# Unitary Model Operator Approach

## to Magic Nuclei and Open-shell Nuclei


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# Plan

- Introduction
- Formulation of UMOA with NN interaction
- Discussion of the results

    the results in isospin basis

    the results in particle basis
- Some challenges
- Summary

# Introduction

Deeper understanding of nuclear structure,
ex., ***Foundations of independent particle motion***,
starting from the free NN interaction
→ Description of short-range repulsive core and tensor force

***Changes in the effective nuclear interaction***
Sizable effects of three-nucleon interactions on the two-body interaction
in not only light but also *medium, medium-heavy nuclei*
→ effective vs genuine three-body interactions

**Evolution of the Nuclear Interaction**
old generation NN int. → Chiral Nuclear Interaction,…

***Excitation and decay properties of weakly bound systems***
→ The same footing description of correlations and continuum states

**Different realization of the short range nuclear interaction**

→the Skyrme and Gogny interactions

Effective theories employ a set of parameters that are adjusted to selected exp. observables. ⇒**a multitudes of different force parametrizations**.

(to do that)

….**the correct single particle states, that are so crucial for high accuracy calculations**,

are in general coupled to collective motion and therefore difficult to determine.

A. Bhagwat, R. Wyss, W. Satula, J. Meng, Y. K. Gambhir,

*Deficiency of Spin Orbit Interaction in Relativistic Mean Filed Theory*

nucl-th/0605009

**In a fully microscopic approach,**

**the single-particle basis should be constructed**

**from the free nucleon-nucleon interaction**

**or more complicated three- and/or many-body interactions.**

G. Hagen, M. Hjorth-Jensen, N. Michel, PRC.**73**, 064307(2006).

# Historical background of UMOA

## development of *microscopic* effective interaction theory

J. H. Van Vleck, Phys. Rev.**33**(1929), 467.

S. Okubo, Prog. Theor.Phys. **12**(1954), 603: decoupling eq. ,
elimination of meson degrees of freedom.

F. Villars, *Enrico Fermi course* **23**(1964),1.: extension of HF in unitary transformation

J. da Providencia and C. M. Shakin, Ann. Phys.**30**(1964), 95.
cluster expansion, medium effect

C. M. Shakin, Y. R. Waghmare, M. H. Hull, Jr., Phys. Rev. 161(1967), 1006.
cal. of $^{16}O$, $^{40}Ca$

K. Suzuki, S. Y. Lee, Prog. Theor. Phys. **64**(1980), 2091: Lee-Suzuki method,
similarity transformation

K. Suzuki, Prog. Theor. Phys. **68**(1982), 246.hermitization of effective interaction

# Extension of UMOA

- UMOA in isospin basis(1986-1994)

- UMOA in particle basis(2004- )

*UMOA with Chiral nuclear interaction*(?)

*for complex effective interaction*(?)

# Formulation of UMOA with 2NF

**Hamiltonian of a many nucleon system**

**interacting via a nucleon-nucleon interaction**

$$H = \sum_i t_i + \sum_{i<j} v_{ij} = \sum_i (t_i + u_i) + \left[ \sum_{i<j} v_{ij} - \sum_i u_i \right]$$

$$= \sum_i h_i + \left[ \sum_{i<j} v_{ij} - \sum_i u_i \right]; \quad h_i \equiv t_i + u_i$$

$v_{ij}$ : realistic NN int. $\Leftarrow$ AV8, AV18, CD-Bonn, Nijmegen, NLO, $\cdots$

Coulomb int.

**Medium effect**

***$u_{i:}$*** **auxiliary, but self-consistently determined potential**

**Anti-hermitian *two-body* correlation operator**

$$S^{(2)} = \sum_{i<j} S_{ij}, \; [S^{(2)\dagger} = -S^{(2)}]$$

Second quantization form

$$S^{(2)} = \left( \frac{1}{2!} \right)^2 \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | S_{12} | \gamma\delta \rangle \, c_\alpha^\dagger c_\beta^\dagger c_\delta c_\gamma$$

## *Description* of correlations in similarity transformation

Schroedinger eq. for a many-body system

$$\boxed{H\left|\Psi_0\right\rangle = E_0\left|\Psi_0\right\rangle}$$

$\left|\Psi_0\right\rangle$ **correlated ground state**

$\left|\Phi_0\right\rangle$ **Reference state (uncorrelated state)**

$$\left|\Psi_0\right\rangle = \mathrm{e}^{S^{(2)}}\left|\Phi_0\right\rangle$$ Exp ansatz

$$\Rightarrow \mathrm{e}^{-S^{(2)}} H \,\mathrm{e}^{S^{(2)}} \mathrm{e}^{-S^{(2)}}\left|\Psi_0\right\rangle = E_0\,\mathrm{e}^{-S^{(2)}}\left|\Psi_0\right\rangle$$

$$\Rightarrow \boxed{[\mathrm{e}^{-S^{(2)}} H \mathrm{e}^{S^{(2)}}]\left|\Phi_0\right\rangle = E_0\left|\Phi_0\right\rangle}$$

# Unitary transformation of Hamiltonian and its cluster expansion

$$\widetilde{H} \equiv e^{-S^{(2)}} H e^{S^{(2)}}$$

$$= \widetilde{H}^{(1)} + \widetilde{H}^{(2)} + \widetilde{H}^{(3)} + \cdots$$

**Second quantization form**

$$\widetilde{H}^{(1)} \equiv \sum_{\alpha\beta} \langle \alpha | h_1 | \beta \rangle c_\alpha^\dagger c_\beta ,$$

$$\widetilde{H}^{(2)} \equiv \left(\frac{1}{2!}\right)^2 \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | \tilde{v}_{12} | \gamma\delta \rangle c_\alpha^\dagger c_\beta^\dagger c_\delta c_\gamma - \sum_{\alpha\beta} \langle \alpha | u_1 | \beta \rangle c_\alpha^\dagger c_\beta ,$$

$$\widetilde{H}^{(3)} \equiv \left(\frac{1}{3!}\right)^2 \sum_{\alpha\beta\gamma\lambda\mu\nu} \langle \alpha\beta\gamma | \tilde{v}_{123}^{(2)} | \lambda\mu\nu \rangle c_\alpha^\dagger c_\beta^\dagger c_\gamma^\dagger c_\nu c_\mu c_\lambda - \left(\frac{1}{2!}\right)^2 \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | \tilde{u}_{12} | \gamma\delta \rangle c_\alpha^\dagger c_\beta^\dagger c_\delta c_\gamma ,$$

....................

**Non-perturbative, Correlation expansion**

# Effective *two-body* interaction *Well behaved interaction*

$$\tilde{v}_{12} \equiv e^{-S_{12}}(h_1 + h_2 + v_{12})e^{S_{12}} - (h_1 + h_2)$$

or $$e^{-S_{12}} H_{12}\, e^{S_{12}} = (h_1 + h_2) + \tilde{v}_{12}$$

$$\left[\tilde{v}_{12} \to v_{12} \text{ as } S_{12} \to 0\right]$$

$$H_{12} \equiv h_1 + h_2 + v_{12}$$

Two-body subsystem Hamiltonian

$$\tilde{u}_{12} \equiv e^{-S_{12}}(u_1 + u_2)e^{S_{12}} - (u_1 + u_2)$$

## Effective *three-body* interaction induced by the two-body correlations

$$\tilde{v}_{123}^{(2NF)} \equiv e^{-S_{123}^{(2)}}(h_1 + h_2 + h_3 + v_{12} + v_{23} + v_{31})e^{S_{123}^{(2)}}$$
$$- (h_1 + h_2 + h_3 + \tilde{v}_{12} + \tilde{v}_{23} + \tilde{v}_{31});$$

$$\left[\tilde{v}_{123}^{(2NF)} \to 0 \text{ as } S_{123}^{(2)} \to 0\right]$$

$$S_{123}^{(2)} \equiv S_{12} + S_{23} + S_{31}$$

# Determination of two-body correlation operator

**Projection operators in two-body state space**

$$P^{(2)} + Q^{(2)} = 1,\ {P^{(2)}}^2 = P^{(2)}, {Q^{(2)}}^2 = Q^{(2)}, P^{(2)}Q^{(2)} = Q^{(2)}P^{(2)} = 0$$

**Eigen value equation for the two-body sub-system in an entire many-body system**

$$\boxed{\begin{aligned} &H_{12} \equiv h_1 + h_2 + v_{12}, \\ &\quad H_{12}\left|\psi_k^{(2)}\right\rangle = E_k^{(2)}\left|\psi_k^{(2)}\right\rangle, \quad (k = 1, 2, \cdots, d, d+1, \cdots, n) \end{aligned}}$$

$$\left|\psi_k^{(2)}\right\rangle = \left(P^{(2)} + Q^{(2)}\right)\left|\psi_k^{(2)}\right\rangle = \left|\phi_k^{(2)}\right\rangle + \omega^{(2)}\left|\phi_k^{(2)}\right\rangle, (k = 1, 2, \cdots, d)$$

$$\left|\phi_k^{(2)}\right\rangle \equiv P^{(2)}\left|\psi_k^{(2)}\right\rangle, Q^{(2)}\left|\psi_k^{(2)}\right\rangle = \omega^{(2)}\left|\phi_k^{(2)}\right\rangle$$

**General solution** for the wave operator

$$\left\langle \psi_k^{(2)} \mid \psi_{k'}^{(2)} \right\rangle = \delta_{kk'}$$

$$\boxed{\omega^{(2)} = \sum_{k=1}^{d} Q^{(2)}\left|\psi_k^{(2)}\right\rangle\left\langle\tilde{\phi}_k^{(2)}\right| P^{(2)}} \quad \because \left\langle\tilde{\phi}_k^{(2)}\middle|\phi_{k'}^{(2)}\right\rangle = \delta_{kk'}, \left|\tilde{\phi}_k^{(2)}\right\rangle : \text{bi-orthogonal state}$$

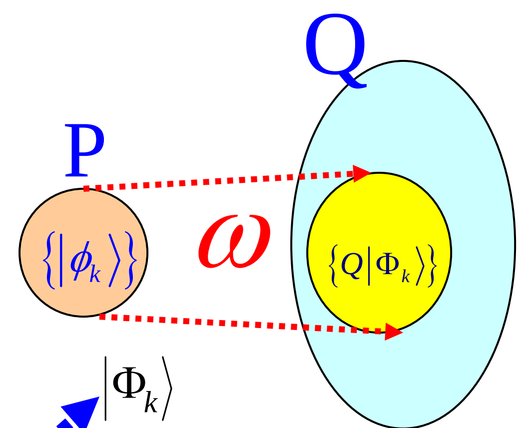
# Projection- and Wave operators

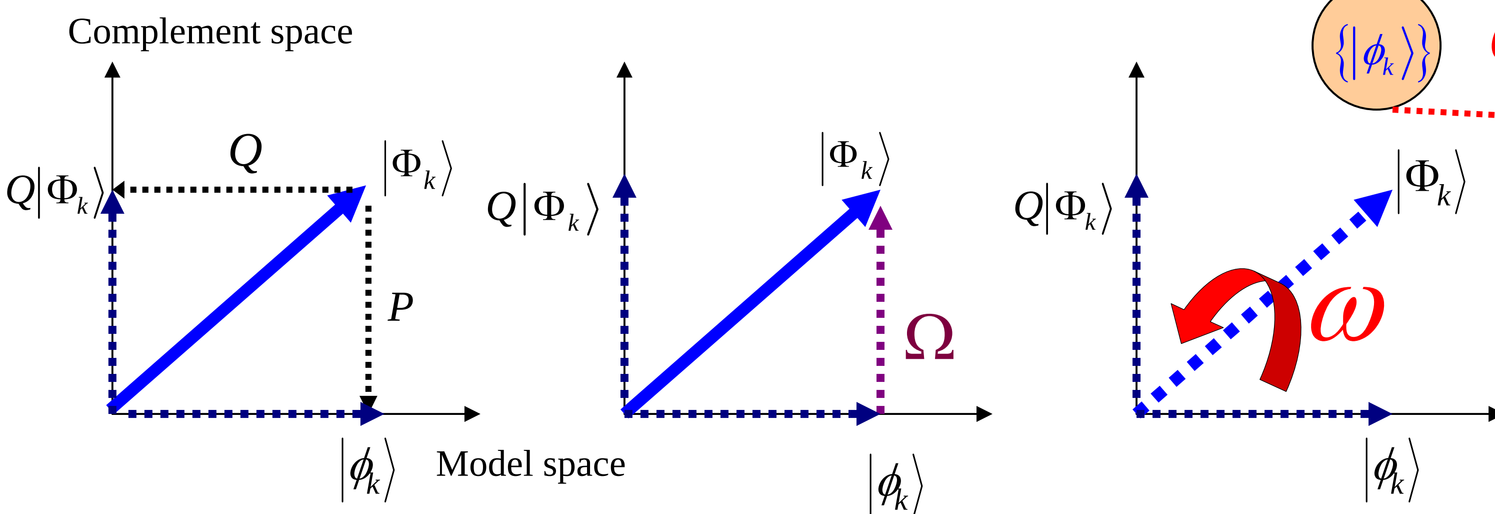
True state

$$
\begin{aligned}
\left|\Phi_k\right\rangle &= (P+Q)\left|\Phi_k\right\rangle \\
&= \left|\phi_k\right\rangle + Q\left|\Phi_k\right\rangle \\
&= \Omega\left|\phi_k\right\rangle \\
&= \left|\phi_k\right\rangle + \omega\left|\phi_k\right\rangle
\end{aligned}
$$

