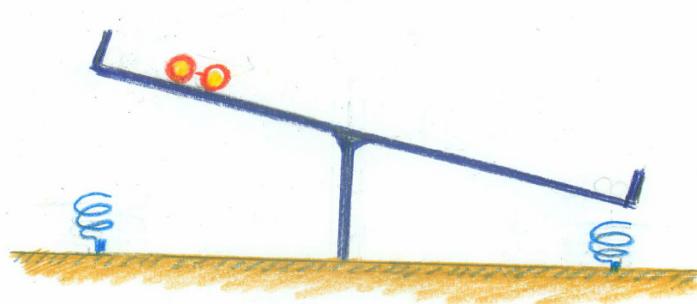


# Nature of excited $0^+$ states in deformed neutron-rich nuclei



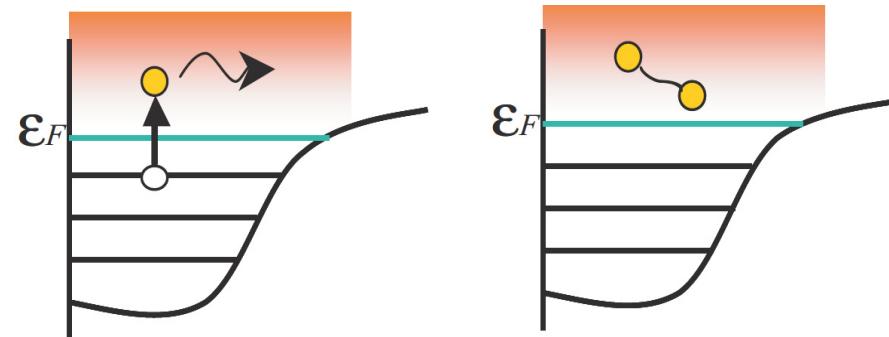
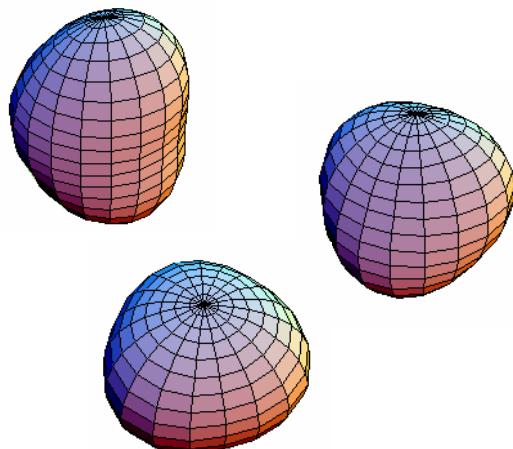
K. Yoshida, M. Yamagami, and K. Matsuyanagi



# Microscopic Approach to Collective Phenomena in Unstable Nuclei

Can we expect new types of collective excitation modes to emerge under the new situation of (deformed) shell structure and pairing correlations in unstable nuclei ???

Let us discuss mechanism of generating collectivity (coherence) in weakly bound systems over from the basics.



# **Neutron-rich nuclei as a unique and new fermion many-body system**

**New environment**

**New degrees of freedom**

Neutron skins  
Di-neutron

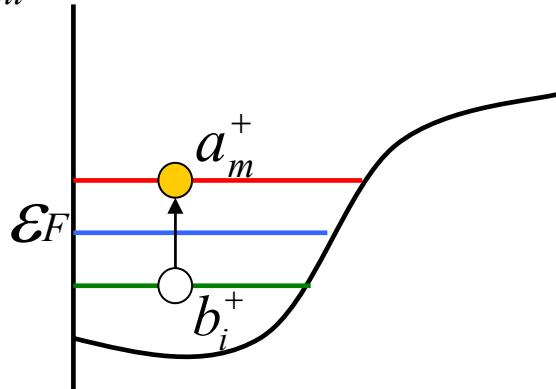
**New types of correlation**

Continuum coupling  
Pairing correlations  
Deformations

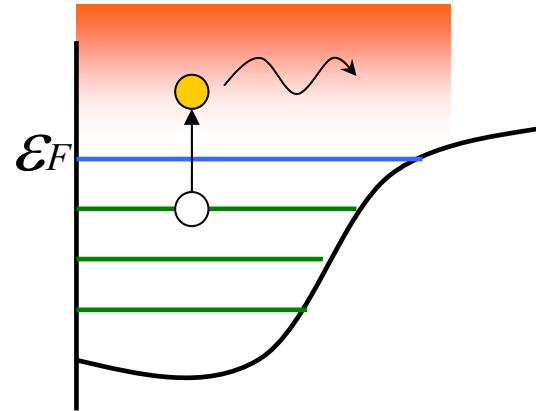
**Unique single-particle and collective motions**

# Collective vibrations $\sim$ superposition of particle-hole excitations

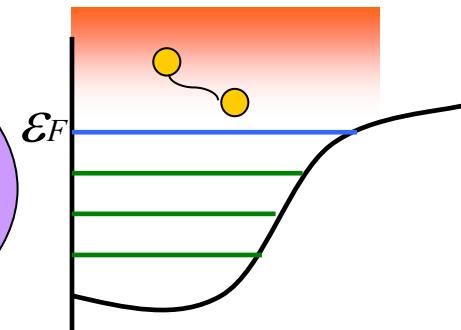
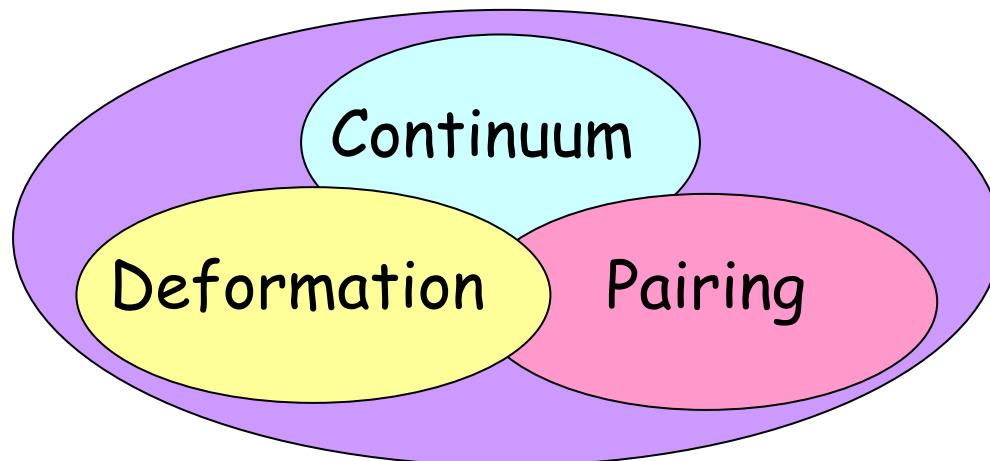
$$| \text{vib.} \rangle = \sum_{mi} X_{mi} a_m^+ b_i^+ - Y_{mi} b_i a_m | \text{gr.} \rangle$$



Stable nuclei



Drip-line nuclei



Mechanism for generation of collective modes under the new environment ?

# Deformed HFB calculation

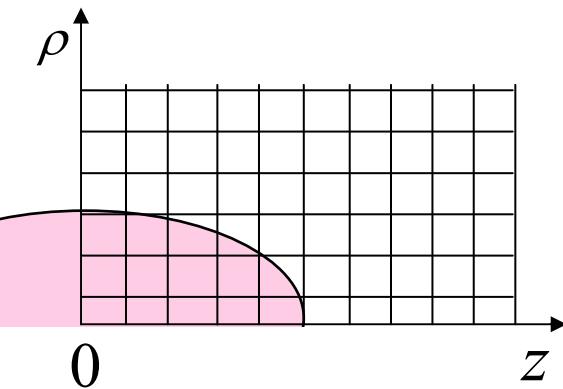
Mean-field    Deformed Woods-Saxon potential

$$V(\rho, z) = V_{\text{WS}} f + V_{\text{SO}} \nabla f \cdot (\sigma \times \mathbf{p})$$

$$f(\rho, z) = (1 + \exp[(r - R(\theta)) / a])^{-1}, \quad r^2 = \rho^2 + z^2$$

Pair-field    Self-consistent pairing

$$\Delta(\rho, z) = \frac{V_0}{2} \left( 1 - \frac{\rho(\mathbf{r})}{\rho_0} \right)$$



Directly solve HFB eq. in coordinate-space mesh-representation

H.O. basis  $\rightarrow$  Spatially extended structure

The equation of motion  $[\hat{H}', \hat{O}_\lambda^+] |\Psi_0^{\text{RPA}}\rangle = \hbar\omega_\lambda \hat{O}_\lambda^+ |\Psi_0^{\text{RPA}}\rangle$



The QRPA equation in the matrix formulation

$$\sum_{\gamma\delta} \begin{pmatrix} A_{\alpha\beta\gamma\delta} & B_{\alpha\beta\gamma\delta} \\ B_{\alpha\beta\gamma\delta} & A_{\alpha\beta\gamma\delta} \end{pmatrix} \begin{pmatrix} X_{\gamma\delta}^\lambda \\ Y_{\gamma\delta}^\lambda \end{pmatrix} = \hbar\omega_\lambda \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X_{\alpha\beta}^\lambda \\ Y_{\alpha\beta}^\lambda \end{pmatrix}$$

Residual interaction

p-h channel  $v_{\text{ph}}(\mathbf{r}, \mathbf{r}') = [t_0(1 + x_o P_\sigma) + \frac{t_3}{6}(1 + x_3 P_\sigma)\rho(\mathbf{r})]\delta(\mathbf{r} - \mathbf{r}')$

