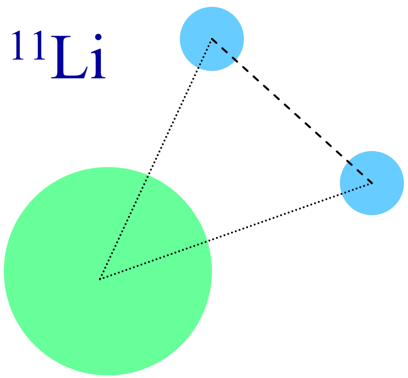
Indication of BCS-BEC crossover behavior in halo nuclei



e-print: nucl-th/0611064

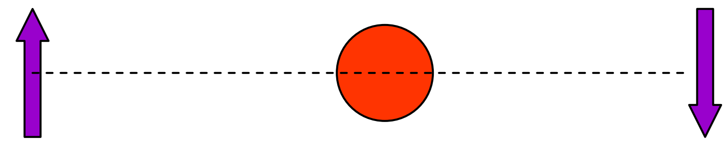
K. Hagino (Tohoku University)
H. Sagawa (University of Aizu)
J. Carbonell (Grenoble)
P. Schuck (IPN, Orsay)

1. *Introduction: Pairing correlations in nuclei*
2. *BCS-BEC crossover behavior in infinite matter*
3. *BCS-BEC crossover behavior in halo nuclei*
4. *Summary*

Introduction: Pairing correlations in nuclei

Coherence length of a Cooper pair:

$$\xi = \frac{\hbar^2 k_F}{m\Delta}$$



much larger than the nuclear size

(note)

$$k_F = 1.36 \ \ \mathrm{fm}^{-1}$$

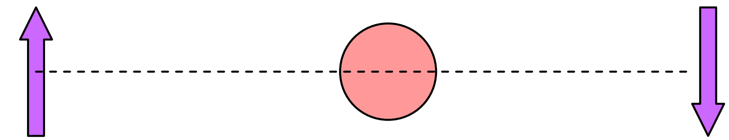
$$\Delta = 12/\sqrt{140} = 1.01 \ \ \mathrm{MeV}$$

(for A=140)

ξ = 55.6 fm

R = 1.2 x 140$^{1/3}$ = 6.23 fm

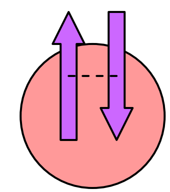
Coherence length of a Cooper pair: $\xi = \dfrac{\hbar^2 k_F}{m\Delta}$



much larger than the nuclear size

?

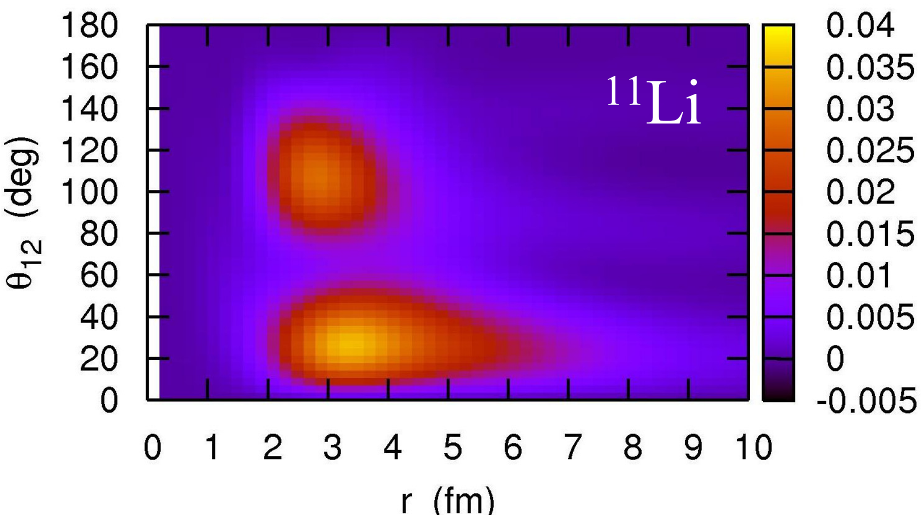
<u>Di-neutron correlations in neutron-rich nuclei</u>

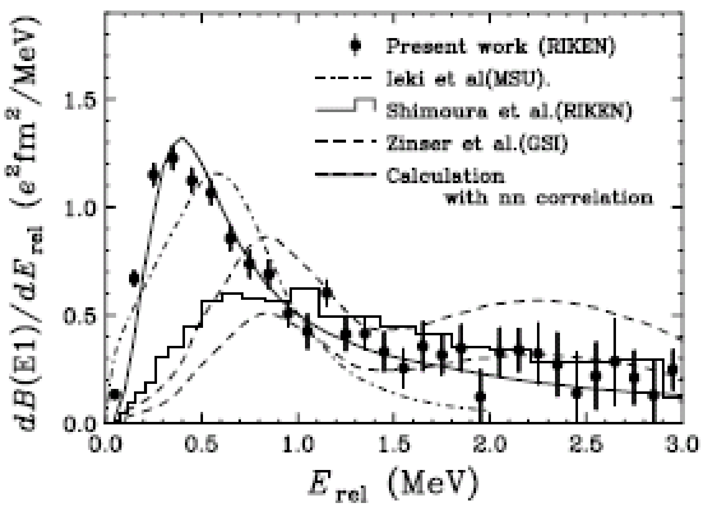


cf. HFB calculations for $^{18\text{-}24}$O, $^{50\text{-}58}$Ca, $^{80\text{-}86}$Ni:

M. Matsuo, K. Mizuyama, Y. Serizawa, PRC71('05)064326

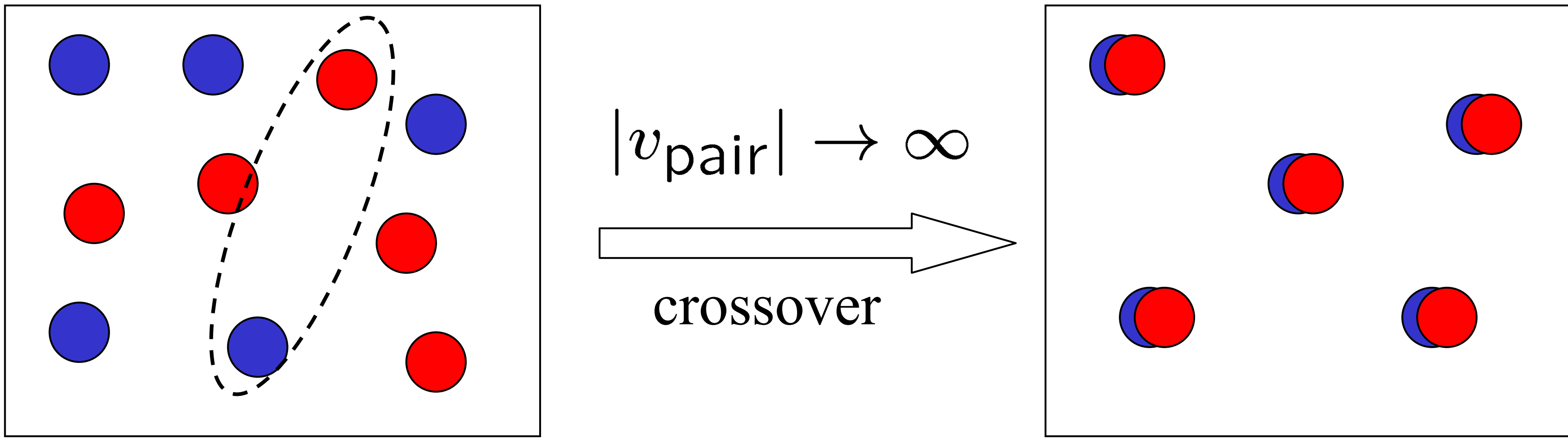
Coul. b.u. of ^{11}Li





K.H. and H. Sagawa, PRC72('05)044321 T. Nakamura et al., PRL96('06)252502

BCS-BEC crossover

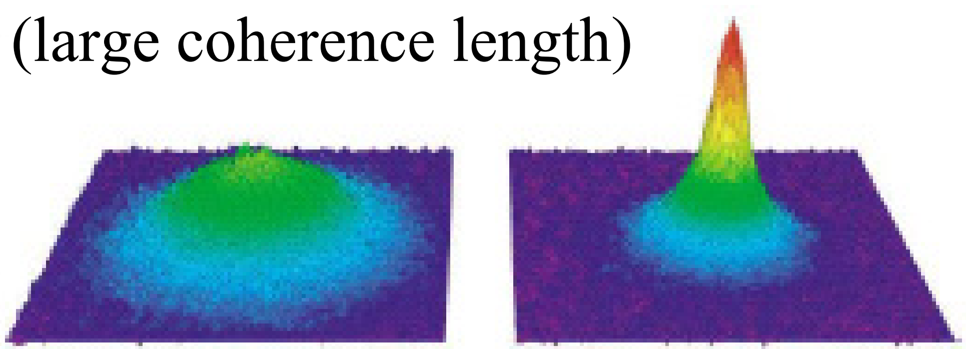


BCS (weak coupling)

- Weakly interacting fermions
- Correlation in p space (large coherence length)

BEC (strong coupling)

- Weakly interacting "diatomic molecules"
- Correlation in r space (small coherence length)



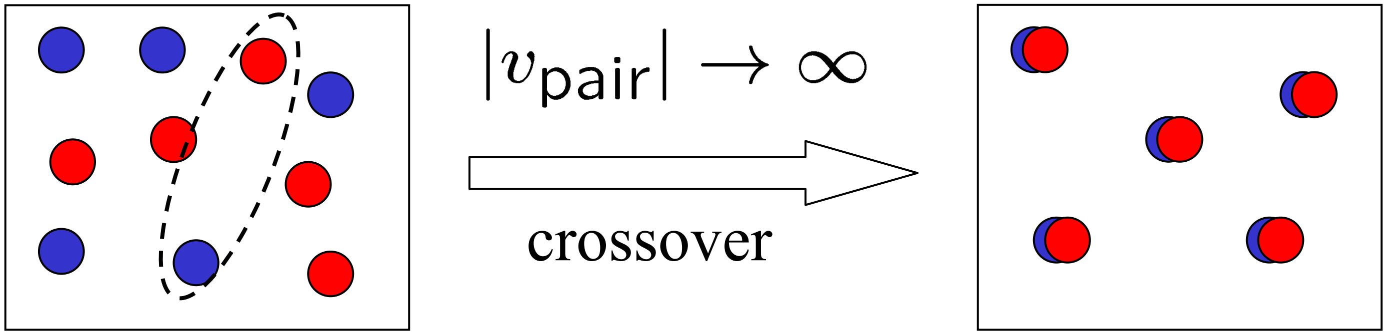
cf. BEC of molecules in ^{40}K

M. Greiner et al., Nature 426('04)537

cf. BCS-BEC crossover in color superconductivity: Y. Nishida and H. Abuki, PRD72('05)096004

BCS-BEC crossover

Cooper pair wave function: $\Psi(\boldsymbol{r},\boldsymbol{r}') \sim \langle \Phi_0 | c^\dagger(\boldsymbol{r},\uparrow)\, c^\dagger(\boldsymbol{r},\downarrow) | \Phi_0 \rangle$

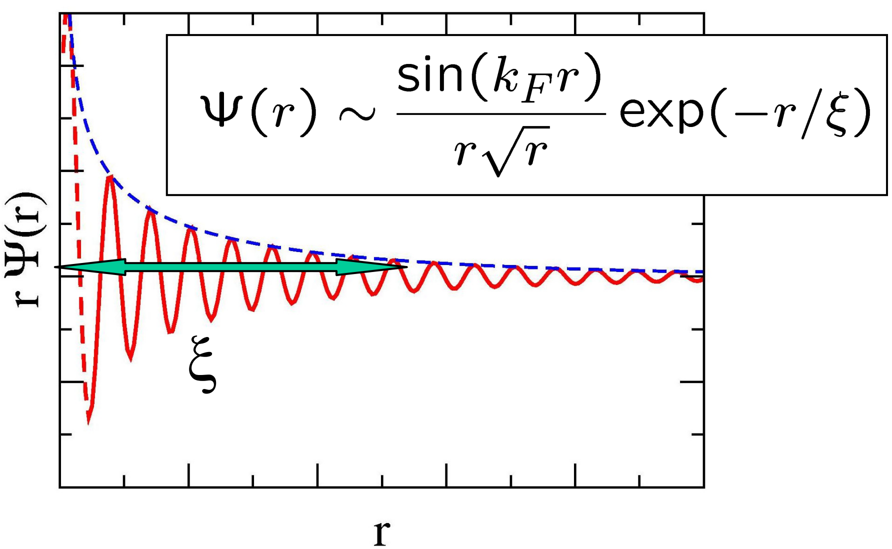


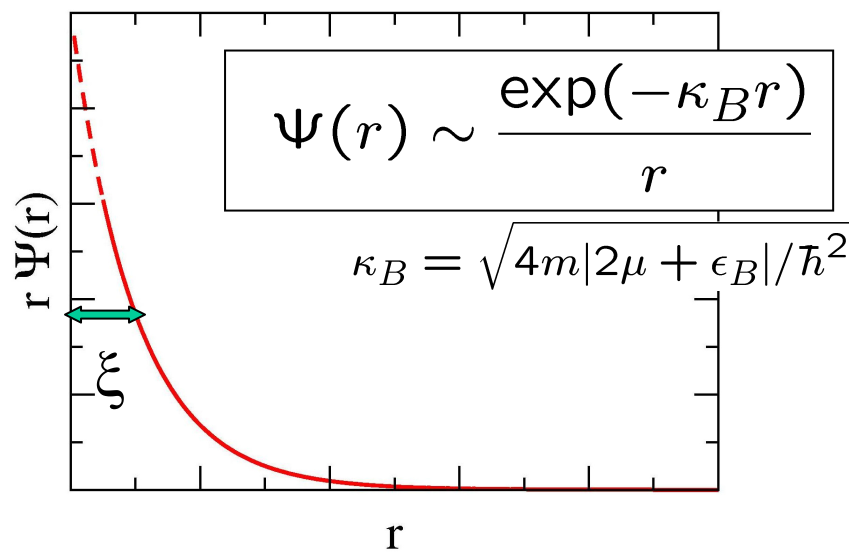
BCS (weak coupling)

Correlation in p space
(large coherence length)

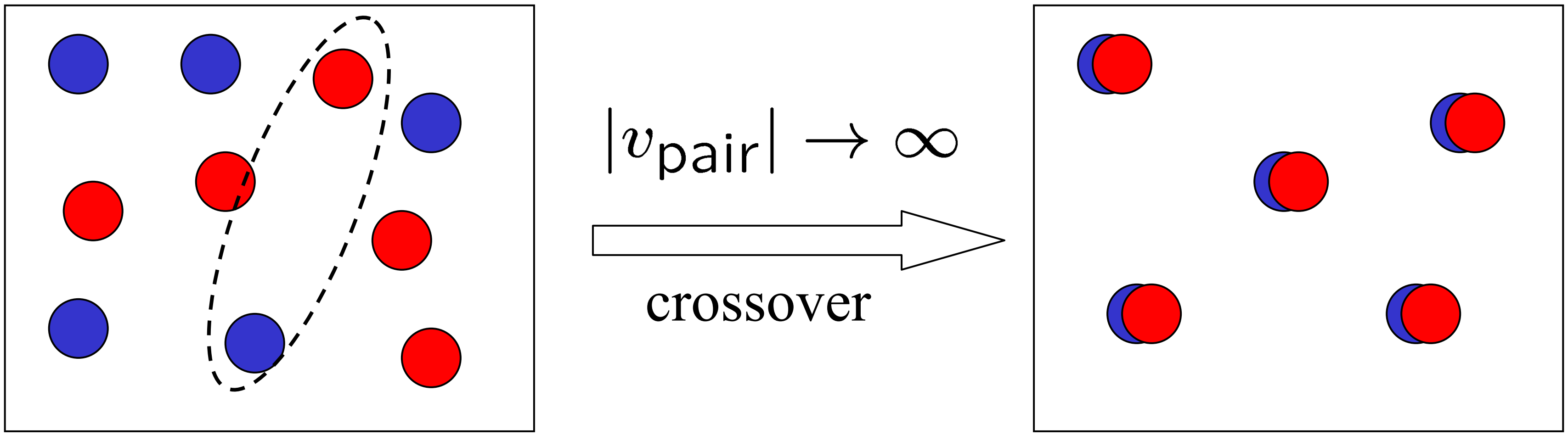
BEC (strong coupling)

Correlation in r space
(small coherence length)





BCS-BEC crossover



BCS (weak coupling)

- Weakly interacting fermions
- Correlation in p space (large coherence length)

??

