Study of the Nickel chain neutron dripline with the spherical Gamow HFB formalism

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Plan

- Gamow states
- Complex scaling method
- Completeness relations
- Gamow HFB definition, quasi-particle states and space
- Densities with Gamow HFB
- Numerical treatment of Gamow HFB
- Description of Nickel calculations
- Nickel densities and pairing densities
- Conclusion and perspectives

Gamow states

- Georg Gamow : α decay

  G.A. Gamow, Zs f. Phys. **51** (1928) 204; **52** (1928) 510

- Definition :

$$u''(r) = \left[\frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2}V(r) - k^2\right] u(r)$$

$$u(r) \sim C_0 \, r^{l+1} \ , \ r \rightarrow 0$$

$$u(r) \sim C_+ \, H^+_{l,\eta}(kr) \ , \ r \rightarrow +\infty \ \text{(bound,resonant)}$$

$$u(r) \sim C_+ \, H^+_{l,\eta}(kr) + C_- \, H^-_{l,\eta}(kr) \ , \ r \rightarrow +\infty \ \text{(scattering)}$$

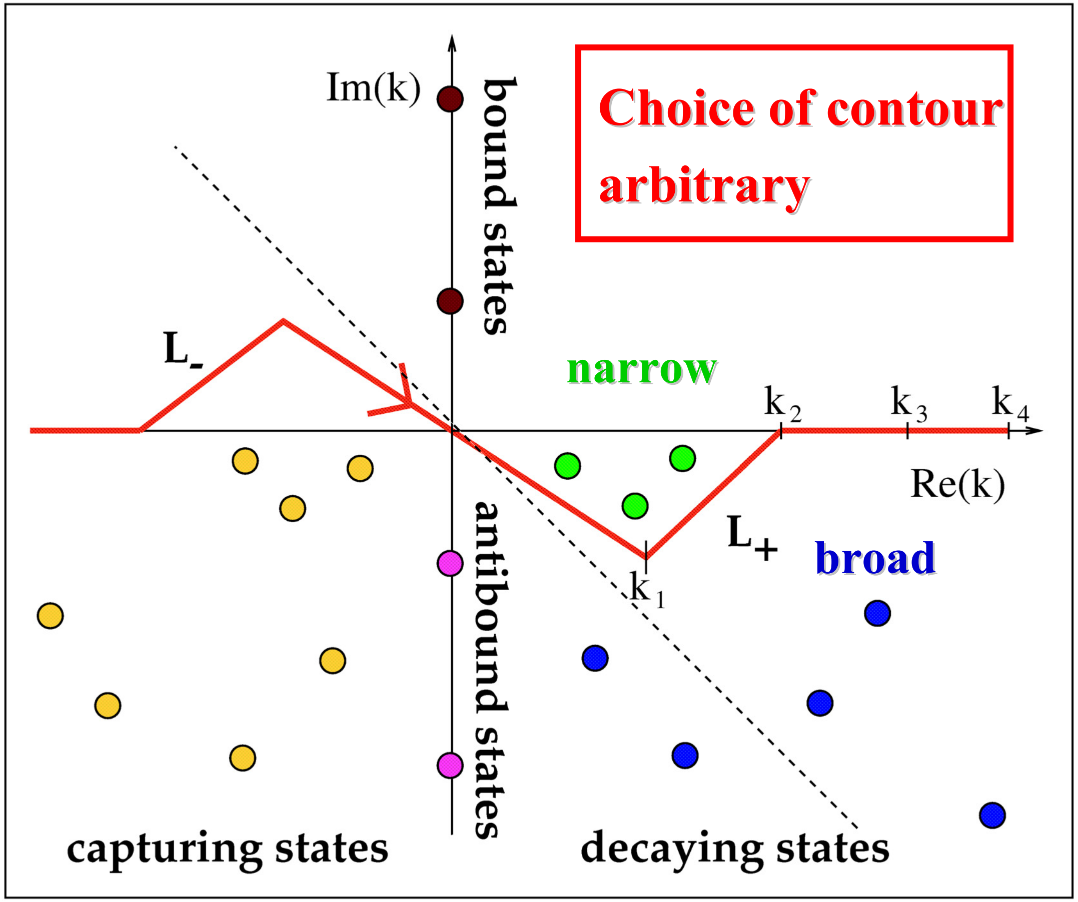
Complex scaling method

- Radial integral calculation : complex scaling

$$\langle u_f|O|u_i\rangle = \int_0^R u_f(r)O(r)u_i(r)\,dr$$
$$+ \quad e^{i\theta} \sum_{\omega_i,\omega_f} \int_0^{+\infty} u_i^{\omega_i}(R+xe^{i\theta})O(R+xe^{i\theta})u_f^{\omega_f}(R+xe^{i\theta})\,dx$$

- Analytic continuation : integral independent of R and θ

Gamow states location



Completeness relations with Gamow states

- Berggren completeness relation (l,j) :

