JUSTIPEN-LACM workshop
Oak Ridge
March 5-8, 2007

Tensor force

in shell and mean-field models

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Outline

1. Tensor force
    "Lowest order" NN force
    Shell evolution everywhere

2. Tensor force in the shell model
    Decomposition of monopoles

3. Tensor force in the mean field models
    GT forces and SBF

4. Summary

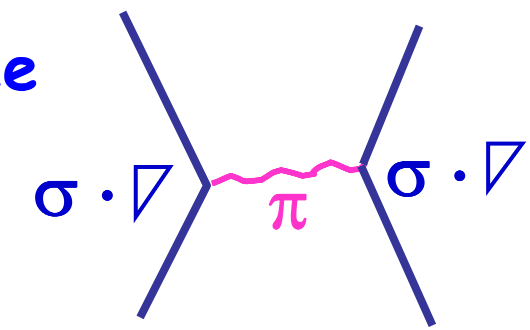
Tensor Interaction

$$V_T = (\tau_1 \tau_2)\,(\,[\sigma_1 \sigma_2]^{(2)}\, Y^{(2)}(\Omega)\,)\,Z(r)$$

contributes only to **S=1** states

relative motion

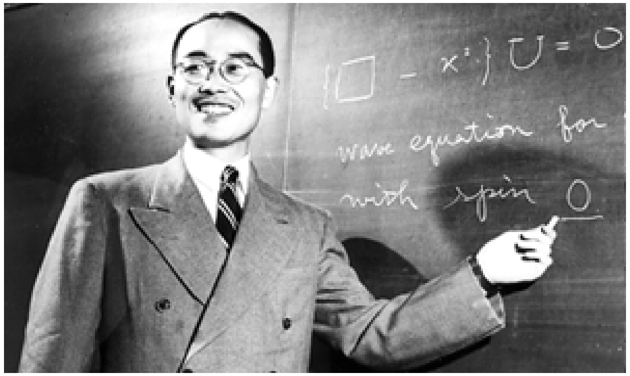
π meson : primary source



ρ meson (~ π+π) : minor (~1/4) cancellation

Ref: Osterfeld, Rev. Mod. Phys. 64, 491 (92)

The atomic nucleus is bound due to meson exchange.
(Yukawa 1935)



Multiple pion exchanges
→ strong effective central forces in *NN* interaction (as represented by σ meson, *etc.*)
→ nuclear binding

Where can we see one pion exchange ?

One pion exchange ~ Tensor force

First-order tensor force effect in spectroscopy
→ manifestation of pions in nuclei

One pion exchange ~ Lowest Order in ChPT

S. Weinberg, PLB 251, 288 (1990)

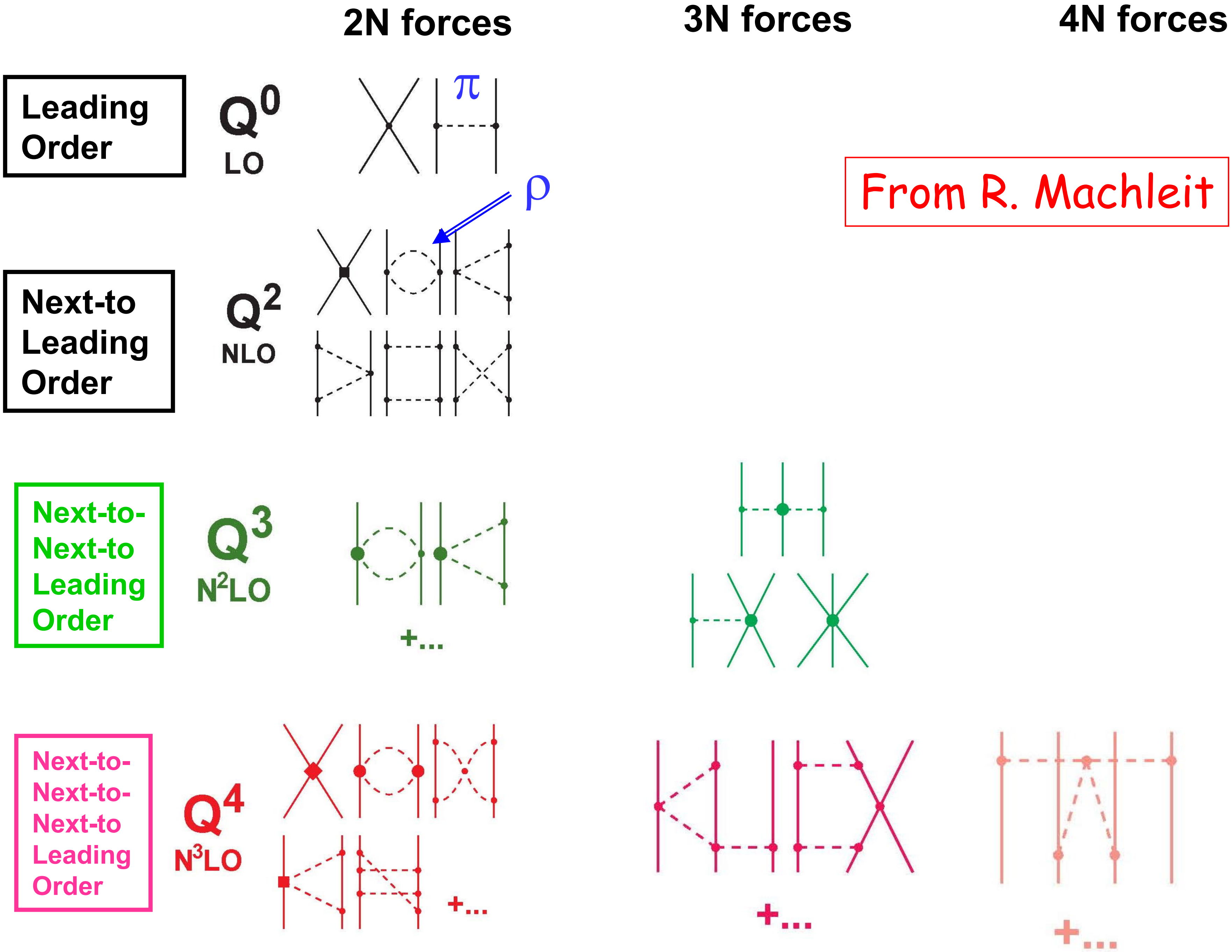
tive potential gives a local coordinate-space two-nucleon potential:

$$V_{2\text{-nucleon}} = 2(C_S + C_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\delta^3(\boldsymbol{x}_1 - \boldsymbol{x}_2)$$

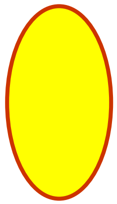
$$-\left(\frac{2g_A}{F_\pi}\right)^2 (\boldsymbol{t}_1 \cdot \boldsymbol{t}_2)(\boldsymbol{\sigma}_1 \cdot \nabla_1)(\boldsymbol{\sigma}_2 \cdot \nabla_2) Y(|\boldsymbol{x}_1 - \boldsymbol{x}_2|)$$

$$-(1' \leftrightarrow 2') , \qquad (8)$$

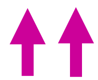
where $Y(r) \equiv \exp(-m_\pi r)/4\pi r$ is the usual Yukawa potential. [Throughout it should be understood that these are local potentials, containing a delta function factor like $\delta^3(\boldsymbol{x}'_1 - \boldsymbol{x}_1)$ for each nucleon.]


2N forces
3N forces
4N forces
Leading Order
Q^0
LO
π
ρ
From R. Machleit
Next-to Leading Order
Q^2
NLO
Next-to-Next-to Leading Order
Q^3
N^2LO
+...
Next-to-Next-to-Next-to Leading Order
Q^4
N^3LO
+...
+...
+...