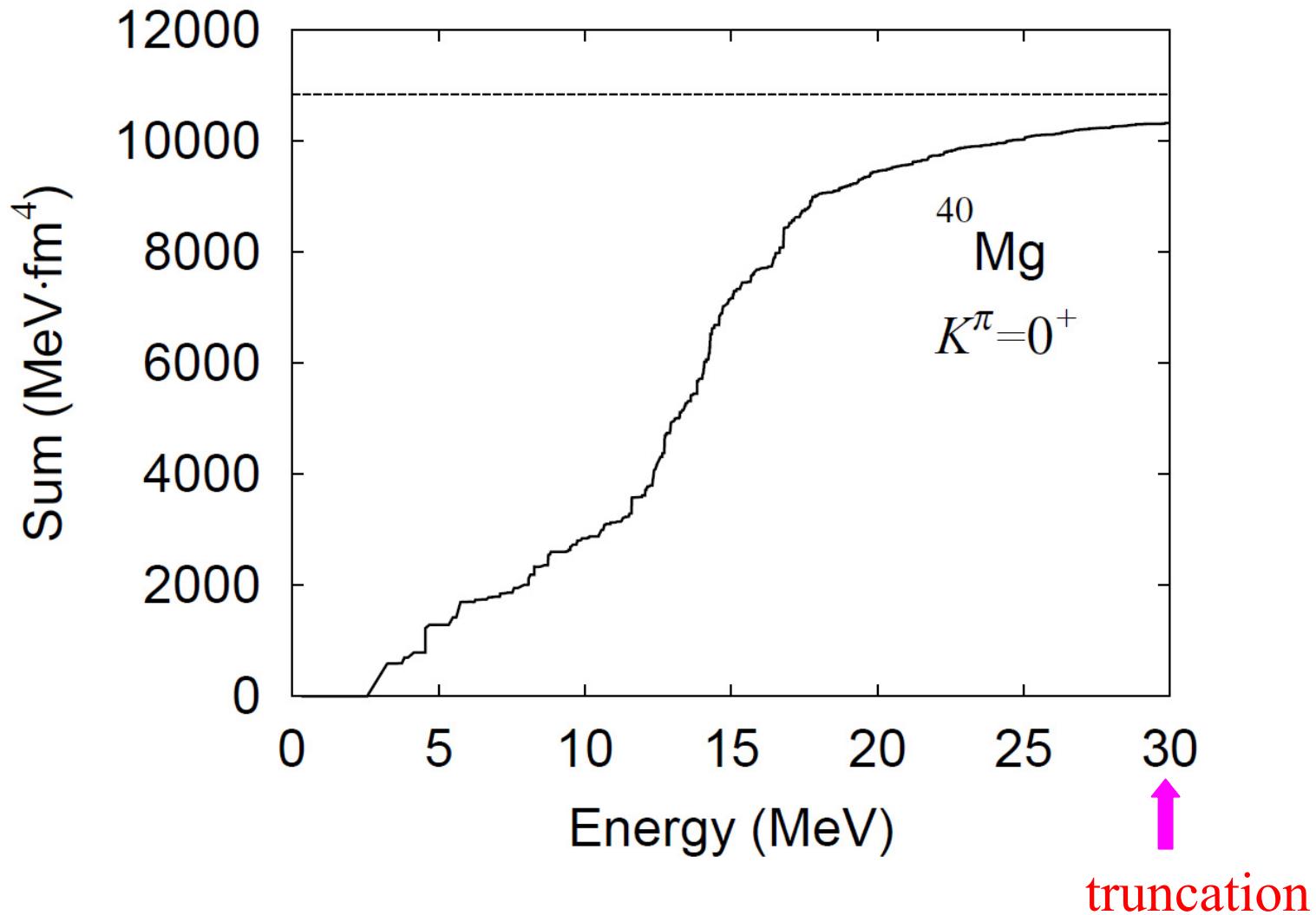
p-p channel  $v_{\text{pp}}(\mathbf{r}, \mathbf{r}') = V_0 \left(1 - \frac{\rho(\mathbf{r})}{\rho_0}\right) \delta(\mathbf{r} - \mathbf{r}')$

Spurious (Nambu-Goldstone) modes are properly removed.

Rotation, Pair rotation, Center-of-mass motion

$$v \rightarrow f \cdot v$$

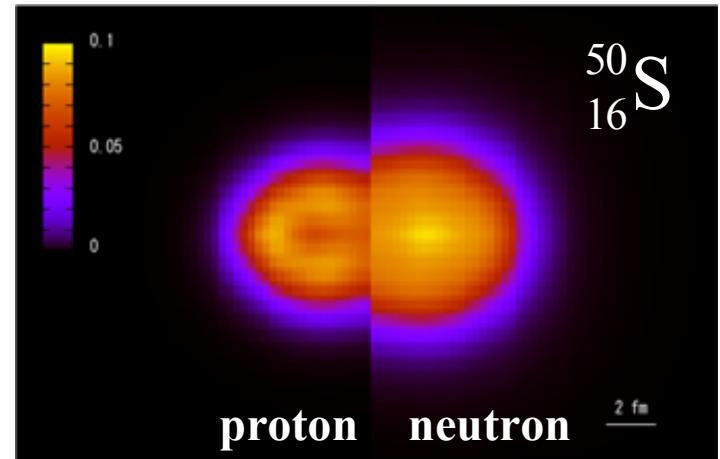
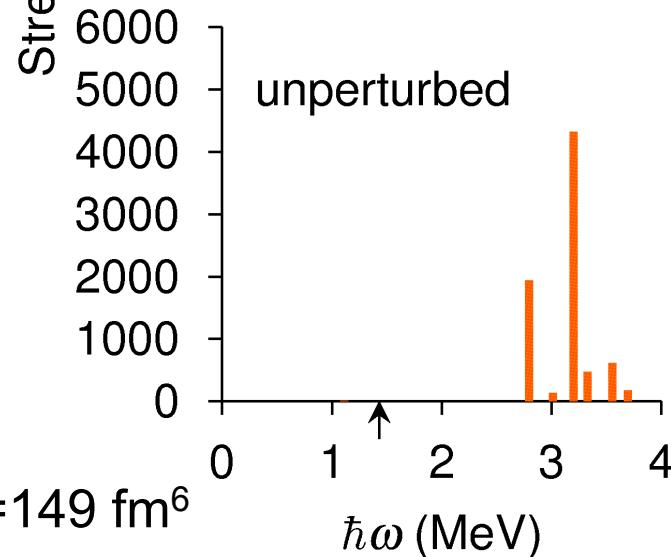
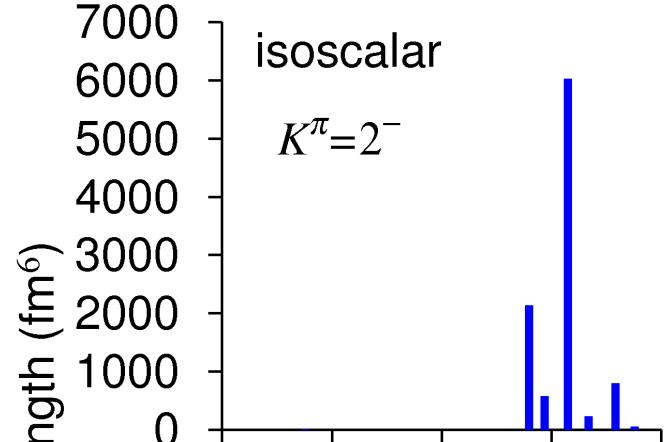
## Test of sum rule



# Octupole excitations on superdeformed state in sulfur isotopes close to the neutron drip line

$$\left| \langle \Psi_{\lambda}^{\text{RPA}} | \hat{Q}_{32}^{\text{IS}} | \Psi_0^{\text{RPA}} \rangle \right|^2$$

$$\hat{Q}_{32}^{\text{IS}} = \sum_{\nu, \pi} \sum_{\sigma} \int d\mathbf{r} r^3 Y_{32} \psi^+(\mathbf{r}, \sigma) \psi(\mathbf{r}, \sigma)$$