Pairing in stable nuclei

BEC (strong coupling)

- Weakly interacting "diatomic molecules"
- Correlation in r space (small coherence length)

??

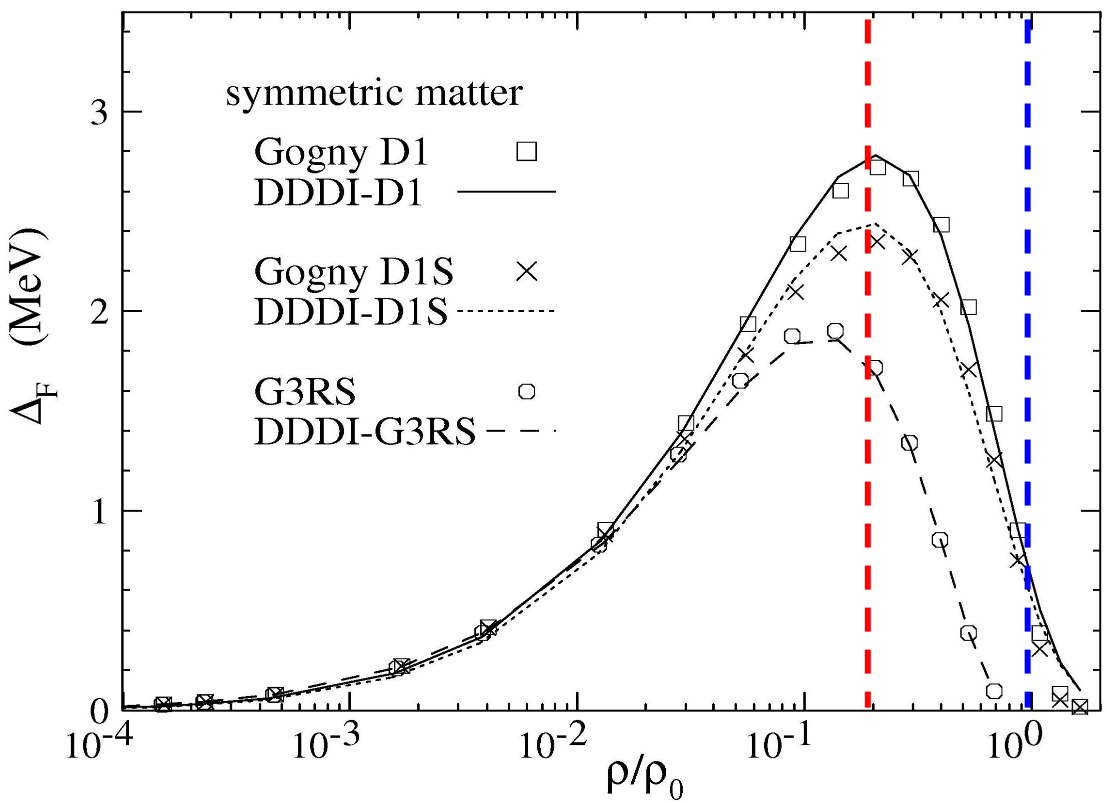
Di-neutron correlations in neutron-rich nuclei

BCS-BEC crossover behavior in infinite nuclear matter

Neutron-rich nuclei ⟺

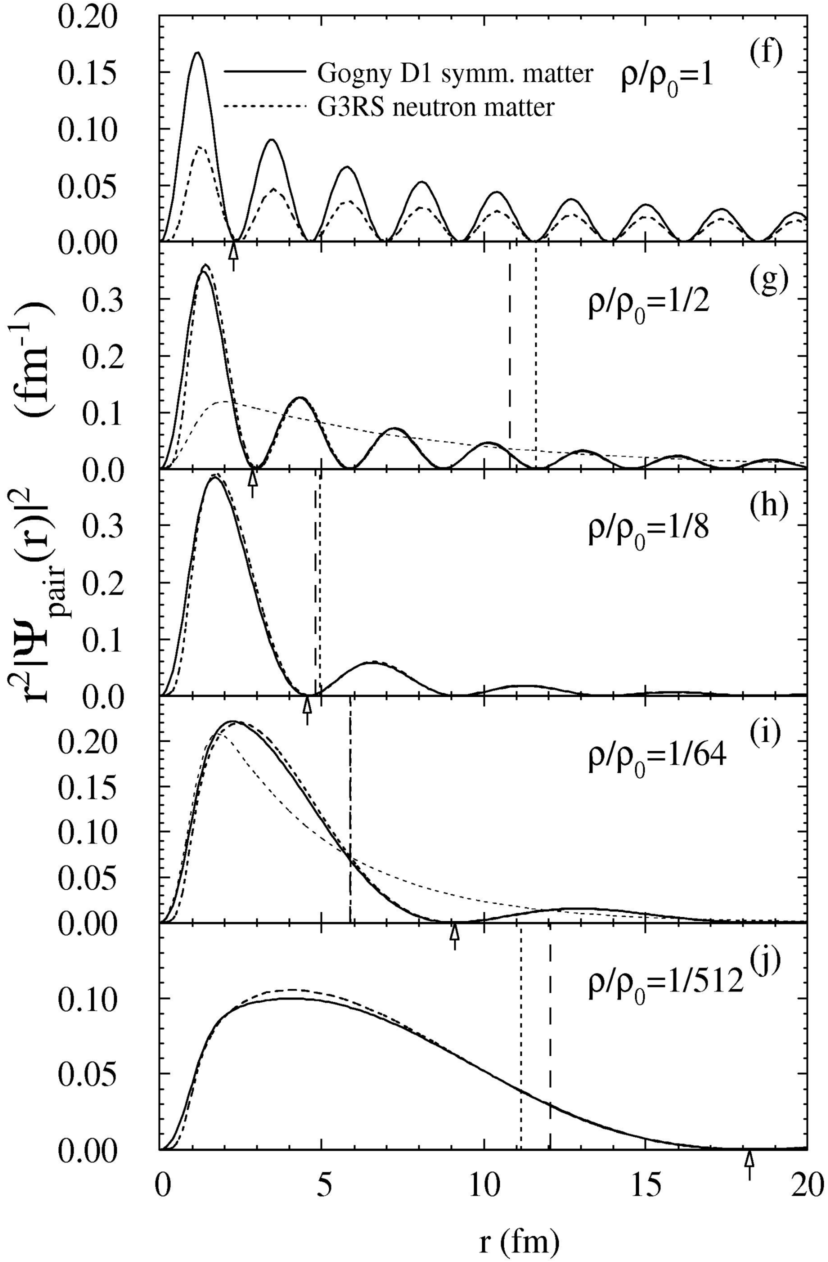
- Weakly bound levels
- Unsaturated density around surface (halo/skin)

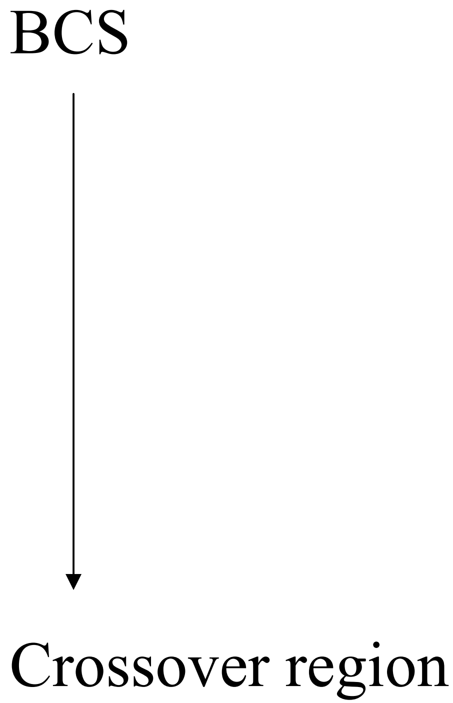
pairing gap in infinite nuclear matter



M. Matsuo, PRC73('06)044309

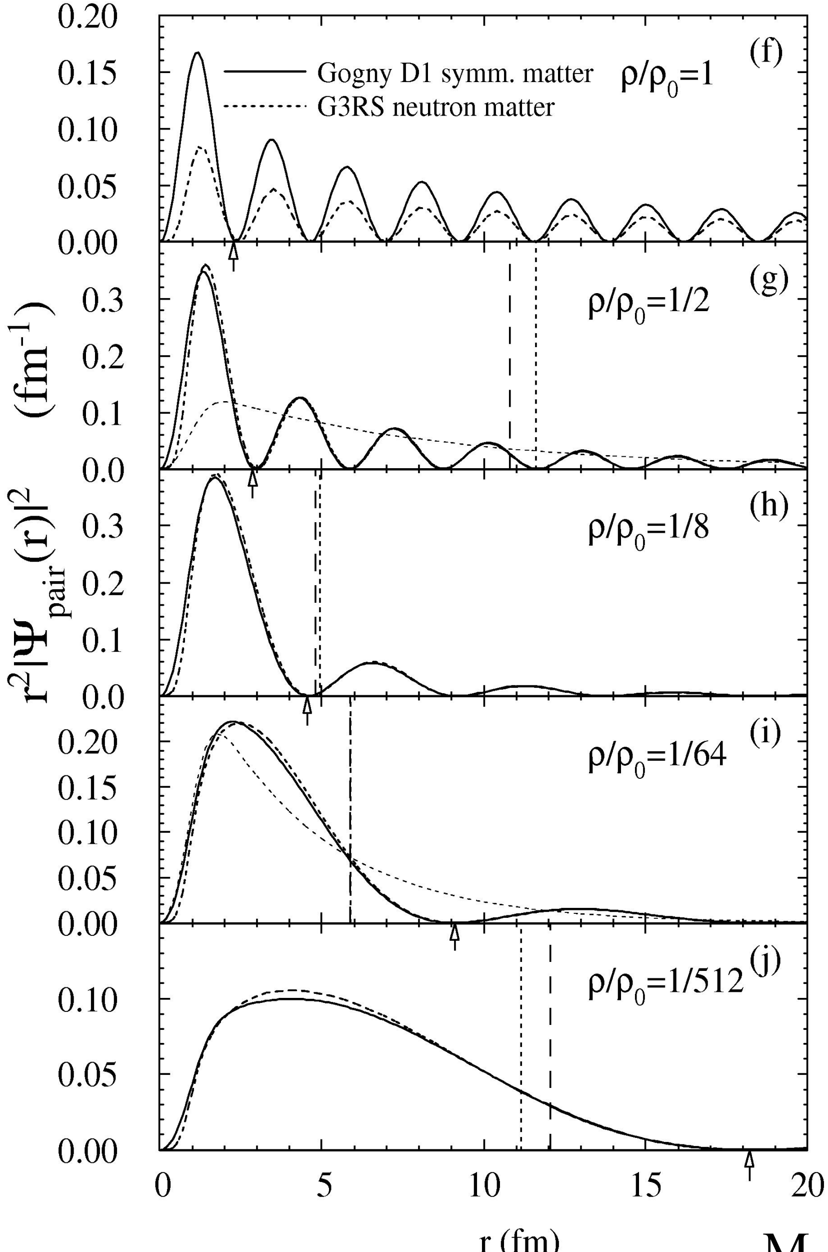
Spatial structure of neutron Cooper pair in infinite matter





M. Matsuo, PRC73('06)044309

Spatial structure of neutron Cooper pair in infinite matter



Our Motivations

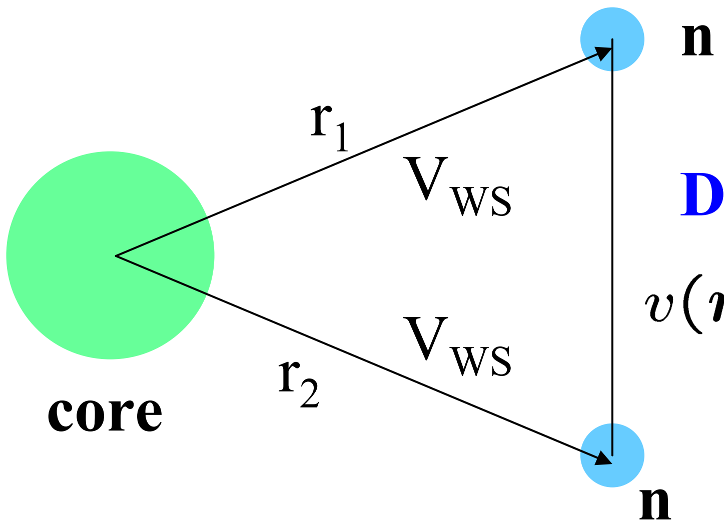
- How can this behavior be seen in *finite* nuclei?
- Relation to di-neutron correlation?