T. Berggren, Nucl. Phys. A **109**, (1967) 205

$$\sum_{n \in (b,d)} |\phi_{nlj}\rangle\langle\phi_{nlj}| + \int_{L+} |\phi_{lj}(k)\rangle\langle\phi_{lj}(k)| \, dk = \mathbb{1}$$

- Completeness beyond Hilbert space

- Continuum discretization : $|\phi_{lj}(k)\rangle \rightarrow \sqrt{\Delta_{k_i}} \cdot |\phi_{lj}(k_i)\rangle$

$$\sum_i |\phi_i\rangle\langle\phi_i| \sim \mathbb{1}$$

Gamow HFB definition

- Complex scaling of densities

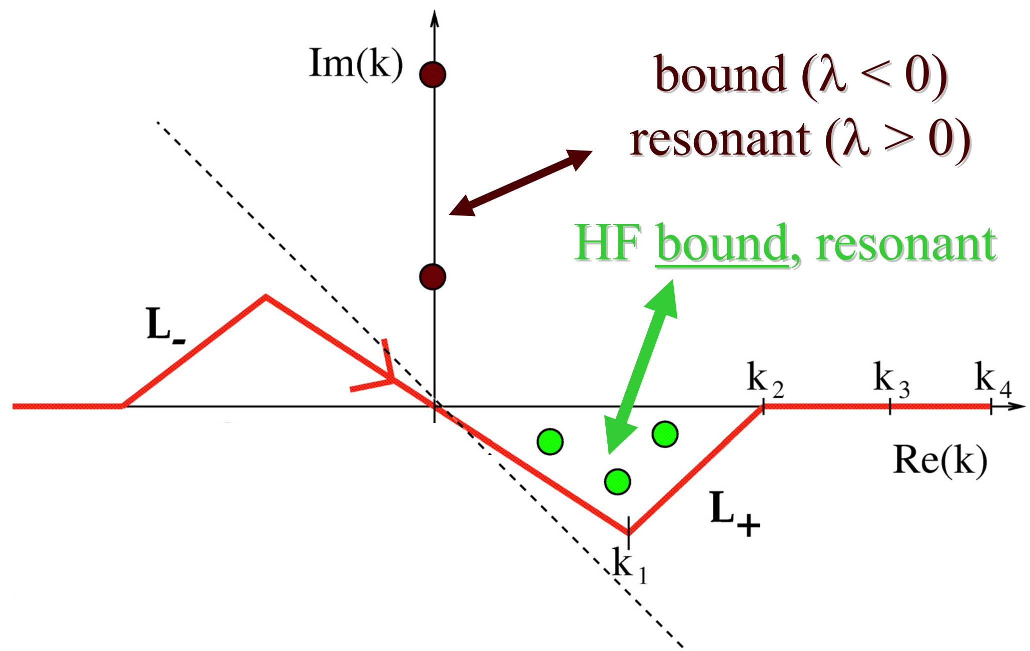
$$\rho(z) \to 0, \quad \tilde{\rho}(z) \to 0 \Rightarrow U_{HFB}(z) \to 0, \quad \tilde{U}_{HFB}(z) \to 0$$
$$z = re^{i\theta}, r \to +\infty, \theta < \frac{\pi}{2}$$

- Problem : Scattering states cannot be calculated in general:

$$|\tilde{U}_{HFB}(z) \cdot u(z)| \to +\infty$$

- Solution: Cut functions for potentials
  Potentials vanish quickly on the real axis
  Hankel/Coulomb wave functions for complex scaling

Gamow HFB space



$$E = E_{qp} - |\lambda|$$

Densities with Gamow HFB

- Complex particle and pairing densities:

$$\rho_{lj}(r) = \sum_{n \in (b,d)} v_{nlj}^2(r) + \int_{L+} v_{lj}^2(k,r)\, dk$$

$$\widetilde{\rho_{lj}}(r) = -\sum_{n \in (b,d)} u_{nlj}(r) v_{nlj}(r) - \int_{L+} u_{lj}(k,r) v_{lj}(k,r)\, dk$$

$$\rho(r) = \sum_{lj} \rho_{lj}(r),\ \tilde{\rho}(r) = \sum_{lj} \widetilde{\rho_{lj}}(r)$$

- HF associated bound and narrow resonant states in discrete sum
- Smoothly varying continuum
- Much smaller continuum contributions

Gamow quasi-particle states (1)

- Quasi-particles : bound, resonant and scattering states

- Bound, resonant states:

S matrix poles => outgoing wave function behavior

$$u(r) \sim C_u^0 \, r^{l+1}, \; r \to 0$$
$$v(r) \sim C_v^0 \, r^{l+1}, \; r \to 0$$
$$u(r) \sim C_u^+ \, H^+_{l,\eta_u}(k_u r), \; r \to +\infty$$
$$v(r) \sim C_v^+ \, H^+_{l,\eta_v}(k_v r), \; r \to +\infty$$
$$k_u \propto \sqrt{\lambda + E}, \; k_v \propto \sqrt{\lambda - E}$$