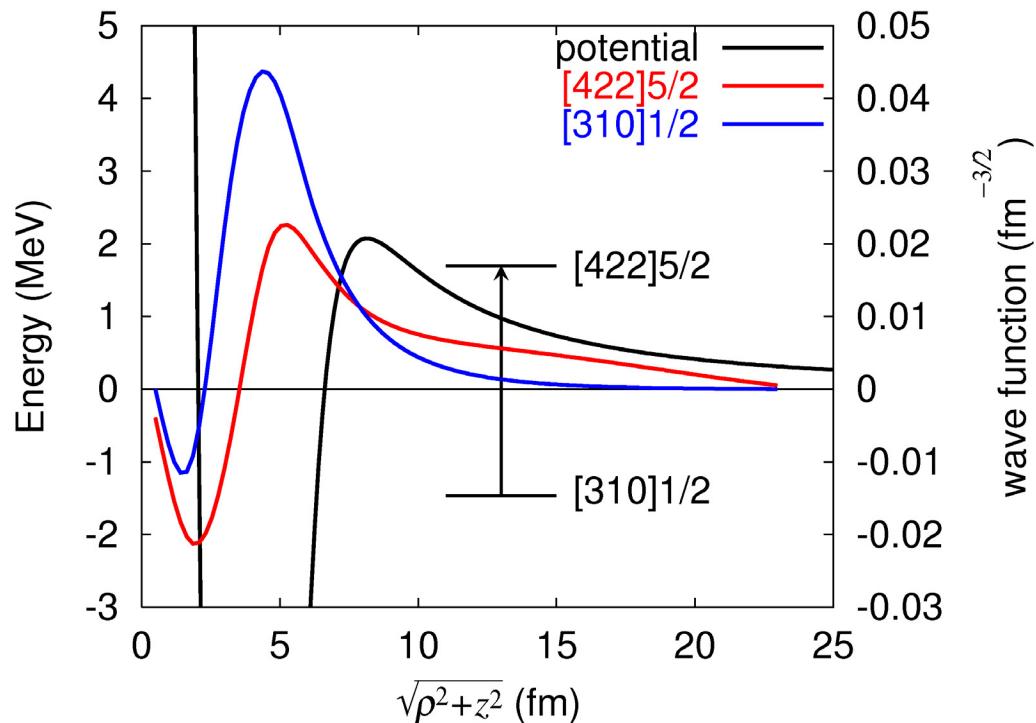
Deformed RPA  
without pairing

K.Yoshida *et al.*, PTP113 (2005) 1251

# Single-particle excitation to the resonance

Single-particle excitation  
from weakly bound to  
resonant state

$$X_{mi} \approx 0.97$$



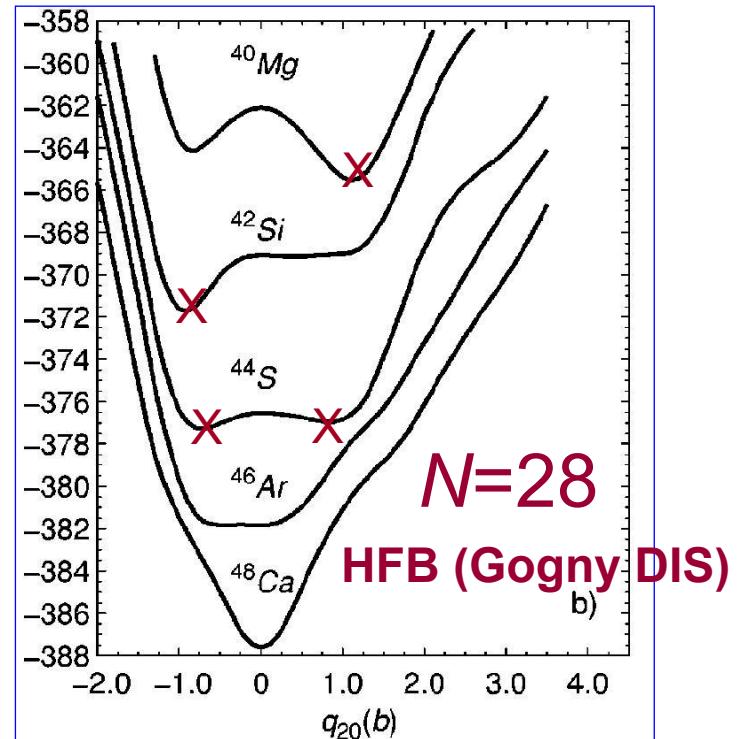
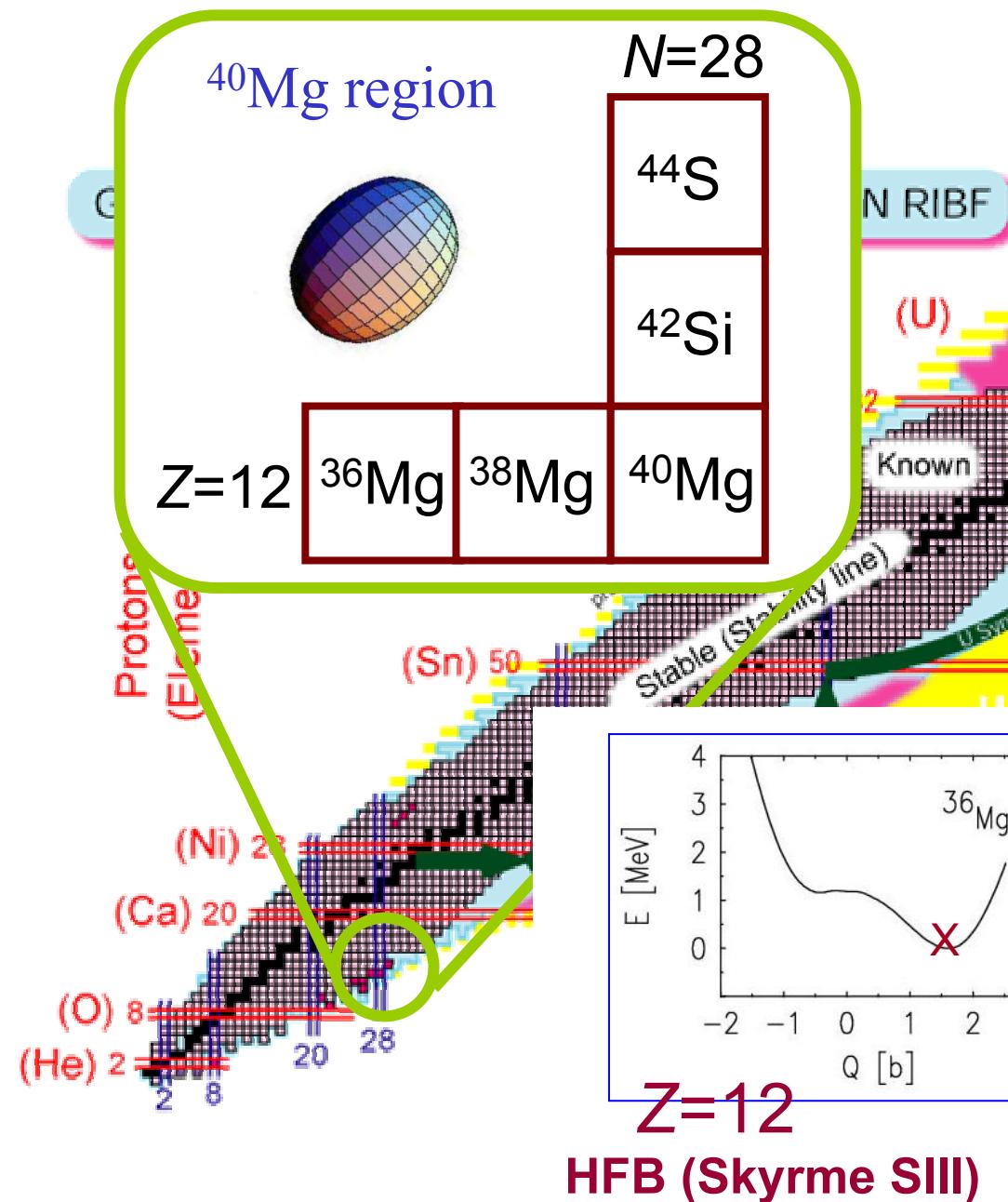
In neutron drip-line nuclei

Collective modes disappear ??

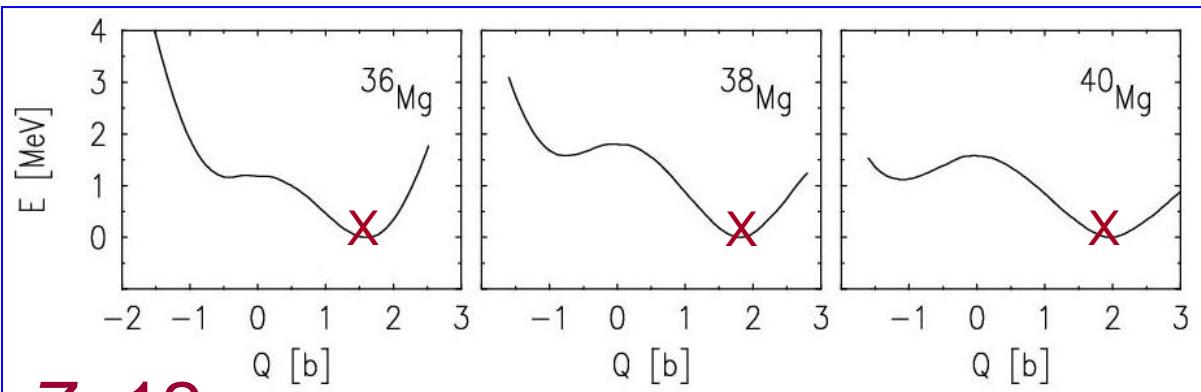


We have to investigate the  
effect of pairing

# Deformed weakly bound systems



R.Rodriguez-Guzman, J.L.Egido, L.M.Robledo  
Phys. Rev. C65, 024304 (2002)



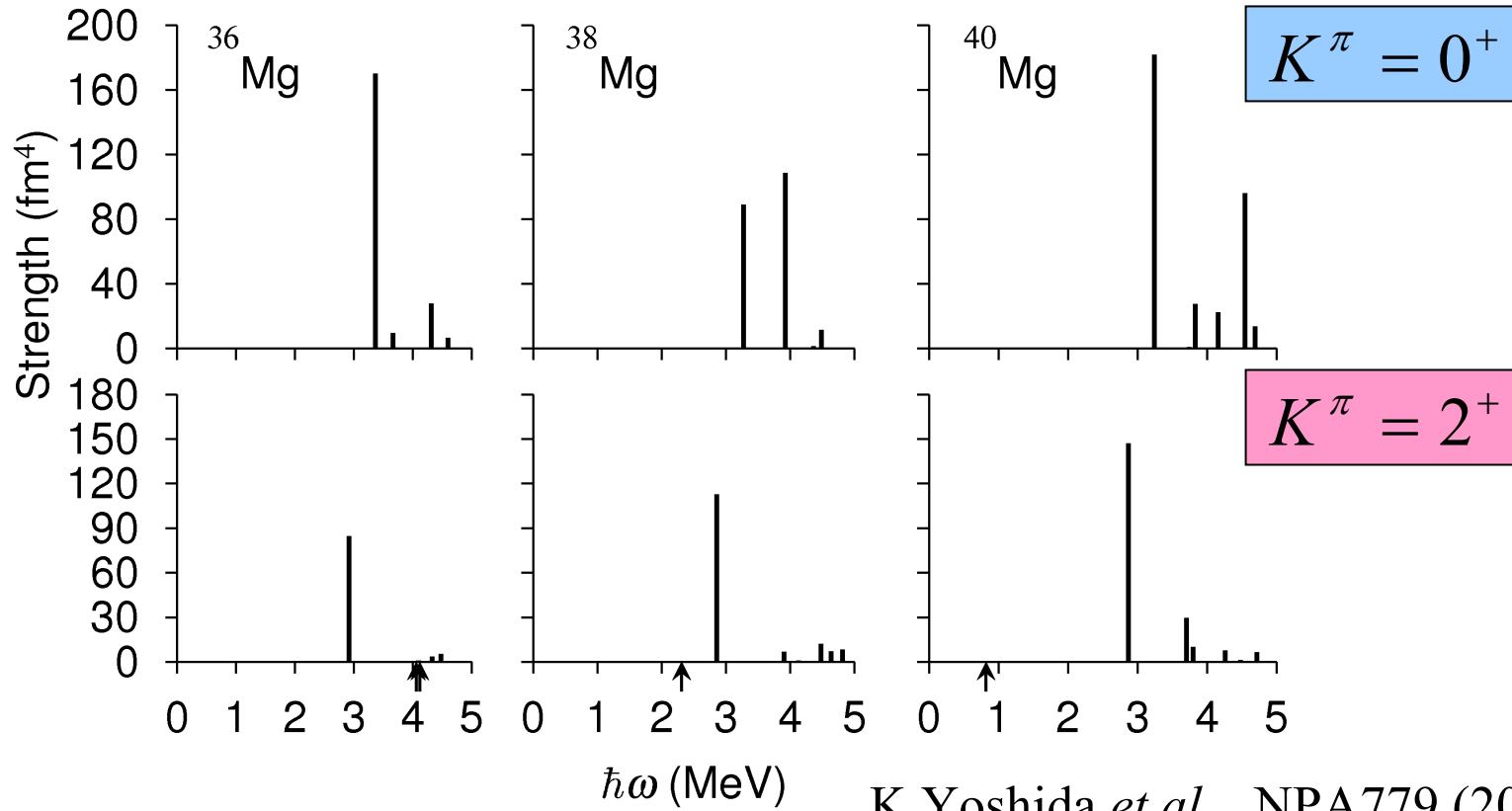
**$Z=12$**

**HFB (Skyrme SIII)**

J.Terasaki, H.Flocard, P.-H.Heenen, P.Bonche  
Nucl.Phys. A621, 706 (1997)

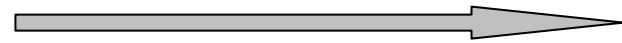
# Quadrupole vibrations in magnesium isotopes close to the neutron drip line

Isoscalar quadrupole transition strengths (intrinsic)