model state

$$\left|\phi_k\right\rangle = P\left|\Phi_k\right\rangle$$




$$Q\left|\Phi_k\right\rangle = \omega\left|\phi_k\right\rangle$$

$$P+Q=1$$
$$P^2=P, Q^2=Q,$$
$$PQ=QP=0$$

**Projection operators**

M$\phi$ller' wave operator

$$\Omega = P+\omega$$

wave operator

$$\omega,\ \omega = Q\omega P$$

$$\omega^2 = \omega^3 = \cdots = 0$$

# Decoupling equation in two-body system

**Relation between wave operator and correlation operator**

$$S^{(2)} = \mathrm{arctanh}(\omega^{(2)} \text{-} \omega^{(2)^{\dagger}})$$

I. Shavitt, L.T. Redman, J. Chem. Phys. **73**(1980), 5711

P. Westhouse, J. Quantum Chem.20(1981), 1243.

K. Suzuki, Prog.Theor.Phys.**68**(1982),246

**Decoupling equation** for transformed Hamiltonian

of two-body subsystem

$$Q^{(2)} \mathrm{e}^{-S_{12}} H_{12} \mathrm{e}^{S_{12}} P^{(2)} = 0$$

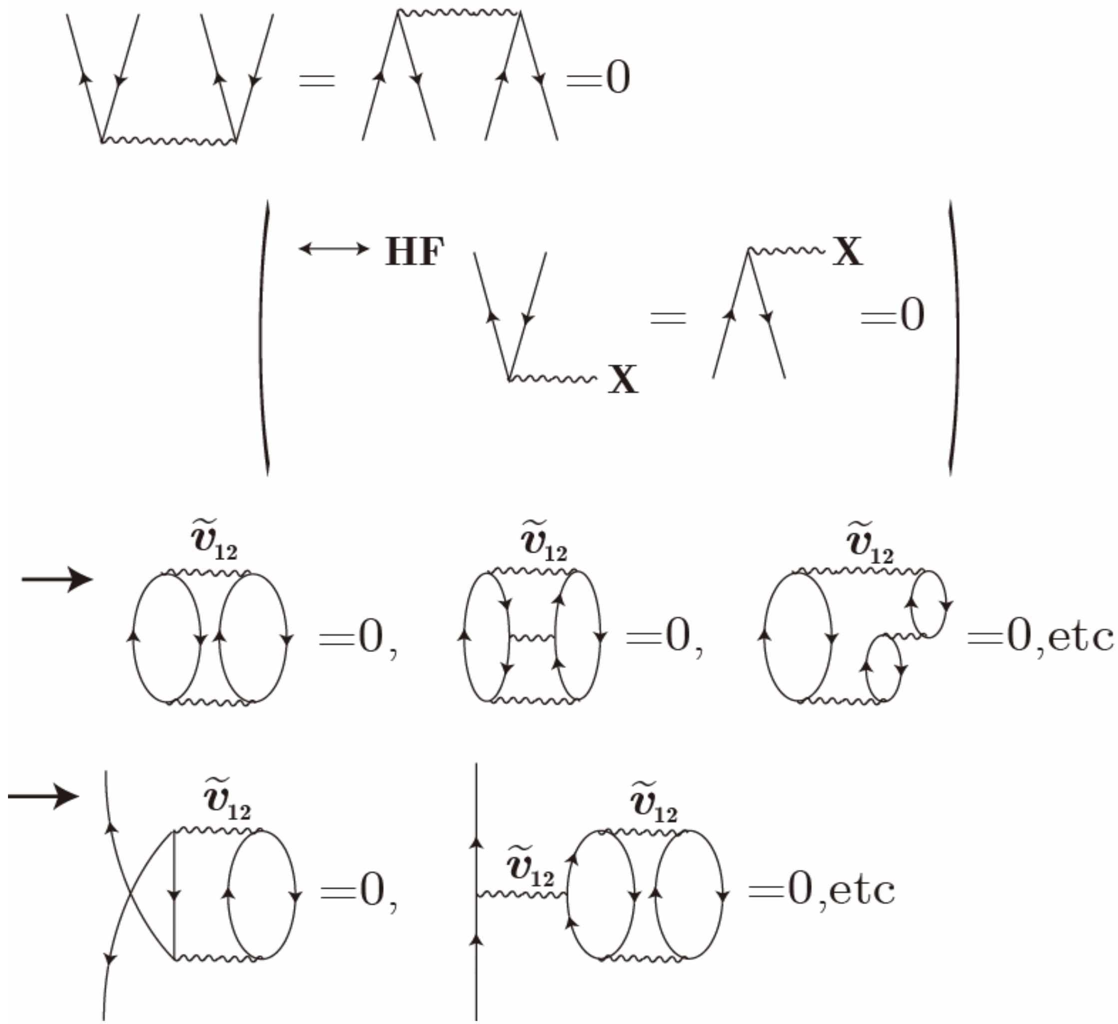
$$\rightarrow Q^{(2)} (h_1 + h_2 + \tilde{v}_{12}) P^{(2)} = 0$$

$$\text{if } Q^{(2)} (h_1 + h_2) P^{(2)} = 0, \quad \text{then}$$

$$\boxed{Q^{(2)} \tilde{v}_{12} P^{(2)} = 0}$$

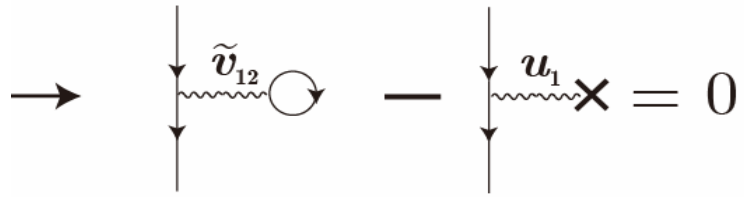
# Decoupling property of effective two-body interaction

$$Q\tilde{v}_{12}P = P\tilde{v}_{12}Q = 0$$

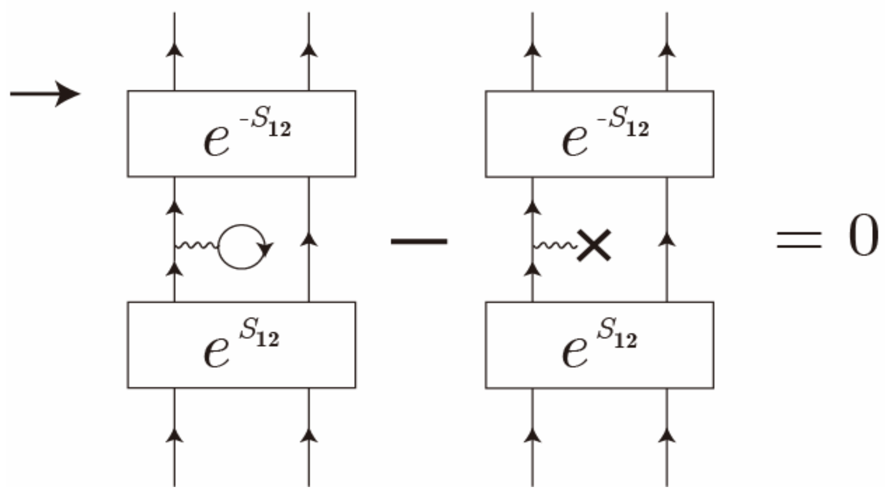


# Self-consistency between
## single-particle potential and effective interaction

$$\langle \alpha | u_1 | \beta \rangle = \sum_{\lambda \leq \rho_F} \langle \alpha\lambda | \tilde{v}_{12} | \beta\lambda \rangle$$




cancellation of one-body and bubble-diagram contribution




J. da Providencia and C. M. Shakin, Ann. Phys.**30**(1964), 95.

# Physical meaning of $S^{(2)}$

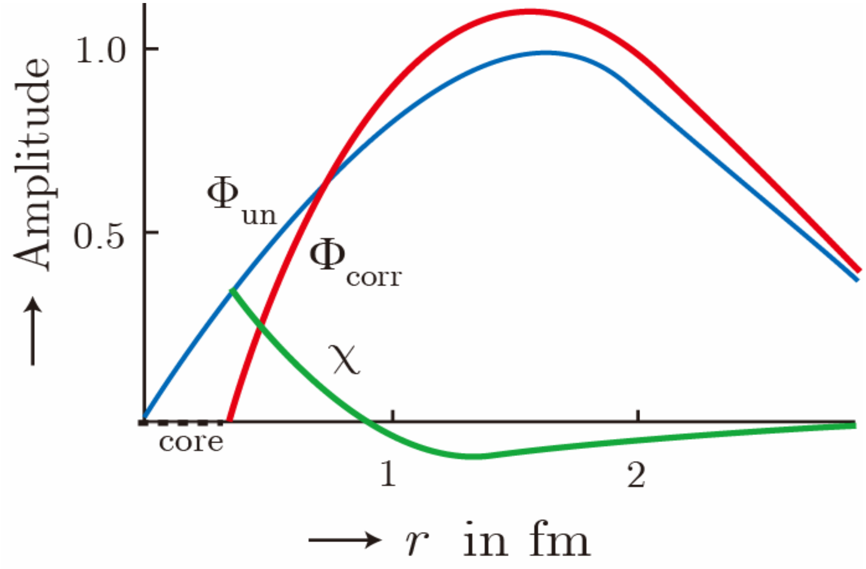
**defect function** $$|\chi\rangle \equiv |\phi\rangle - |\psi\rangle$$

$|\phi\rangle$ : uncorrelated(unperturbed) relative w.f. of 2N system

$|\psi\rangle$ : correlated relative w.f. of 2N system

$$|\psi\rangle = e^{S^{(2)}} |\phi\rangle = \left[1 + S^{(2)} + \tfrac{1}{2} S^{(2)^2} + \cdots\right] |\phi\rangle$$

**Wound integral** $$\rightarrow \langle\phi|\chi\rangle \approx -\tfrac{1}{2}\langle\phi| S^{(2)^2} |\phi\rangle \quad \because \langle\phi| S^{(2)} |\phi\rangle = 0$$



# Removal of Center-of-mass motion effect

Intrinsic Hamiltonian
$$H_{in} = H - \frac{\boldsymbol{P}^2}{2M}$$
$$= \sum_i t_i + \sum_{i<j} v_{ij} - \frac{\boldsymbol{P}^2}{2M},$$

The A-dependent Hamiltonian which is composed of the intrinsic motion and CM motion confined in H.O. pot.

$$H(A) = H_{in} + \left( \frac{\boldsymbol{P}^2}{2M} + \frac{1}{2} M\omega^2 \boldsymbol{R}^2 - \frac{3}{2}\hbar\omega \right)$$
$$= \sum_i t_i + \sum_{i<j} v_{ij} + \frac{1}{2} M\omega^2 \boldsymbol{R}^2 - \frac{3}{2}\hbar\omega$$
$$= \sum_i t_i + \sum_{i<j} \left\{ v_{ij} + \frac{1}{A-1} X_{ij} - \frac{A-2}{A(A-1)} x_{ij} \right\} - \frac{3}{2}\hbar\omega,$$

where

$$X_{ij} \equiv \frac{1}{2}(2m)\omega^2 \boldsymbol{R}_{ij}^2, \left( \boldsymbol{R}_{ij} \equiv \frac{\boldsymbol{r}_i + \boldsymbol{r}_j}{2} \right)$$

$$x_{ij} \equiv \frac{1}{2}\left(\frac{m}{2}\right)\omega^2 \boldsymbol{r}_{ij}^2, \left( \boldsymbol{r}_{ij} \equiv \boldsymbol{r}_i - \boldsymbol{r}_j \right)$$