Gamow quasi-particle states (2)

- Scattering states:

u(r): incoming and outgoing components
v(r): outgoing wave function behavior

$$u(r) \sim C_u^0 \, r^{l+1}, \; r \to 0$$
$$v(r) \sim C_v^0 \, r^{l+1}, \; r \to 0$$
$$u(r) \sim C_u^+ \; H^+_{l,\eta_u}(k_u r) + C_u^- \; H^-_{l,\eta_u}(k_u r), \; r \to +\infty$$
$$v(r) \sim C_v^+ \; H^+_{l,\eta_v}(k_v r), \; r \to +\infty$$
$$k_u \propto \sqrt{\lambda + E}, \; k_v \propto \sqrt{\lambda - E}$$

Gamow quasi-particle states (3)

- Matching conditions:

$$\frac{u'(R_0^+)}{u(R_0^+)} - \frac{u'(R_0^-)}{u(R_0^-)} = 0 \quad (1) : \text{u(r), u'(r) continue}$$

$$\frac{v'(R_0^+)}{v(R_0^+)} - \frac{v'(R_0^-)}{v(R_0^-)} = 0 \quad (2) : \text{v(r), v'(r) continue}$$

$$\frac{u(R_0^+)}{v(R_0^+)} - \frac{u(R_0^-)}{v(R_0^-)} = 0 \quad (3) : \text{u(r) continue} \Leftrightarrow \text{v(r) continue}$$

Gamow quasi-particle states (4)

- Matching constants:

  S-matrix poles: (1)-(2)-(3) $\Leftrightarrow$ 3 constants: $\left(E, \frac{C_v^0}{C_u^0}, \frac{C_v^+}{C_u^+}\right)$

  Scattering states: No box => energy E fixed

  (1) always verified

  (2)-(3) $\Leftrightarrow$ 2 constants: $\left(\frac{C_v^0}{C_u^0}, C_v^+\right)$

- Method of resolution:

  (2,3)-dimensional Newton method

  Very good starting points essential

Gamow quasi-particle states (5)

- Normalization:

S-matrix poles: complex scaling

$$\text{Reg}\left[\int_0^{+\infty}\left(u(r)^2+v(r)^2\right)\, dr\right]=1$$

Scattering states: Dirac delta normalization

$$\int (u_k u_{k'} + v_k v_{k'}) = \delta(k-k') \Leftrightarrow C_u^+ \cdot C_u^- = \frac{1}{2\pi}$$

Continuum discretization: $\begin{pmatrix} u(k) \\ v(k) \end{pmatrix} \rightarrow \sqrt{\Delta_{k_i}} \begin{pmatrix} u(k_i) \\ v(k_i) \end{pmatrix}$

Numerical treatment of Gamow HFB (1)

- Self-consistent procedure:

(1) HF with HO basis : zero-th order ph potential

(2) BCS on HF/HO : zero-th order pp potential and λ

(3) HFB with HO basis: nearly exact potentials

localization of quasi-particle resonances

(4) Gamow-HFB in coordinate space

Numerical treatment of Gamow HFB (2)

- Matching conditions:

$$\phi_u^0 \to \begin{pmatrix} r^{l+1} \\ 0 \end{pmatrix} \quad \phi_v^0 \to \begin{pmatrix} 0 \\ r^{l+1} \end{pmatrix} \quad \phi_u^+ \to \begin{pmatrix} H^+_{l,\eta_u} \\ 0 \end{pmatrix} \quad \phi_v^+ \to \begin{pmatrix} 0 \\ H^+_{l,\eta_v} \end{pmatrix}$$

$$\begin{aligned} & C_u^0 \phi_u^0(R_0) + C_v^0 \phi_v^0(R_0) \\ = \quad & C_u^+ \phi_u^+(R_0) + C_v^+ \phi_v^+(R_0) \end{aligned}$$

- 4x4 determinant : zero at bound states
- Similar conditions for scattering states
- But: determinant not zero for resonant states

Numerical treatment of Gamow HFB (3)

- Quasi-particles S-matrix poles: one small component

  No pairing approximation to localize pole energies

- Quasi-particles S-matrix poles: no small component

  Bisection with 4x4 determinant to find energy ($e \sim |\lambda|$)

- Quasi-particles scattering states: no energy matching
- Constants from 4x4 determinant conditions
- Refinement with Jost functions Newton method