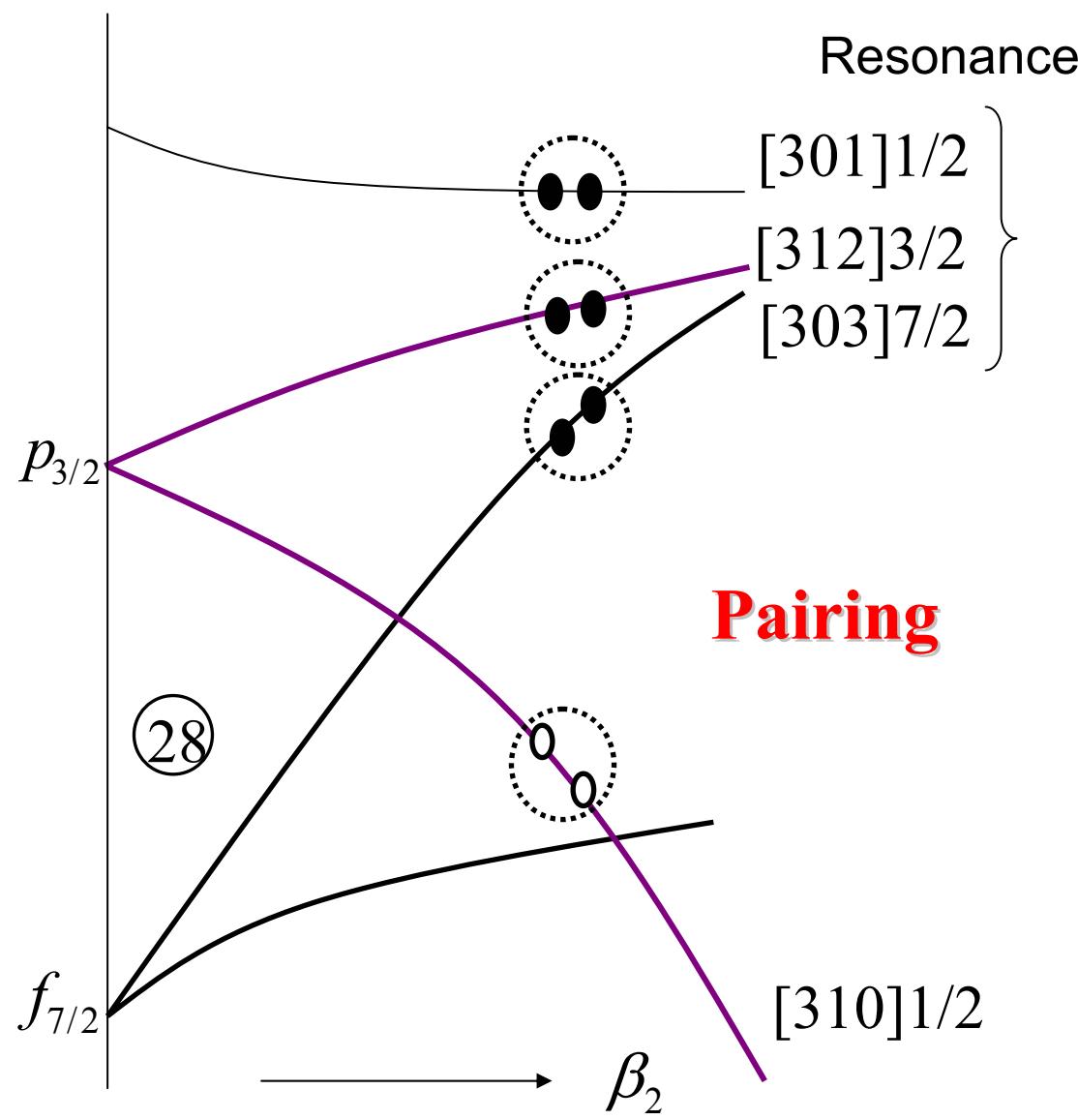
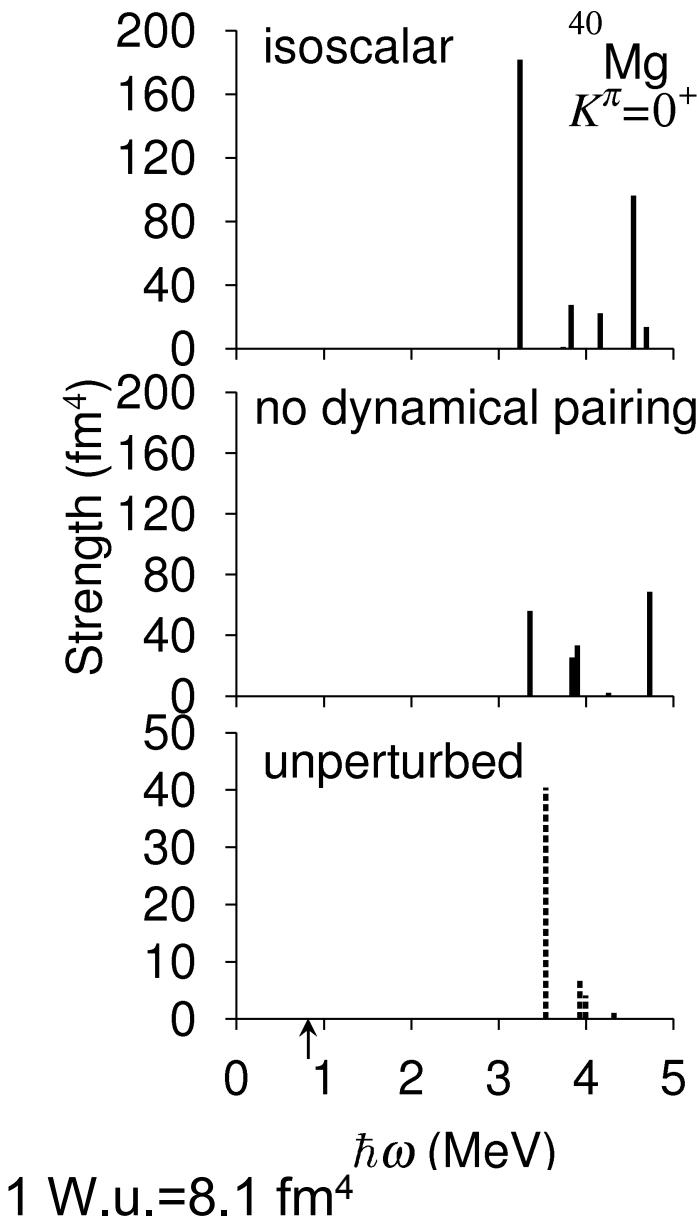
K.Yoshida *et al.*, NPA779 (2006) 99

Neutron number increasing



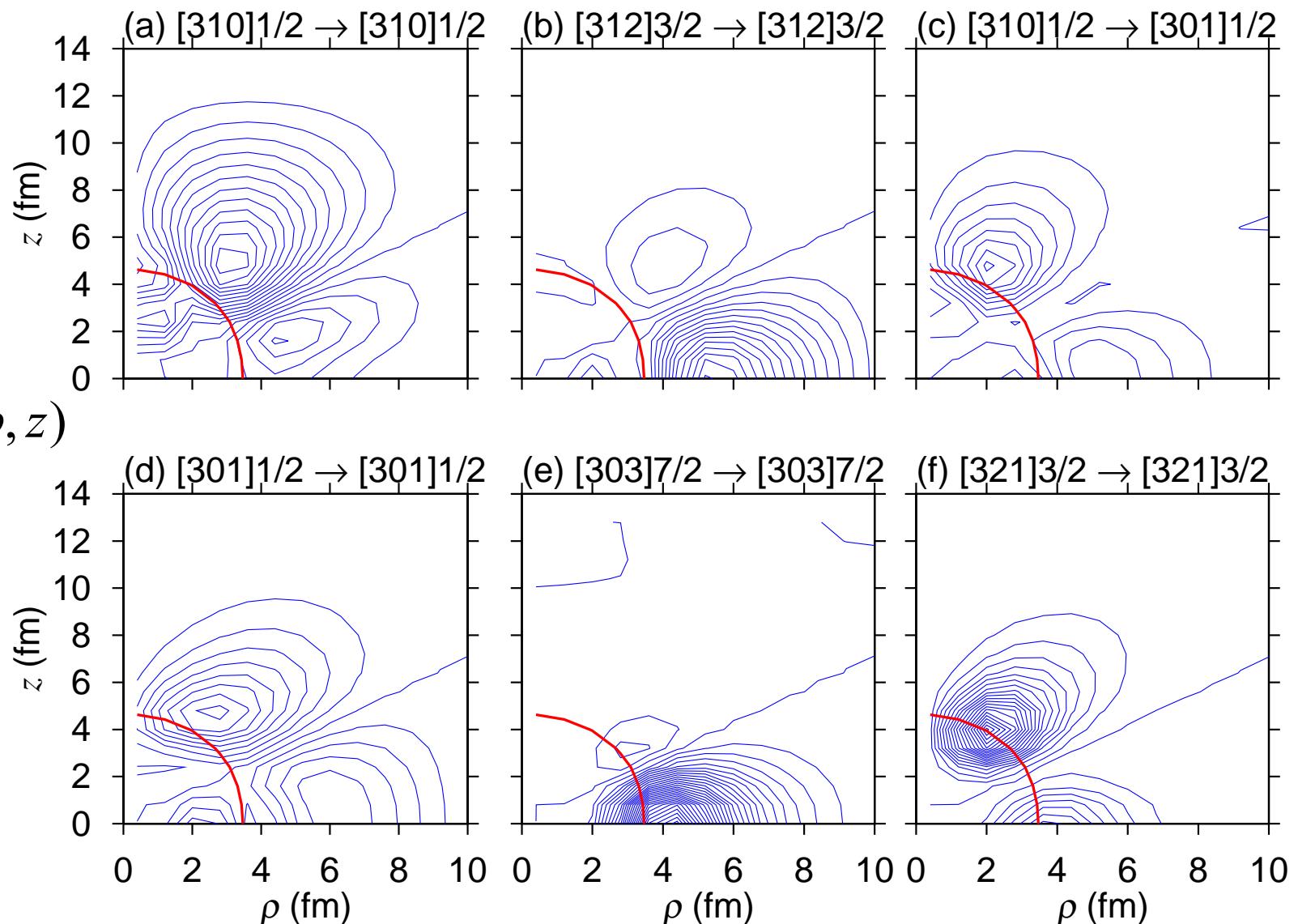
Enhancement of neutron excitation

# Microscopic Structure of the $K=0^+$ modes in $^{40}\text{Mg}$



# Spatial structure of 2qp excitations (p-h channel)

$$\langle \alpha\beta | \hat{Q}_{20} | \text{HFB} \rangle \equiv \int d\rho dz Q_{20,\alpha\beta}^{(\text{uv})}(\rho, z) \quad \hat{Q}_{20} = \sum_{\sigma} \int d\mathbf{r} r^2 Y_{20} \psi^+(\mathbf{r}, \sigma) \psi(\mathbf{r}, \sigma)$$

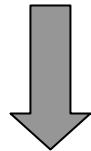


# Mechanism for generation of soft $K=0^+$ mode

**Collective both in p-h and in p-p channel**

How to generate the coherent mode?