$^{11}\mathrm{Li} = {}^{9}\mathrm{Li} + \mathrm{n} + \mathrm{n}$

M. Matsuo, PRC73('06)044309

BCS-BEC crossover behavior in halo nuclei

Three-body model



G.F. Bertsch and H. Esbensen, *Ann. of Phys. 209('91)327*
H. Esbensen, G.F. Bertsch, K. Hencken, *Phys. Rev. C56('99)3054*

Density-dependent delta-force

$$v(\boldsymbol{r}_1,\boldsymbol{r}_2) = v_0(1+\alpha\rho(r)) \times\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)$$

$$\boxed{H=\frac{\boldsymbol{p}_1^2}{2m}+\frac{\boldsymbol{p}_2^2}{2m}+V_{nC}(r_1)+V_{nC}(r_2)+V_{nn}+\frac{(\boldsymbol{p}_1+\boldsymbol{p}_2)^2}{2A_cm}}$$

Diagonalization: $\boxed{\Psi_{gs}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\mathcal{A}\sum_{nn'lj}\alpha_{nn'lj}\,\Psi^{(2)}_{nn'lj}(\boldsymbol{r}_1,\boldsymbol{r}_2)}$

$$\Psi^{(2)}_{nn'lj}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\sum_m\langle jmj-m|00\rangle\psi_{nljm}(\boldsymbol{r}_1)\psi_{n'lj-m}(\boldsymbol{r}_2)$$

Application to ^{11}Li

$$V_{nn}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\left(v_0+\frac{v_\rho}{1+\exp[(r_1-R_\rho)/a_\rho]}\right)$$

$$a_{nn}=-15\ \mathrm{fm},\ \ E_{\mathrm{cut}}=30\ \mathrm{MeV},\ \ R_{box}=40\ \mathrm{fm}$$

$$\Rightarrow\quad v_0=\frac{2\pi^2\hbar^2}{m}\cdot\frac{2a_{nn}}{\pi-2k_c\,a_{nn}}$$

$v_\rho,\ R_\rho,\ a_\rho$: adjusted so that S_{2n} can be reproduced

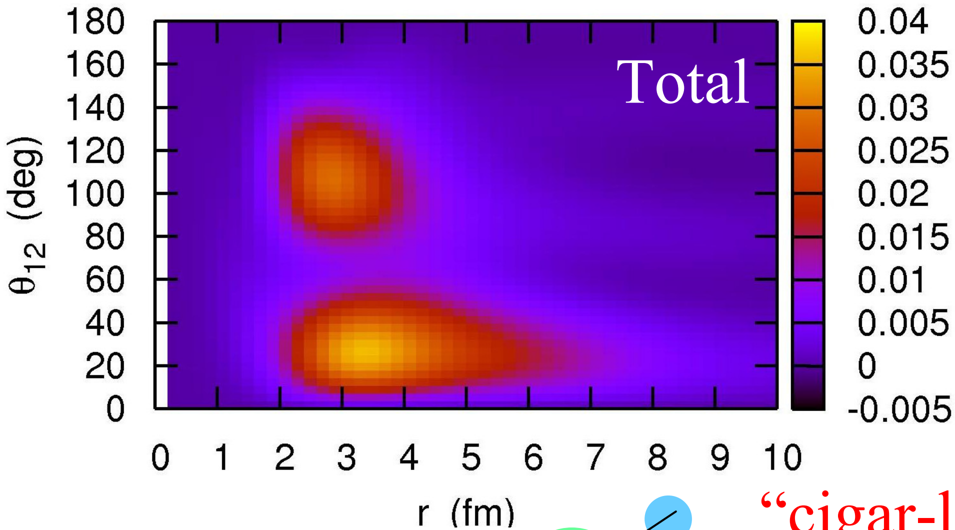
WS: adjusted to $p_{3/2}$ energy in ^{8}Li & n-^{9}Li elastic scattering (maximum: near 540 keV)

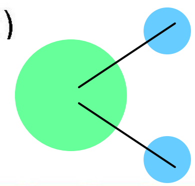
Parity-dependence $\longleftarrow$ to increase the s-wave component

S_{2n} (MeV)	$\langle r^2_{nn}\rangle$ (fm^2)	$\langle r^2_{c-2n}\rangle$ (fm^2)	dominant config.	fraction (%)	$S=0$ (%)
0.295	41.4	26.3	$(p_{1/2})^2$	59.1	60.6
			$(s_{1/2})^2$	22.7	

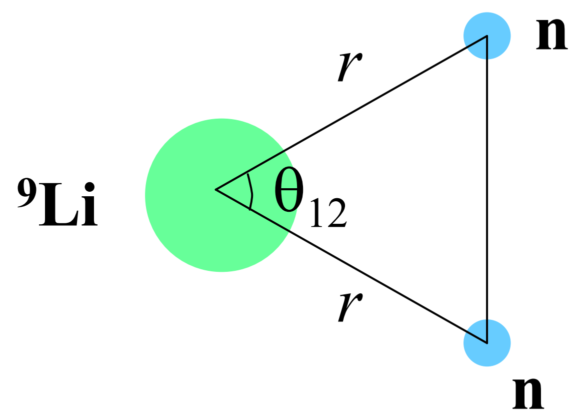
Two-particle density for ^{11}Li

K.H. and H. Sagawa, PRC72('05)044321

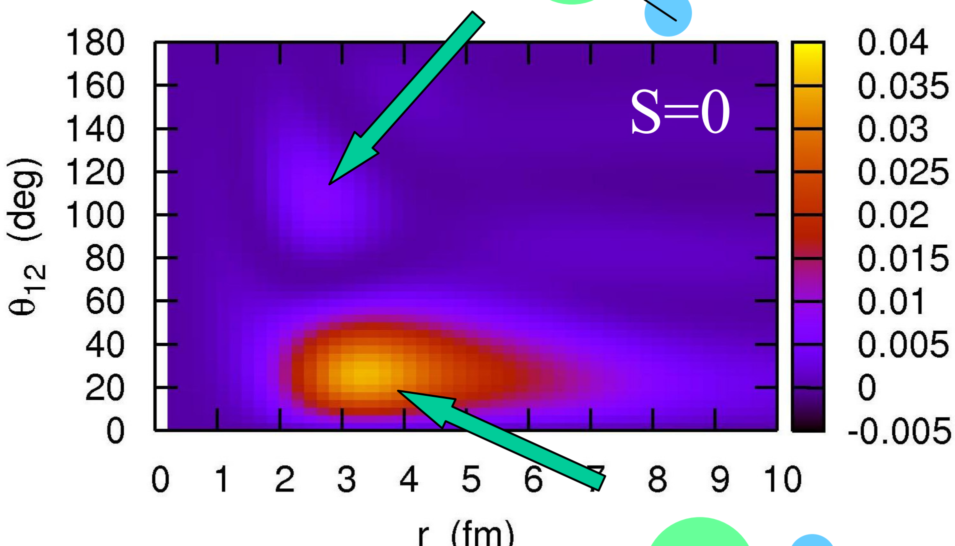


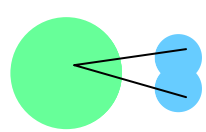


"cigar-like" configuration



$$8\pi^2 r^4 \sin\theta_{12} \cdot \rho_2(r, r, \theta_{12})$$

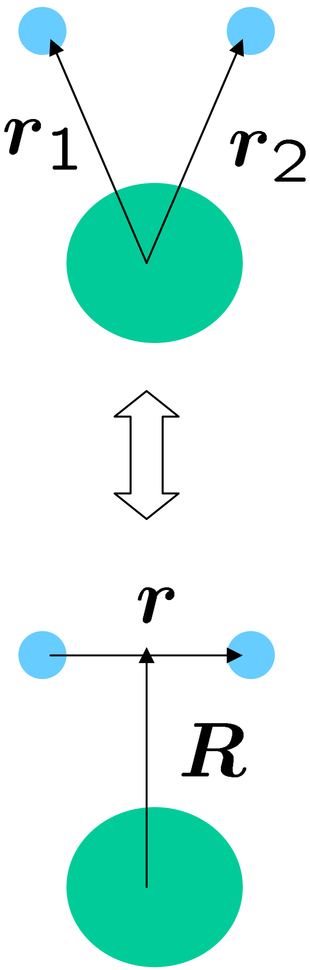




"di-neutron" configuration

Di-neutron wave function in Borromean nuclei

$$\boxed{\Psi^{(S=0)}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\sum_L f_L(r,R)[Y_L(\hat{\boldsymbol{r}})Y_L(\hat{\boldsymbol{R}})]^{(00)}}$$



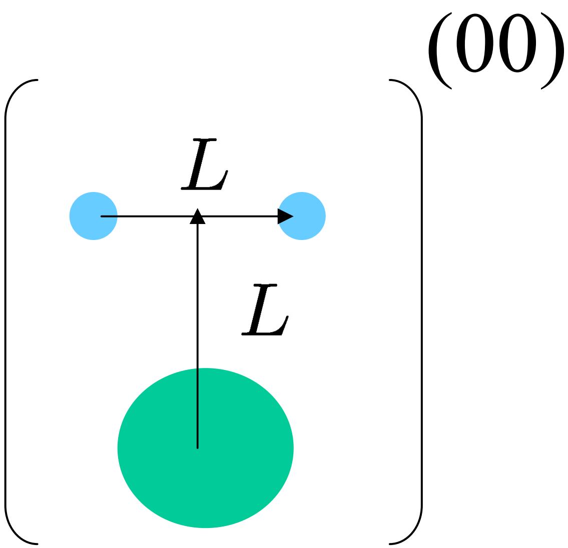
$$\begin{aligned}
f_L(r,R) &= \sum_{n'\leq n}\sum_{l,j}\alpha_{nn'lj}\,(-)^{l+L}\frac{\sqrt{2\pi(2j+1)}}{\sqrt{2(1+\delta_{n,n'})}}\\
&\times\int_0^\pi \sin\theta d\theta\, Y_{L0}(\theta)\,\sum_m(-)^m\frac{(l-m)!}{(l+m)!}\\
&\times P_l^m(\cos\theta_1)P_l^m(\cos\theta_2)\\
&\times\phi_{nlj}(r_1)\phi_{n'lj}(r_2)\\
r_1 &= \sqrt{R^2+r^2/4+Rr\cos\theta}\\
r_2 &= \sqrt{R^2+r^2/4-Rr\cos\theta}\\
\cos\theta_1 &= (R+r\cos\theta/2)/r_1\\
\cos\theta_2 &= (R-r\cos\theta/2)/r_2
\end{aligned}$$

B.F. Bayman and A. Kallio, Phys. Rev. 156('56)1121

Di-neutron wave function in Borromean nuclei

$$\boxed{\Psi^{(S=0)}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\sum_L f_L(r,R)[Y_L(\hat{\boldsymbol{r}})Y_L(\hat{\boldsymbol{R}})]^{(00)}}$$

$$P_L=\int r^2dr\int R^2dR\,|f_L(r,R)|^2$$

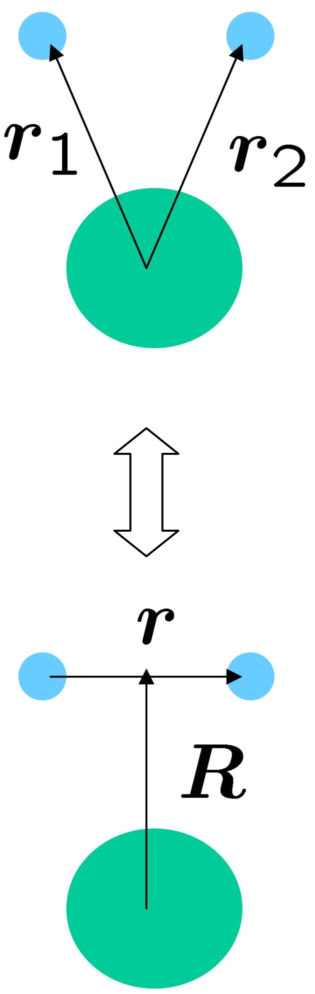


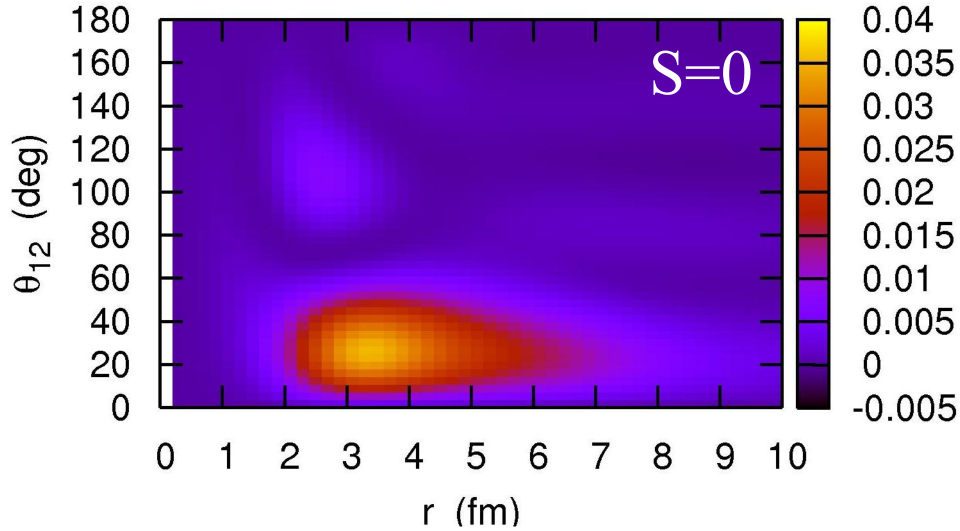
$$\begin{array}{|lcl|}\hline P_{L=0} & = & 0.578 \\ P_{L=2} & = & 0.020 \\ P_{L=4} & = & 0.00452 \\ \hline\end{array}$$

Sum = 0.603

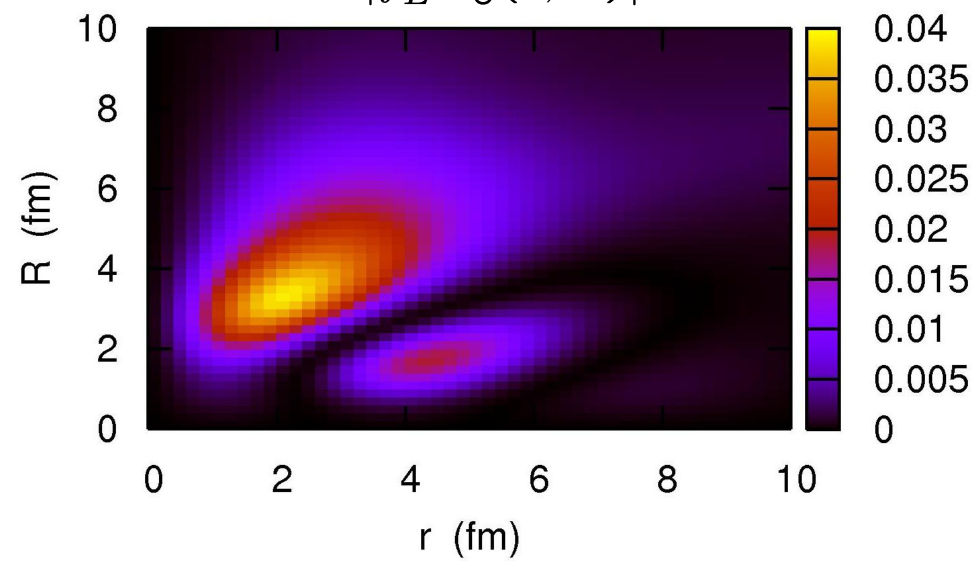
$\Longleftrightarrow$ $P_{S=0} = 0.606$

$$\Psi^{(S=0)}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\sum_L f_L(r,R)[Y_L(\hat{\boldsymbol{r}})Y_L(\hat{\boldsymbol{R}})]^{(00)}$$