# Particle-hole transformation of transformed Hamiltonian

$$
\begin{aligned}
\tilde{H} \approx E_0 \\
&+ \sum_{\alpha\beta} \langle \alpha | \tilde{h} | \beta \rangle a_\alpha^\dagger a_\beta - \sum_{\alpha'\beta'} \langle \alpha' | \tilde{h} | \beta' \rangle s'_{\underline{\alpha'}} s'_{\underline{\beta'}} b_{\alpha'}^\dagger b_{\beta'} \\
&+ \sum_{\alpha\beta'} \langle \alpha | \tilde{h} | \beta' \rangle s'_{\underline{\beta'}} a_\alpha^\dagger b_{\beta'}^\dagger + \sum_{\alpha'\beta} \langle \alpha' | \tilde{h} | \beta \rangle s'_{\underline{\alpha'}} b_{\alpha'} a_\beta \\
&+ \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | \tilde{V}_p | \gamma\delta \rangle a_\alpha^\dagger a_\beta^\dagger a_\gamma a_\delta \\
&+ \frac{1}{4} \sum_{\alpha'\beta'\gamma'\delta'} \langle \alpha'\beta' | \tilde{V}_h | \gamma'\delta' \rangle s'_{\underline{\alpha'}} s'_{\underline{\beta'}} s'_{\underline{\gamma'}} s'_{\underline{\delta'}} b_{\alpha'}^\dagger b_{\beta'}^\dagger b_{\gamma'} b_{\delta'} \\
&+ \sum_{\alpha\beta'\gamma\delta'} \langle \alpha\beta' | \tilde{V}_{ph} | \gamma'\delta \rangle s'_{\underline{\beta'}} s'_{\underline{\delta'}} a_\alpha^\dagger b_{\beta'}^\dagger b_{\delta'} a_\gamma + \cdots
\end{aligned}
$$

**Ground-state energy**

$$
\begin{aligned}
E_0 = \sum_{\lambda \le \rho F} \langle \lambda | \tilde{h}_1 | \lambda \rangle - \frac{1}{2!} \sum_{\lambda\mu \le \rho F} \langle \lambda\mu | \tilde{v}_{12} | \lambda\mu \rangle + \frac{1}{3!} \sum_{\lambda\mu\nu \le \rho F} \langle \lambda\mu\nu | \tilde{v}_{123} | \lambda\mu\nu \rangle \\
- \frac{1}{4!} \sum_{\lambda\mu\nu\phi \le \rho F} \langle \lambda\mu\nu\phi | \tilde{v}_{1234} | \lambda\mu\nu\phi \rangle
\end{aligned}
$$

# *Effective* single-particle matrix elements

**3-body cluster terms**

4-body cluster terms

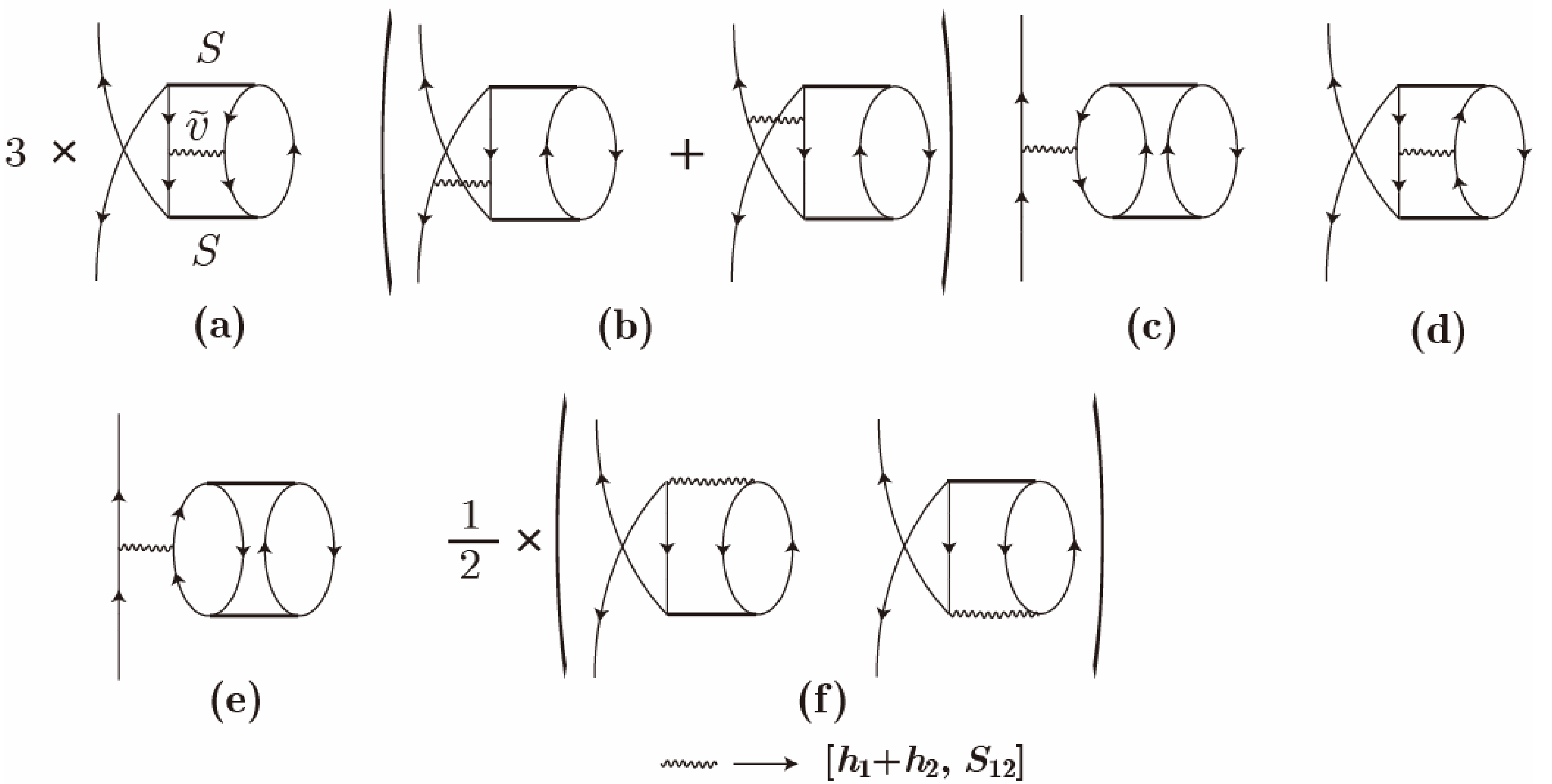
$$\langle\alpha|\tilde{h}|\beta\rangle \equiv \langle\alpha|h|\beta\rangle + \langle\alpha|\tilde{h}^{(3BC)}|\beta\rangle + \langle\alpha|\tilde{h}^{(4BC)}|\beta\rangle,$$

**2-body corr. amplitude**

2-body eff.int.

$$\begin{aligned}
&= \langle\alpha|h|\beta\rangle \\
&\quad + \frac{3}{4}\sum_{\lambda'\mu'\chi'\nu'\varepsilon} \langle\lambda'\mu'|\tilde{v}|\chi'\nu'\rangle\langle\chi'\nu'|S|\beta\varepsilon\rangle\langle\alpha\varepsilon|S|\lambda'\mu'\rangle \\
&\quad - \sum_{\lambda'\mu'\chi'} \Big[\langle\alpha\lambda'|\tilde{v}|\varepsilon\chi'\rangle\langle\mu'\chi'|S|\beta\varphi\rangle\langle\varepsilon\varphi|S|\mu'\lambda'\rangle + (\alpha\leftrightarrow\beta)\Big] \\
&\quad + \frac{1}{2}\sum_{\lambda'\mu'\chi'} \langle\alpha\lambda'|\tilde{v}|\beta\chi'\rangle\langle\mu'\chi'|S|\varepsilon\varphi\rangle\langle\varepsilon\varphi|S|\mu'\lambda'\rangle \\
&\quad - \sum_{\lambda'\mu'\nu'} \langle\lambda'\varphi|\tilde{v}|\nu'\varepsilon\rangle\langle\mu'\nu'|S|\varphi\beta\rangle\langle\varepsilon\alpha|S|\mu'\lambda'\rangle \\
&\quad - \frac{1}{2}\sum_{\lambda'\mu'\varepsilon\pi\varphi} \langle\alpha\varepsilon|\tilde{v}|\beta\pi\rangle\langle\lambda'\mu'|S|\varepsilon\varphi\rangle\langle\pi\varphi|S|\lambda'\mu'\rangle \\
&\quad + \frac{1}{4}\sum_{\lambda'\mu'\varepsilon} \Big[\langle\lambda'\mu'|T|\alpha\varepsilon\rangle\langle\alpha\varepsilon|S|\lambda'\mu'\rangle + (\alpha\leftrightarrow\beta)\Big]
\end{aligned}$$

# Contributions of 3-,4-body cluster terms to s.p. Hamiltonian



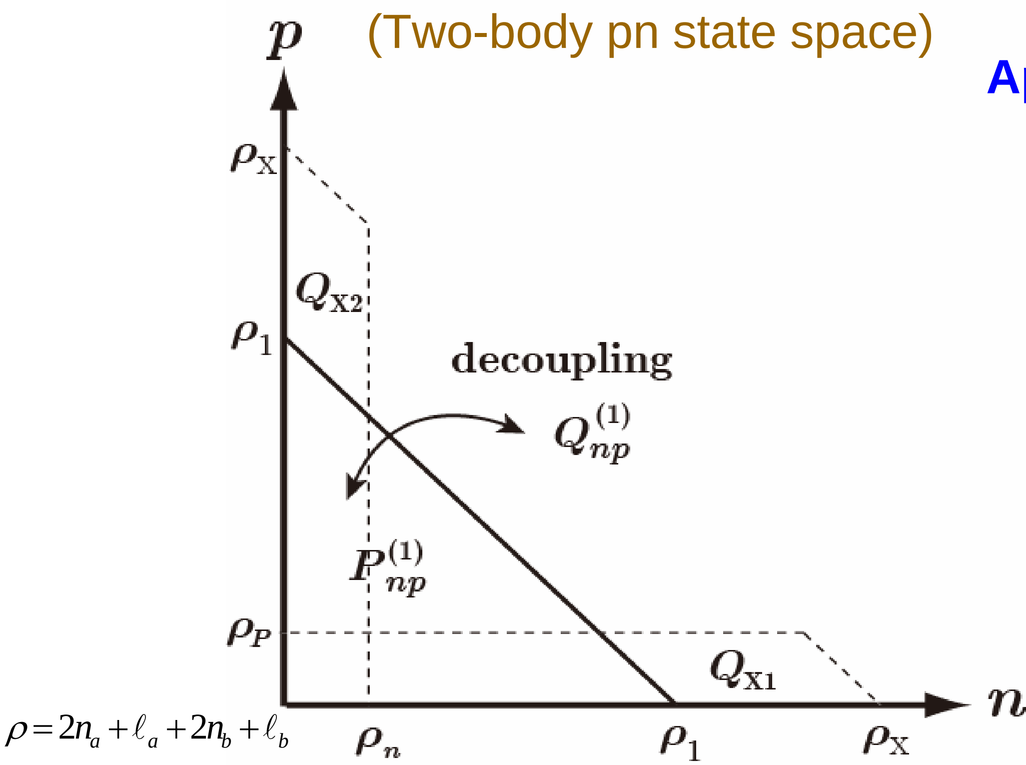
# Two-body interaction matrix element

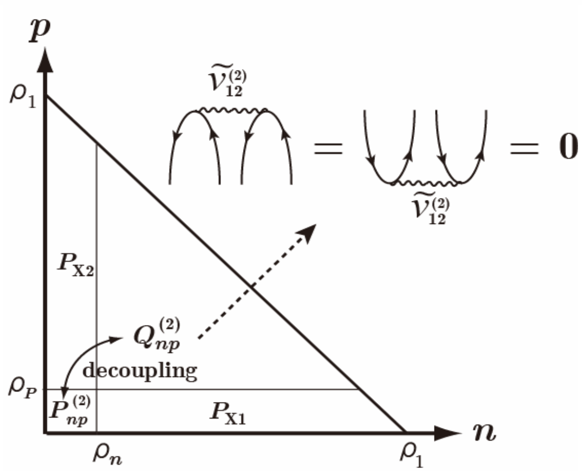
$$\left\langle \alpha\beta \middle| \tilde{V} \middle| \gamma\delta \right\rangle \equiv \left\langle \alpha\beta \middle| \tilde{v}_{12} \middle| \gamma\delta \right\rangle + \frac{1}{2!} \sum_{\lambda \le \rho_F} \left\langle \alpha\beta\lambda \middle| \tilde{v}_{123}^{(2NF)} \middle| \gamma\delta\lambda \right\rangle$$

Effective two-body int.