Intuitive Picture

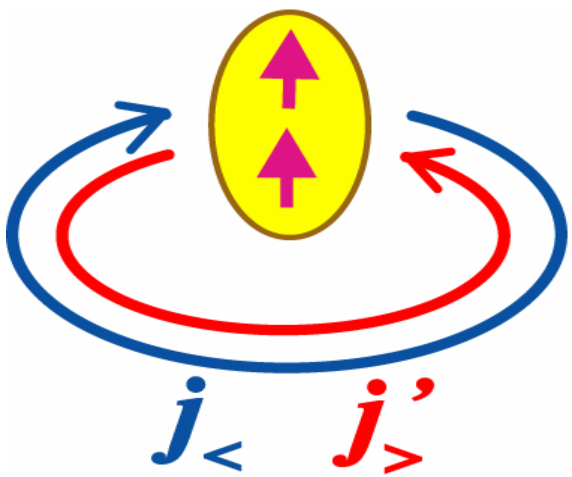


wave function of relative motion

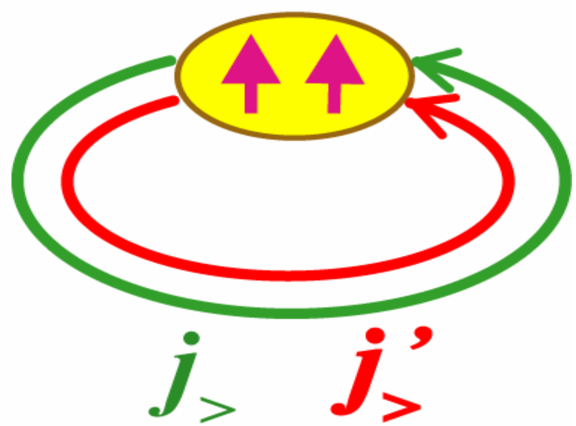


spin of nucleon

large relative momentum



small relative momentum



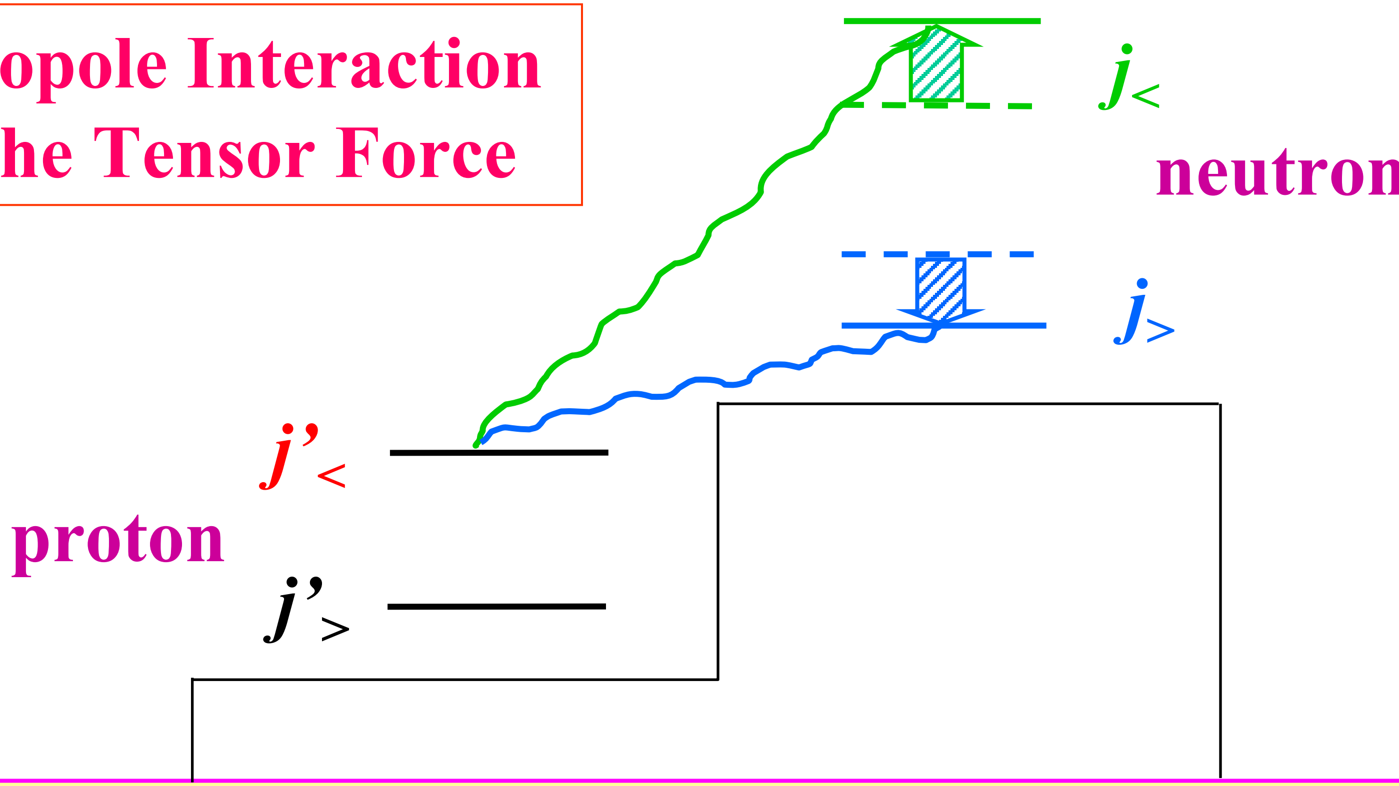
deuteron ⇨ attractive

repulsive

TO *et al.*, Phys. Rev. Lett. 95, 232502 (2005)

TO *et al.*, Phys. Rev. Lett. 95, 232502 (2005)

Monopole Interaction of the Tensor Force



Identity for tensor monopole interaction

$$(2j_> + 1)\ v_{m,T}^{(j' j_>)} + (2j_< + 1)\ v_{m,T}^{(j' j_<)} = 0$$

$v_{m,T}$: monopole strength for isospin T

In the shell model, single-particle properties are considered by the following quantities …….

Effective single particle energy

- **Monopole part of the *NN* interaction**

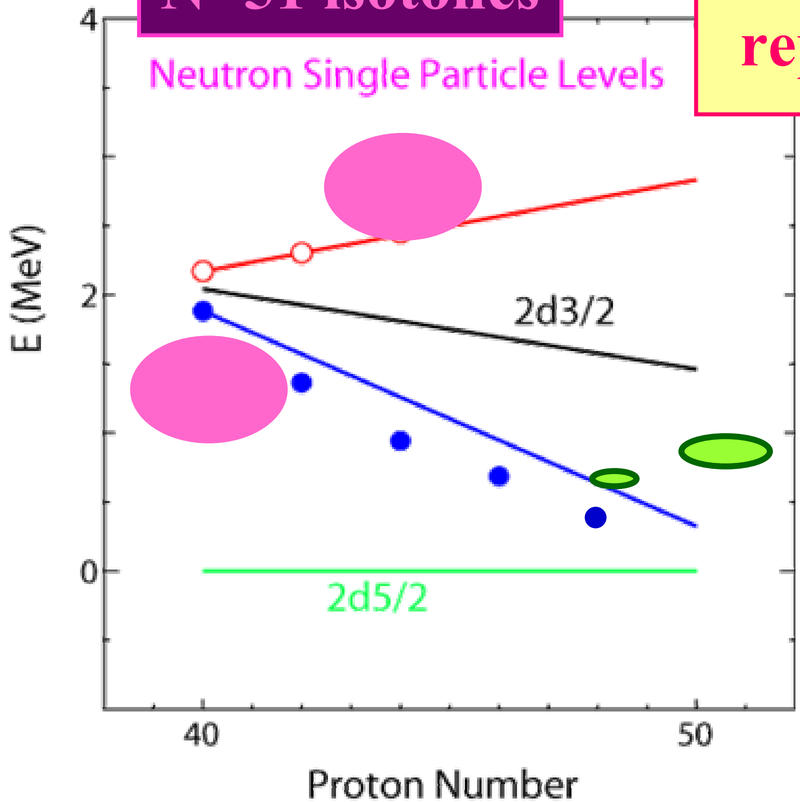
$$V_{ab}^{T} = \frac{\sum_J (2J+1) V_{abab}^{JT}}{\sum_J (2J+1)}$$

→ **Angular averaged interaction**

Isotropic component is extracted from a general interaction.

Changes of N=51 neutron effective single-particle energies from Zr to Sn

N=51 isotones



repulsion between $g_{7/2}$ and $h_{11/2}$

Federman-Pittel mechanism

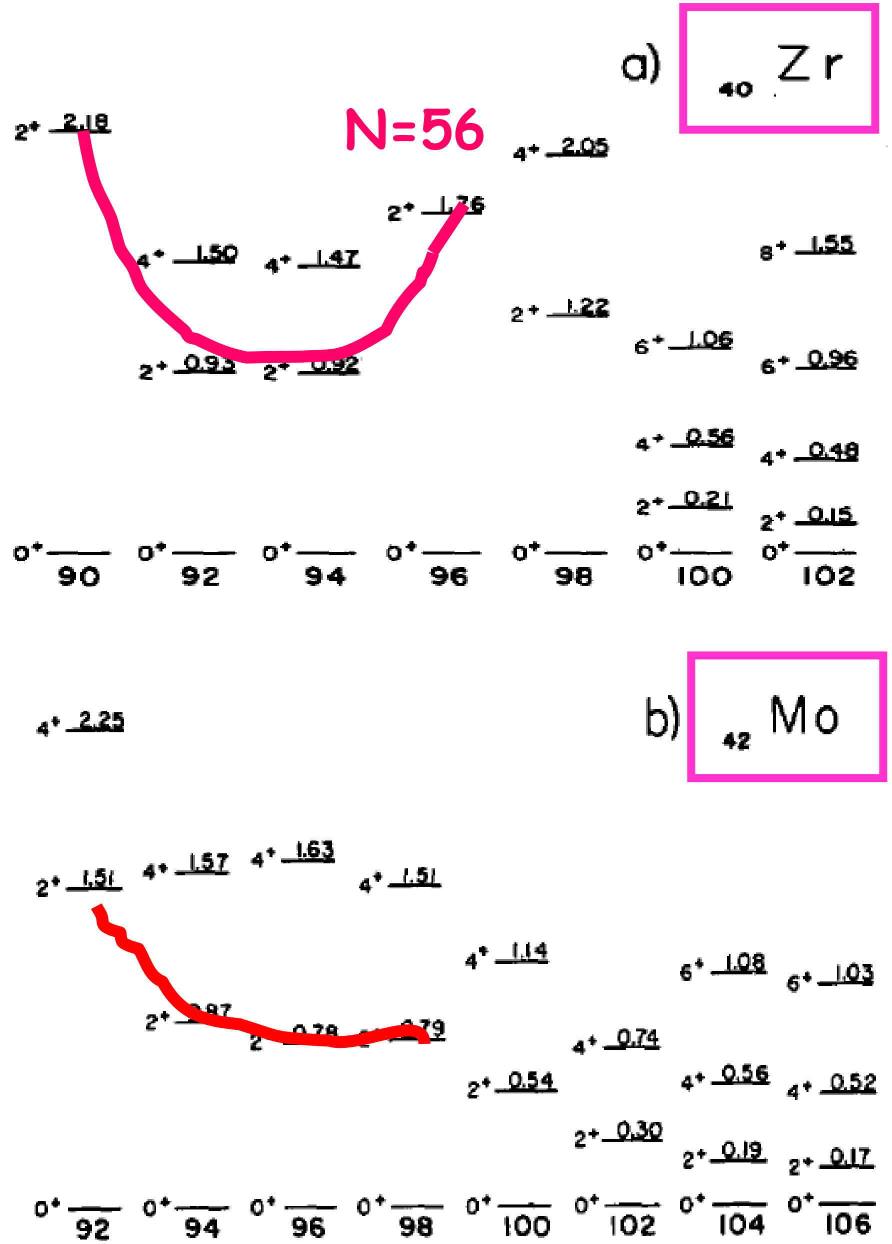
Lines : $\pi + \rho$ meson tensor

Points : exp. level

Zr → Sn

protons in

shown relative to $d_{5/2}$



Z=40

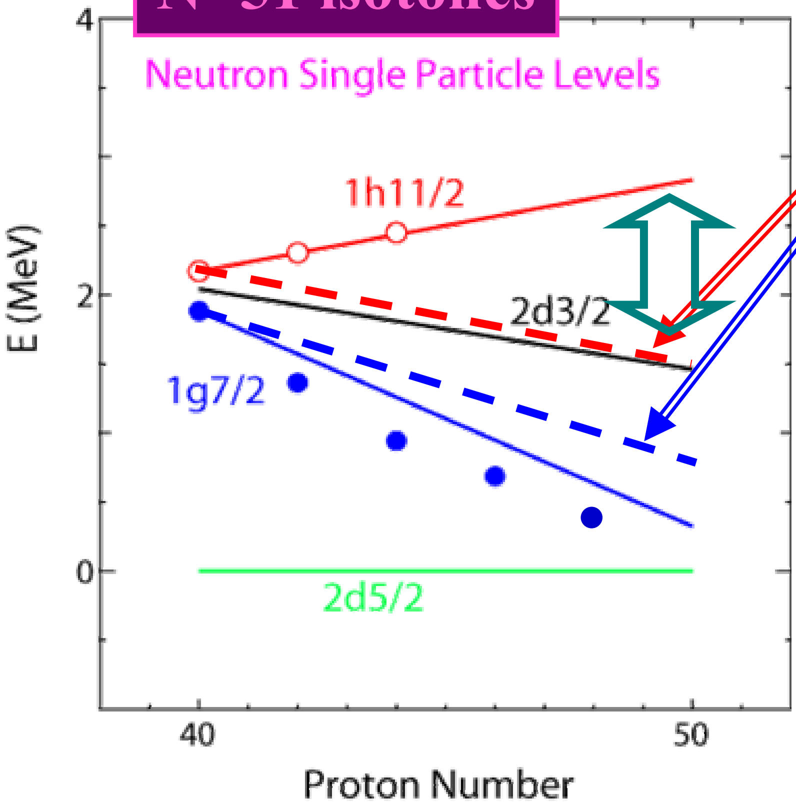
Neutron g7/2 is lowered by 2 protons in g9/2 ?