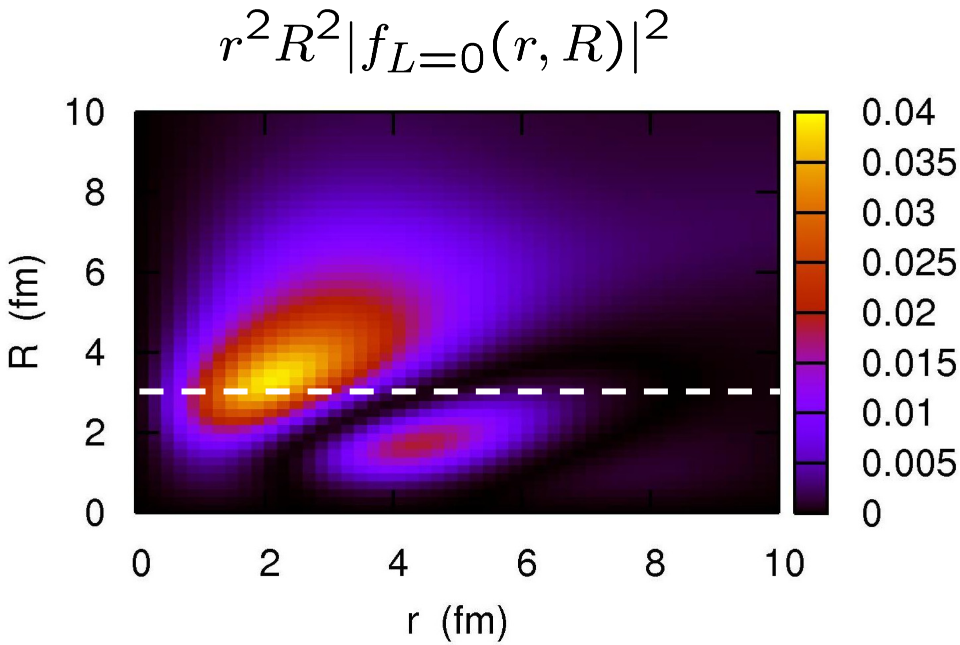


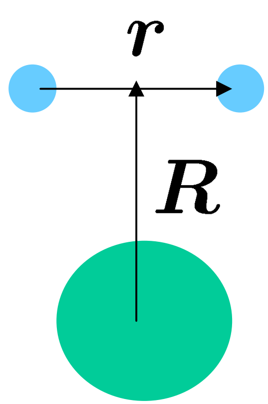


$r^2R^2|f_{L=0}(r,R)|^2$

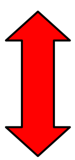


Plot the wf at several values of *R*

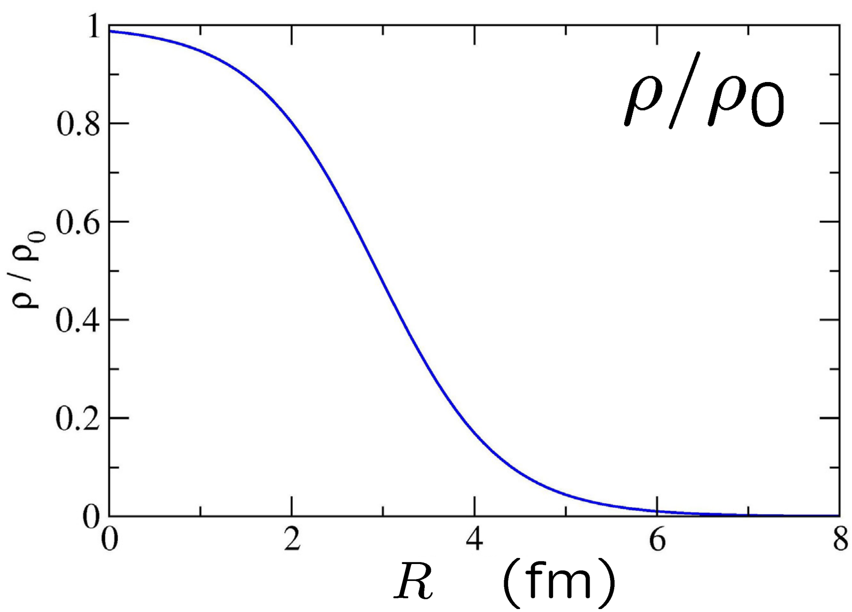




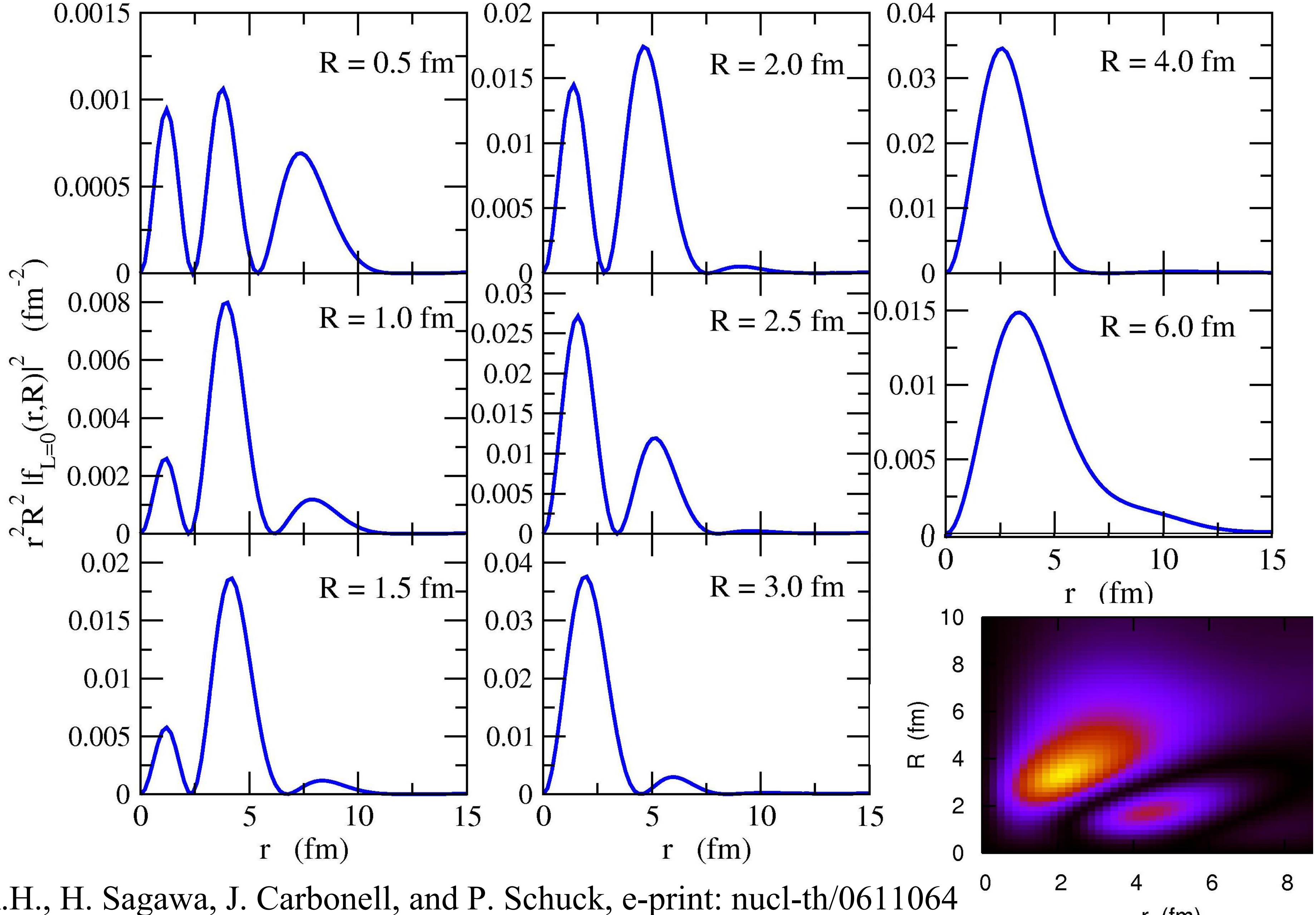
$$v_{nn} = F[\rho(R)] \cdot \delta(\boldsymbol{r})$$



Probing the behavior at several densities

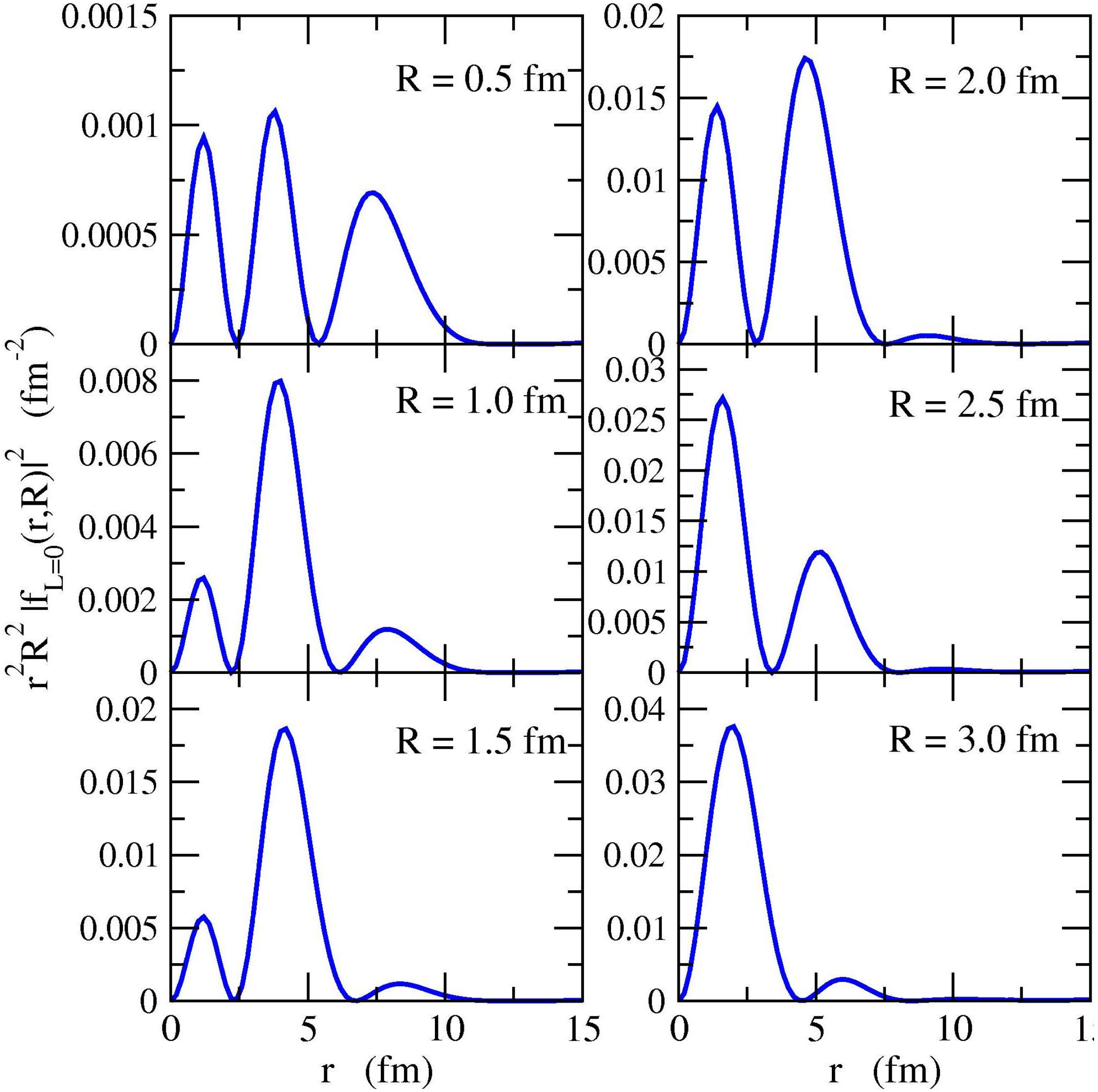


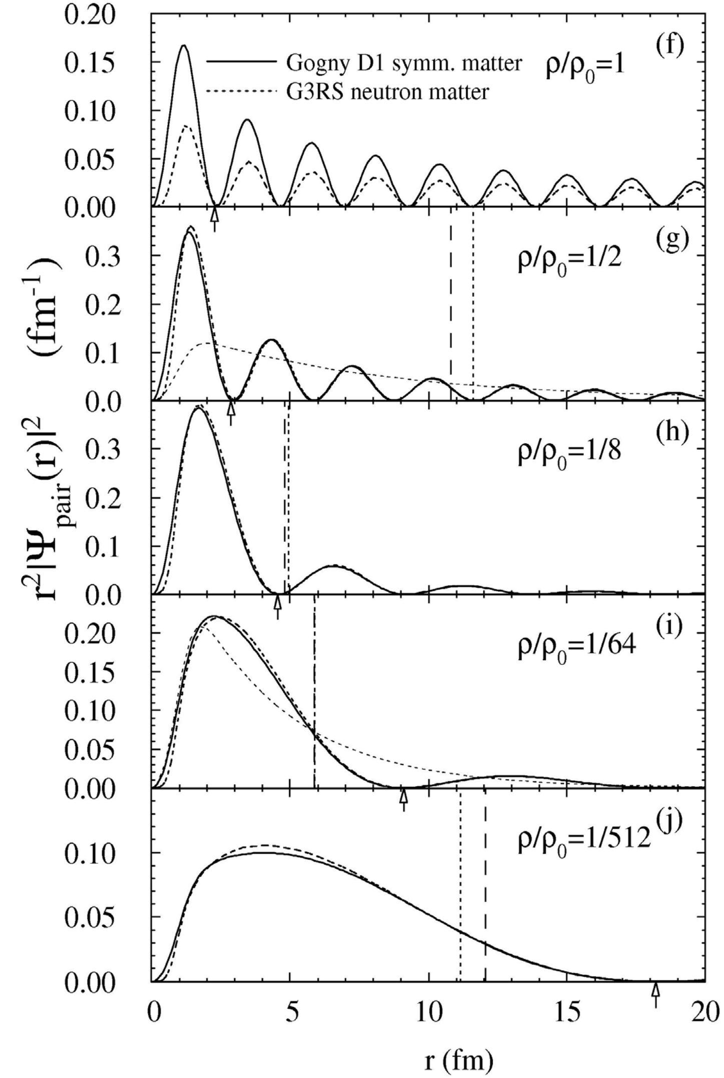
$$r^2 R^2 |f_{L=0}(r, R)|^2$$



K.H., H. Sagawa, J. Carbonell, and P. Schuck, e-print: nucl-th/0611064 good correspondence