Description of Nickel calculations

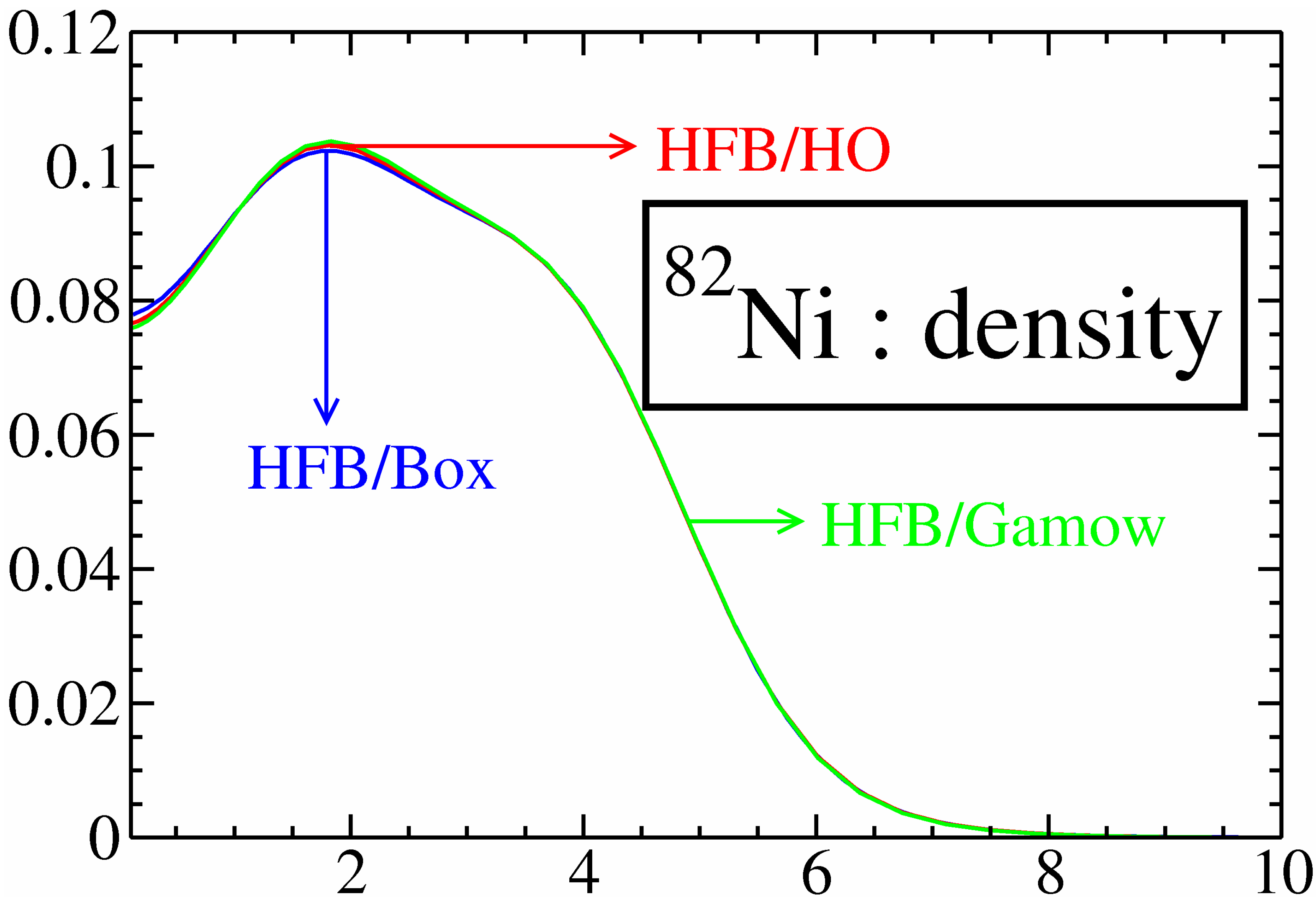
- Considered nuclei: ^{82}Ni, ^{84}Ni, ^{86}Ni (^{88}Ni unbound)

- Interaction and space:
  Skyrme interaction Sly4
  E_{cut} = 60 MeV, l: 0 → 20 (10 for box calculations)
  Gamow states for $l \leq 5$, HO states for $l \geq 6$

  Comparison with box and full HO calculations

- Interest: Pairing energy decreases without box treatment
  M. Grasso et al., Phys. Rev. C, **64** (064321)