Z=42
(two protons in g9/2)

From Federman and Pittel, Phys. Lett. B 69, 385 (1977)

Can a short range attraction explain behaviors of these orbits ?

N=51 isotones



Skyrme SIII results
2-body ~ δ-function

$g_{7/2}$ goes down slowly

$h_{11/2}$ does NOT go up

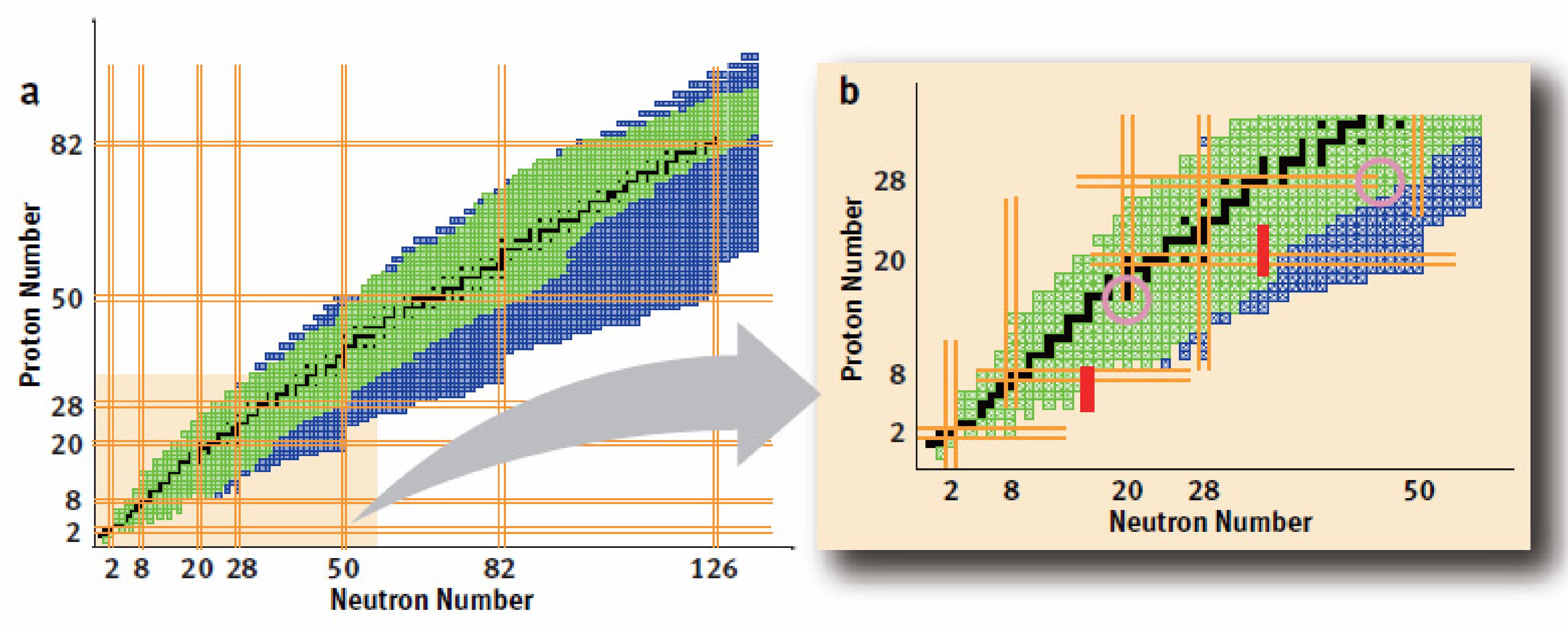
Lines : π + ρ meson

Points : corresponding exp. level

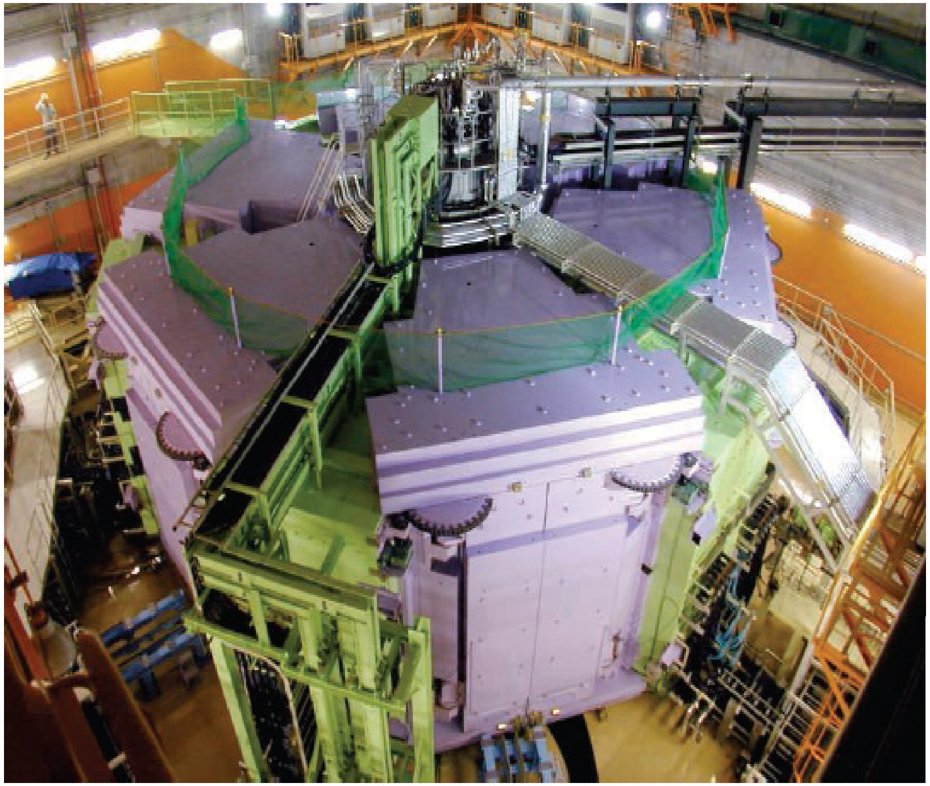
protons in $g_{9/2}$

A force reveals its magic

The inclusion of the long-neglected tensor force into theoretical models revises our understanding of 'magic numbers' in the atomic nucleus



A city works its magics. … N.Y.



NUCLEAR PHYSICS

Japan Gets Head Start in Race to Build Exotic Isotope Accelerators

A new facility begins to explore the structure of the nucleus as Europe awaits two machines and the United States revises its plans

"Science" magazine issued on Dec. 20, 2006

expected. However, recent findings suggest that the known magic numbers—2, 8, 20, 28, 50, 82, and 126—may not apply to nuclei with an extreme excess or deficiency of

Revving up. Japan's new exotic isotope accelerator should come on line within weeks.

First beam on Dec. 28, 2006

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An example from pf shell ($f_{7/2}$, $f_{5/2}$, $p_{3/2}$, $p_{1/2}$)

Microscopic | **Phenomenological**

G-matrix + polarization correction + **empirical refinement**

- **Start from a realistic microscopic interaction**

  M. Hjorth-Jensen, et al., Phys. Repts. 261 (1995) 125

  – Bonn-C potential

  – 3rd order Q-box + folded diagram

- 195 two-body matrix elements (TBME) and 4 single-particle energies (SPE) are calculated

  → *Not completely good* (theory imperfect)

- Vary 70 Linear Combinations of 195 TBME and 4 SPE
- **Fit** to **699 experimental energy data of 87 nuclei**