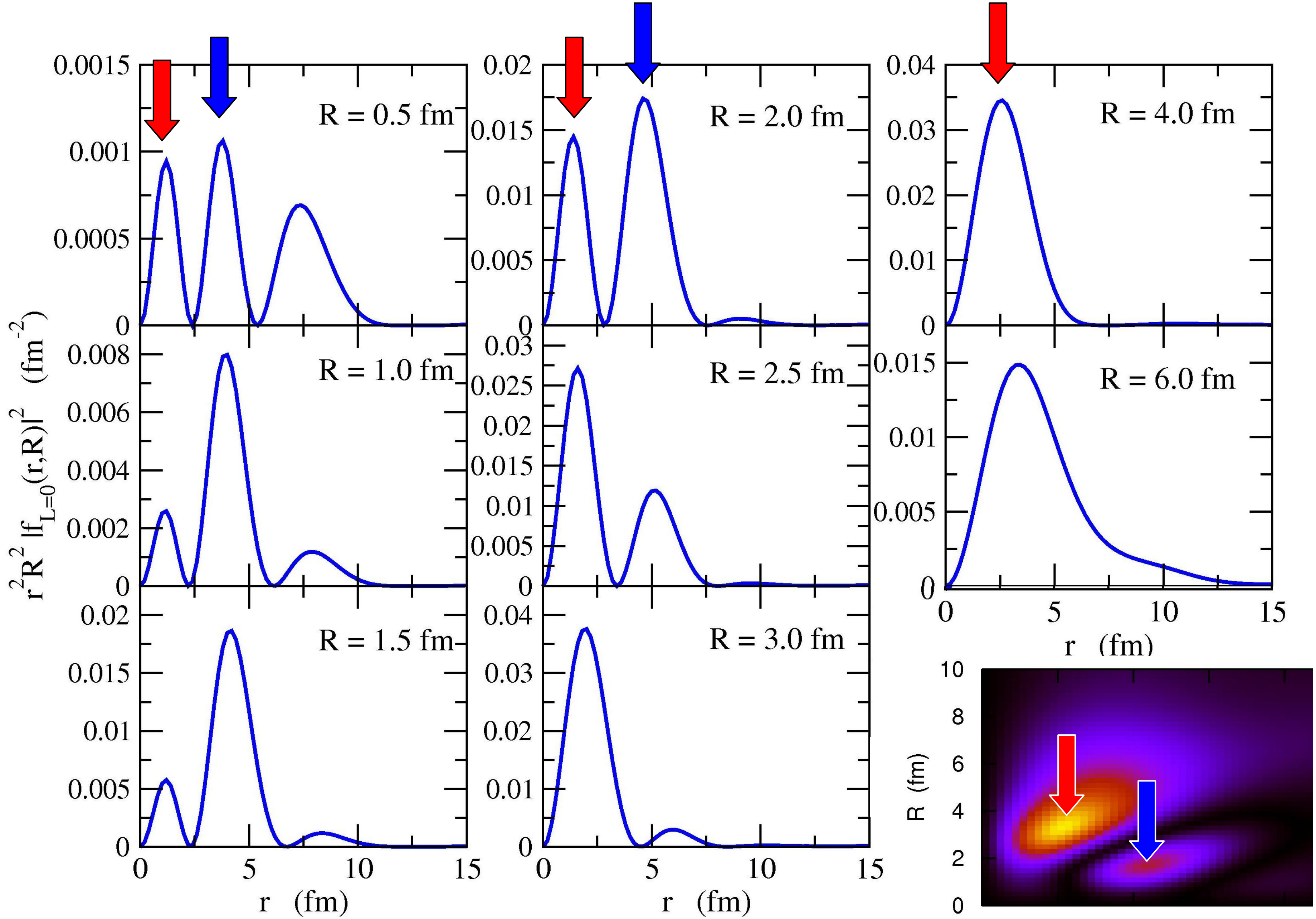
^{11}Li ⟷ Nulcear Matter Calc.





K.H., H. Sagawa, J. Carbonell, and P. Schuck, e-print: nucl-th/0611064

M. Matsuo, PRC73('06)044309

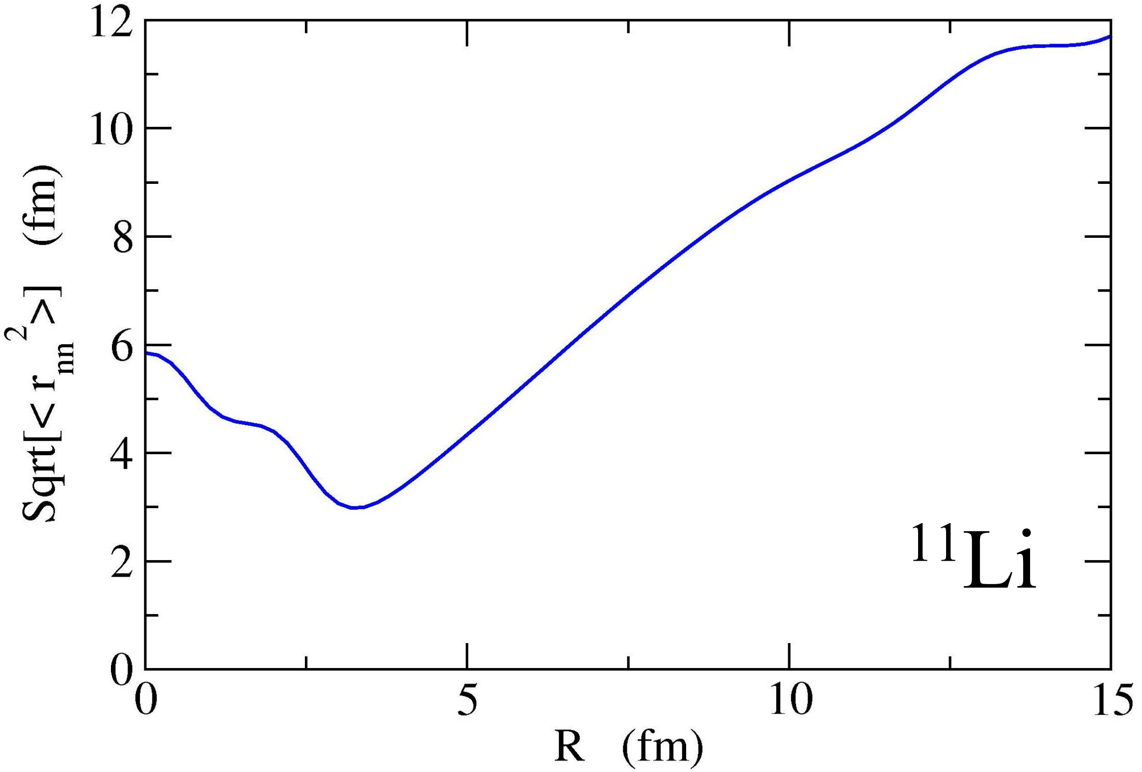


: di-neutron
configurations
: cigar-like

2-neutron rms distance

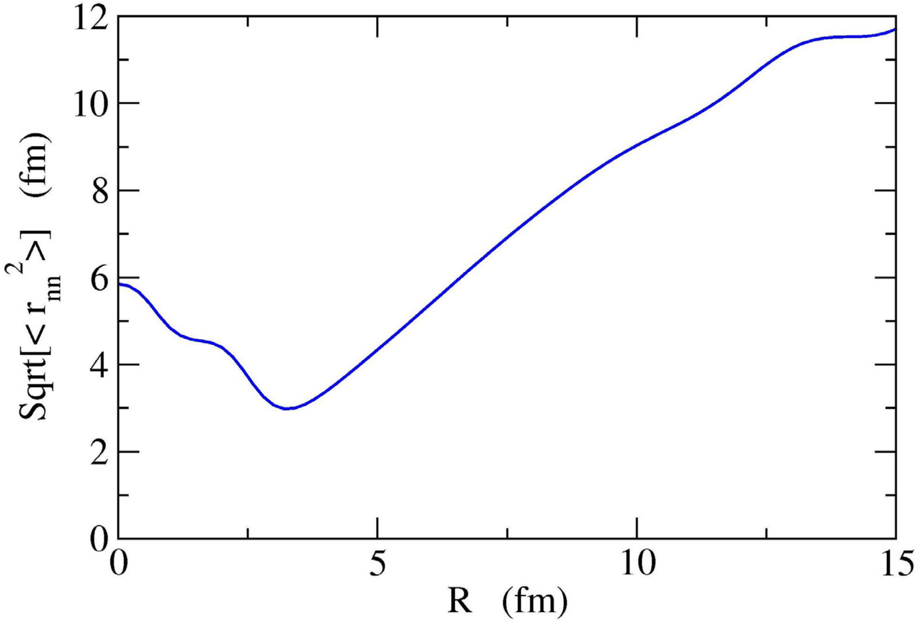
$$\sqrt{\langle r_{nn}^2 \rangle}(R) = \sqrt{\frac{\int r^4 dr\, |f_{L=0}(r,R)|^2}{\int r^2 dr\, |f_{L=0}(r,R)|^2}}$$

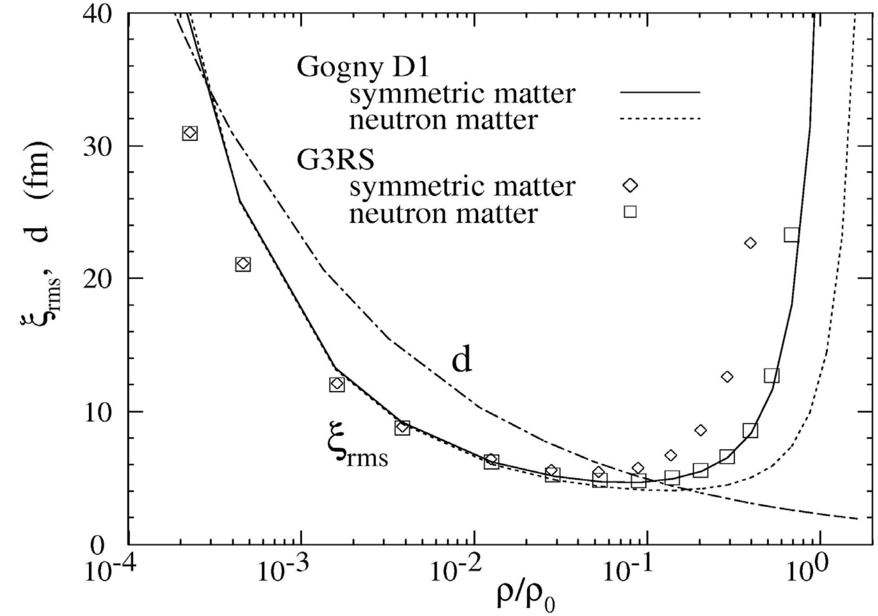
$$\Psi^{(S=0)}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \sum_L f_L(r,R)[Y_L(\hat{\boldsymbol{r}})Y_L(\hat{\boldsymbol{R}})]^{(00)}$$



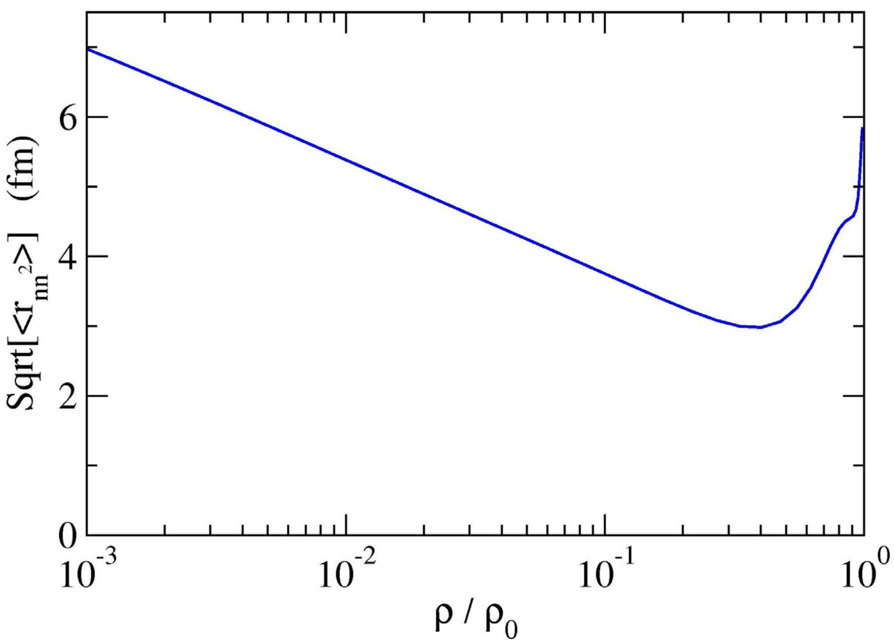
$$\sqrt{\langle r_{nn}^2 \rangle}(R) = \sqrt{\frac{\int r^4 dr\, |f_{L=0}(r,R)|^2}{\int r^2 dr\, |f_{L=0}(r,R)|^2}}$$

Matter Calc.