Effective three-body int.
induced by the two-body
correlations

# Two-step determination of the effective interactions




**Approximate decoupling** ;

1) average over $j_a, L_{\mathrm{CM}}$

 in two-body relative-CM system

2) diagonal in CM q.n.

3) $u_1$= 0 for Q $^{(1)}$ space

$$\widetilde{H}^{(\mathrm{I})} \equiv e^{-S^{(\mathrm{I})}} H e^{S^{(\mathrm{I})}}$$

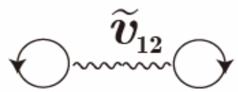
$$\widetilde{H}^{(\mathrm{II})} \equiv e^{-S^{(\mathrm{II})}} \widetilde{H}^{(\mathrm{I})} e^{S^{(\mathrm{II})}}$$

**Exact decoupling** in shell-model basis

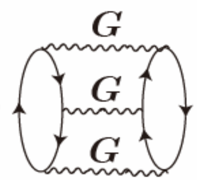
in two-body space

# Diagrammatic expression of the ground-state energy in terms of G-matrix

$$\Delta E_0^{(2BCT)} \equiv \frac{1}{2}\sum_{a'b' \leq \rho_F} \langle a'b' | \tilde{v}_{12} | a'b' \rangle$$

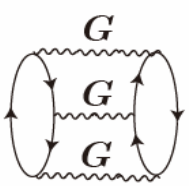
$=$ 

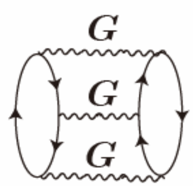

$\simeq$ 

$+ \ (-)$ 

$+ \begin{pmatrix} \text{higher-order} \\ \text{terms} \end{pmatrix}$

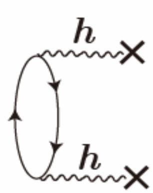
$$\Delta E_0^{(3BCT)} \equiv \text{three-body-cluster terms} \approx \langle \tilde{v} \rangle \langle S \rangle^2$$

$\simeq 2 \cdot$ 

$+$ 

$+ \begin{pmatrix} \text{higher-order} \\ \text{terms} \end{pmatrix}$

$$\begin{pmatrix} \tilde{v} = & G + \text{higher-order terms} \\ S \approx & \dfrac{G}{e} + \text{higher-order terms} \end{pmatrix}$$

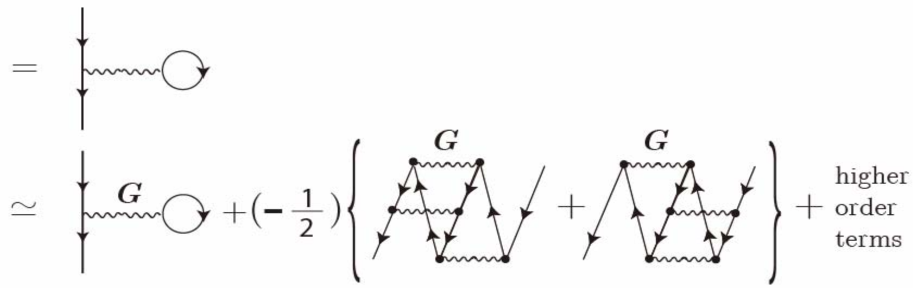
$\Delta E_0^{(h)} \equiv$ 


$$h \equiv t_1 + u_1 \longleftarrow \langle \tilde{v}_{12} \rangle$$

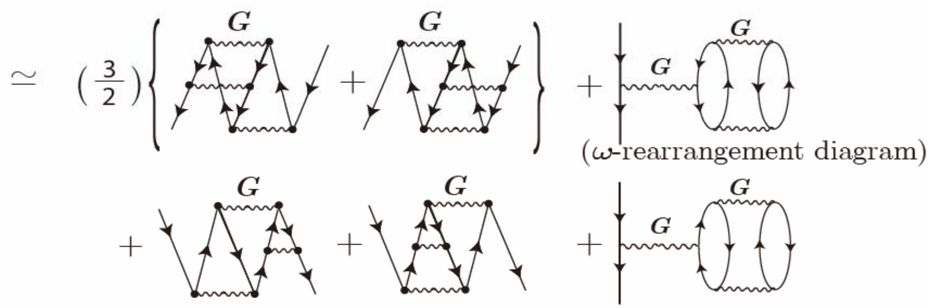
$$\langle \alpha | u_1 | \beta \rangle = \sum_{\lambda' \leq \rho_F} \langle \alpha\lambda' | \tilde{v}_{12} | \beta\lambda' \rangle$$

# Diagrammatic expression of the single hole energies in terms of G-matrix

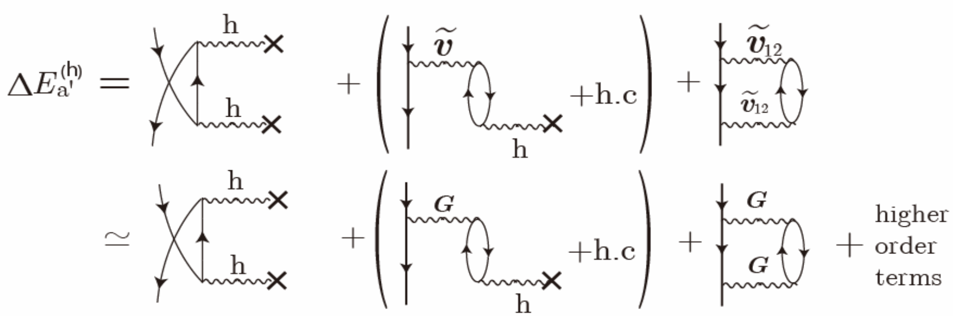
$$\Delta E^{(2BCT)}_{a'} \equiv \sum_{\lambda' \leq \rho_F} \langle a'\lambda' | \tilde{v}_{12} | a'\lambda' \rangle \equiv \langle a' | u_1 | a' \rangle$$



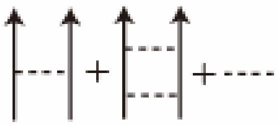
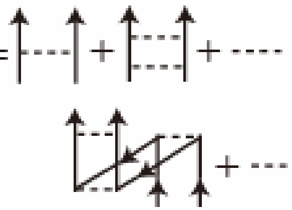

$$\Delta E^{(3BCT)}_{a'} \equiv \text{three-body-cluster terms} \approx \langle \tilde{v} \rangle \langle S \rangle^2$$




+ higher order terms

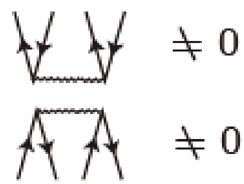
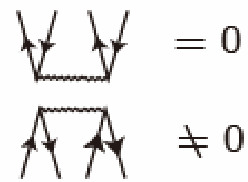
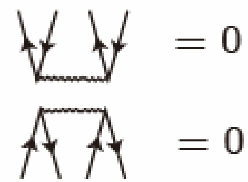
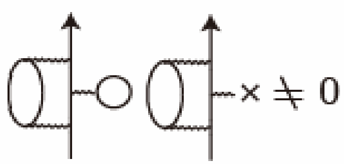
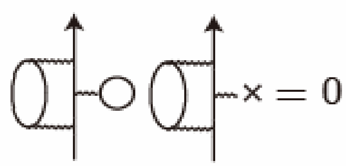
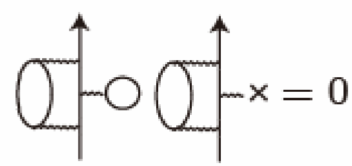
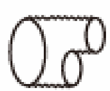


# Comparison of G-matrix, CCM and UMOA (1)

| | $G$-matrix theory | CCM | UMOA |
|---|---|---|---|
| Basic Element | $G =$  =Sum of Ladders | $\tilde{V}_{12}=$  $=G+G_1G+G_2GG+\cdots$ <br> $=G+$Folded diagrams | $W_{12}=G+1/2(G_1G+GG_1)$ <br> $+\cdots\cdots$ <br> =Herimitian counter part of $\tilde{V}_{12}$. |
| Hermitisity | Non-Hermitian (Hermitian if the starting energies are all degenerate) | Non-Herimitian | Herimitian |
| $E$-dependence | $E$-dependent | $E$-independent | $E$-independent |

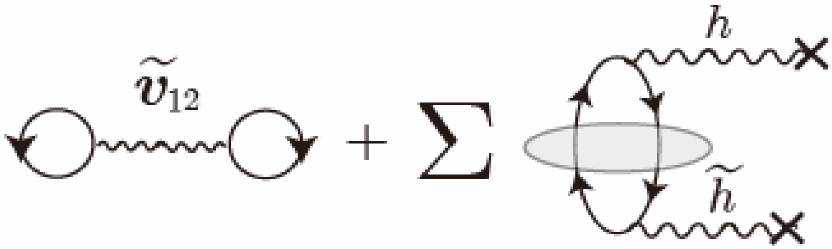
$$G_n \equiv \frac{1}{n!}\frac{d^nG(\varepsilon)}{d\varepsilon^n}$$

# Comparison of G-matrix, CCM and  UMOA (2)

| | $G$-matrix theory | CCM | UMOA |
|---|---|---|---|
| Decoupling Property | Non-decoupled  | Half-decoupled  | Decoupled  |
| Self-consistency | Generally impossible  | Possible  | Possible  |
| Ground-state energy (Potential energy of a Closed-Shell Core) | $E_0=$  $+$  $+ \cdots$ | $E_0=$  (No other contributions) | $E_0=$  +(Contributions from three-or-more -body Cluster trems) |

# Calculation of the ground-state energy

$$E_0 = E_0^{(0)} + \Delta E^{(ph)} + \Delta E^{(3\mathrm{BC})}$$

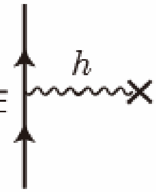
$=$  $+ \Delta E^{(3\mathrm{BC})}$

# Calculation of the "single-particle energies'

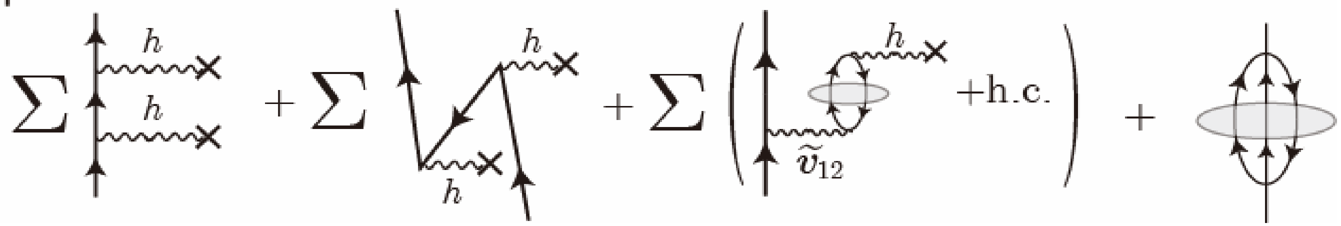
$$\varepsilon_{\mathrm{a}} = \varepsilon_{\mathrm{a}}^{(0)} + \Delta\varepsilon_{\mathrm{a}} + (\text{Folded diagrams}) + \Delta\varepsilon_{\mathrm{a}}^{(3\mathrm{BC})}$$