Why are the transition strengths large ?  $\sim 10-20$  W.u. (intrinsic)



**Two key points**

**Pair correlations**

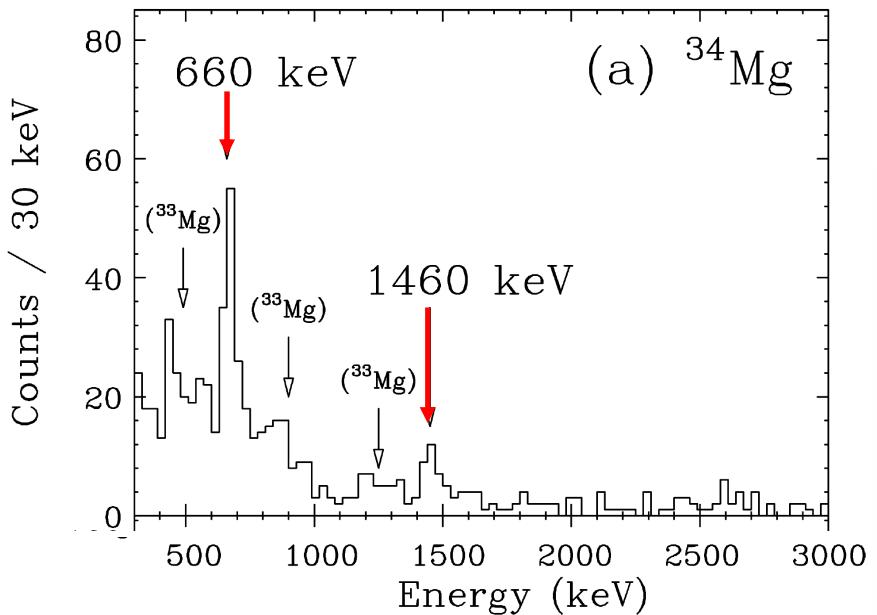
Effect of dynamical pairing

**Weakly bound system**

Spatially extended structure of quasiparticle wave functions

# Deformations of neutron-rich Mg isotopes

K.Yoneda *et al.*, PLB499(2001)233



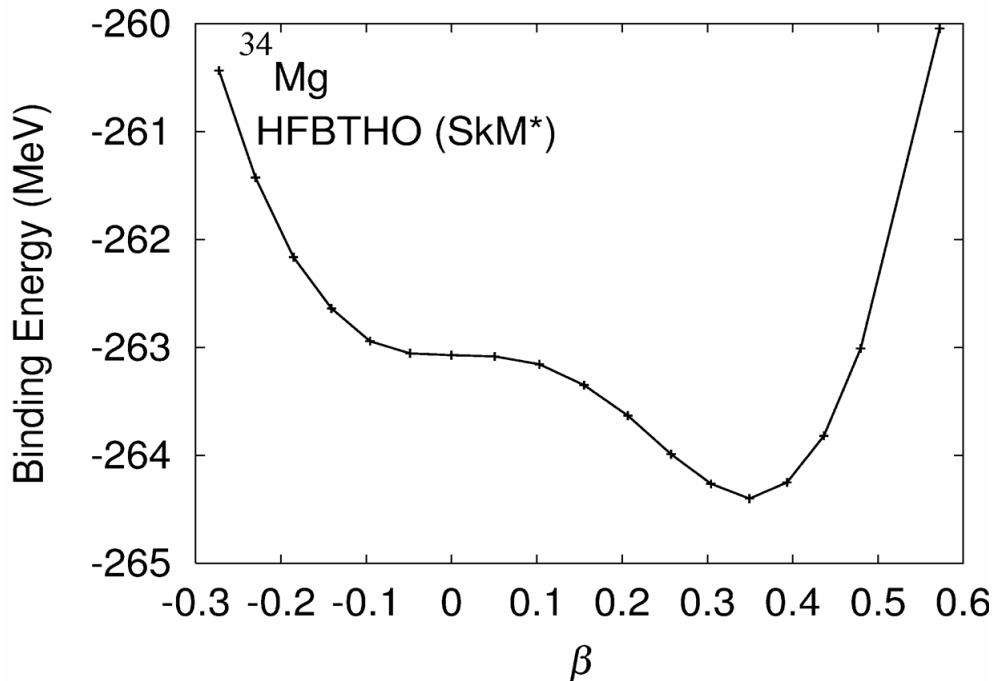
$$\frac{E(4_1^+)}{E(2_1^+)} = \frac{2120(\text{keV})}{660(\text{keV})} = 3.2$$



$^{34}\text{Mg}$  is well deformed.

“Skyrme-HFB deformed nuclear mass table”

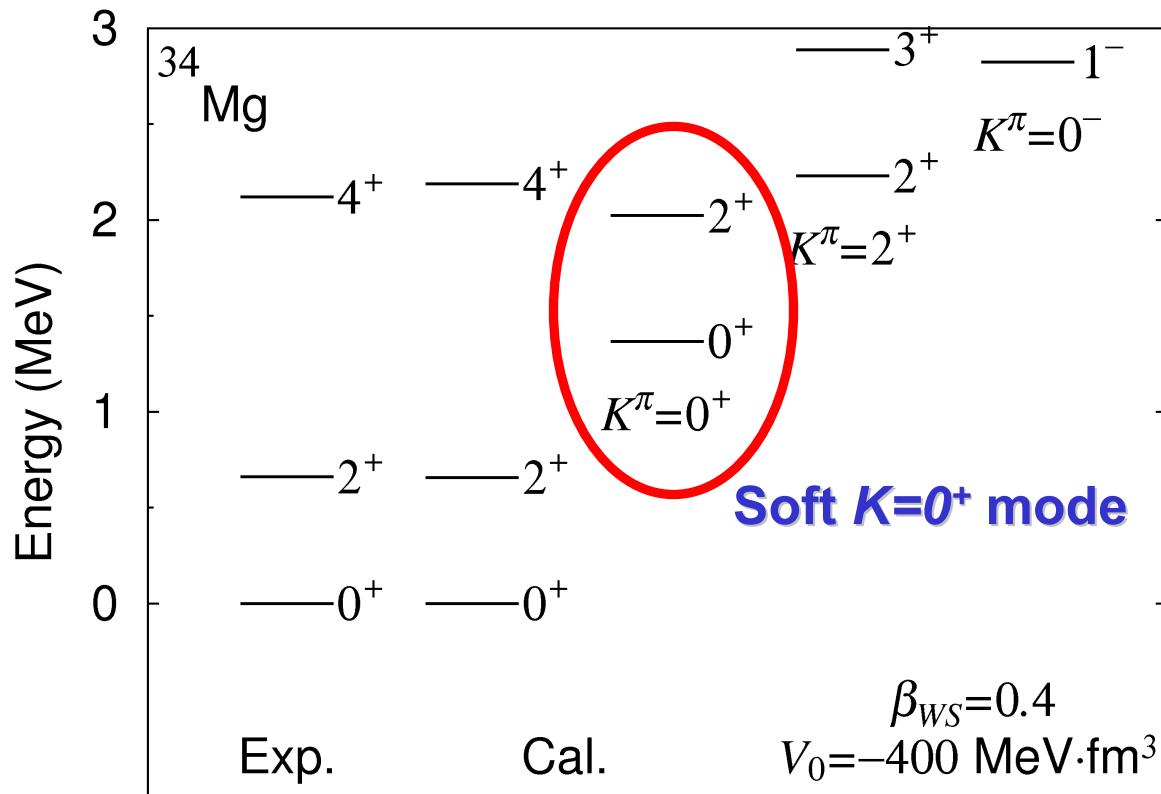
M.Stoitsov *et al.*, PRC68(2003)054312



The program “HFBTHO” (v1.66p)

M.Stoitsov *et al.*, Comp.Phys.Comm.167(2005)43

# Low-lying levels in $^{34}\text{Mg}$



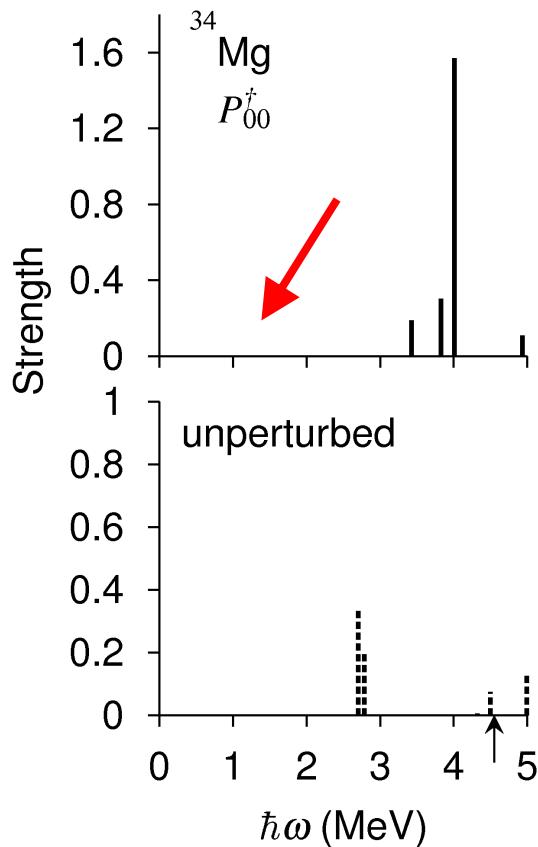
$$E(I, K) = \hbar\omega_{\text{RPA}} + \frac{\hbar^2}{2J_{\text{TV}}} (I(I+1) - K^2)$$

Microscopically calculated

# Two neutron pair transition strengths

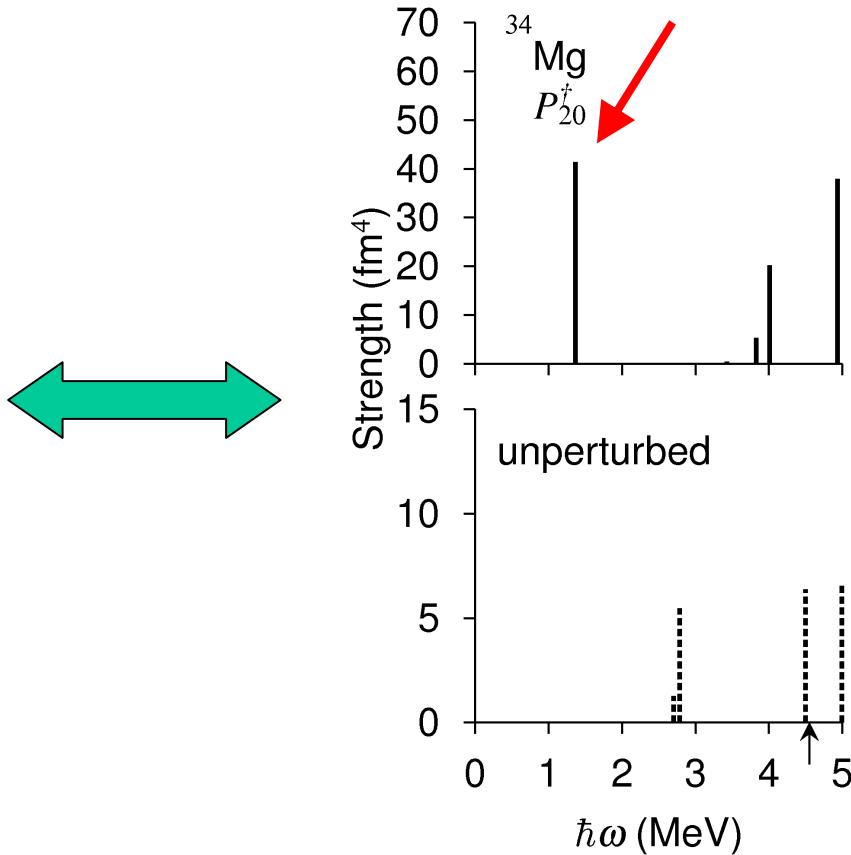
## Monopole pairing

$$P_{00}^+ = \int d\mathbf{r} \psi^+(\mathbf{r}, \uparrow) \psi^+(\mathbf{r}, \downarrow)$$



## Quadrupole pairing

$$P_{20}^+ = \int d\mathbf{r} r^2 Y_{20}(\hat{\mathbf{r}}) \psi^+(\mathbf{r}, \uparrow) \psi^+(\mathbf{r}, \downarrow)$$