GXPF1 interaction

M. Honma et al., PRC65 (2002) 061301(R)

G-matrix vs. GXPF1

G : input to fit
GXPF1 : output

two-body matrix element

$\langle ab; JT \,|\, V \,|\, cd\,; JT \rangle$

$7 = f_{7/2},\ 3 = p_{3/2},\ 5 = f_{5/2},\ 1 = p_{1/2}$

T=0 … more attractive

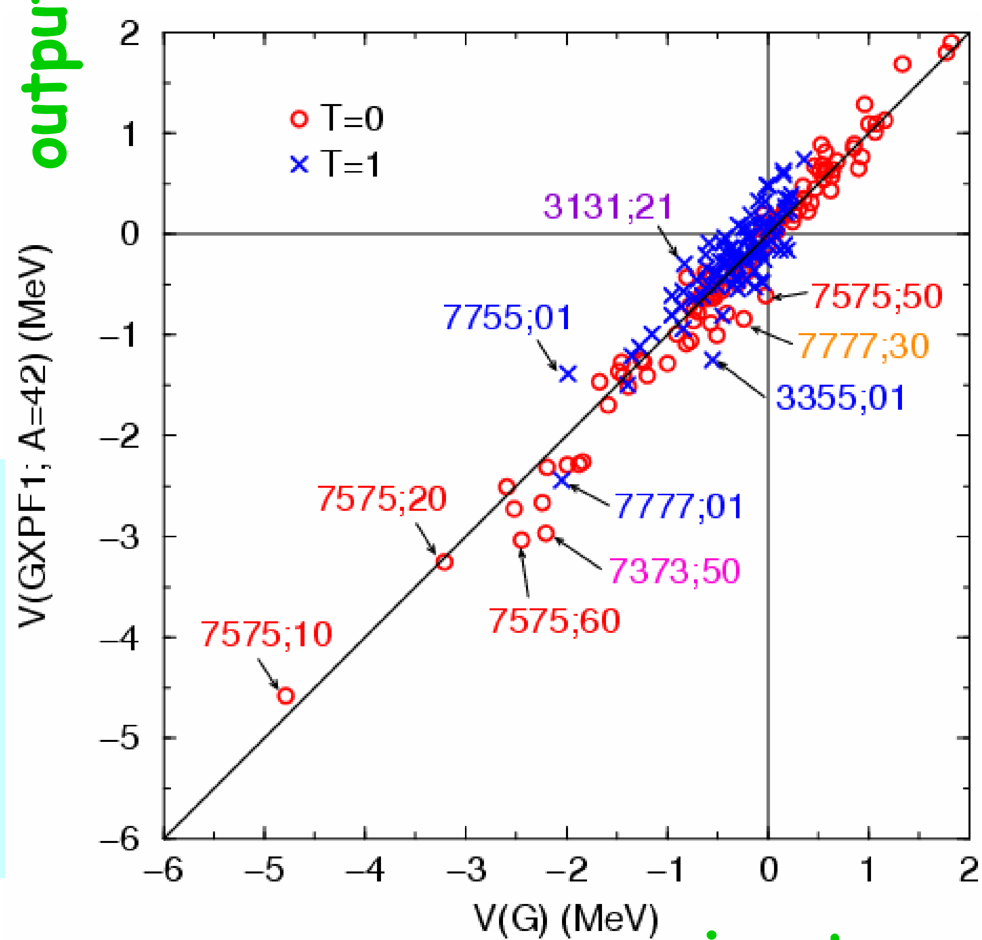
T=1 … more repulsive

- Relatively large modifications in

  $V(abab\,;J0\,)$ with large J

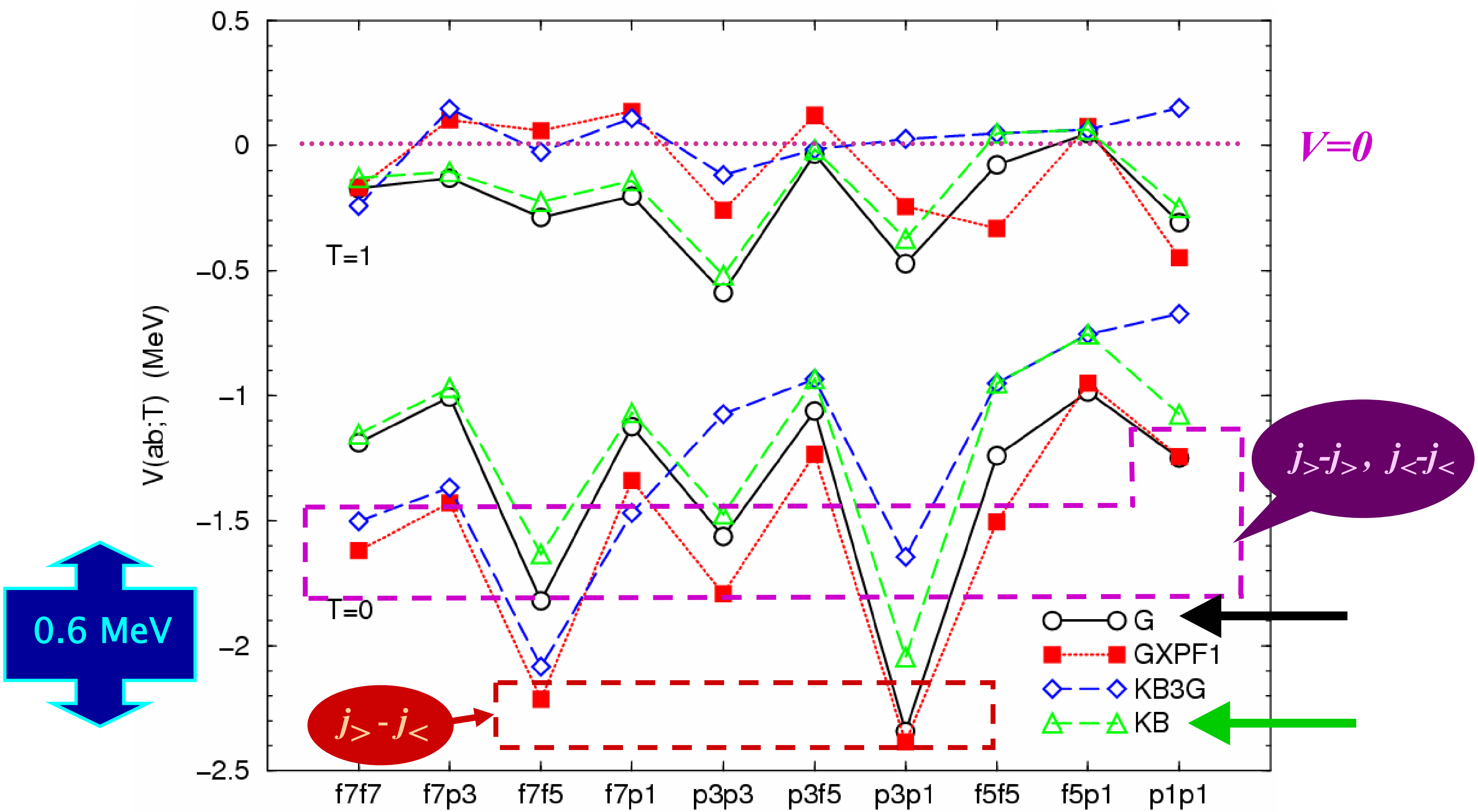
  $V(aabb\,;J1\,)$ pairing

output



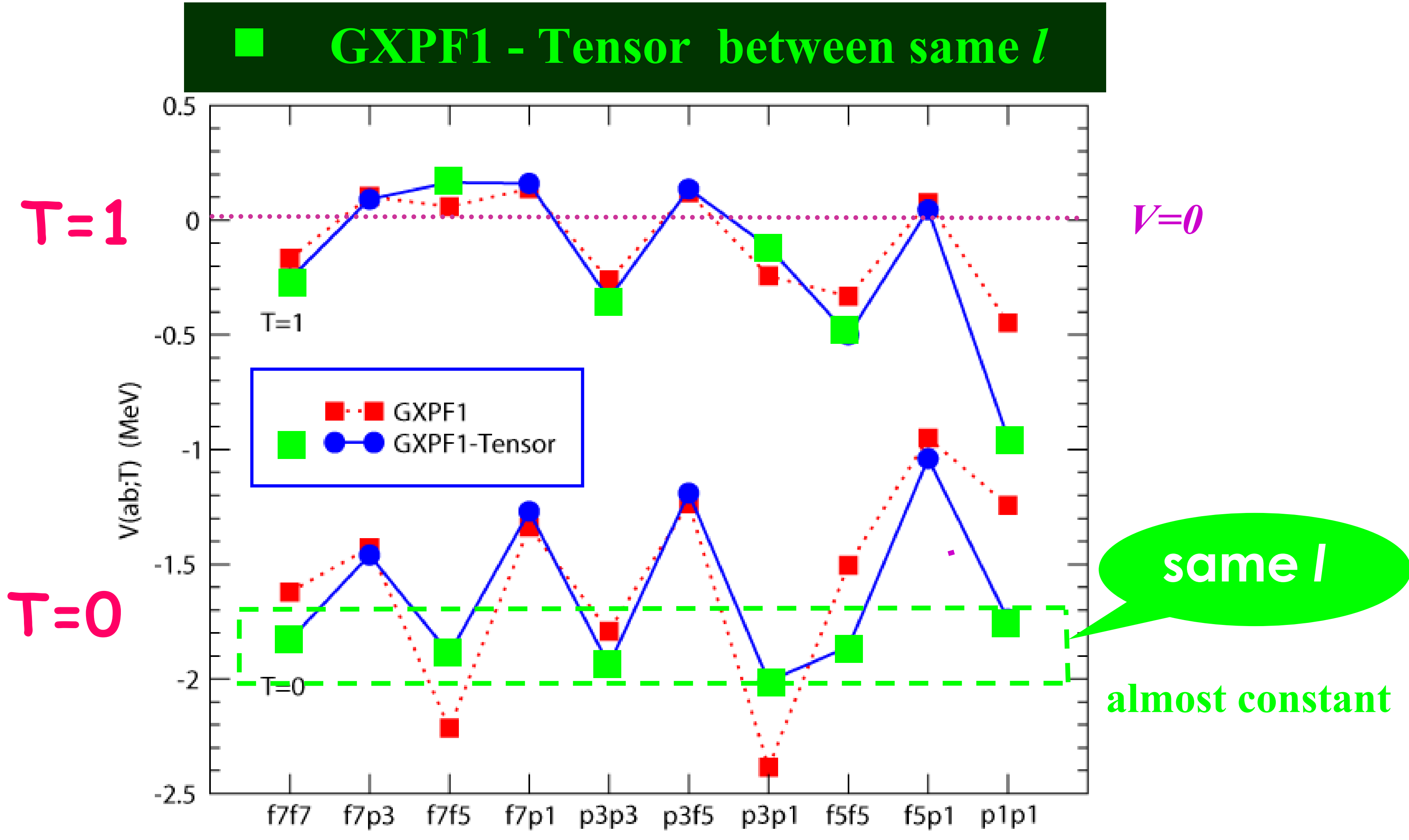
input

Monopole part of various interactions



KB interactions : Poves, Sanchez-Solano, Caurier and Nowacki, Nucl. Phys. A694, 157 (01)

Monopole interaction of GXPF1 :
full (original) and
part after subtraction of tensor part (π + ρ)



Monopole interaction
determined by from G-matrix-based empirical fit

T=0

average part (primarily due to central force)

l, l' dependence ← G-matrix can do

global shift from G-matrix … more attraction

fit for now. 3NF or density-dependent 2NF.

tensor force -> variation (clear, robust, no fit)