M. Matsuo, PRC73('06)044309



cf. Free n-n system

virtual state around zero energy

⇨ $\langle r \rangle \sim 12$ fm

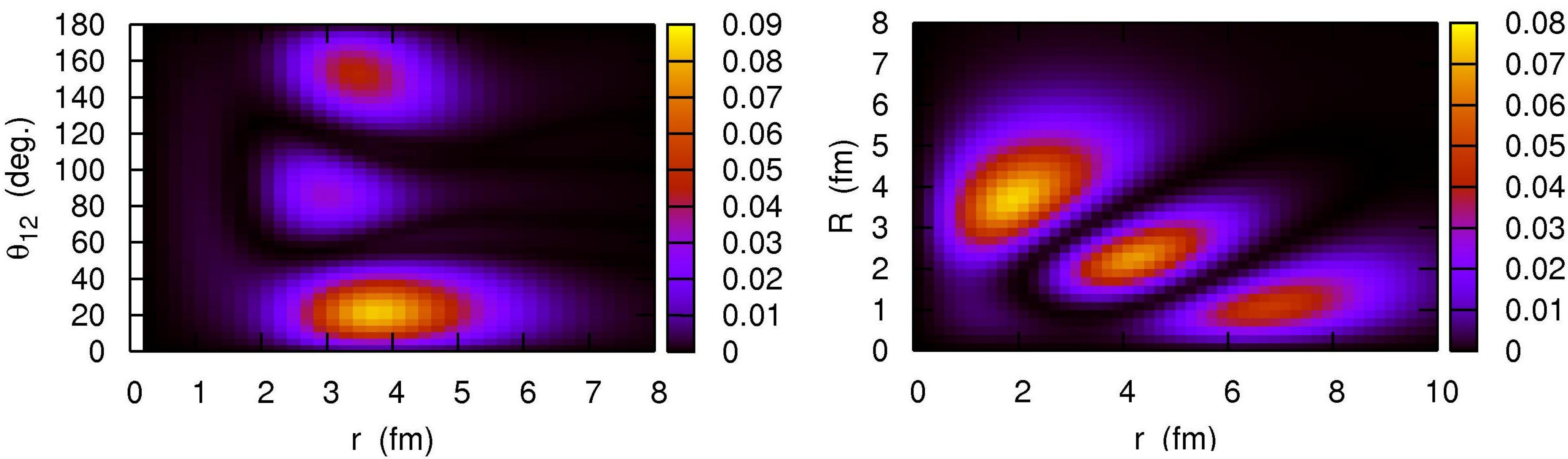
(Nijmegen potential)

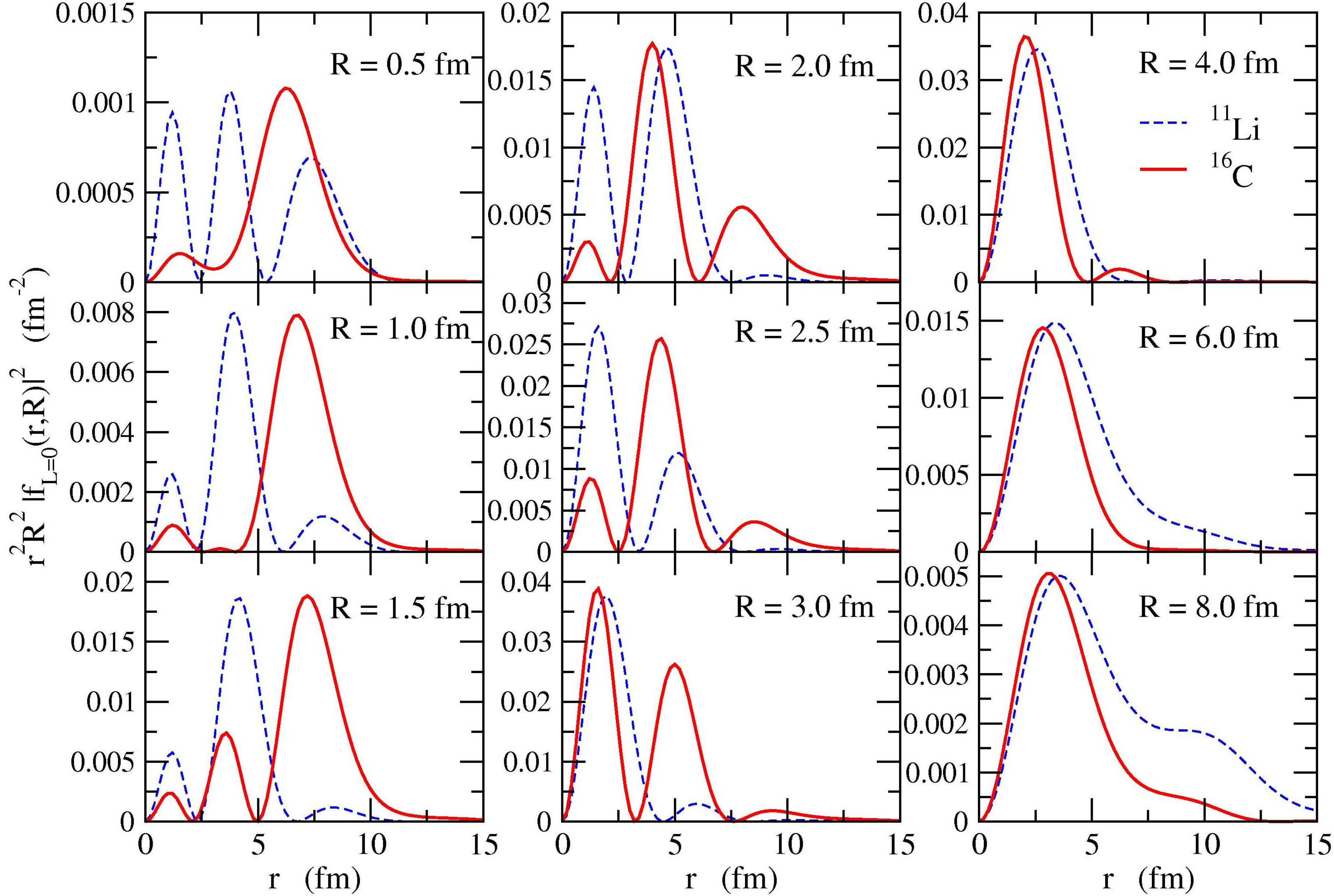
Universality? : Di-neutron wave function in ^{16}C

$^{14}C + n + n$ model for ^{16}C

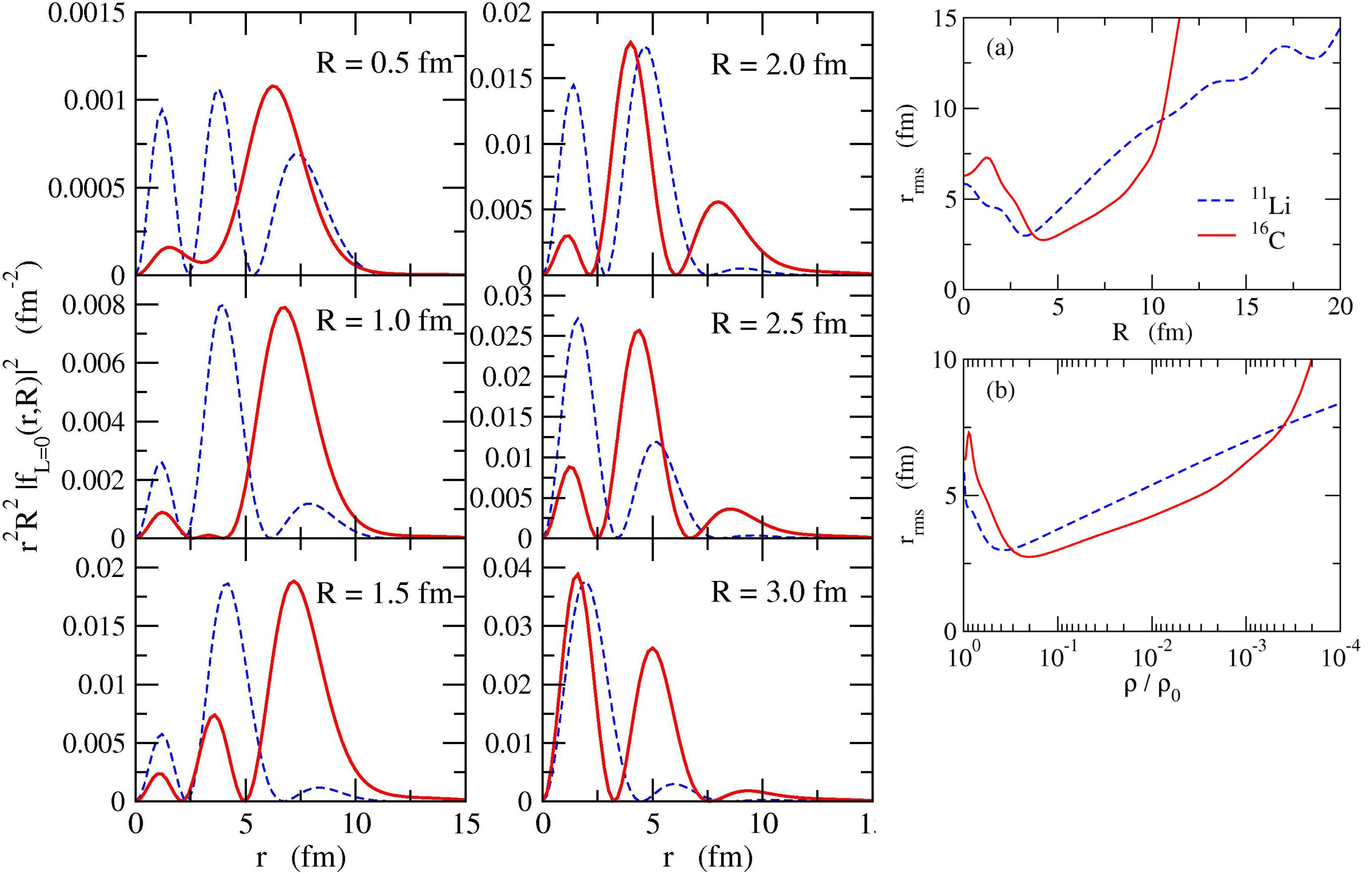
Y. Suzuki, H. Matsumura, and B. Abu-Ibrahim, PRC70('04)051302
W. Horiuchi and Y. Suzuki, PRC73('06)037304
K.H. and H. Sagawa, PRC in press ('07)

E_{2+} = 1.63 MeV   [exp: 1.77 MeV]
B(E2)=0.899 e^2fm^4 [exp: 0.63 +/- 0.27 e^2fm^4]
rms radius = 2.64 fm [exp: 2.64 +/- 0.05 fm]
also longitudinal mom. distribution for $^{16}C+^{12}C \longrightarrow ^{15}C + X$



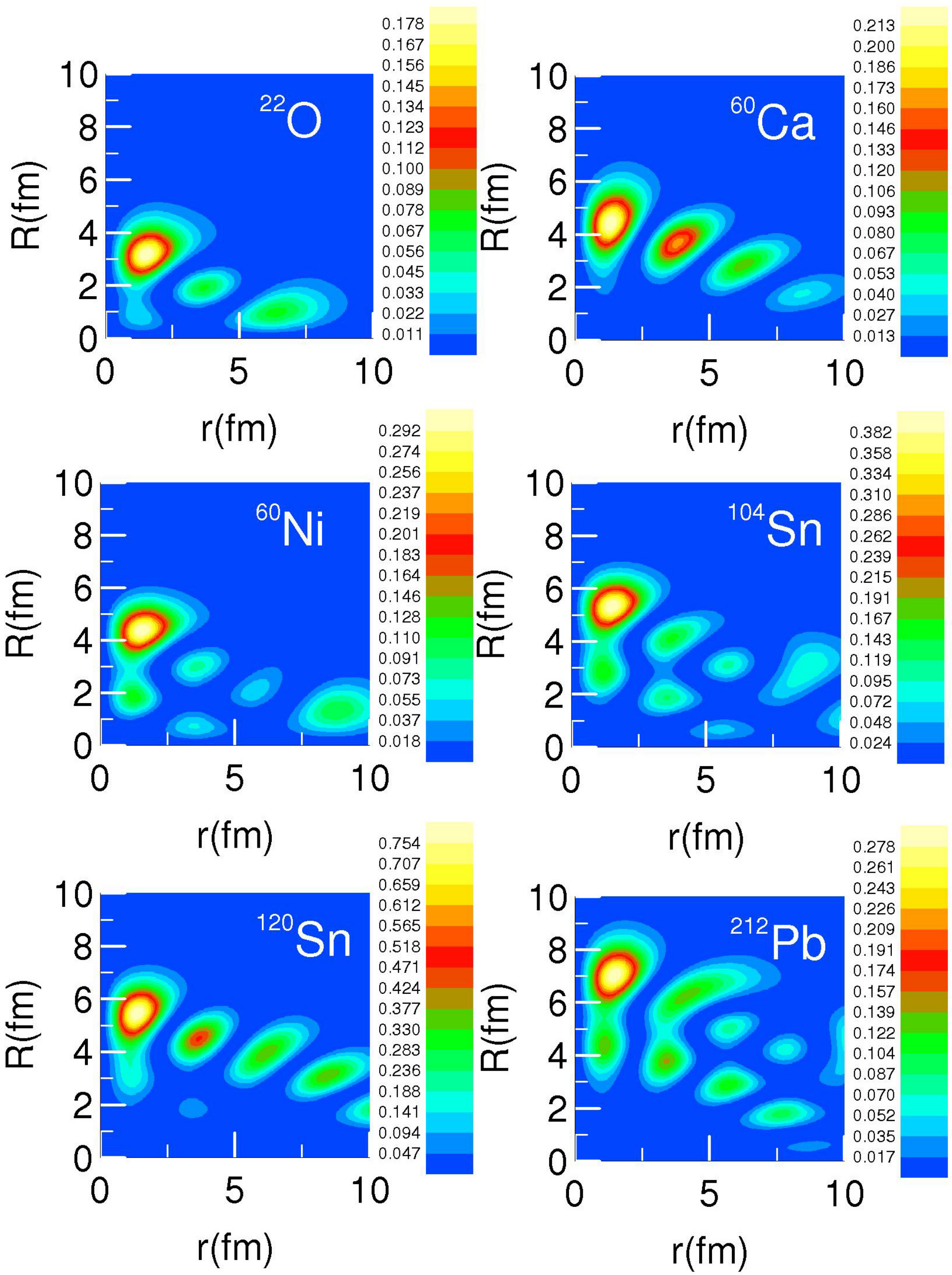


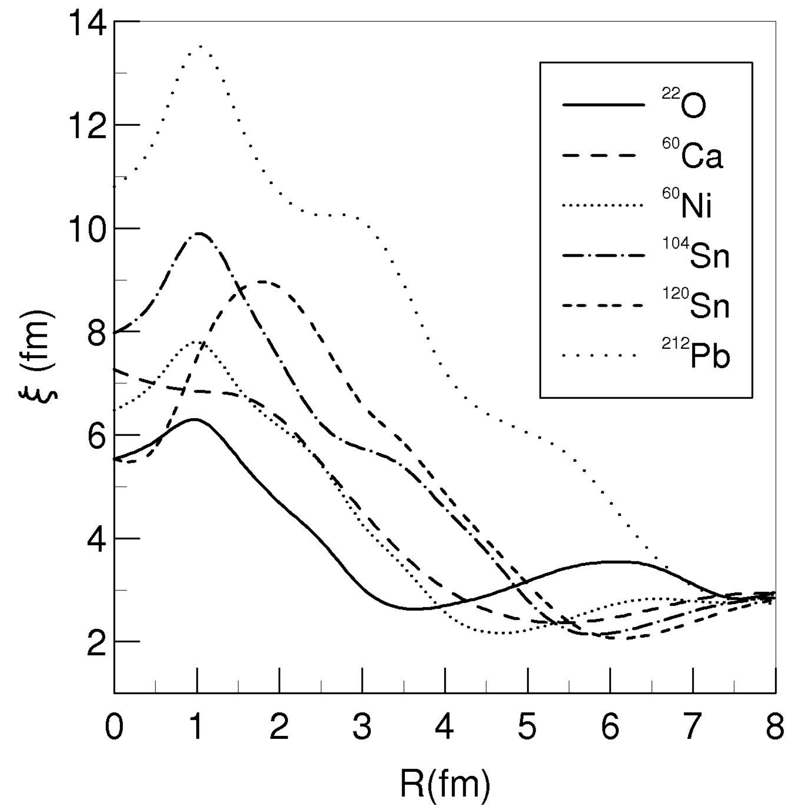
R = 0.5 fm
R = 2.0 fm
R = 4.0 fm
^{11}Li
^{16}C
R = 1.0 fm
R = 2.5 fm
R = 6.0 fm
R = 1.5 fm
R = 3.0 fm
R = 8.0 fm
$r^2R^2\,|f_{L=0}(r,R)|^2\ \ (\mathrm{fm}^{-2})$
r (fm)
r (fm)
r (fm)