**Particle energies**

$\varepsilon_{\mathrm{a}}^{(0)} \equiv$



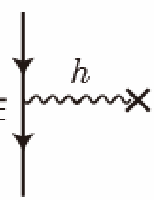

$\Delta\varepsilon_{\mathrm{a}} \equiv$



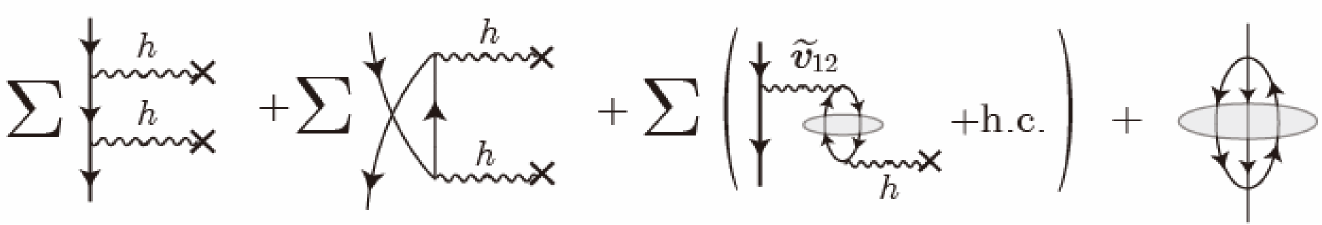

$$\varepsilon_{\mathrm{a'}} = \varepsilon_{\mathrm{a'}}^{(0)} + \Delta\varepsilon_{\mathrm{a'}} + (\text{Folded diagrams}) + \Delta\varepsilon_{\mathrm{a'}}^{(3\mathrm{BC})}$$

**Hole energies**

$\varepsilon_{\mathrm{a'}}^{(0)} \equiv$




$\Delta\varepsilon_{\mathrm{a'}} \equiv$



Results of calculations in isospin basis
with *old-generation* NN interactions
(Reid, Paris, SSC, Bonn-A, -B, -C)

1986-1994

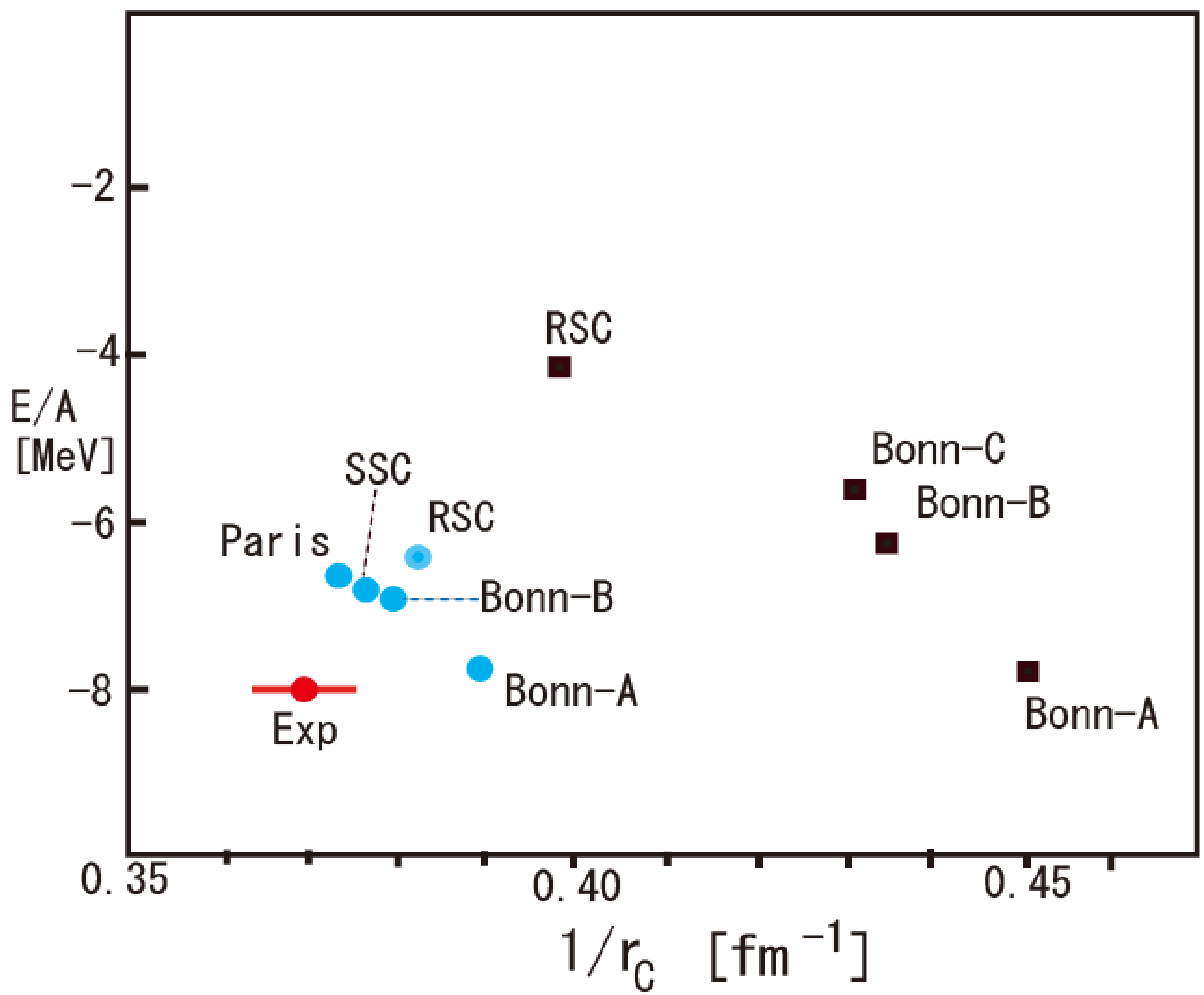
NN int. for np (T=0)
pp,np,nn (T=1)
(1/3) Coulomb int. for pp,np,nn

$^{16}$O; binding energy
charge radius,
*absolute* single-particle energies
spin-orbit splittings

$^{40}$Ca; binding energy
charge radius,
*absolute* single-particle energies
spin-orbit splittings

($^{4}$He); binding energy -- *unpublished* --

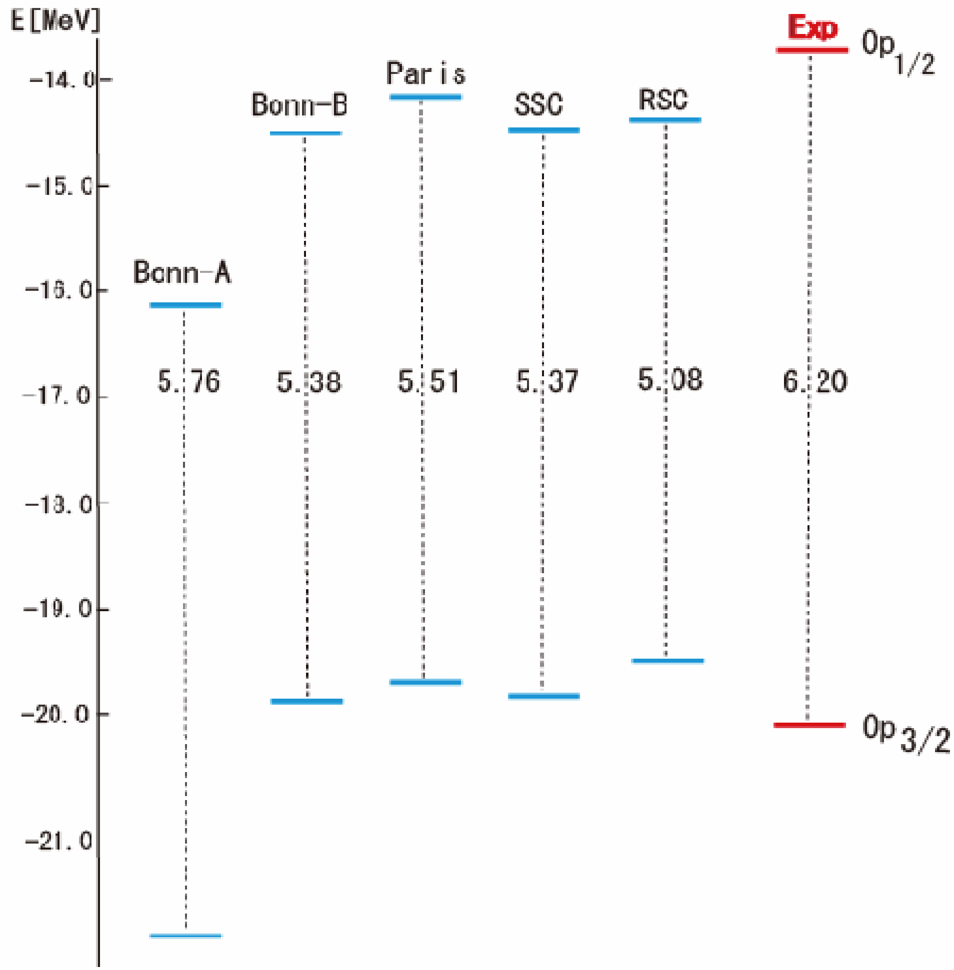
# Binding Energy vs Inverse Charge Radius in $^{16}O$