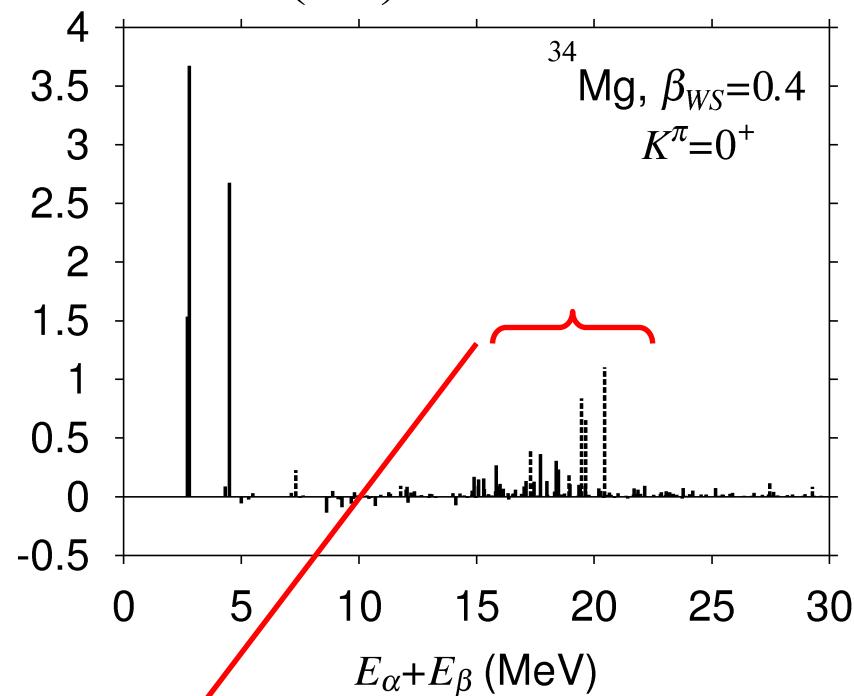
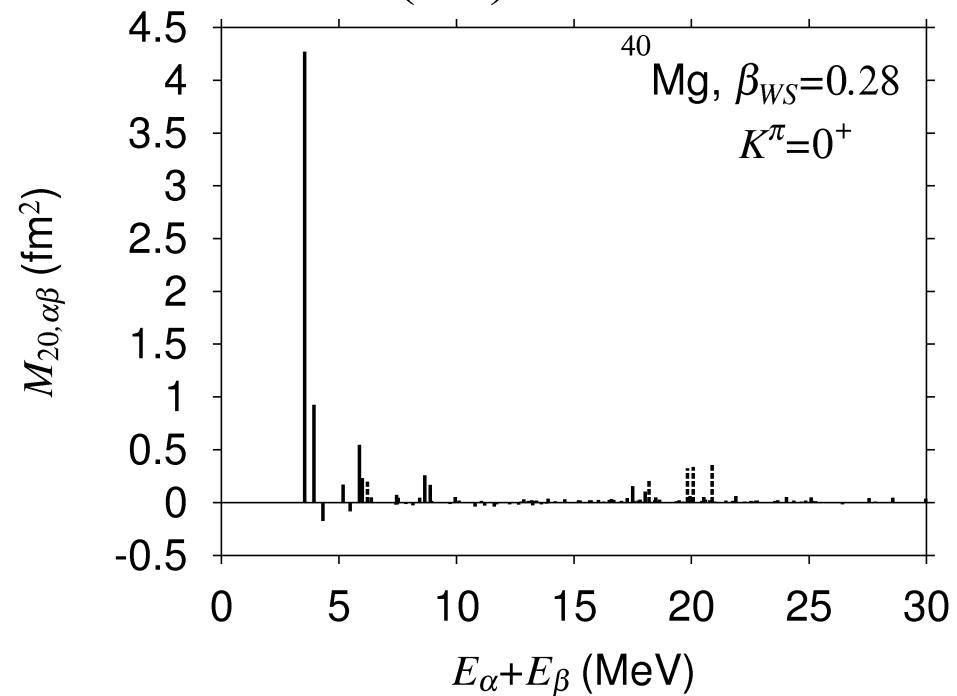
# Comparison of the $K=0^+$ modes in $^{40}\text{Mg}$ and $^{34}\text{Mg}$

## Matrix elements of 2qp excitations generating the $K=0^+$ mode

$$\left\langle \Psi_{\lambda}^{\text{RPA}} \left| \hat{Q}_{20} \right| \Psi_0^{\text{RPA}} \right\rangle = \sum_{\alpha\beta} M_{20,\alpha\beta}^{\text{(uv)}}$$

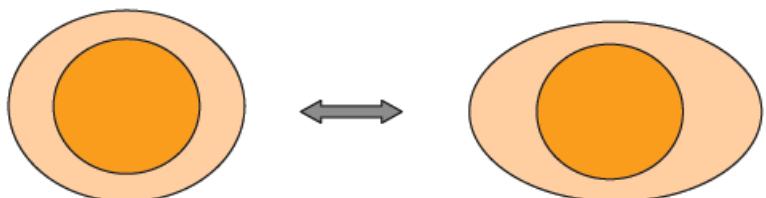
$$B(E2) = 3.4 e^2 \text{fm}^4$$

$$B(E2) = 15.9 e^2 \text{fm}^4$$



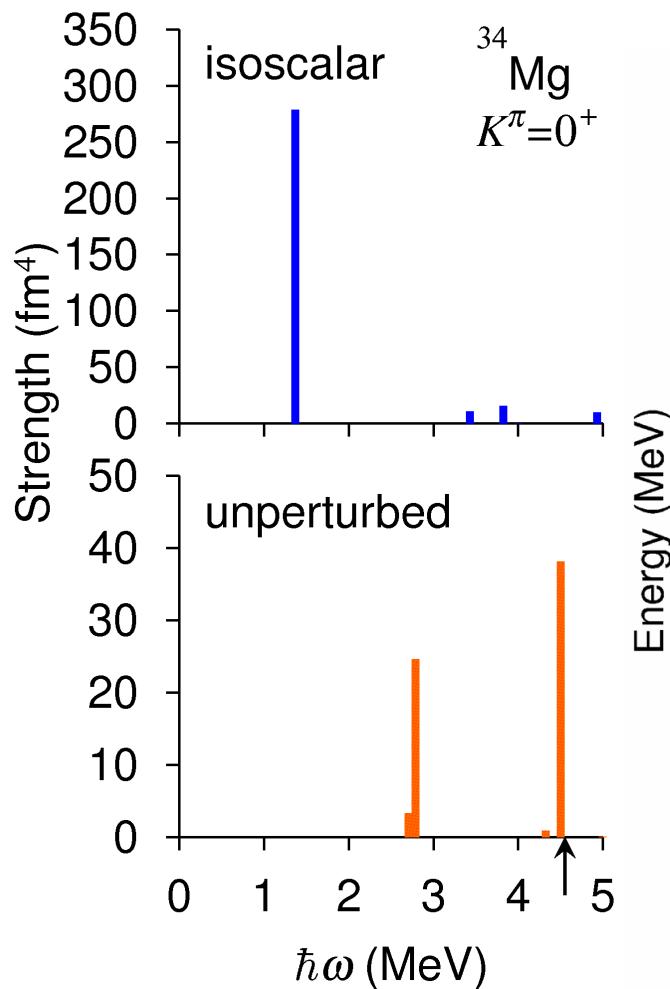
2 $\hbar\omega$  excitations of proton

- $\pi[211]3/2 \rightarrow \pi[431]3/2$  (19.5 MeV)
- $\pi[101]3/2 \rightarrow \pi[321]3/2$  (17.3 MeV)
- $\pi[110]1/2 \rightarrow \pi[330]1/2$  (19.6 MeV)
- $\pi[220]1/2 \rightarrow \pi[440]1/2$  (20.1 MeV)

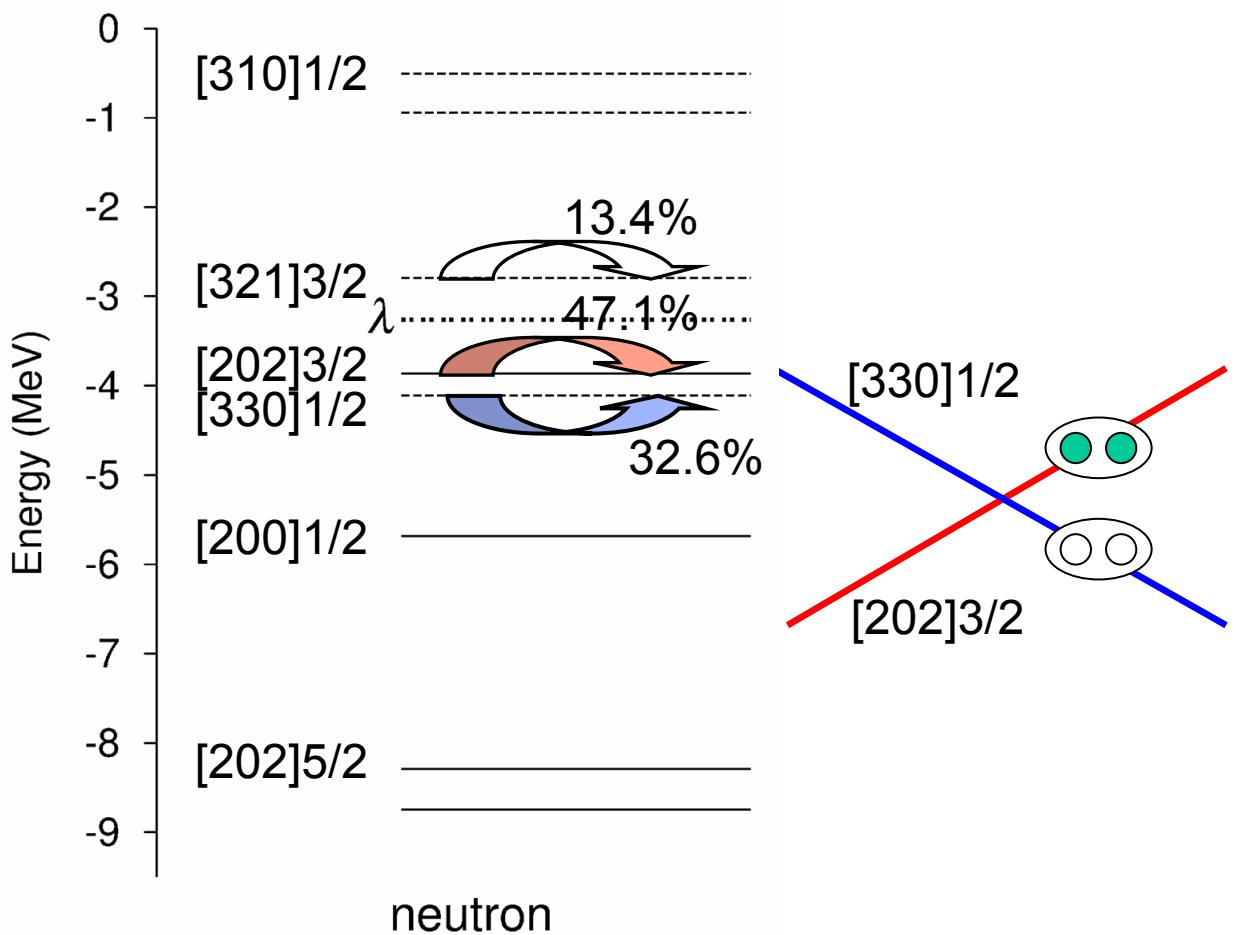


# Soft $K=0^+$ mode in $^{34}\text{Mg}$

$$\beta_2 = 0.4$$



1 W.u.=6.5 fm<sup>4</sup>



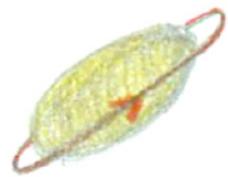
# Quadrupole fluctuation of the neutron pair density