- Preliminary calculations:
  All new code
  Complex contour dependence, spurious imaginary part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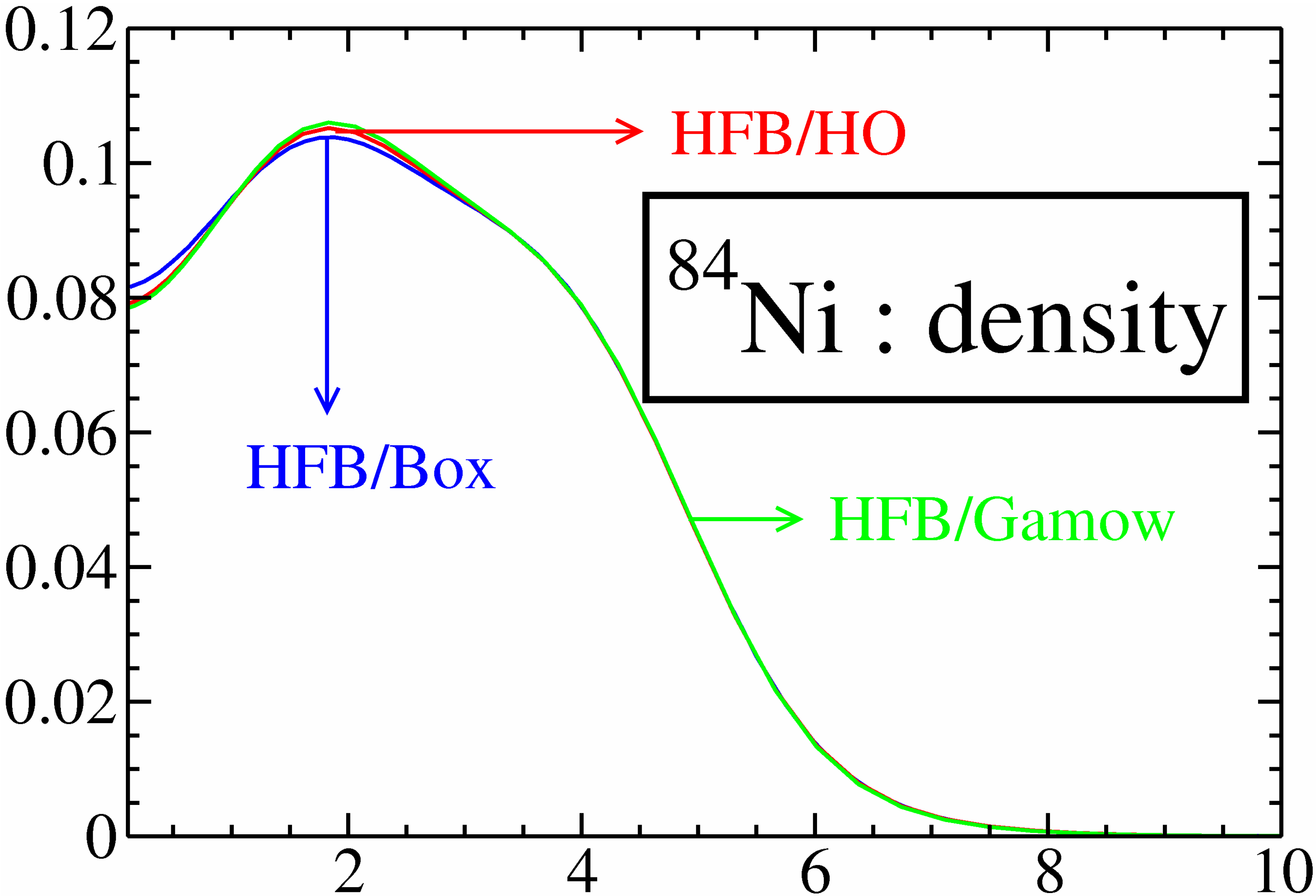