● UMOA Suzuki, Okamoto, Kumagai, Phys. Rev C36(1987), 804

■ BHF Schmid, Muether, Machleidt, Nucl. Phys. A530(1991), 14

# Single-Particle Energies in $^{16}O$

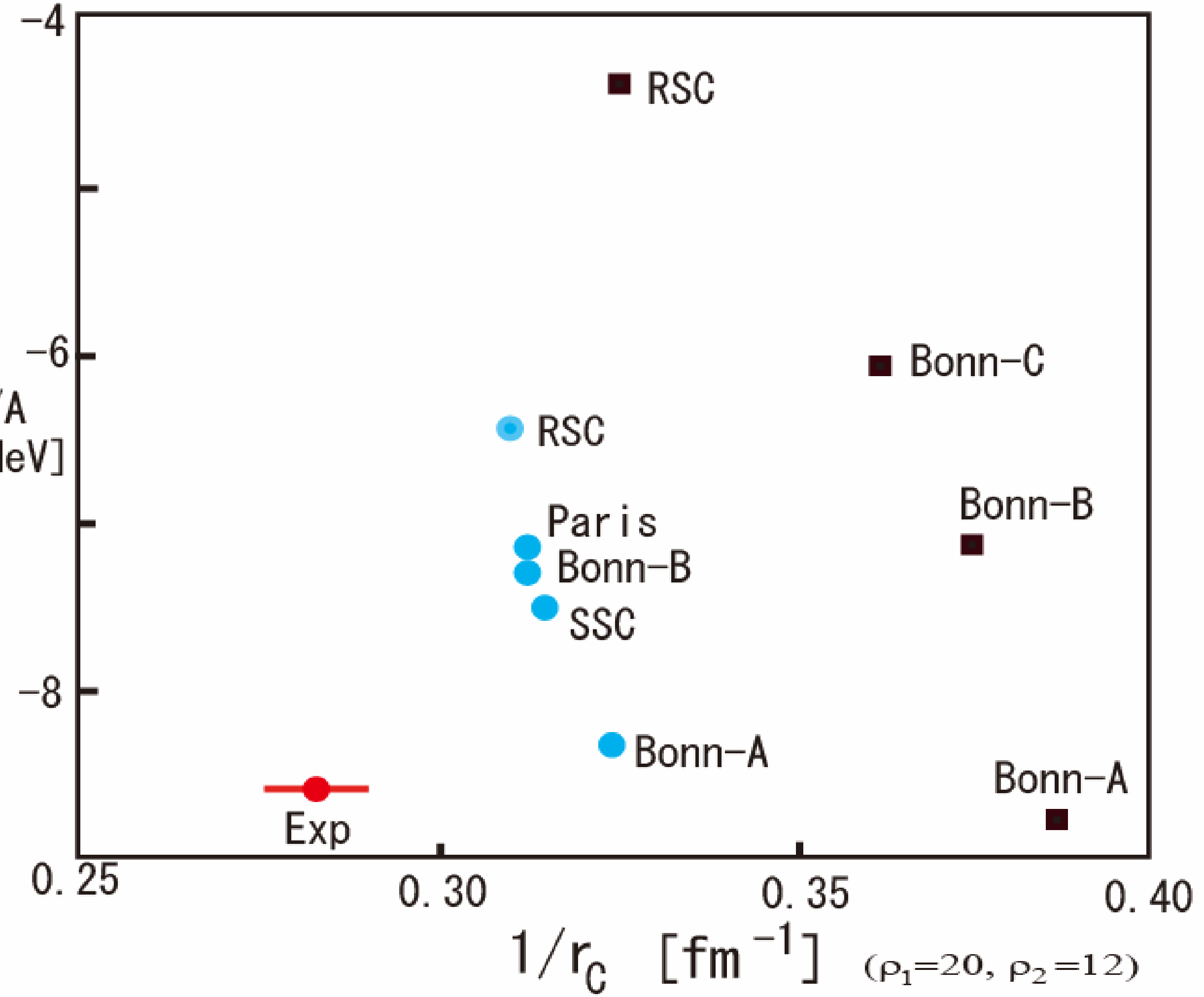



Suzuki, Okamoto, Prog. Theor. Phys. 92 (1994), 1045

references-therein

UMOA calculation isospin basis

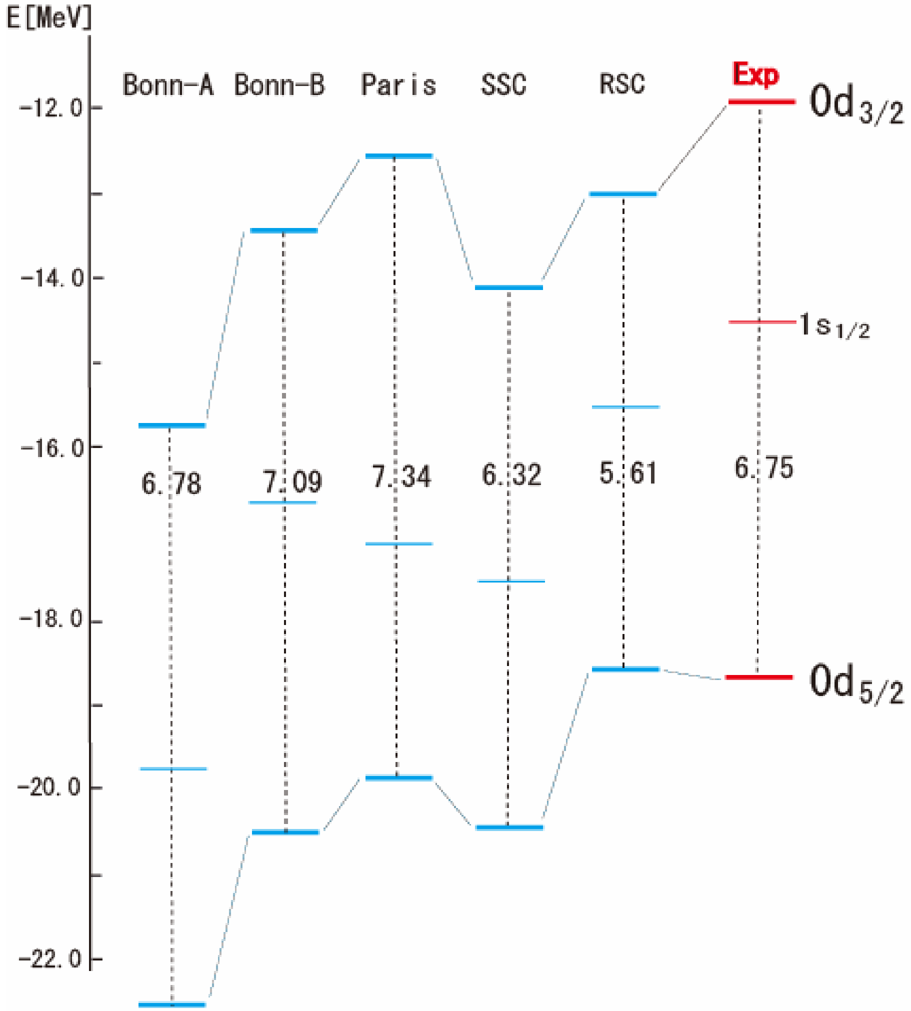
# Binding Energy vs Inverse Charge Radius in $^{40}Ca$




● UMOA Kumagai, Suzuki, Okamoto, Prog. Theor. Phys. 97(1997), 1023

■ BHF Schmid, Muether, Machleidt, Nucl. Phys. A530(1991), 14

# Single-Particle Energies in $^{40}$Ca




Suzuki, Kumagai, Okamoto, Prog. Theor. Phys. 97(1997), 102

UMOA calculation isospin basis

# Calculation of $^{4}$He in UMOA (isospin basis)
## -*unpublished* (1986)-

Ground-state energy

**-28.25 [ MeV ] ( with Reid Soft-Core NN potential)**

**-27.13 [ MeV ] ( with Paris NN potential)**

**-28.12 [ MeV ] ( with Super Soft-Core NN potential)**

$\rho_1 = 20, \rho_2 = 10, \rho_3 = 0, \ell_{\max} = 4, \mathrm{Max1{=}20}, \hbar\omega = 20 \text{ MeV}$

With coulomb interaction,

without spurious CM motion effect

**-28.30 [ MeV ] ( Experiment)**

# Effects of the three-body cluster terms

**Sizable contribution to the ground-state energy**

**Convergence of cluster expansion**

In theory, unitarily transformation does not terminate in its expansion series.

In the actual calculations

$$E_0 = E_0^{(2BC)} + \Delta E_0^{(3BC)} + \cdots;$$

$$\rightarrow \left| \frac{\Delta E_0^{(3BC)}}{E_0^{(2BC)}} \right| \times 100 \approx 3\,(\%)$$

*almost converges*

**Reducing of the dependence of the calculated results on s.p. H.O. basis**

**A significant effects  in reproducing the correct nuclear size**

K.Suzuki, R. Okamoto, Prog. Theor. Phys.76(1986),127-142.

## Results of calculation in particle basis with high precision NN interactions

(CD-Bonn, Nijmegen-I, Nijmegen93, N3LO)

NN int. for np,pp,nn,

Coulomb int. for pp

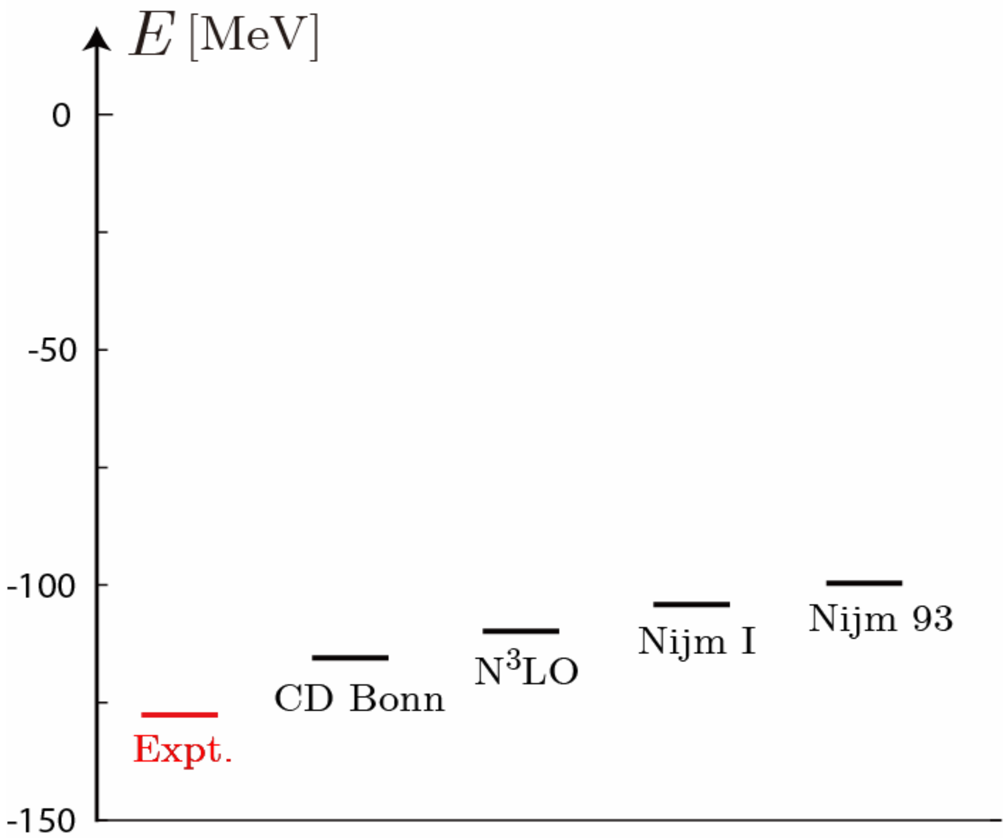
$^{16}$O, $^{15}$O, $^{15}$N, $^{17}$O, $^{17}$F

