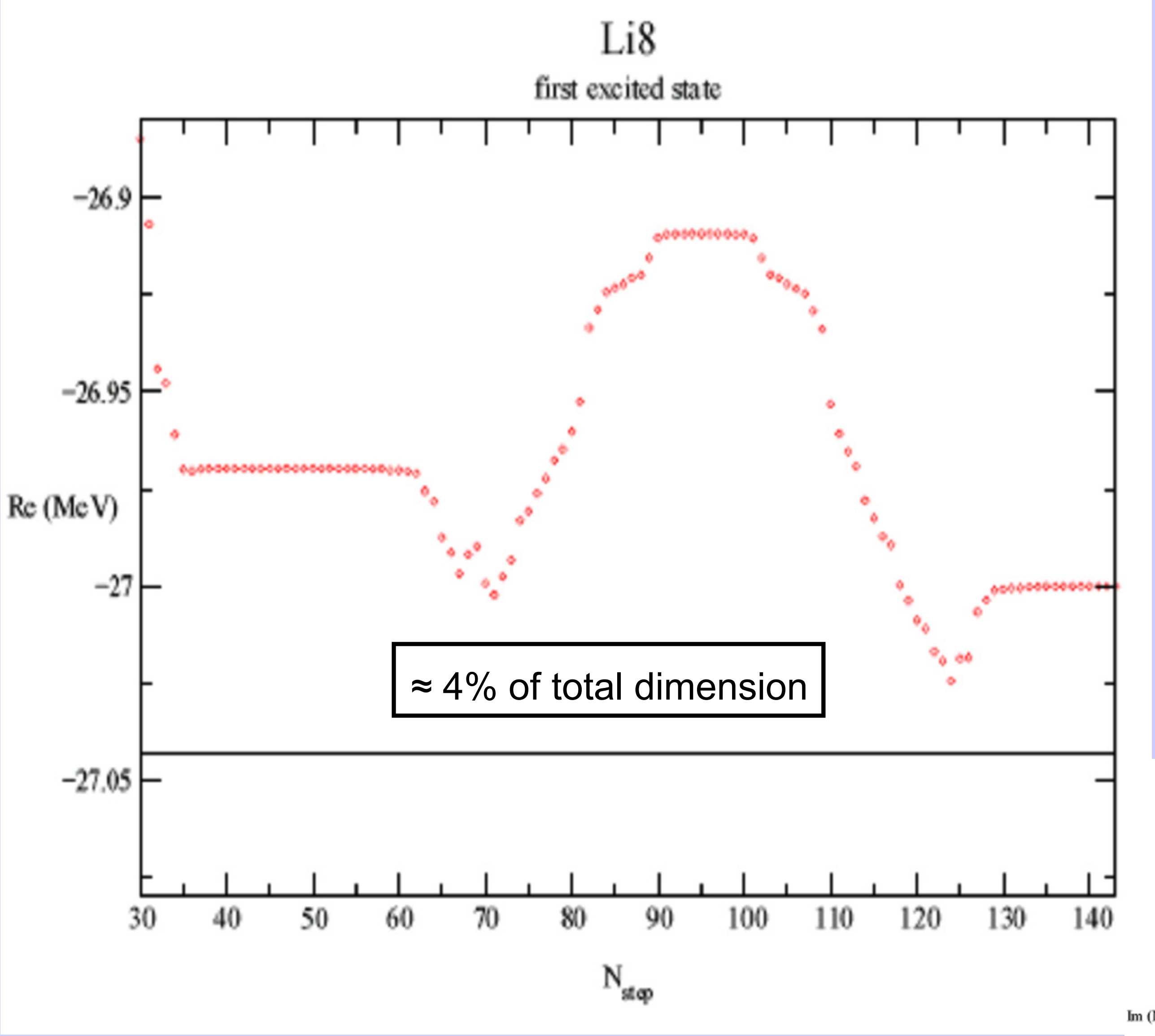
- ✗ 52 shells in total
- ✗ total dimension=8540

→ ≈ 20% of total valence space needed to reach exact value.

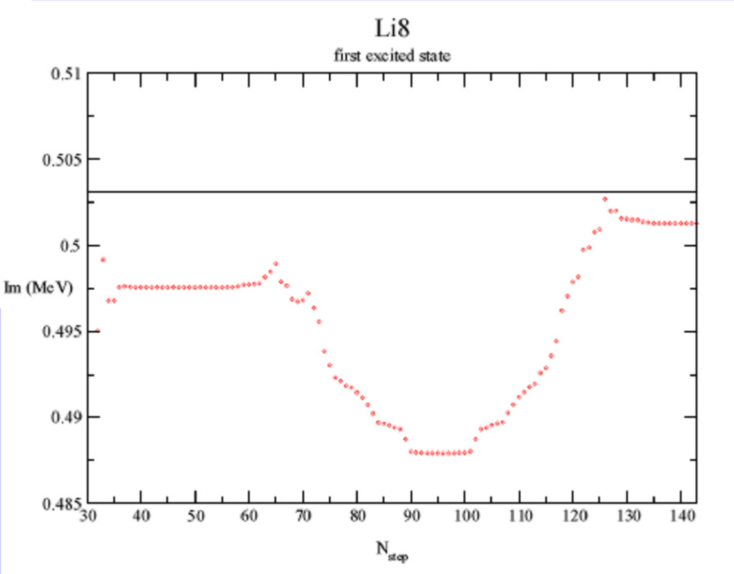


- ✗ 3 neutrons +1 proton
- ✗ ***Hartree Fock basis***
- ✗ pole space : $0p_{3/2}$, $0p_{1/2}$ (p/n)
- ✗ continuum space :
  - $p_{3/2}$, $p_{1/2}$ complex continuum
  - $s_{1/2}$, $d_{5/2}$ real continuum
- ✗ 52 shells in total
- ✗ total dimension=190616