HFB/HO
HFB/Box
HFB/Gamow
82Ni : density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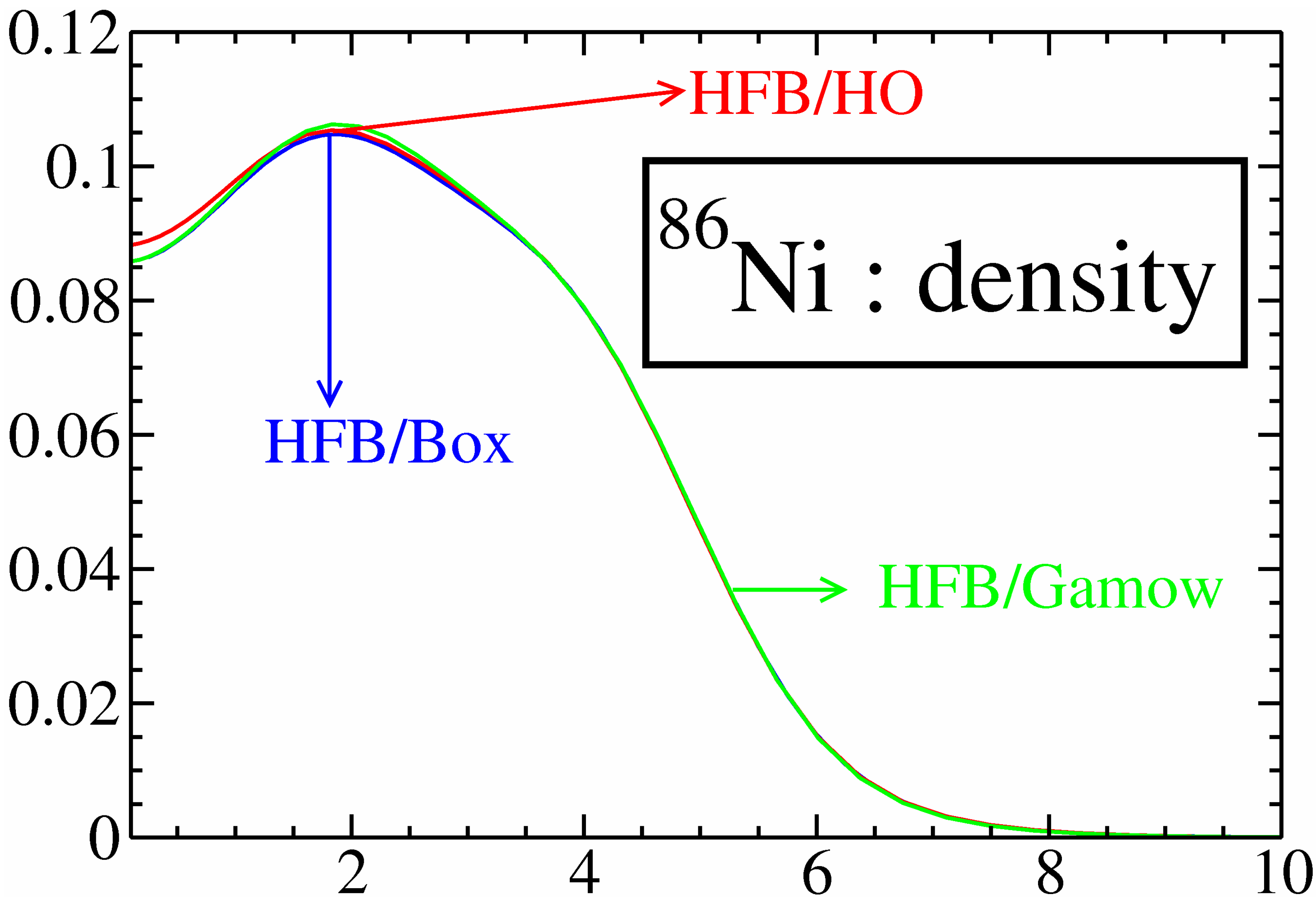
HFB/HO
HFB/Box
HFB/Gamow
84Ni : density