$^{22,23,24,25}$O

($^{40}$Ca) ;almost converging results

($^{4}$He, $^{3}$He, $^{3}$H);preliminary

# Calculation of $^{16}$O in particle basis

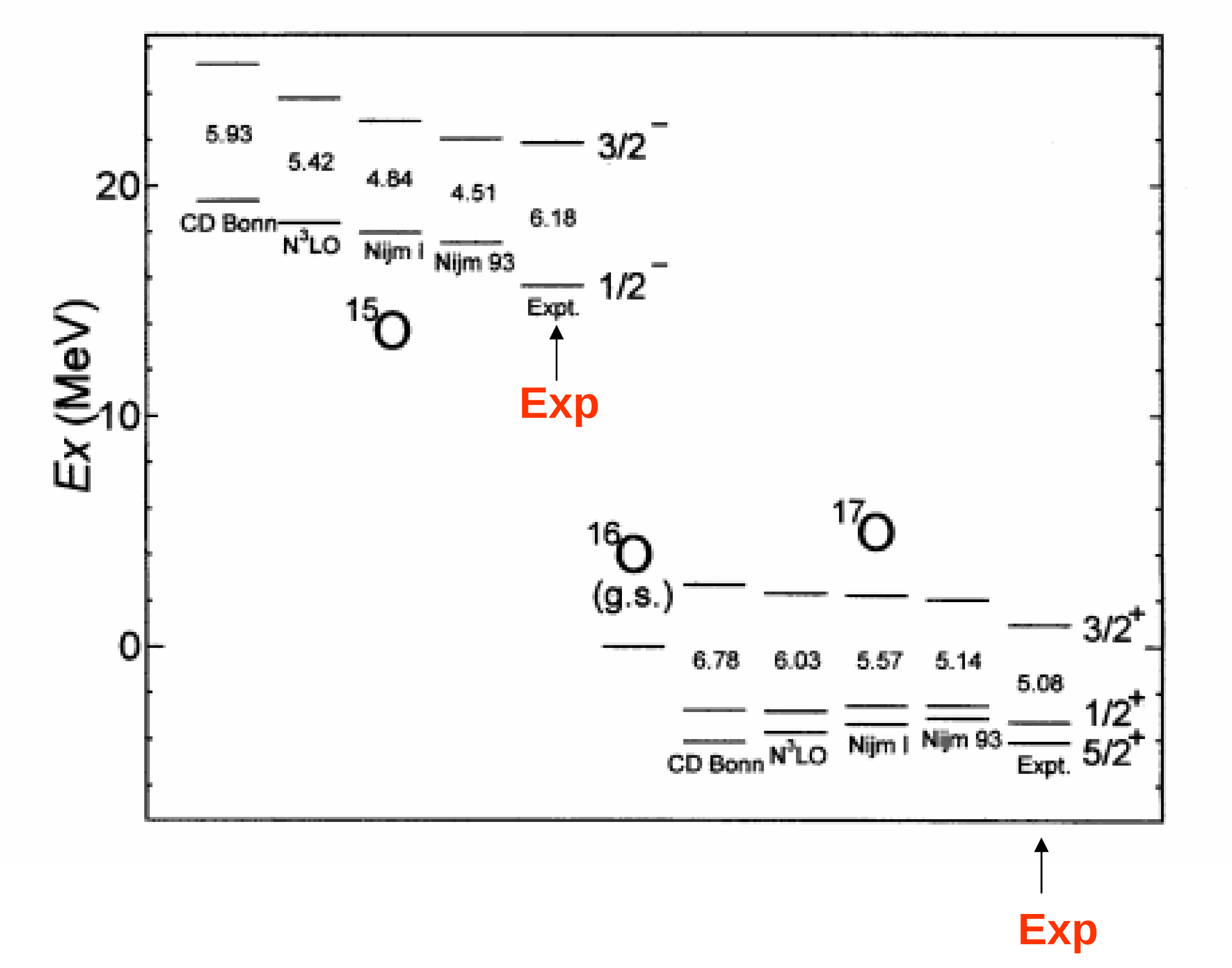



**-127.62 -115.61 -110.00 -104.25 -99.69**

**(9% missing!)**

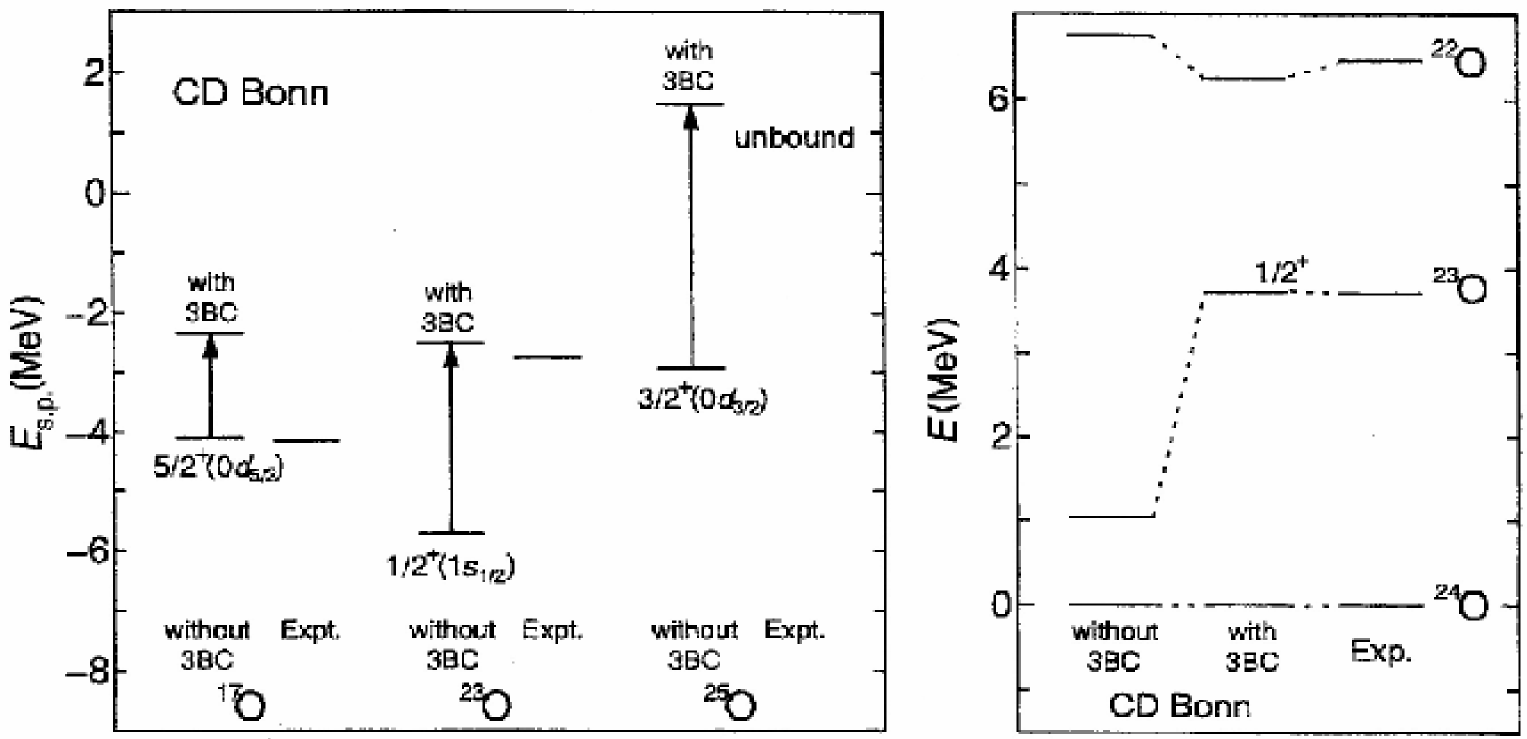
S. Fujii, R. Okamoto, K. Suzuki, Phys.Rev.**C69**(2004),034328

# Results of $^{15,17}O$



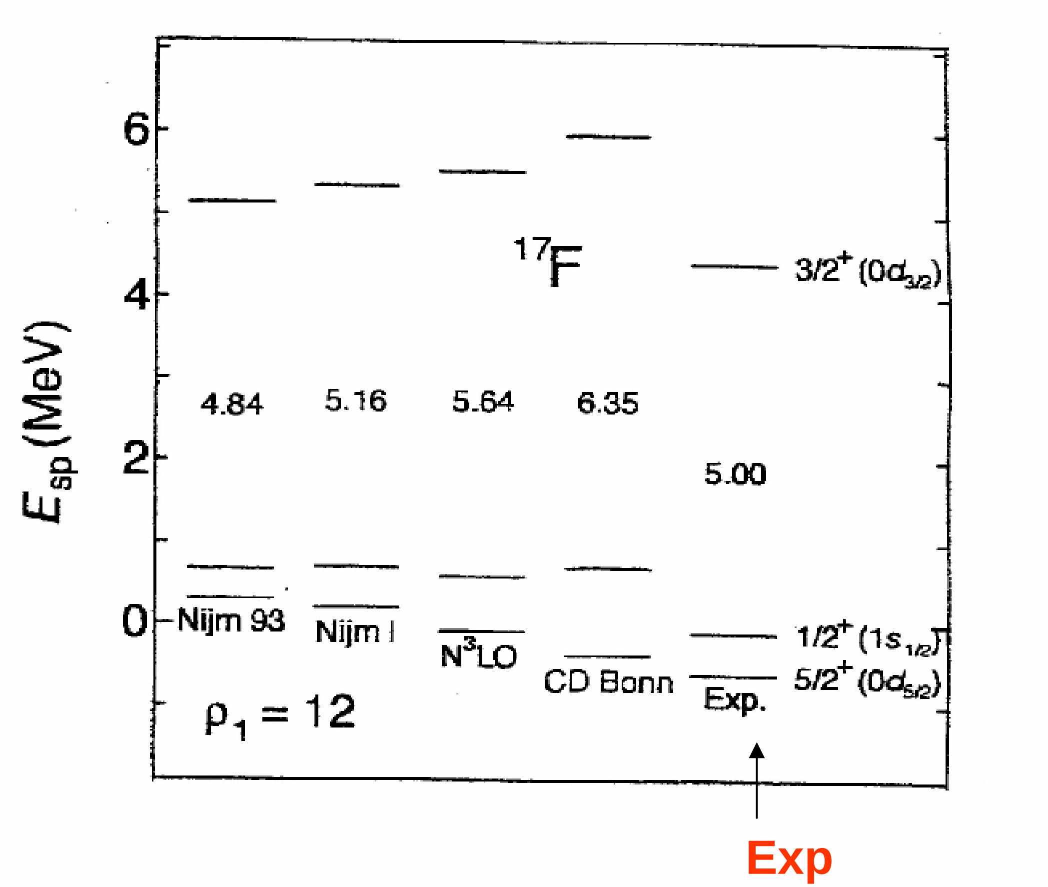

S. Fujii, R. Okamoto, K. Suzuki,
J. Phys. Conference Series , 20 (2005), 83-88

# Calculations of neutron-rich O isotopes



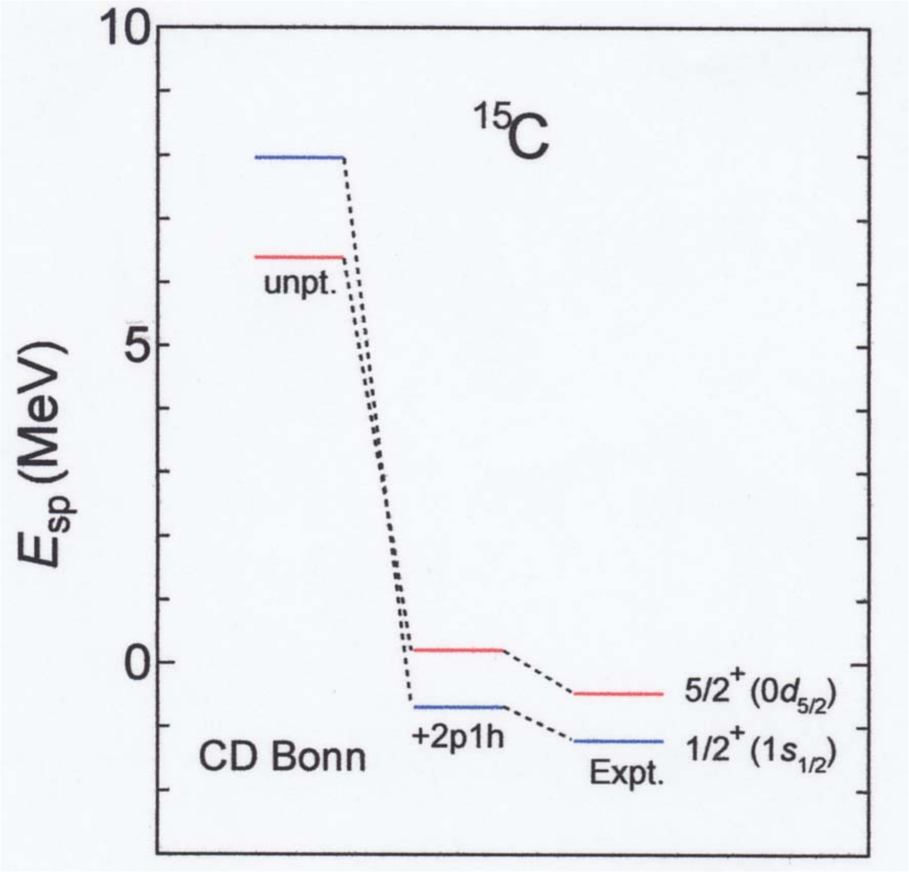

S. Fujii, R. Okamoto, K. Suzuki,
Proceedings of the International Symposium on A New Era of Nuclear Structure Physics (NENS03),Niigata, November 2003 (World Scientific, Singapore, 2004, pp.~70-74)

# Calculation of $^{17}F$



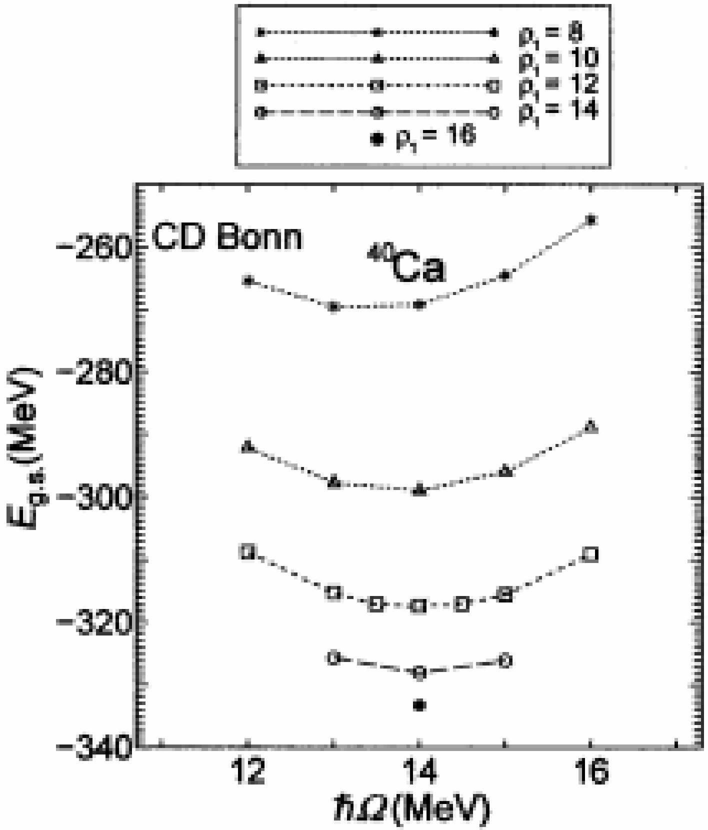

S. Fujii, R. Okamoto, K. Suzuki, PRC**69**(2004),034328

# Calculation of $^{15}C$




S. Fujii, R. Okamoto, K. Suzuki,
J. Phys. Conference Series , 20 (2005), 83-88

# Results of $^{40}Ca$




Very weak dependence on the H.O. size
adopted s.p.basis

Almost converges

S. Fujii, R. Okamoto, K. Suzuki,
J. Phys. Conference Series , 20 (2005), 83-88

# Results of $^4$He (in particle basis)

UMOA calculation with CD-Bon NN potential

$$E_0^{(unperturbed)} = -23.23 \text{ MeV}$$

$$E_0^{(unperturbed)} + \Delta E_0^{(\text{1p-1h})} = -26.15 \text{ MeV}$$

$$E_0^{(unperturbed)} + \Delta E_0^{(\text{1p-1h})} + \Delta E_0^{(3BC)} = -27.72 \text{ MeV}$$

$$\frac{\Delta E_0^{(3BC)}}{E_0^{(unperturbed)}} \times 100 = 0.032 \, [\%]$$

A bit worse cluster expansion in contrast to $^{16}$O case

$$\frac{\Delta E_0^{(3BC)}}{E_0^{(unperturbed)}} \times 100 = \begin{cases} 1.3 \, [\%] \text{ in } {}^{16}\text{O for Paris NN pot.} \\ 1.2 \, [\%] \text{ in } {}^{16}\text{O for Reid NN pot.} \\ 1.4 \, [\%] \text{ in } {}^{16}\text{O for SSC NN pot.} \end{cases}$$

**Faddeev-Yakubovski calculation with CD-Bon NN potential**

$$E_0 = -27.74 \text{ MeV (s-wave approximation)}$$

S. Fujii, et al., PRC70, 024003 (2004)

# Some challenges

1) incorporation of 3NF (← Chiral nuclear interaction)

2) microscopic effective interactions for weakly bound nuclei;
a generalization of effective interactions
from real to ***complex* effective interactions**
**G. Hagen, M. Hjorth-Jensen, J.S. Vagen**
***Effective interaction techniques for Gamow shell model***
**PRC71,044314(2006)**.