- ✗ 3 neutrons +1 proton
- ✗ ***Hartree Fock basis***
- ✗ pole space : $0p_{3/2}$, $0p_{1/2}$ (p/n)
- ✗ continuum space :
  - $p_{3/2}$, $p_{1/2}$ complex continuum
  - $s_{1/2}$, $d_{5/2}$ real continuum
- ✗ 52 shells in total
- ✗ total dimension=151530



Conclusion

◆ no advantage in including the antibound state fo description of halo in ^{11}Li.

◆ application of the Density Matrix Renormalization Group method in the Gamow Shell Model:

✗ non hermitian problem.

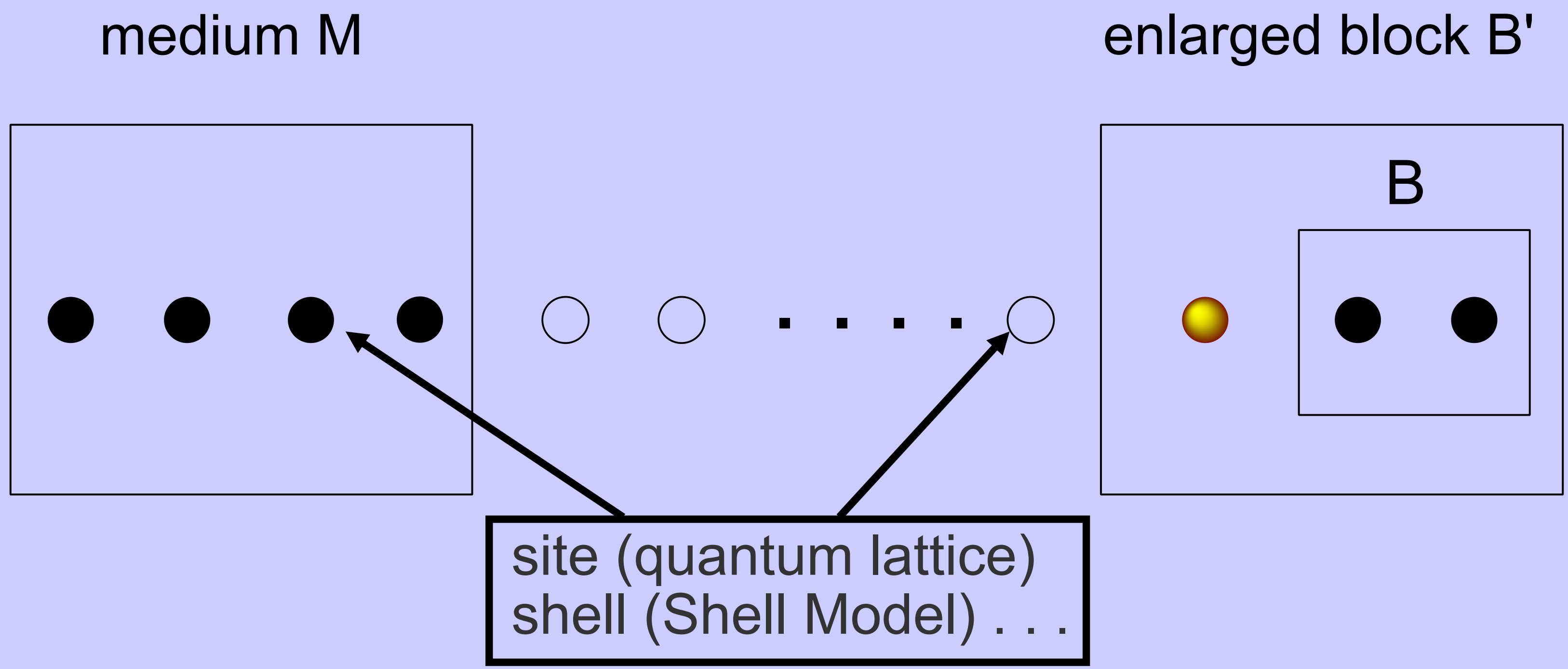
✗ rotational invariance is conserved.

✗ ground and excited states.

✗ for a given nucleus minimal N_{opt} is independent on N_{sh} .

✗ DMRG-dimension also practically independent on N_{sh} .

◆ parallelization in progress . . .



✗ resolution in enlarged space M⇘B' :
$$|\Psi\rangle = \sum_{i=1}^{\mathrm{dim M}} \sum_{j=1}^{\mathrm{dim B'}} \Psi_{i,j} |i^{\mathrm{M}}\rangle |j^{\mathrm{B'}}\rangle$$

✗ truncation in B' ➡ density matrix
$$\rho^{B'}_{j,j'} = \sum_{i=1}^{\mathrm{dim M}} \Psi_{ij} \Psi^{*}_{i,j'}$$

✗ one keeps $N_{\mathbf{opt}}$ eigenstates of $\rho^{B'}$ (largest eigenvalues).