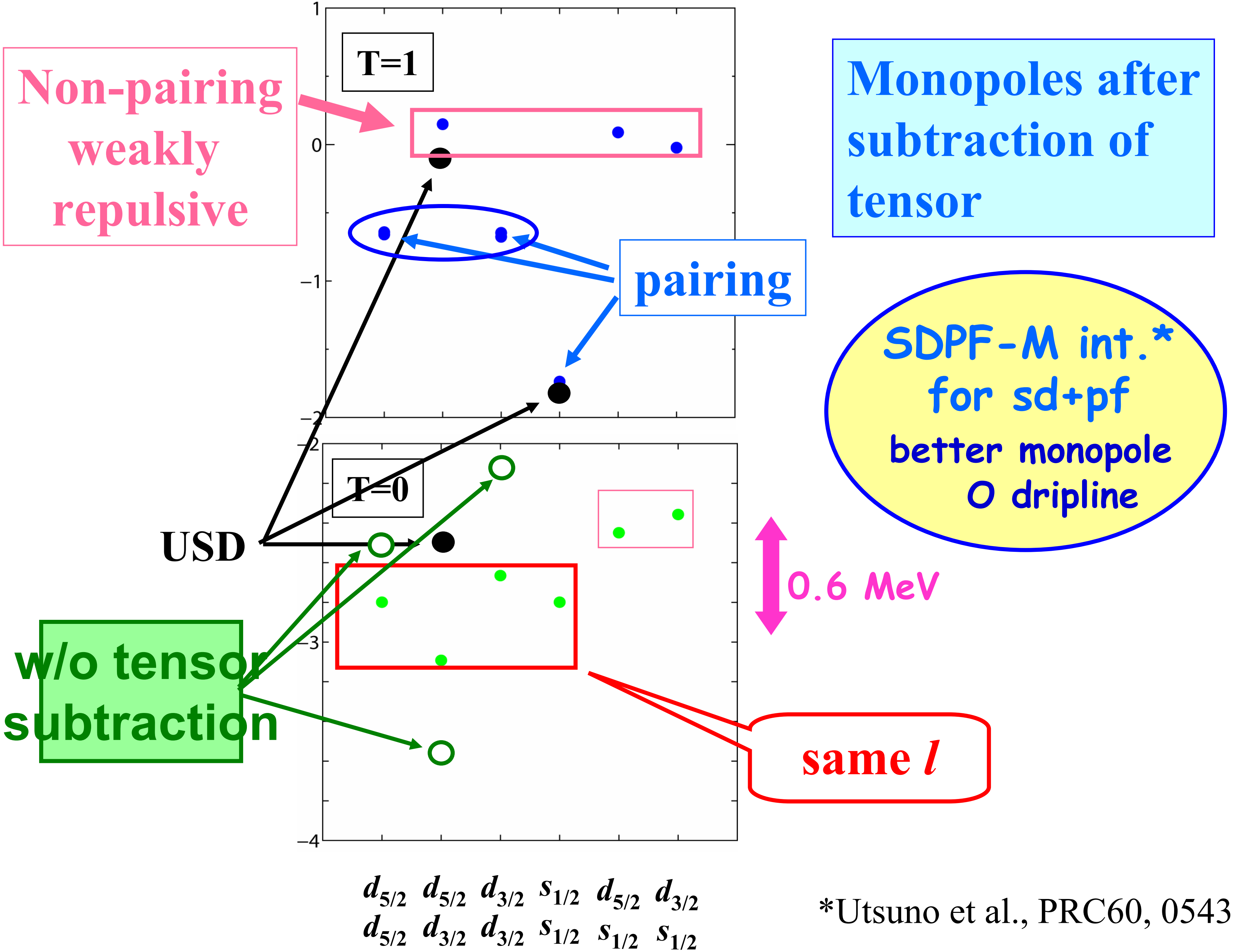
T=1

if j = j', pairing -> attractive

if not -> ~ 0 or slightly repulsive

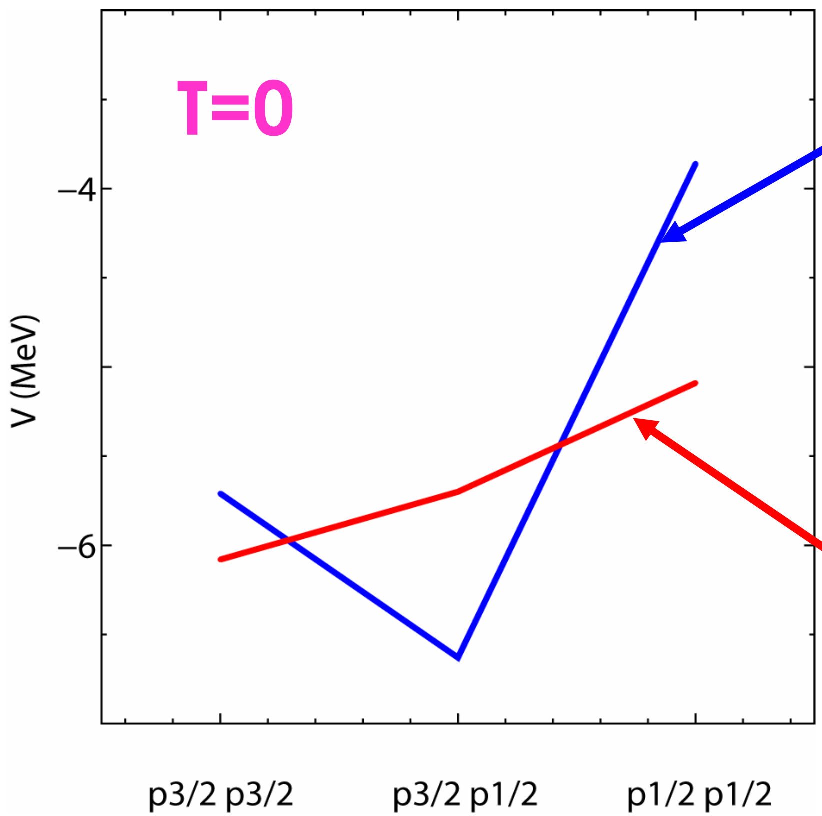
global shift opposite to T=0, fit for now also

What about sd shell ?



*Utsuno et al., PRC60, 054315

Monopole interaction in p-shell



SFO*

Spin-isospin improved interaction based on Cohen-Kurath

*) Suzuki, Fujimoto and Otsuka, Phys. Rev. C 67, 044302 (03)

SFO - tensor

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Implementation of tensor interaction into mean field calculations

Gogny interaction (J. Decharge and D. Gogny, 1980)

(1+σσ+ττ+σστ) (Gauss1 + Gauss2) + Density Dep.

finite range **zero range**

Successful descriptions of various properties with D1S interaction (J.F. Berger et al., Nucl. Phys. A428, 23c (84))