3) the structure of other effective operators, such as the radius, the quadrupole moment, etc.

4) Description of excited states; cf. CCM-eq-of-motion-method

# Formulation of UMOA with 2NF and 3NF

K. Suzuki, Prog. Theor. Phys.79 (1998), 330.

Hamiltonian with 2NF and 3NF

$$H = \sum_i t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk} = \sum_i (t_i + u_i) + \left[ \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk} - \sum_i u_i \right]$$

$$= \sum_i h_i + \left[ \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk} - \sum_i u_i \right]; \quad h_i \equiv t_i + u_i$$

Genuine three-nucleon interaction(3NF)

$\left(V_{ij}, V_{ijk}\right) \Leftarrow$ Chiral nuclear interaction

Chiral perturbation theory

**Three-body sub-system Hamiltonian**

$$H_{123} \equiv (h_1 + h_2 + h_3) + (v_{12} + v_{23} + v_{31}) + v_{123},$$

**Three-body sub-system Hamiltonian *dressed with two-body correlations***

**in an entire many-body system**

$$\boxed{\begin{aligned} \widetilde{H}_{123} &\equiv e^{-S_{123}^{(2)}} H_{123}\, e^{S_{123}^{(2)}} \\ &= (h_1 + h_2 + h_3) + (\tilde{v}_{12} + \tilde{v}_{23} + \tilde{v}_{31}) + \tilde{v}_{123}^{(2)}, \end{aligned}}$$

$$\begin{aligned} \tilde{v}_{123}^{(2)} \equiv\ & e^{-S_{123}^{(2)}} (h_1 + h_2 + h_3 + v_{12} + v_{23} + v_{31} + v_{123}) e^{S_{123}^{(2)}} \\ & - (h_1 + h_2 + h_3 + \tilde{v}_{12} + \tilde{v}_{23} + \tilde{v}_{31}); \end{aligned}$$

$$\Rightarrow \tilde{v}_{123}^{(2)} = \tilde{v}_{123}^{(2NF)} + e^{-S_{123}^{(2)}} v_{123} e^{S_{123}^{(2)}}; \quad S_{123}^{(2)} \equiv S_{12} + S_{23} + S_{31}$$

$$\left[ \tilde{v}_{123}^{(2)} \to v_{123} \text{ as } S_{123}^{(2)} \to 0 \right]$$

three-body interaction
*induced by*
*the two-body correlations*

three-body interaction
*dressed with the two-body correlations*

# Calculation of three-body correlation operator

Projection operators in three-body state space

$$P^{(3)} + Q^{(3)} = 1,\ P^{(3)^2} = P^{(3)}, Q^{(3)^2} = Q^{(3)}, P^{(3)}Q^{(3)} = Q^{(3)}P^{(3)} = 0$$

Solution for the three-body subsystem Hamiltonian

$$\boxed{\widetilde{H}_{123} \left| \psi_k^{(3)} \right\rangle = E_k^{(3)} \left| \psi_k^{(3)} \right\rangle}, \quad (k = 1, 2, \cdots, d^{(3)}, d^{(3)} + 1, \cdots, n^{(3)})$$

$$\left| \psi_k^{(3)} \right\rangle = \left( P^{(3)} + Q^{(3)} \right) \left| \psi_k^{(3)} \right\rangle = \left| \phi_k^{(3)} \right\rangle + \omega^{(3)} \left| \phi_k^{(3)} \right\rangle, (k = 1, 2, \cdots, d^{(3)})$$

$$\left| \phi_k^{(3)} \right\rangle \equiv P^{(3)} \left| \psi_k^{(3)} \right\rangle, Q^{(3)} \left| \psi_k^{(3)} \right\rangle = \omega^{(3)} \left| \phi_k^{(3)} \right\rangle$$

$$\left\langle \psi_k^{(3)} \mid \psi_{k'}^{(3)} \right\rangle = \delta_{kk'}$$

General solution for the wave operator

$$\omega^{(3)} = \sum_{k=1}^{d^{(3)}} Q^{(3)} \left| \psi_k^{(3)} \right\rangle \left\langle \tilde{\phi}_k^{(3)} \right| P^{(3)} \quad \because \left\langle \tilde{\phi}_k^{(3)} \middle| \phi_{k'}^{(3)} \right\rangle = \delta_{kk'}, \left| \tilde{\phi}_k^{(3)} \right\rangle : \text{bi-orthogonal state}$$

Relation between mapping operator and correlation operator

$$S^{(3)} = \text{arctanh}(\omega^{(3)} - \omega^{(3)^\dagger})$$

## Transformed Hamiltonian in terms of three-body correlations

$$\widetilde{\widetilde{H}} \equiv e^{-S^{(3)}} \widetilde{H} e^{S^{(3)}} = e^{-S^{(3)}} [e^{-S^{(2)}} H e^{S^{(2)}}] e^{S^{(3)}}$$

Anti-hermitian *three-body* correlation operator

$$S^{(3)} = \sum_{i<j<k} S_{ijk} , [S^{(3)\dagger} = -S^{(3)}]$$

Second quantization form

$$S^{(3)} = \left(\frac{1}{3!}\right)^2 \sum_{\alpha\beta\gamma\lambda\mu\nu} \langle \alpha\beta\gamma | S_{123} | \lambda\mu\nu \rangle c_\alpha^\dagger c_\beta^\dagger c_\gamma^\dagger c_\nu c_\mu c_\lambda$$

# Decoupling equation for transformed Hamiltonian of three-body subsystem

$$\boxed{Q^{(3)} \cdot e^{-S_{123}} \widetilde{H}_{123} \, e^{S_{123}} \cdot P^{(3)} = 0}$$

$$\rightarrow Q^{(3)} \cdot e^{-S_{123}} \left[ (h_1 + h_2 + h_3) + (\tilde{v}_{12} + \tilde{v}_{23} + \tilde{v}_{31}) + \tilde{v}^{(2)}_{123} \right] e^{S_{123}} \cdot P^{(3)} = 0$$

If $\quad Q^{(3)}(h_1 + h_2 + h_3)P^{(3)} = Q^{(3)}(\tilde{v}_{12} + \tilde{v}_{23} + \tilde{v}_{31})P^{(3)} = 0,$

$$Q^{(3)} \tilde{v}^{(2)}_{123} \, P^{(3)} = 0$$

# Effective three-body interaction

***Due to 2NF and 3NF***

$$\tilde{v}_{123} \equiv e^{-S_{123}} \left[ e^{-S_{123}^{(2)}} (h_1 + h_2 + h_3 + v_{12} + v_{23} + v_{31} + v_{123}) e^{S_{123}^{(2)}} \right] e^{S_{123}}$$

$$- (h_1 + h_2 + h_3 + \tilde{v}_{12} + \tilde{v}_{23} + \tilde{v}_{31})$$

$$= e^{-S_{123}} \left[ \tilde{v}_{123}^{(2)} + (h_1 + h_2 + h_3 + \tilde{v}_{12} + \tilde{v}_{23} + \tilde{v}_{31}) \right] e^{S_{123}}$$

$$- (h_1 + h_2 + h_3 + \tilde{v}_{12} + \tilde{v}_{23} + \tilde{V}_{31})$$

$$\boxed{\therefore \tilde{v}_{123} = e^{-S_{123}} \left[ \left\{ \tilde{v}_{123}^{(2NF)} + e^{-S_{123}^{(2)}} v_{123} e^{S_{123}^{(2)}} \right\} + (h_1 + h_2 + h_3 + \tilde{v}_{12} + \tilde{v}_{23} + \tilde{v}_{31}) \right] e^{S_{123}} - (h_1 + h_2 + h_3 + \tilde{v}_{12} + \tilde{v}_{23} + \tilde{v}_{31}),}$$

**three-body interaction** ***induced by the two-body correlations***

$$\left[ \tilde{v}_{123} \to \tilde{v}_{123}^{(2NF)} + e^{-S_{123}^{(2)}} v_{123} e^{S_{123}^{(2)}} \quad \text{as } S_{123} \to 0 \right]$$

**three-body interaction** ***dressed with the two-body correlations***

# Two-body interaction matrix element due to 2NF and 3NF

**2NF** 3NF

$$\left\{ \tilde{V}_{123}^{(2NF)} + e^{-S_{123}^{(2)}} v_{123} e^{S_{123}^{(2)}} \right\}$$

$$\langle \alpha\beta | \tilde{V} | \gamma\delta \rangle \equiv \langle \alpha\beta | \tilde{v}_{12} | \gamma\delta \rangle + \frac{1}{2!}\sum_{\lambda\le\rho_F} \langle \alpha\beta\lambda | \tilde{v}_{123} | \gamma\delta\lambda\rangle$$

???

$\tilde{V}_{123}^{(2NF)}$ **A significant effects in reproducing the correct nuclear size ( in $^{16}$O )**

***Microscopic origin*** **of**

**Effective NN interaction with density dependence**

# Short summary

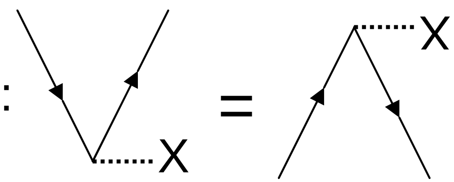
$$Q^{(1)}h_1\ P^{(1)}=0;\ \mathrm{HF{=}UMOA\text{-}}S^{(1)}\Leftrightarrow \mathrm{CCM\text{-}S}$$

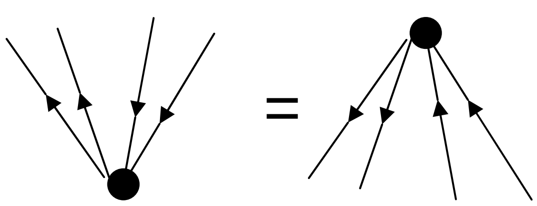
$$Q^{(2)}\tilde{v}_{12}\ P^{(2)}=0;\ \mathrm{UMOA\text{-}}S^{(1)}S^{(2)}\Leftrightarrow \mathrm{CCM\text{-}SD}$$

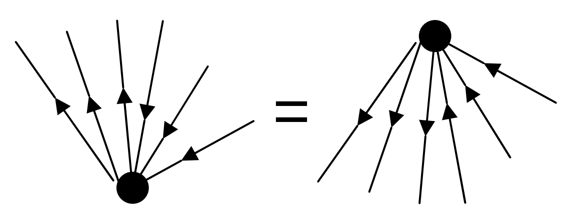
$$Q^{(3)}\tilde{v}_{123}\ P^{(3)}=0;\ \mathrm{UMOA\text{-}}S^{(1)}S^{(2)}S^{(3)}\Leftrightarrow \mathrm{CCM\text{-}SDT}$$

unitary

Non-unitary

HF:  = 0

 = 0

 = 0

**Overall, the nature of the ‘real’ and ‘effective’ three-body forces  remains quite complicated and elusive.**

Fayache, Vary, Barrett, Navratil, Aroua,

*An initio No-Core Shell Model with Many-Body Forces*

nucl-th/0112066

**Not always so.**

**It might be possible to formulate it rather transparently.**

# Summary

- UMOA as an application of effective interaction theory
  a *unitary* coupled cluster method
  an algebraic approach
  with systematic approximation schemes

- UMOA has been proved to be
  applicable to He, O, Ca isotopes
  with high-precision NN interactions

- Challenges;
  incorporation of 3NF
  microscopic effective interactions for weakly bound nuclei
  a generalization of effective interactions
  from real to *complex* effective interactions