**Ground state**

$$|0\rangle = \frac{1}{\sqrt{a^2 + b^2}} (a |\nu_1 \bar{\nu}_1\rangle + b |\nu_2 \bar{\nu}_2\rangle)$$

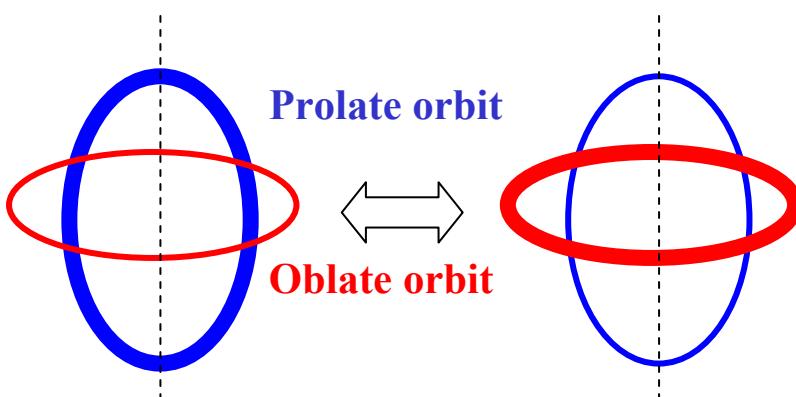
**Excited state**

$$|0'\rangle = \frac{1}{\sqrt{a^2 + b^2}} (-b |\nu_1 \bar{\nu}_1\rangle + a |\nu_2 \bar{\nu}_2\rangle)$$



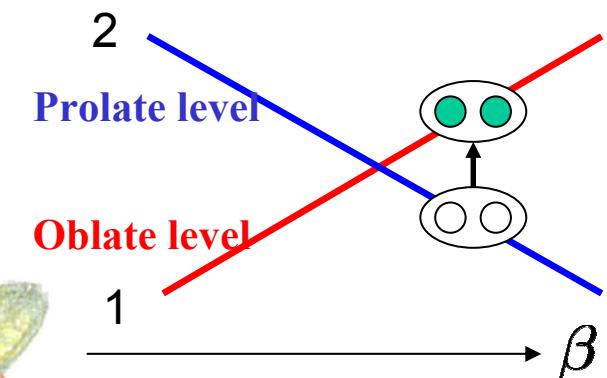
**Transition matrix element**

$$\rightarrow \langle 0' | r^2 Y_{20} | 0 \rangle = \frac{2ab}{a^2 + b^2} \left\{ \langle \nu_2 | r^2 Y_{20} | \bar{\nu}_2 \rangle - \langle \nu_1 | r^2 Y_{20} | \bar{\nu}_1 \rangle \right\}$$



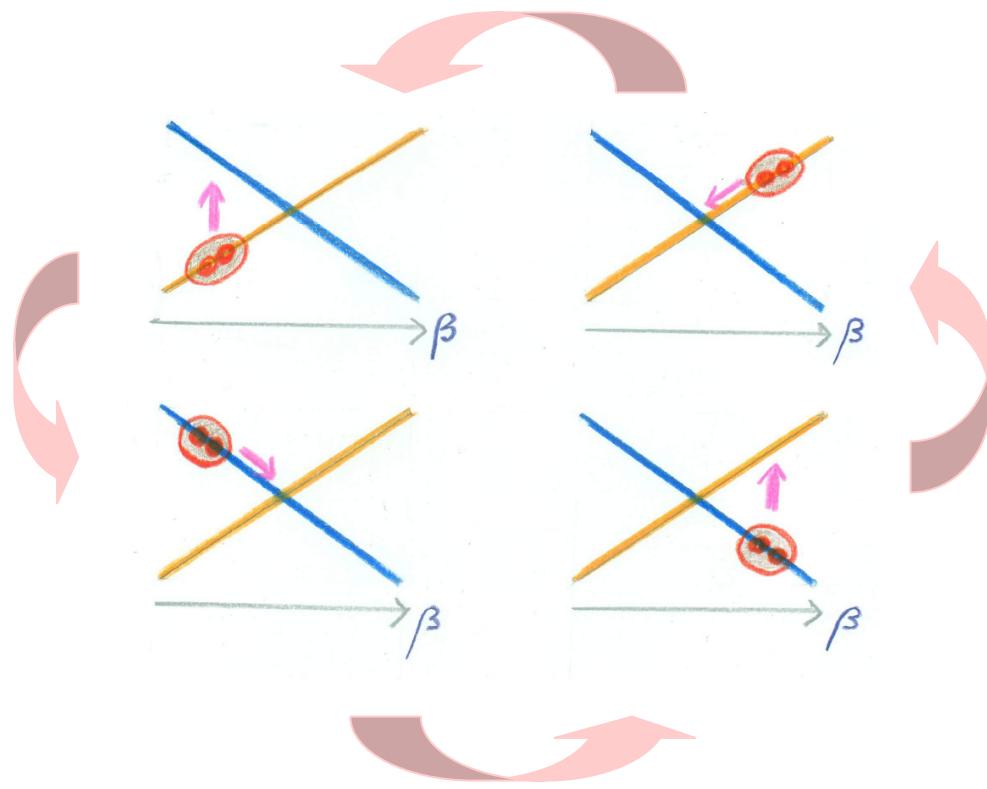
**Coherent for opposite sign**

**(Bohr-Mottelson)**



# Quadrupole oscillation of pair density

Mechanism of coupling between the pair-density fluctuation and the quadrupole shape vibration