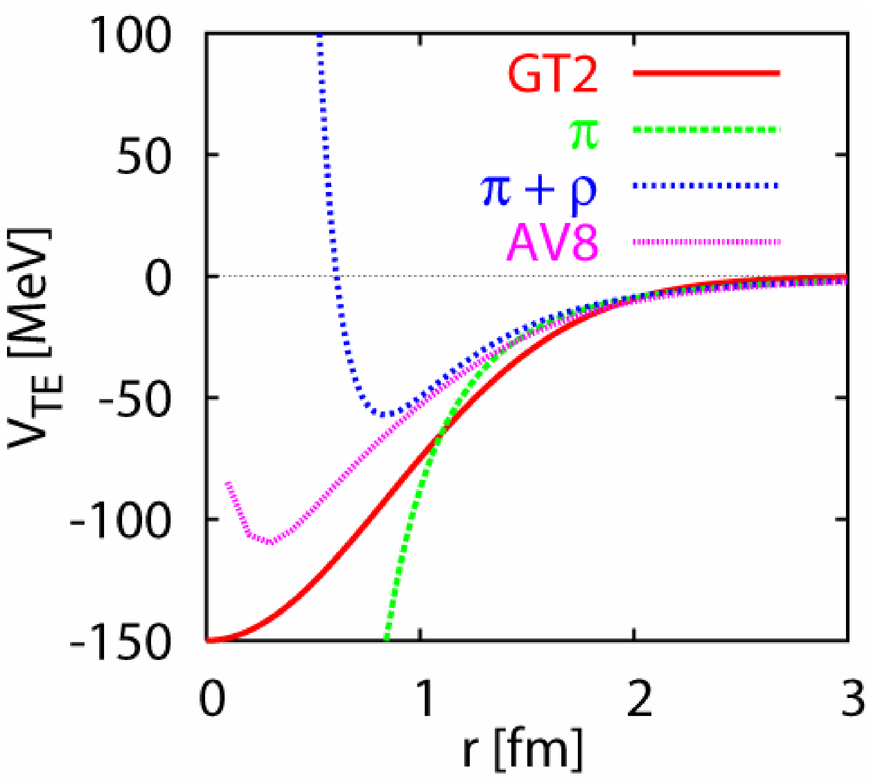
Tensor interaction is added

All parameters are readjusted

Nuclear matter properties reproduced with improvement of imcompressibility

Gogny-Tokyo interaction – 2, 3 (GT2, 3)
Pairing treated well by GT3 → Abe

Triplet-Even potential due to the Tensor force

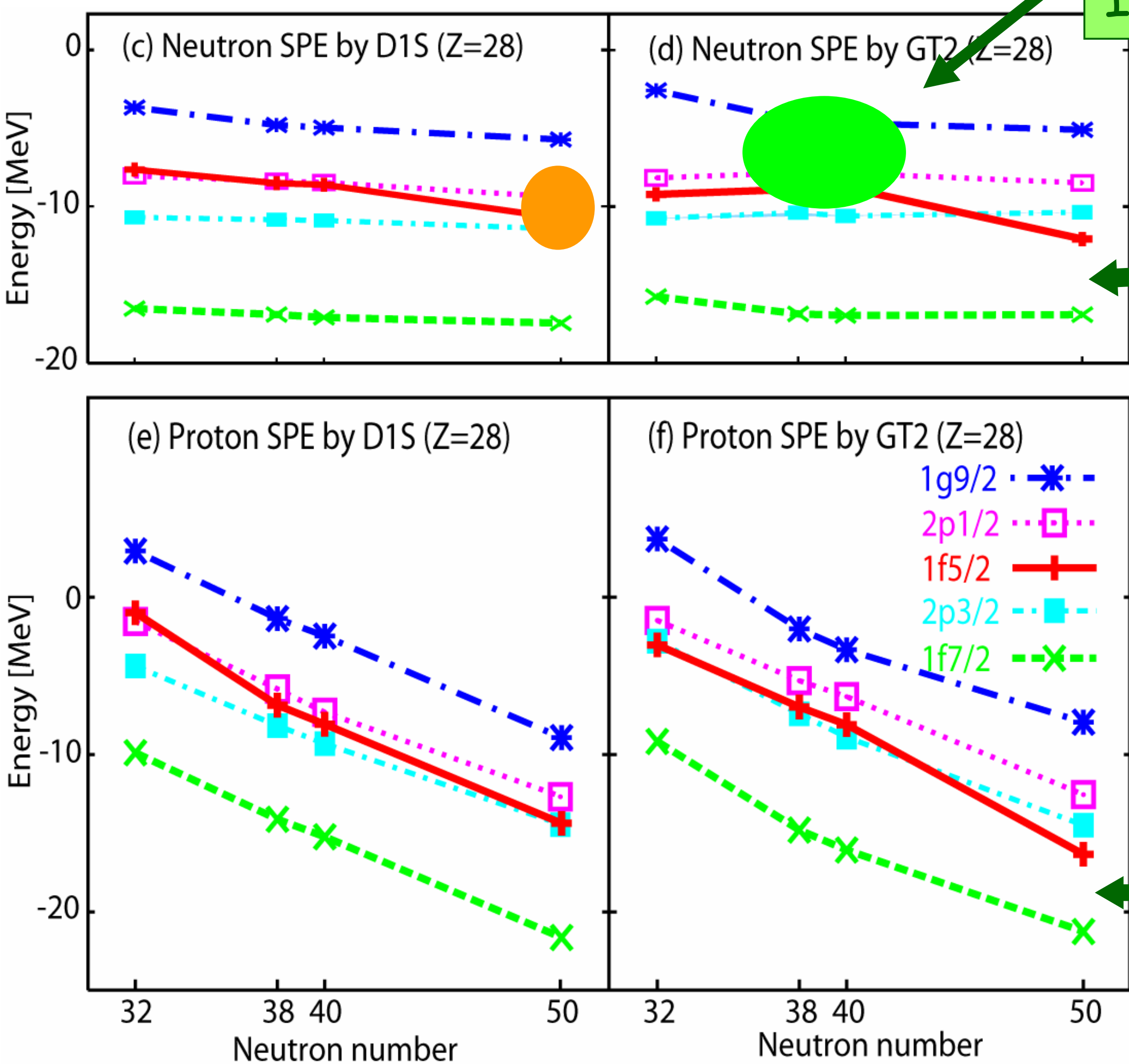


Abe : HFB calculations including this tensor force

Single-particle energies of exotic Ni isotopes (HF calculation)

w/o tensor | with tensor

another region of Intruder ground state



N=28 weakened but not relevant ?

Z=28 weakened

TO, Matsuo, Abe, Phys. Rev. Lett. 97, 162501 (2006)

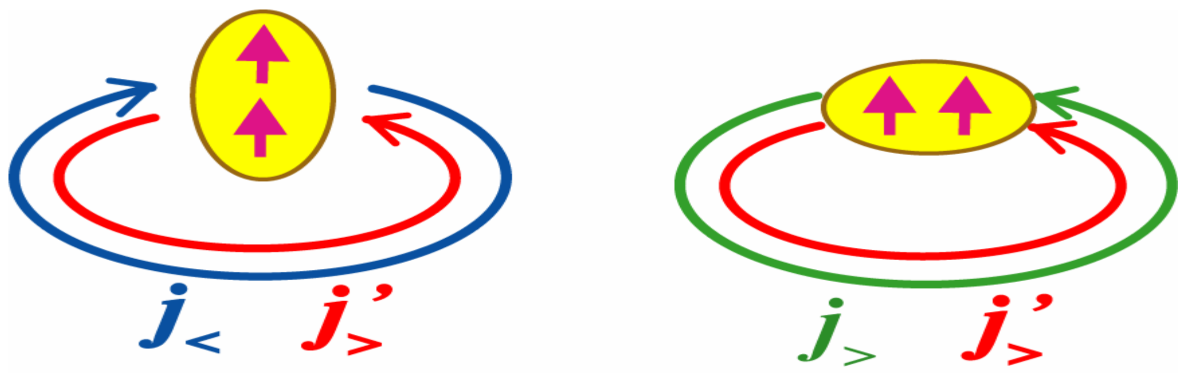
Stancu, Brink and Flocard, Phys. Lett. 68B, 108 (1977)

Zero-range spin-momentum tensor coupling term (*a la* Skyrme)

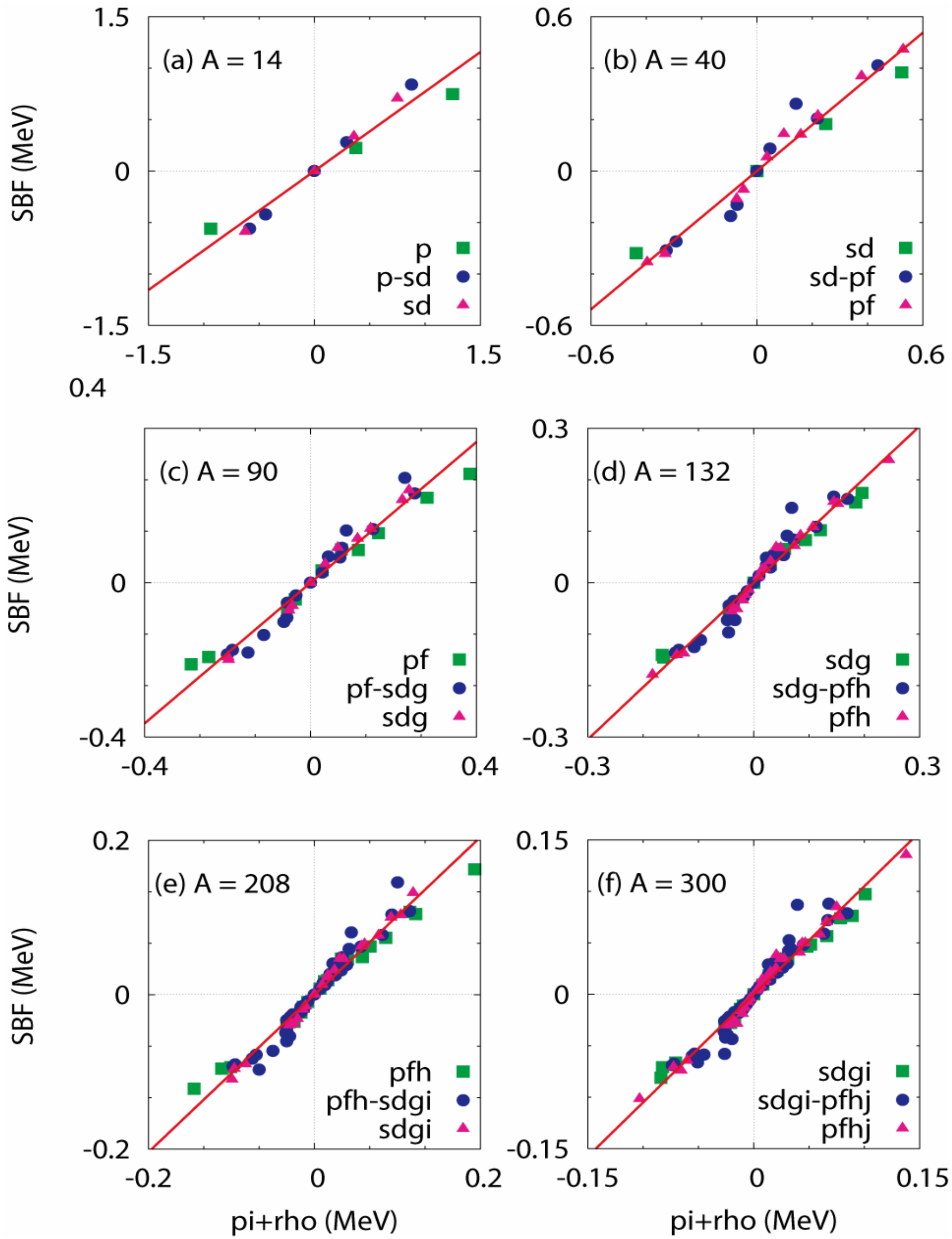
$$
\begin{aligned}
v_{\mathrm{T}} = \tfrac{1}{2}T\{&[(\sigma_1\cdot k')(\sigma_2\cdot k') - \tfrac{1}{3}(\sigma_1\cdot\sigma_2)k'^2]\delta(r_1-r_2) \\
&+ \delta(r_1-r_2)[(\sigma_1\cdot k)(\sigma_2\cdot k) - \tfrac{1}{3}(\sigma_1\cdot\sigma_2)k^2]\} \\
&+ U\{(\sigma_1\cdot k')\delta(r_1-r_2)(\sigma_1\cdot k) \\
&- \tfrac{1}{3}(\sigma_1\cdot\sigma_2)[k'\cdot\delta(r_1-r_2)k]\}, \qquad (1)
\end{aligned}
$$

This is not be a good approximation to the tensor force itself, but may provide us with a good simulation of the monopole effect of the tensor shown below (basic idea of MF/DFT).