R = 0.5 fm
R = 1.0 fm
R = 1.5 fm
R = 2.0 fm
R = 2.5 fm
R = 3.0 fm
$r^2R^2\,|f_{L=0}(r,R)|^2$ (fm^{-2})
r (fm)
(a)
(b)
r_{rms} (fm)
R (fm)
ρ / ρ_0
^{11}Li
^{16}C

Gogny HFB calculations



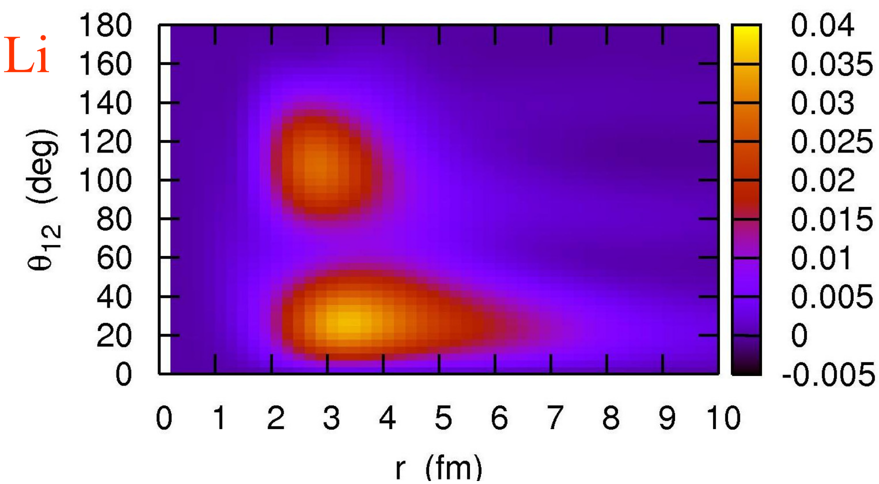


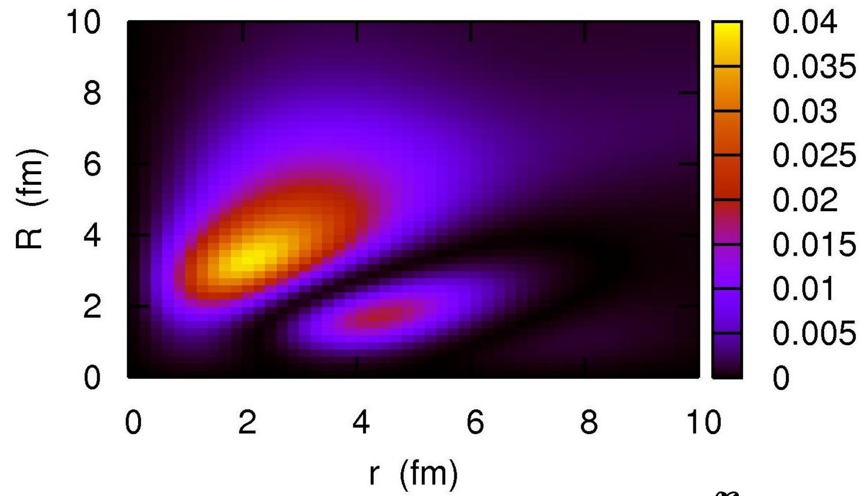
N. Pillet, N. Sandulescu, and P. Schuck, e-print: nucl-th/0701086

Summary

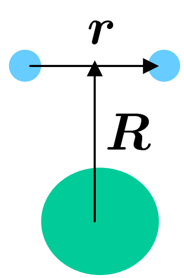
➢Application of three-body model to Borromean nuclei

^{11}Li





➢Di-neutron wave function for each *R*



- Close correspondence to the matter calculations
- BCS/BEC crossover phenomenon
- Concentration of a Cooper pair on the nuclear surface
- Unified understanding of di-neutron and cigar configurations
- Also in other superfulid nuclei (universality)

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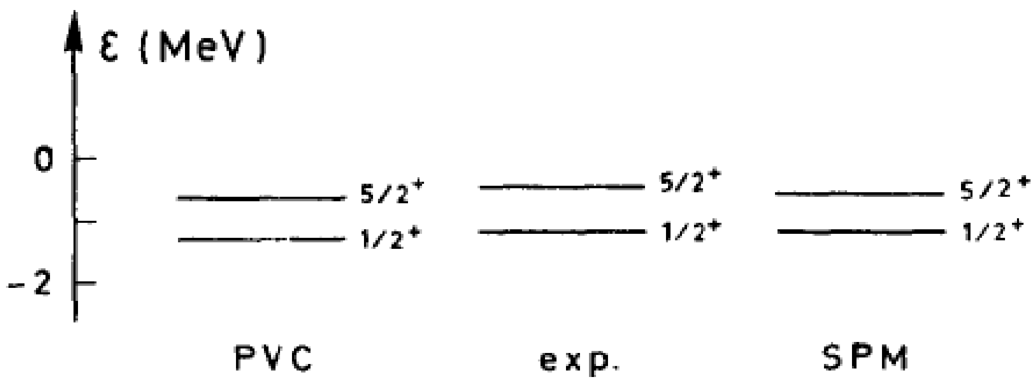


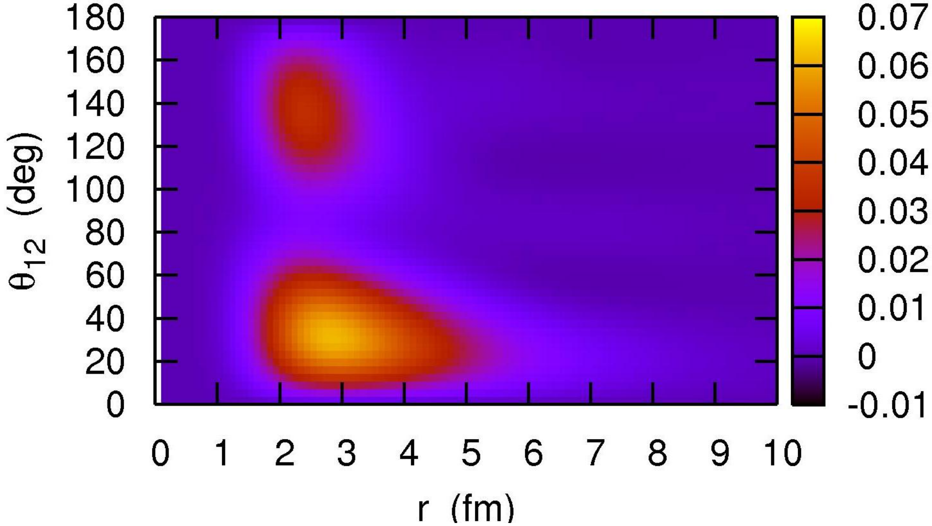
Fig. 4. Same as Fig. 2 for ^{15}C.

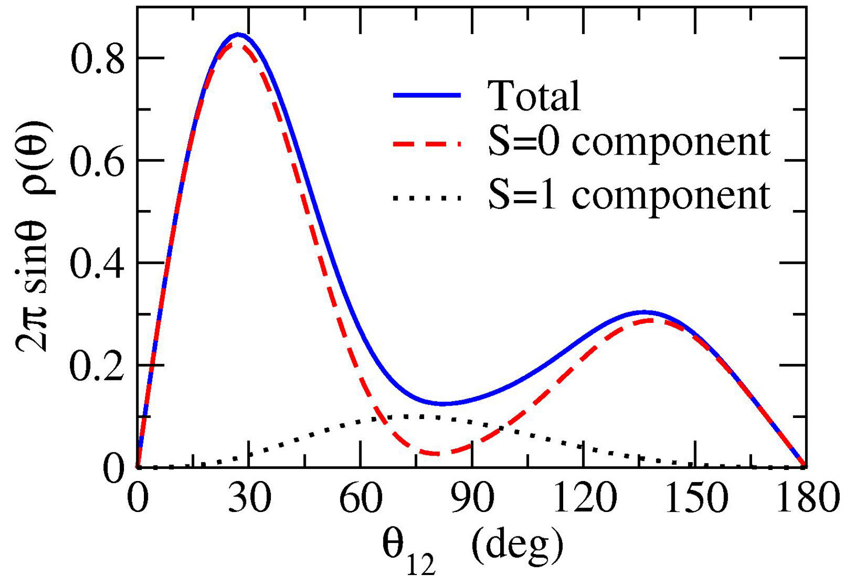
WS + particle-vibration coupling

Woods-Saxon(WS)

Two-particle density for ^{6}He

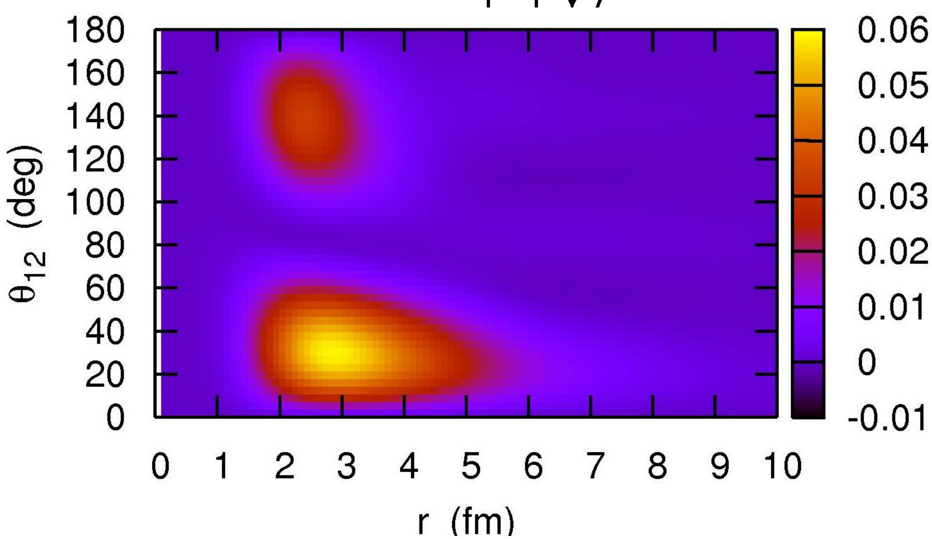
Total



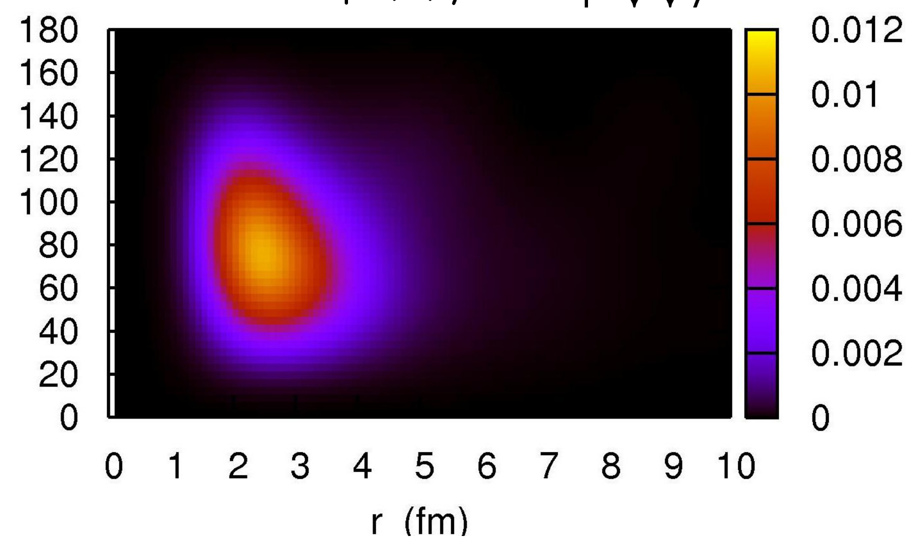


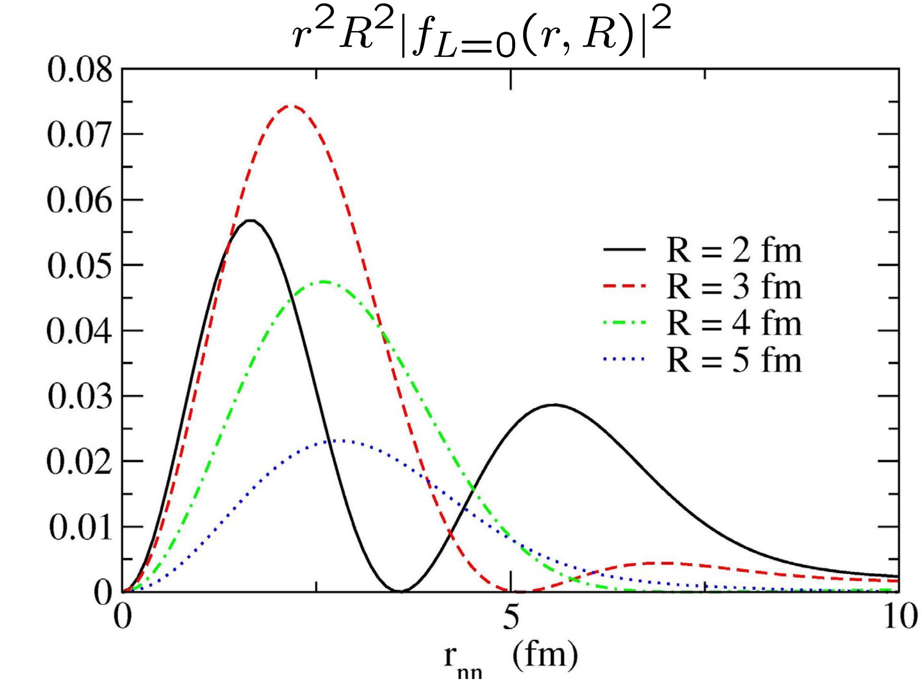
$\rightarrow \langle \theta_{12} \rangle = 66.33 \ \text{deg.}$