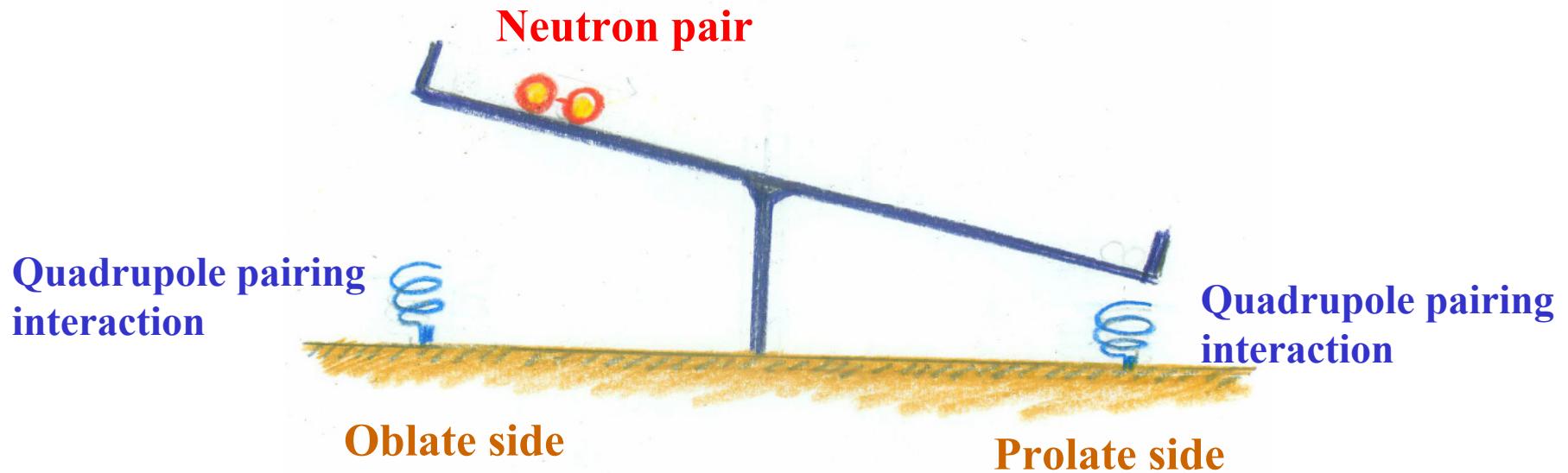
BCS picture



Di-neutron picture

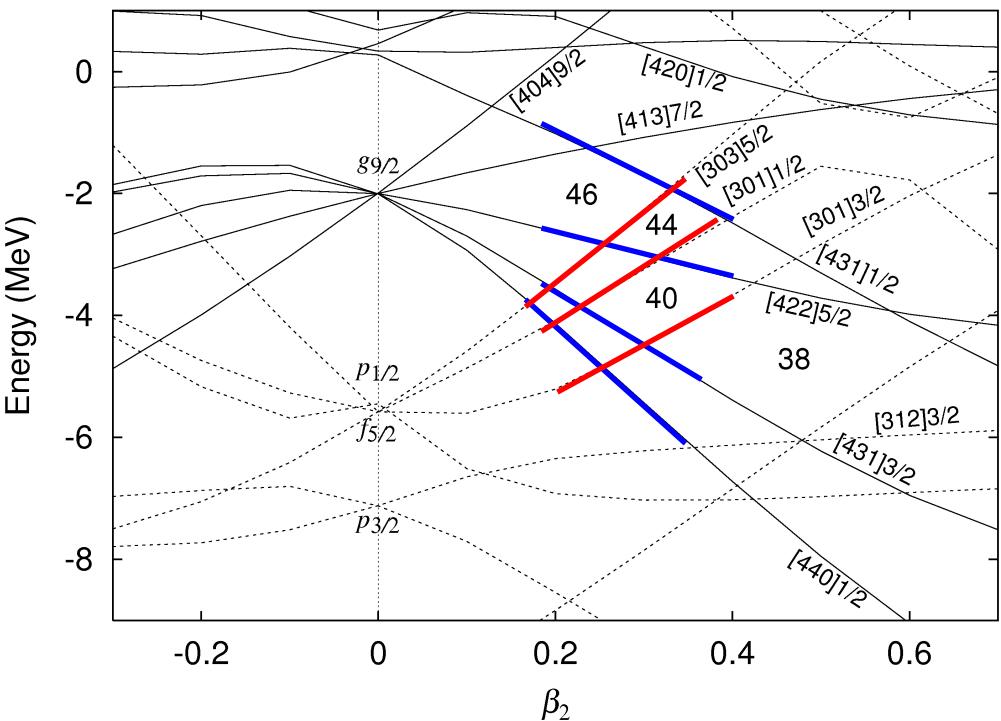


# See-saw Mechanism

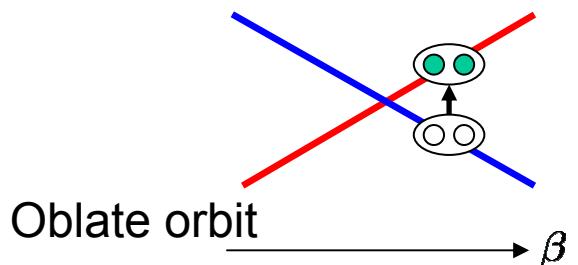


# Neutron-rich Cr and Fe isotopes

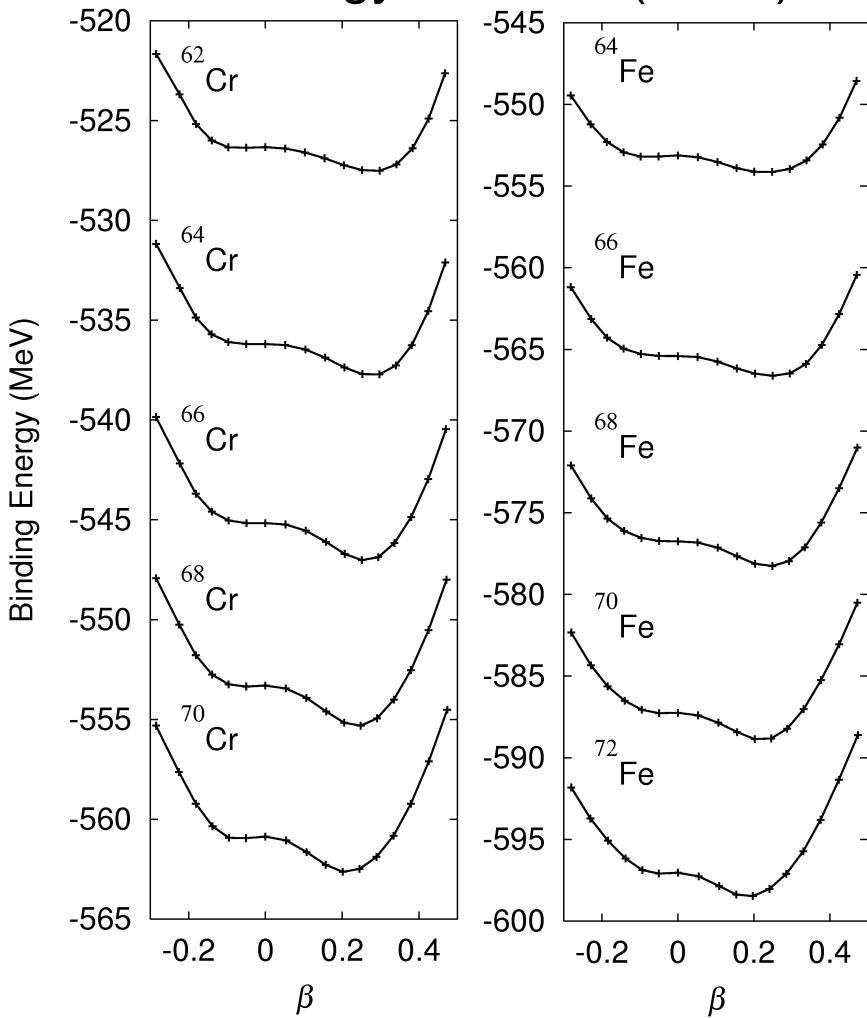
Neutron single-particle energies of  $^{64}\text{Cr}$



Prolate orbit



Potential energy surfaces (SkM\*)



The program “HFBTHO” (v1.66p)

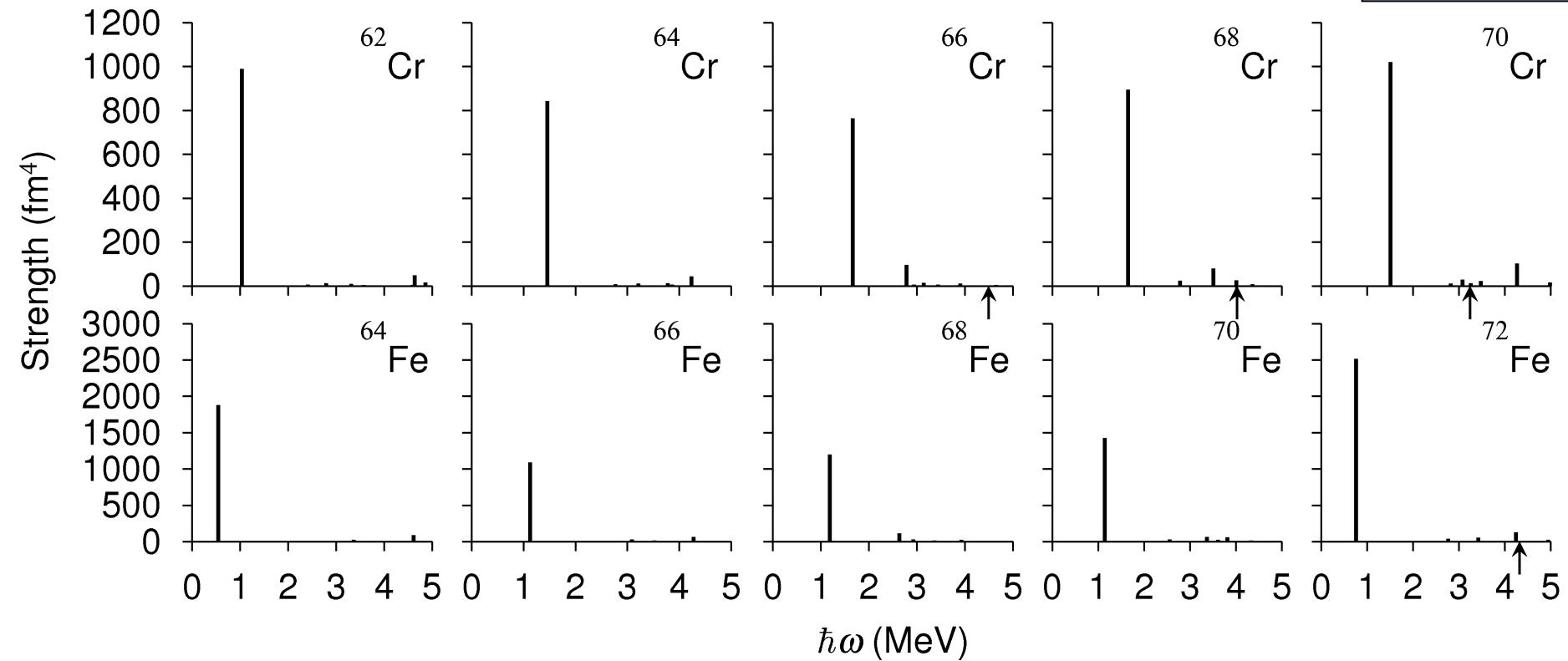
M.Stoitsov *et al.*, Comp.Phys.Comm.167(2005)43

# Soft $K=0^+$ modes in deformed neutron-rich nuclei

$\beta_2 = 0.3$

$N=40$

$K^\pi = 0^+$

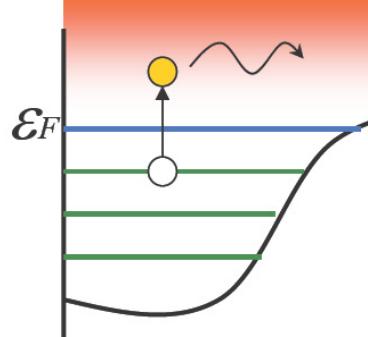


1 W.u.=15~18 fm<sup>4</sup>

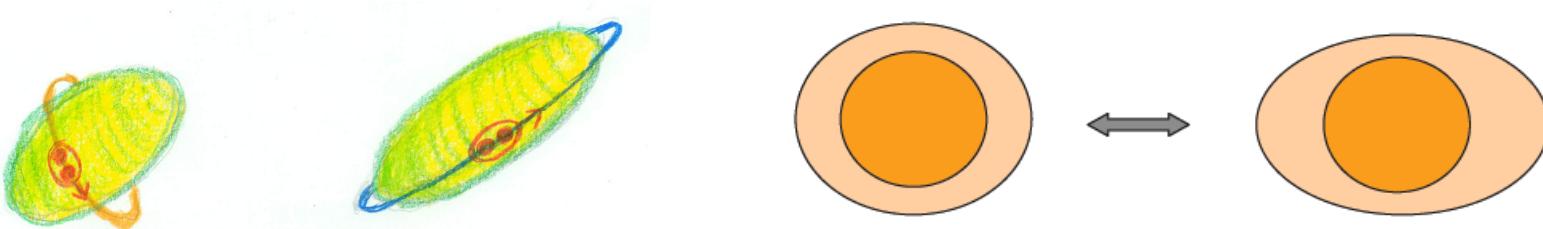
# Conclusion

On the basis of **the deformed QRPA** calculation,  
we have suggested that

**Soft  $K^\pi = 0^+$  modes in deformed neutron-rich nuclei indicate**



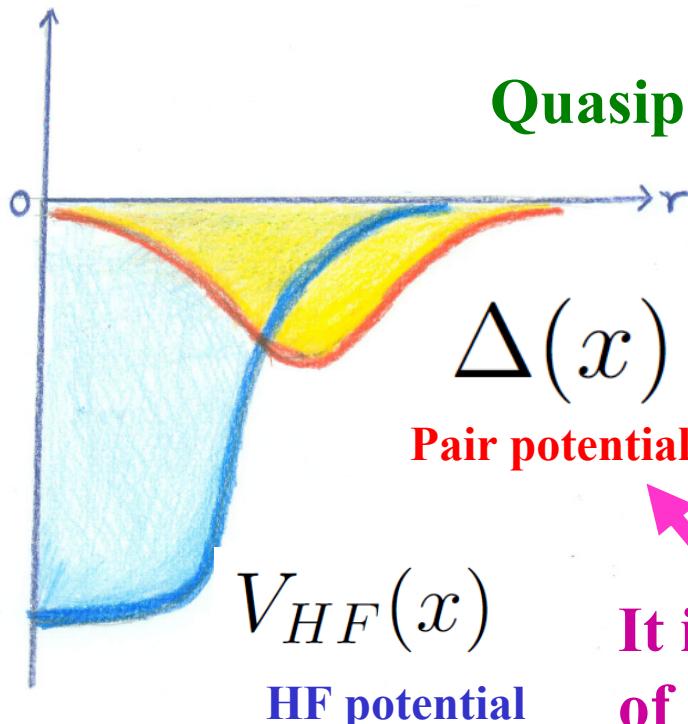
- ♥ Importance of **anisotropic** spacial structure of pairing correlations (**quadrupole pairing**) in generating their collectivities
- ♥ Striking enhancement of transition strengths associated with **very extended** wave functions (**loosely bound and resonance states**)



# New quasiparticle picture different from the BCS quasiparticle

Hartree-Fock-Bogoliubov mean field

$$\begin{pmatrix} t + V_{HF}(x) - \lambda & \Delta(x) \\ -\Delta^*(x) & -t - V_{HF}(x) + \lambda \end{pmatrix} \begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = E \begin{pmatrix} u(x) \\ v(x) \end{pmatrix}$$



Quasiparticle wave function for  $E > |\lambda|$

upper component  $u(x)$  non-localized  
lower component  $v(x)$  localized

It is extended outside of the surface  
of the HF density distribution !!