large relative momentum   small relative momentum



deuteron ⇨ attractive   **repulsive**



Monopole matrix elements

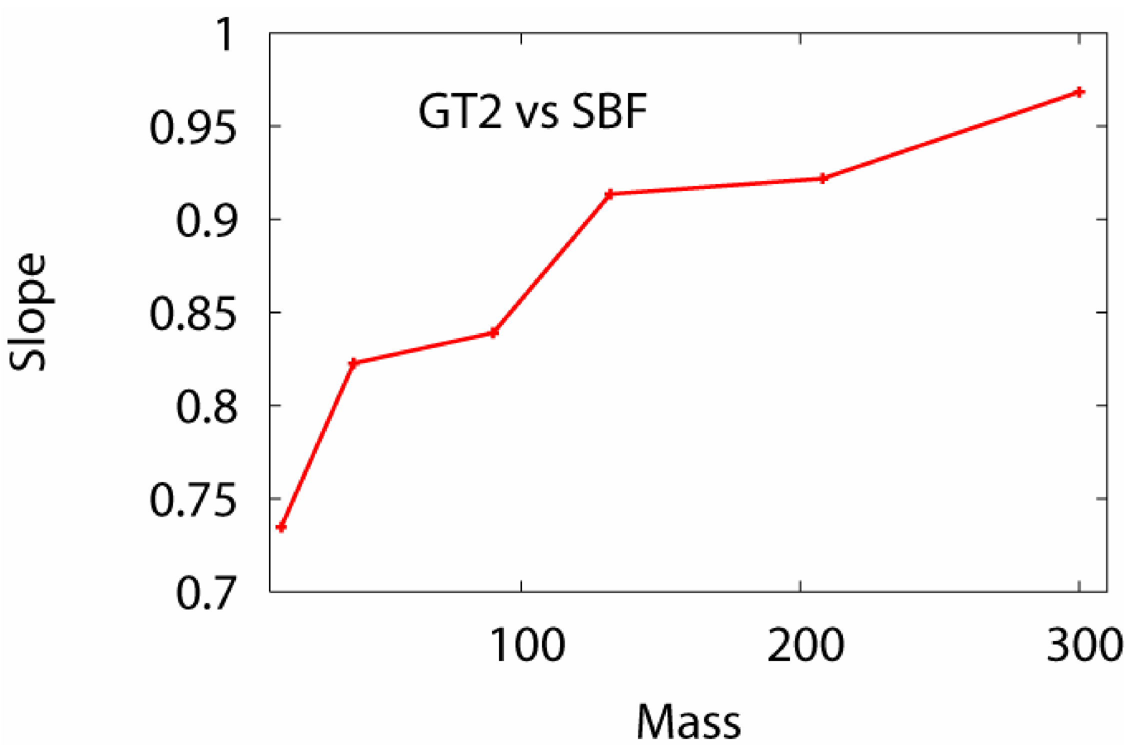
π+ρ exchange tensor
vs
SBF

SBF :Stancu-Brink-Flocard

We used, in this comparison,
T = 450, U=150

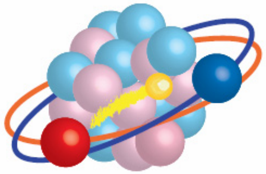
About half values
as compared to
the short range
approximation of SBF,
closer to q=1 fm^{-1}

The slope changes



Even at this point, a factor of two off in the short range approximation (Stancu, Brink and Flocard, Phys. Lett. 68B, 108 (1977))

Summary



Nuclear shell evolves in unique ways as compared to other physical systems, particularly in exotic nuclei (note that ΔN must be >10 to see this → **RNB !**).

- **Shell evolution due to tensor interactions**
  - **-drives $j_>$ or $j_<$ levels in a specific and robust way**
    intuitive picture → many cases expected from p-shell to superheavies
  - **- is the dominant origin of shell evolution**

Simpler origin of Tensor force :
dominated by one pion exchange (of course ρ, higher order …)

←→ consistent with Chiral Perturbation (Weinberg)

NN = short range + lower-order pion exchange

Shell model :

- Tensor force is properly included in G-matrix.
  (not necessarily in old phenomenological interactions)
- Central part → dependence on nodal structure
  no effect on spin-orbit splitting
- Overall monopole (T=0, 1) seem to be affected
  by some origin(s) beyond the two-body interaction

Mean-field models … *central parts still to be revised*

- Gogny-type + Tensor (Gaussian) → Abe's talk on HFB
- SBF (zero range)
  is not good approximation to the tensor force itself.
  can simulate with an overall strength parameter.

Tensor part of the interaction can be shared by the shell model and MF model.

Collaborators

T. Suzuki	Nihon U.
R. Fujimoto	U. Tokyo
H. Grawe	GSI
Y. Akaishi	KEK
D. Abe	Tokyo
T. Matsuo	Hitachi Ltd.
M. Honma	U. Aizu
Y. Utsuno	JAEA

END

Equivalent expressions of tensor force

The standard expression of tensor force may be

$$V_T = (\vec{\tau}_1 \cdot \vec{\tau}_2)\, S_{12}\, V(r)$$

$$S_{12} = 3\,(\vec{s}_1 \cdot \vec{r}/r)\,(\vec{s}_2 \cdot \vec{r}/r) - (\vec{s}_1 \cdot \vec{s}_2)$$

$$= \; 3\,(\,[\vec{s}_1 \times \vec{s}_2]^{(2)} \cdot [\vec{r} \times \vec{r}]^{(2)}/r^2)$$

By using $[\vec{r} \times \vec{r}]^{(2)}/r^2 = \sqrt{8\pi/15}\; Y^{(2)}$**, we get**

$$S_{12} = \sqrt{24\pi/5}\,(\,[\vec{s}_1 \times \vec{s}_2]^{(2)} \cdot Y^{(2)})$$

We thus obtain an equivalent expression

$$V_T = \sqrt{24\pi/5}\;(\vec{\tau}_1 \cdot \vec{\tau}_2)\,(\,[\vec{s}_1 \times \vec{s}_2]^{(2)} \cdot Y^{(2)})V(r)$$

Federman and Pittel, Phys. Lett. B 69, 385 (1977)

- Overlap of radial wave functions is emphasized -

they can simultaneously fill the $1g_{9/2}$ proton and $1g_{7/2}$ neutron orbitals. The strong overlap of these spin-orbit-partner orbitals can lead to important n-p correlations in this region and thus to deformation.

At this point it is useful to generalize our earlier remarks as to when strong n-p correlations should occur. As noted earlier, the crucial criterion is that the neutrons and protons occupy orbitals with good overlap. It was pointed out long ago [8] that the overlap between two orbitals $(n_N\, l_N\, j_N)$ and $(n_P\, l_P\, j_P)$ is maximum if $n_N = n_P$ and $l_N \approx l_P$. So far, we have focussed on cases in which $n_N = n_P$ and $l_N = l_P$, although we have emphasized that j_N need not be the same as j_P.

Federman, Pittel and Etchegoyen, PLB 140 (1984)

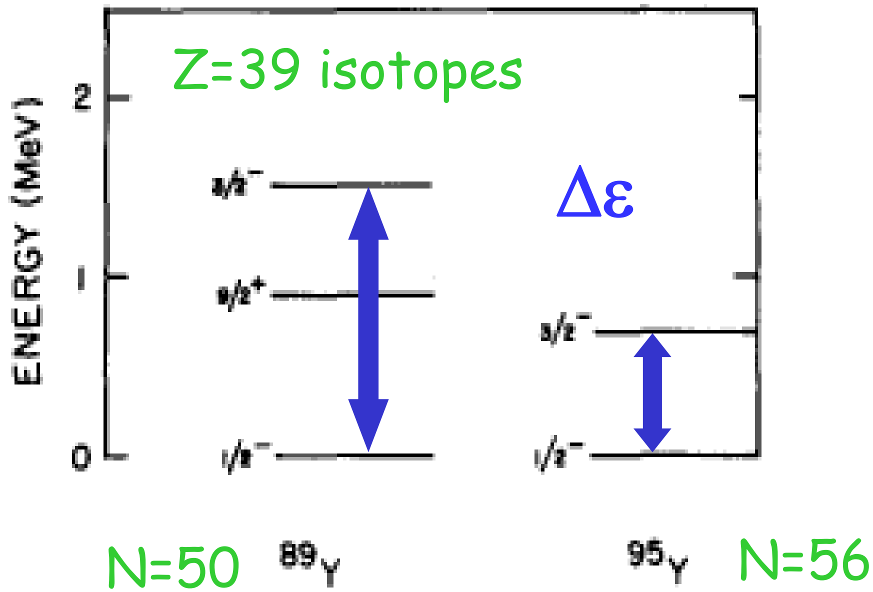
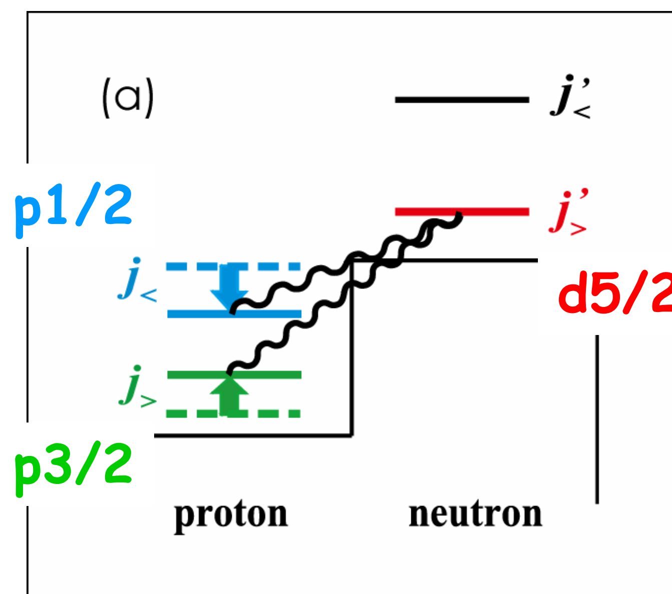


Fig. 2. Experimental spectra of ^{89}Y and ^{95}Y, from ref. [1].

From the π+ρ exchange tensor force :

$\Delta\varepsilon$ = 0.90 MeV



$\Delta\varepsilon$: spacing between $p_{3/2}$ and $p_{1/2}$

Thus, we predict

$$\Delta\epsilon(N = 56) - \Delta\epsilon(N = 50) = -0.54 \text{ MeV} , \qquad (7)$$

in qualitative agreement with the experimental reduction of 0.82 MeV.