HFB/HO
^{86}Ni : density
HFB/Box
HFB/Gamow
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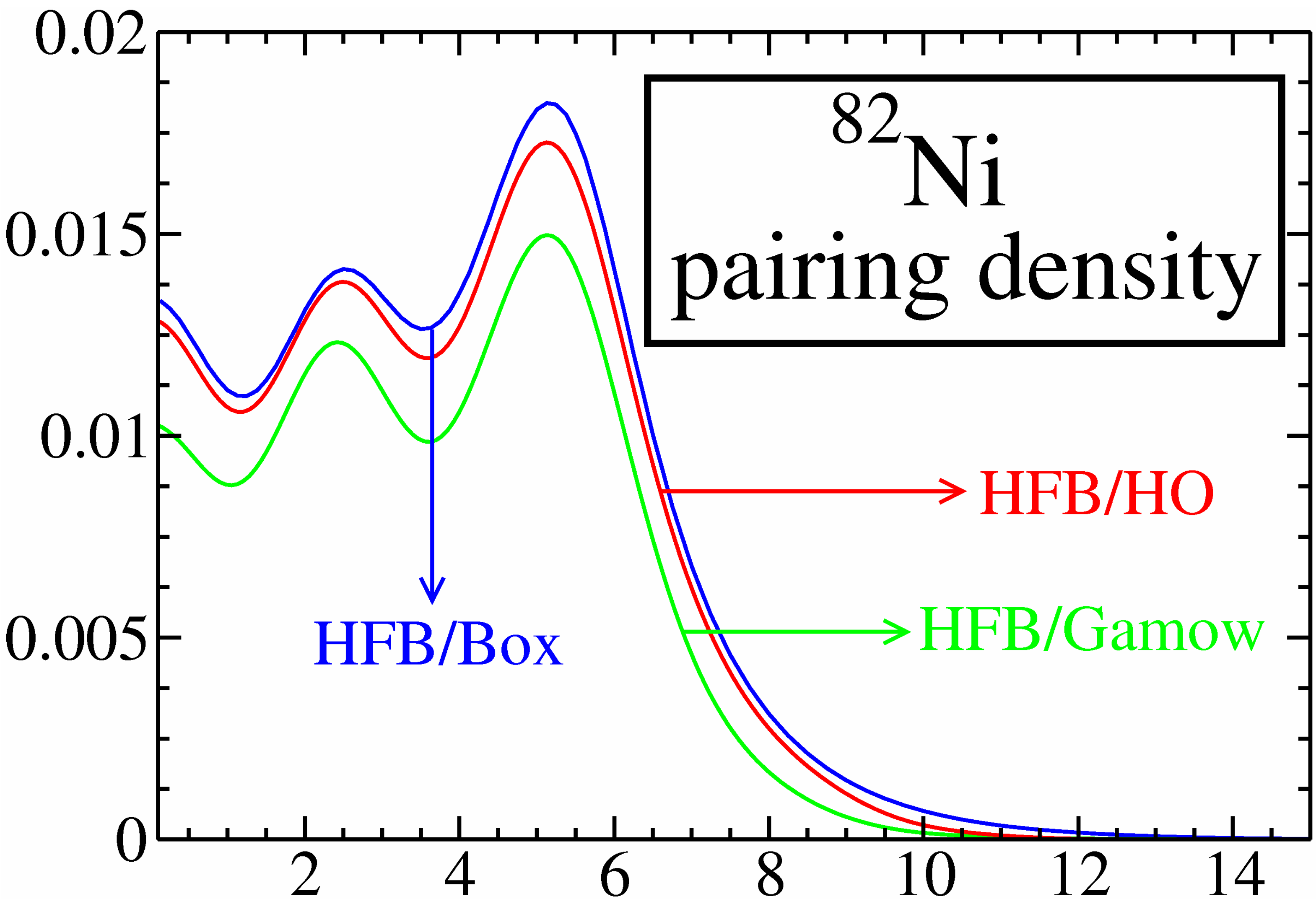
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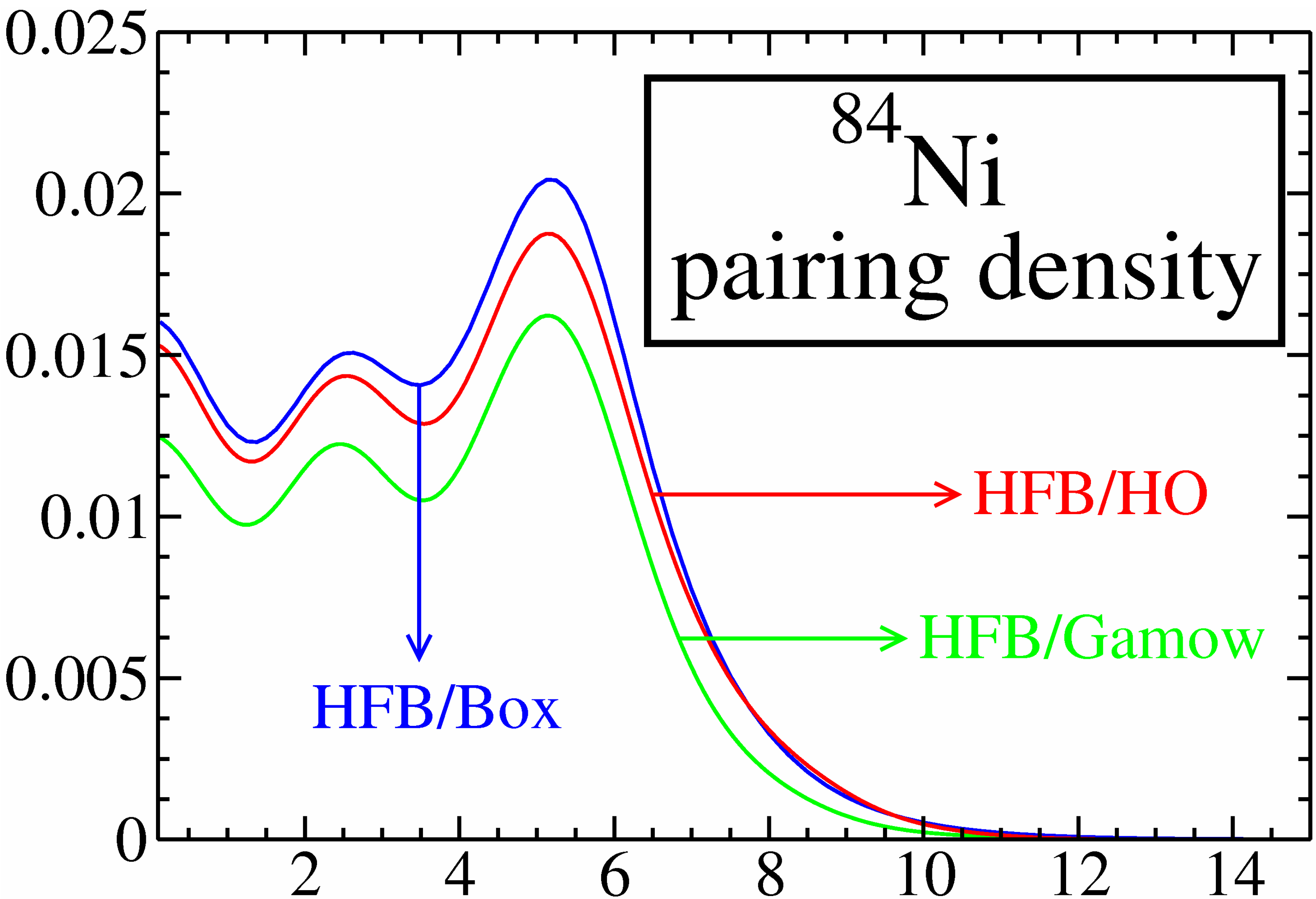
^{82}Ni
pairing density
HFB/HO
HFB/Box
HFB/Gamow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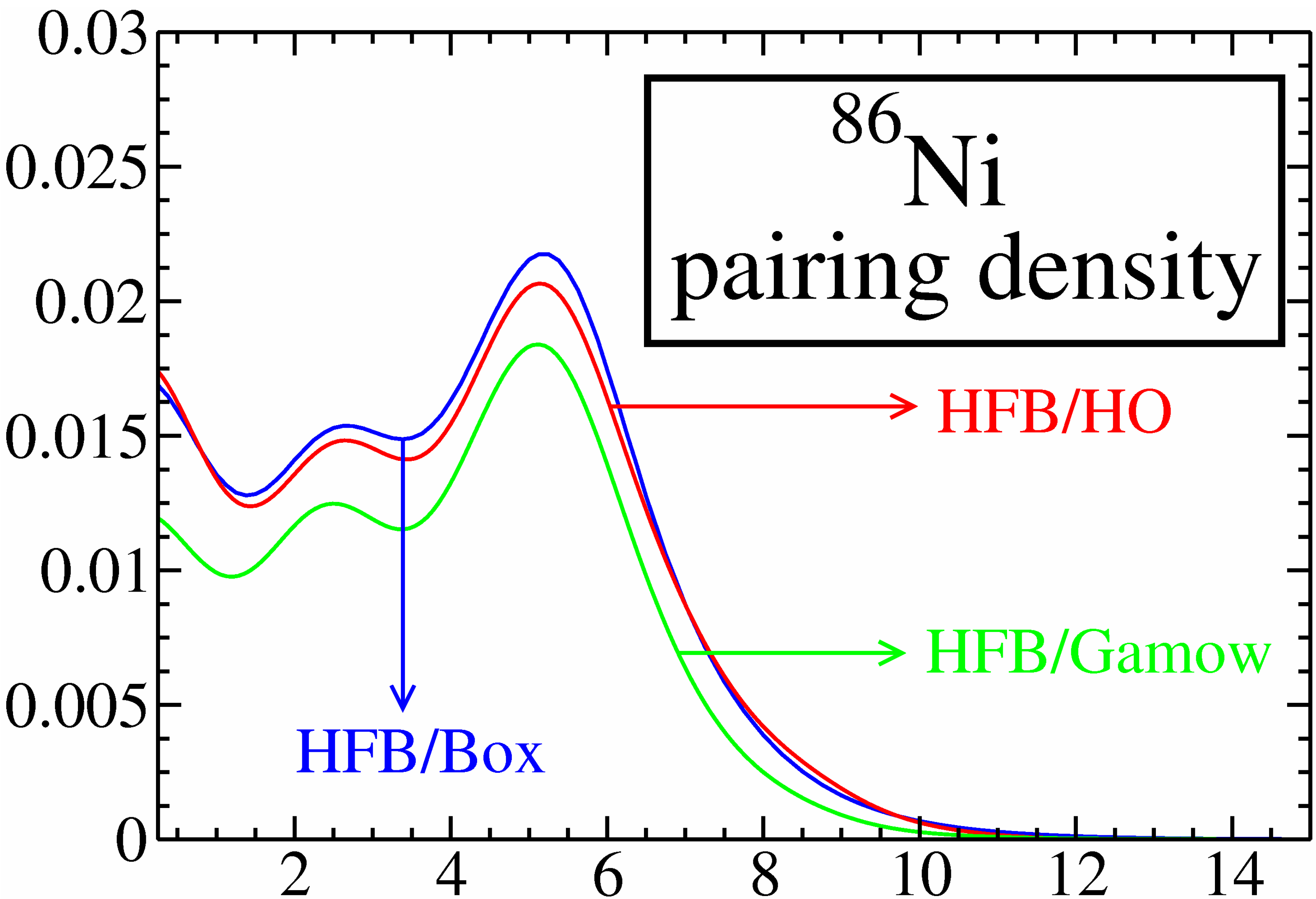
^{84}Ni
pairing density
HFB/HO
HFB/Gamow
HFB/Box



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^{86}Ni
pairing density
HFB/HO
HFB/Gamow
HFB/Box

Conclusion and perspectives

- Gamow HFB model:

  Necessary tool to study dripline heavy nuclei

  Demands new theoretical and computational methods

  Calculations not stable: method not yet fully operational

- First applications

  Nickel chain close to neutron dripline

  Pairing components:

  smaller with Gamow HFB than with box treatment

- Very preliminary results:

  More precise calculations needed