Reduction of the Spin-Orbit Splittings at the $N = 28$ Shell Closure

L. Gaudefroy,[1,2] O. Sorlin,[2,1] D. Beaumel,[1] Y. Blumenfeld,[1] Z. Dombrádi,[3] S. Fortier,[1] S. Franchoo,[1] M. Gélin,[2] J. Gibelin,[1] S. Grévy,[2] F. Hammache,[1] F. Ibrahim,[1] K. W. Kemper,[4] K.-L. Kratz,[5,6] S. M. Lukyanov,[7] C. Monrozeau,[1] L. Nalpas,[8] F. Nowacki,[9] A. N. Ostrowski,[5,6] T. Otsuka,[10] Yu.-E. Penionzhkevich,[7] J. Piekarewicz,[4] E. C. Pollacco,[8] P. Roussel-Chomaz,[2] E. Rich,[1] J. A. Scarpaci,[1] M. G. St. Laurent,[2] D. Sohler,[11] M. Stanoiu,[12] T. Suzuki,[13] E. Tryggestad,[1] and D. Verney[1]

Neutron single-particle energies

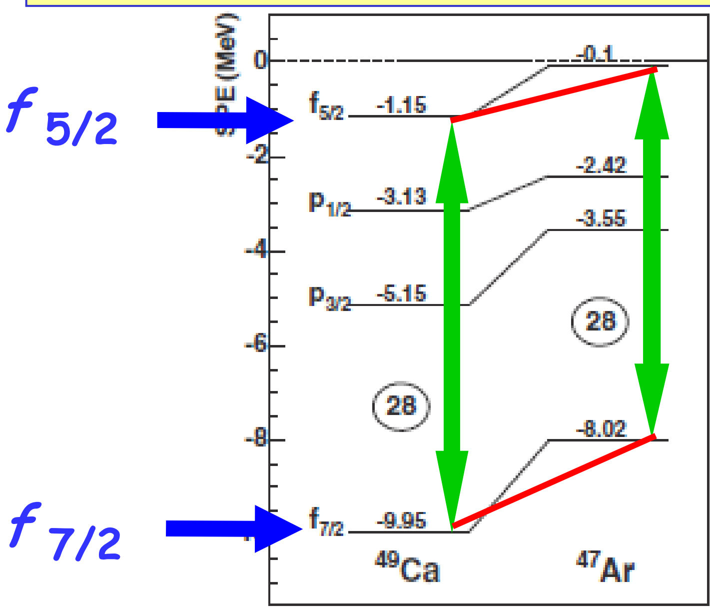


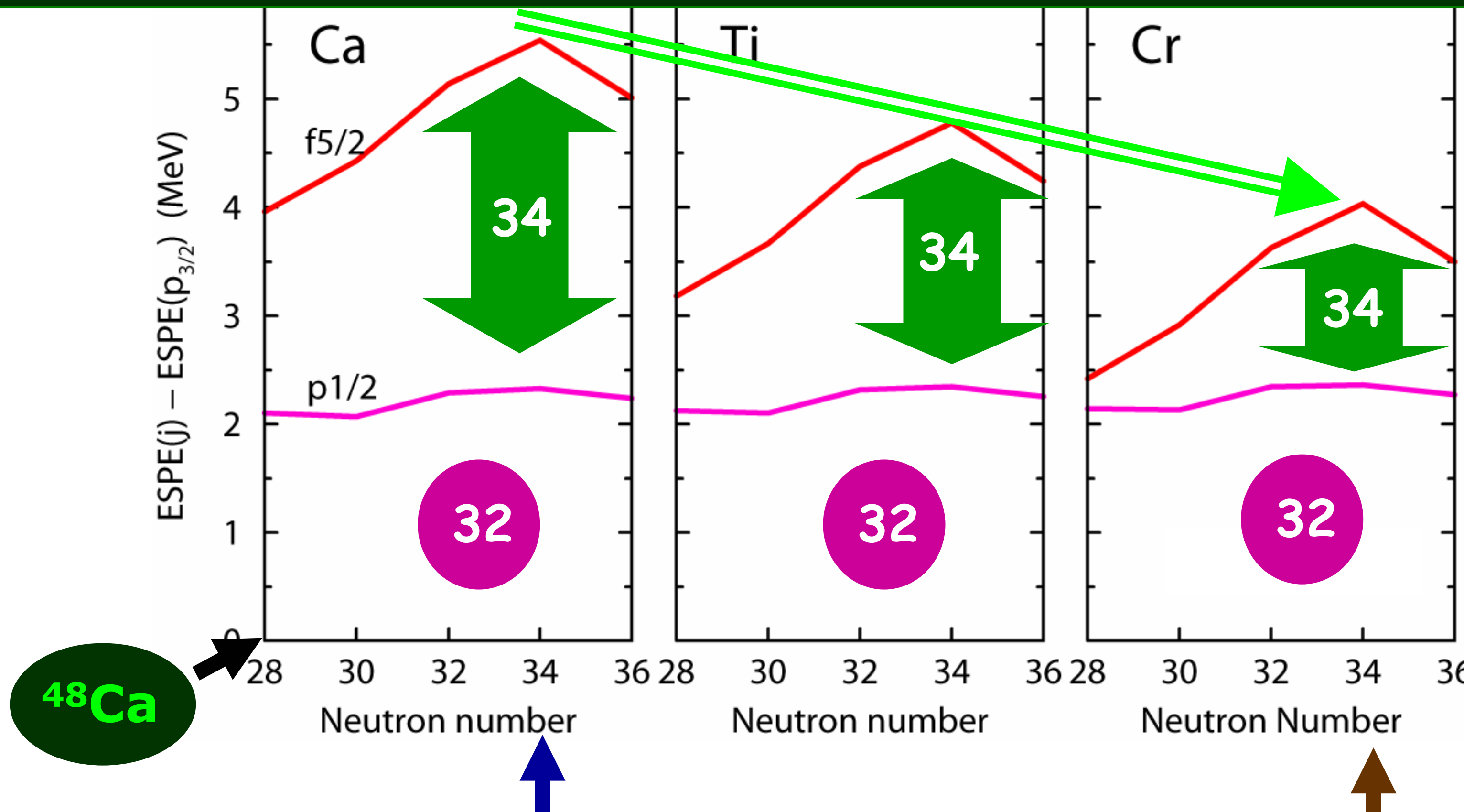
FIG. 3. Neutron single-particle energies (SPE) of the fp orbitals for the $^{47}Ar_{29}$ and $^{49}Ca_{29}$ nuclei (see text for details).

Mean-field models (Skyrme or Gogny) do not reproduce this reduction.

Tensor force effect due to vacancies of proton $d_{3/2}$ in $^{47}_{18}Ar_{29}$: 650 (keV) by $\pi+\rho$ meson exchange.

Effective single-particle energies of Ca, Ti and Cr isotopes calculated by **GXPF1B** interaction

Monopole effect of tensor+central force (f7/2 occupancy)

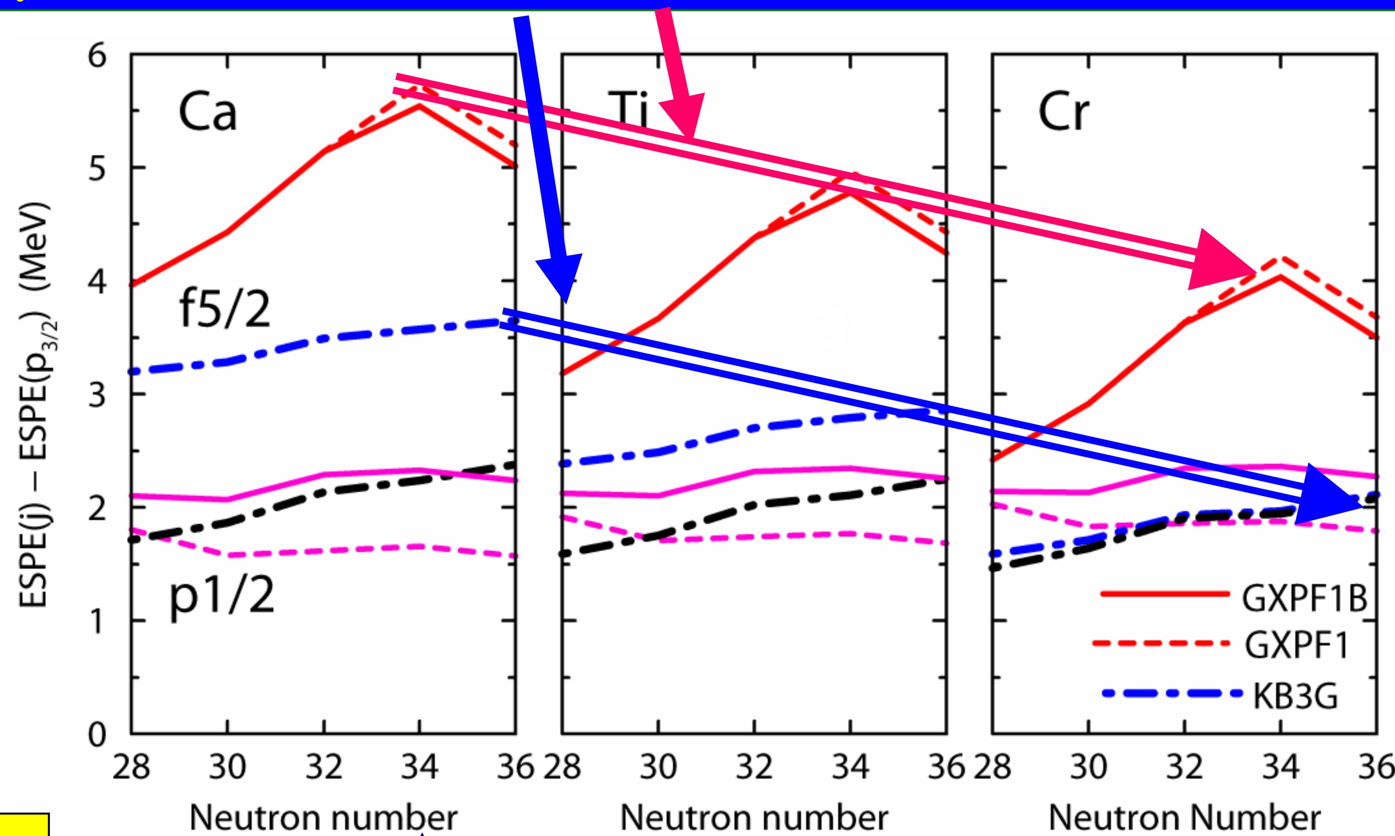


Gap = 2 MeV at N=32 and 3 MeV at N=34

Gap = 2 MeV at N=32 and 1.5 MeV at N=34

Comparison among GXPF1B, GXPF1 and KB3G

Monopole effect of tensor+cent force ~ almost the same



GXPF1B:

Gap = 2 MeV at N=32 and 3 MeV at N=34

Gap = 2 MeV at N=32 and 1.5 MeV at N=34