S=0 $|\uparrow\downarrow\rangle$

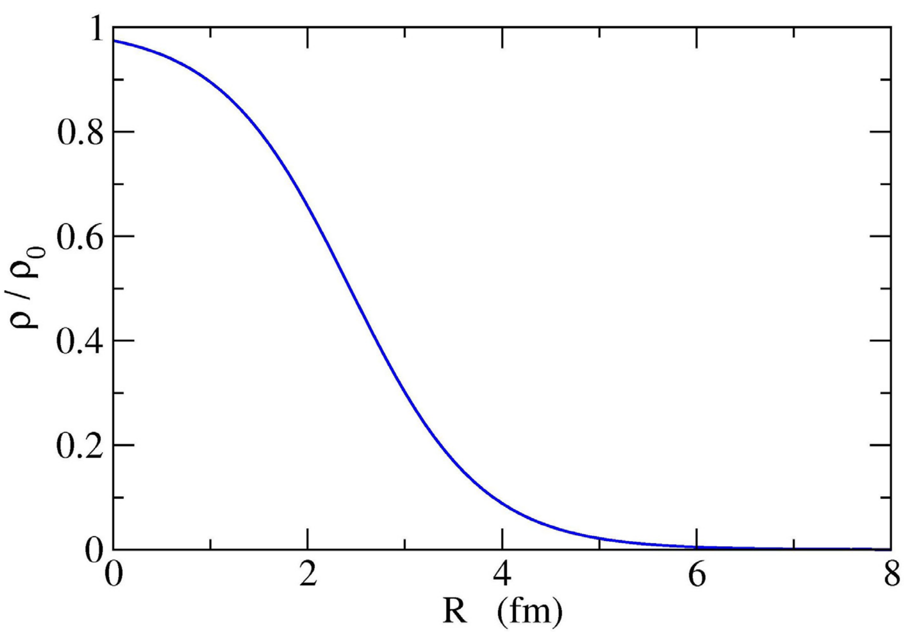


S=1 $|\uparrow\uparrow\rangle$ or $|\downarrow\downarrow\rangle$

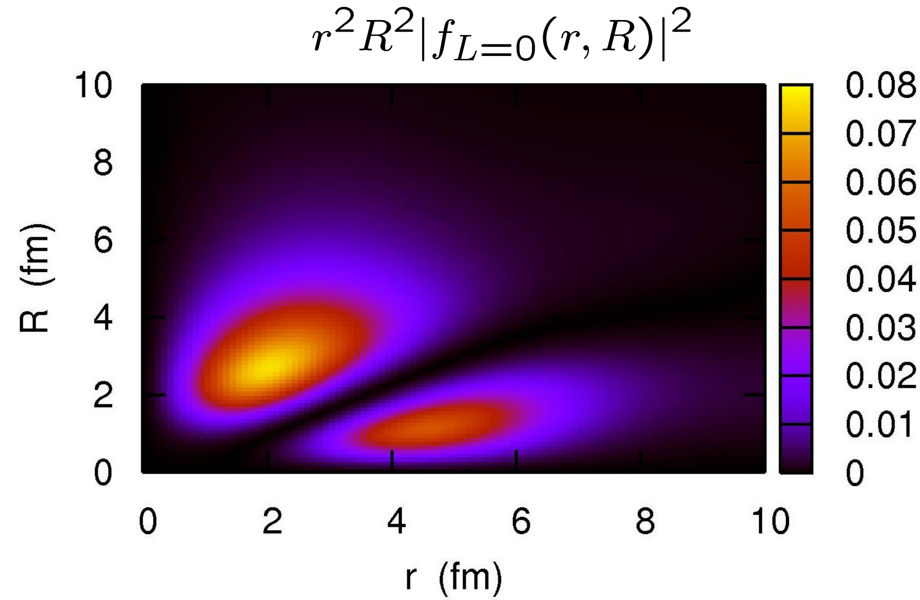




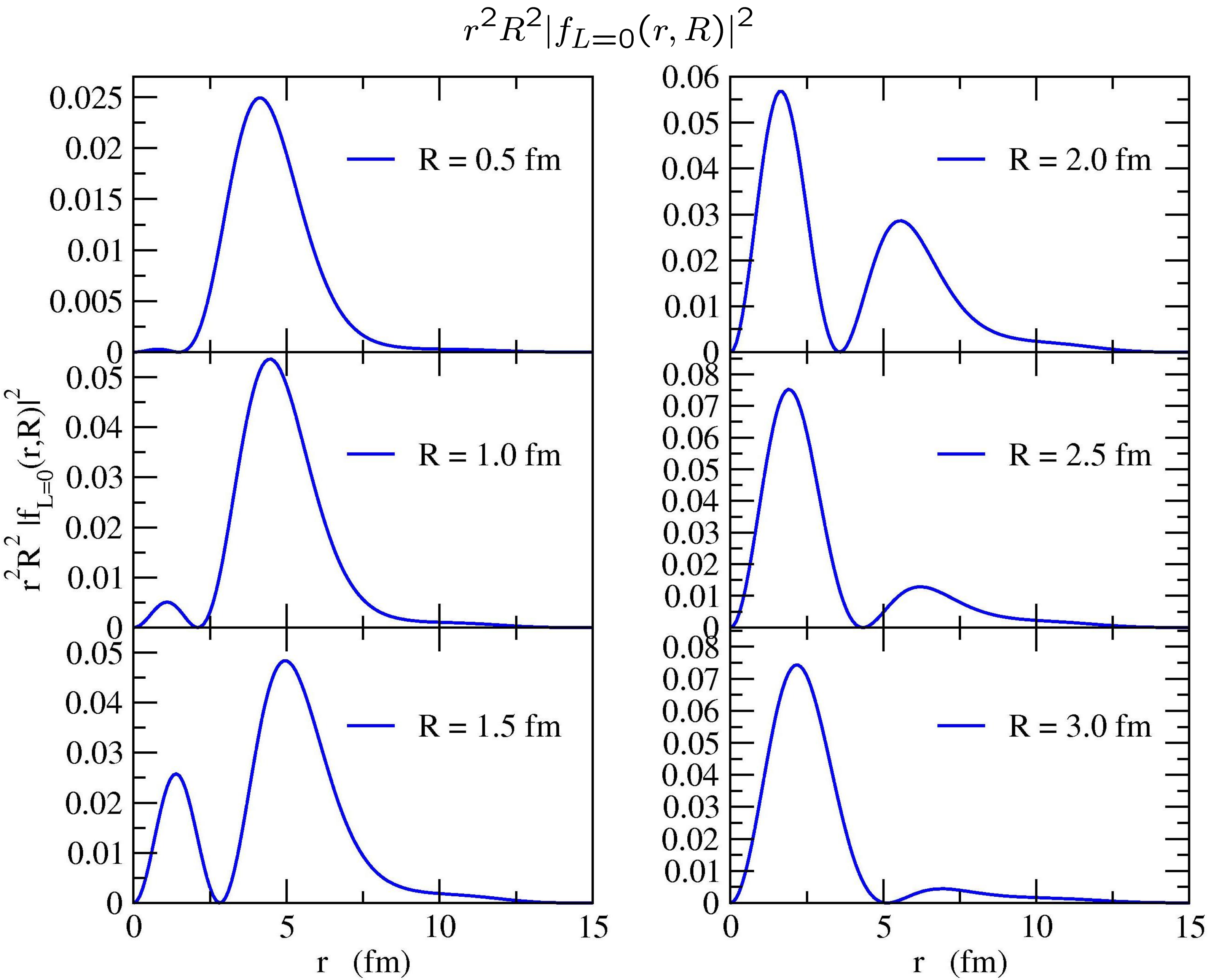
$r^2 R^2 |f_{L=0}(r,R)|^2$
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
R = 2 fm
R = 3 fm
R = 4 fm
R = 5 fm
0
5
10
r_{nn} (fm)



1
0.8
0.6
0.4
0.2
0
ρ / ρ_0
0
2
4
6
8
R (fm)

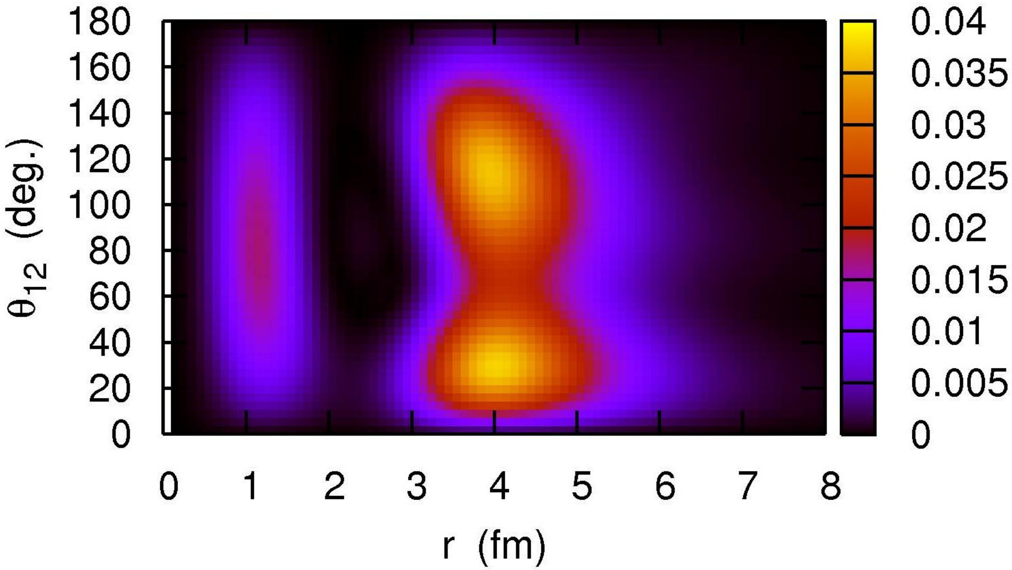


$r^2 R^2 |f_{L=0}(r,R)|^2$
10
8
6
4
2
0
R (fm)
0
2
4
6
8
10
r (fm)
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0

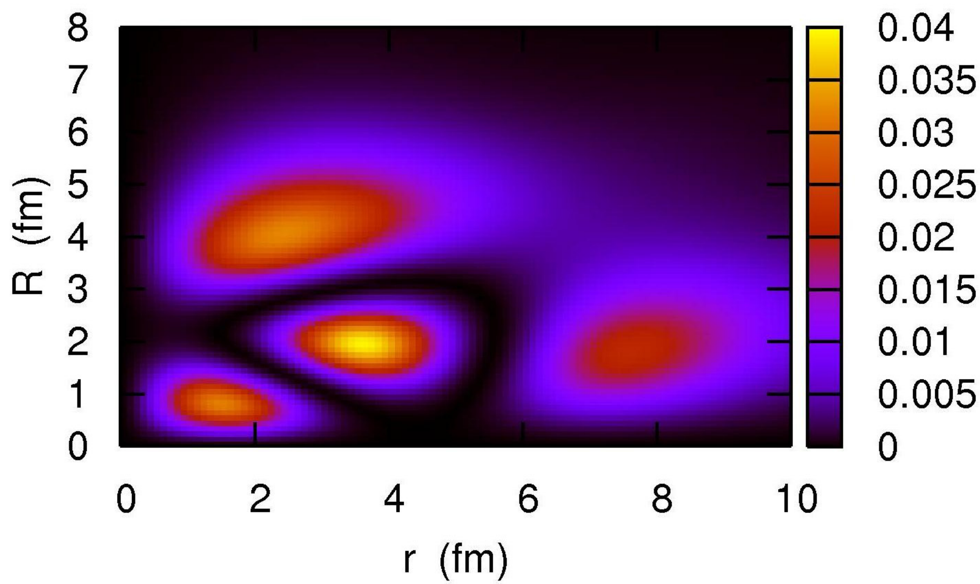


$r^2 R^2 |f_{L=0}(r,R)|^2$
R = 0.5 fm
R = 1.0 fm
R = 1.5 fm
R = 2.0 fm
R = 2.5 fm
R = 3.0 fm
$r^2 R^2 |f_{L=0}(r,R)|^2$
r (fm)
r (fm)

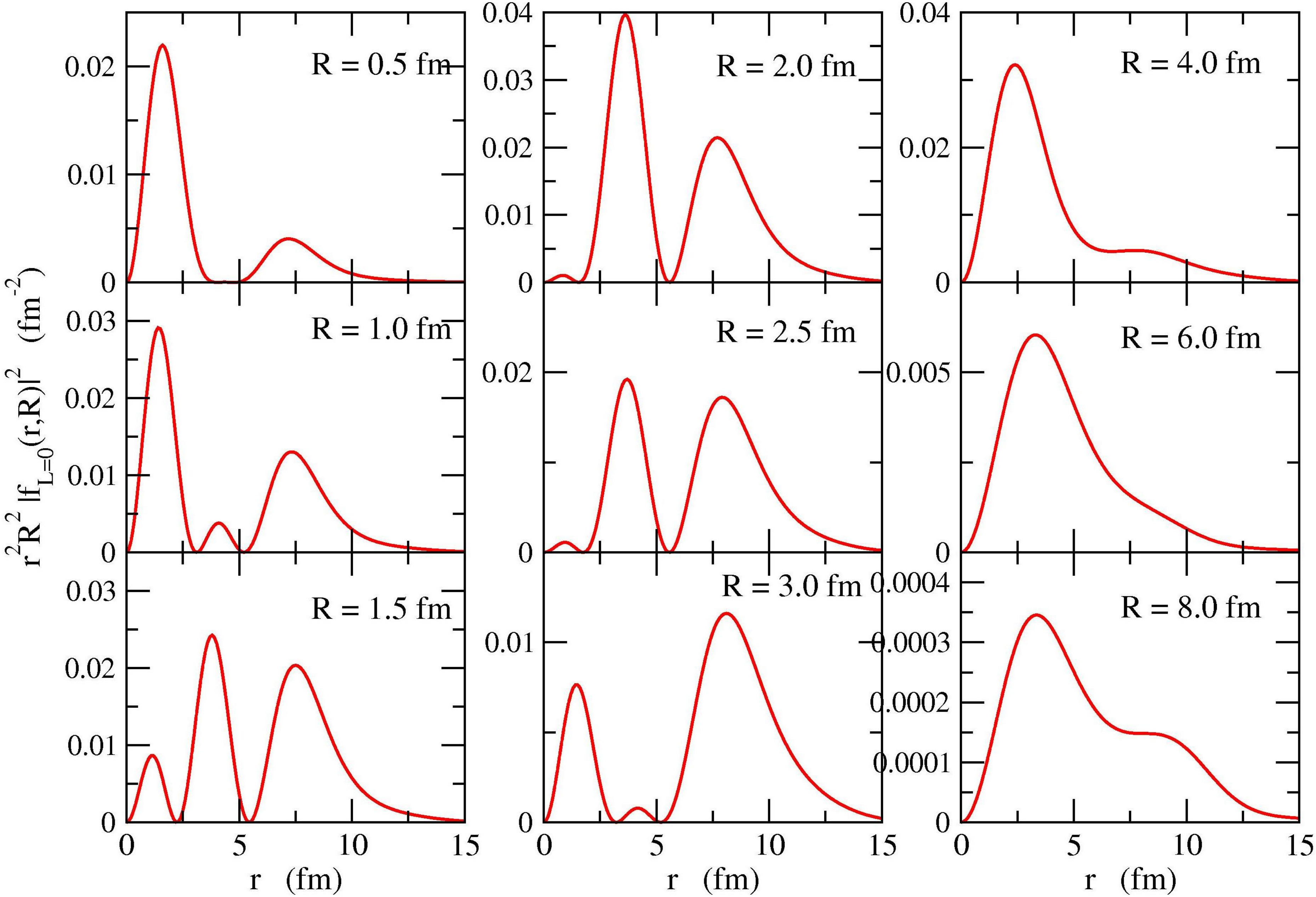
^{24}O



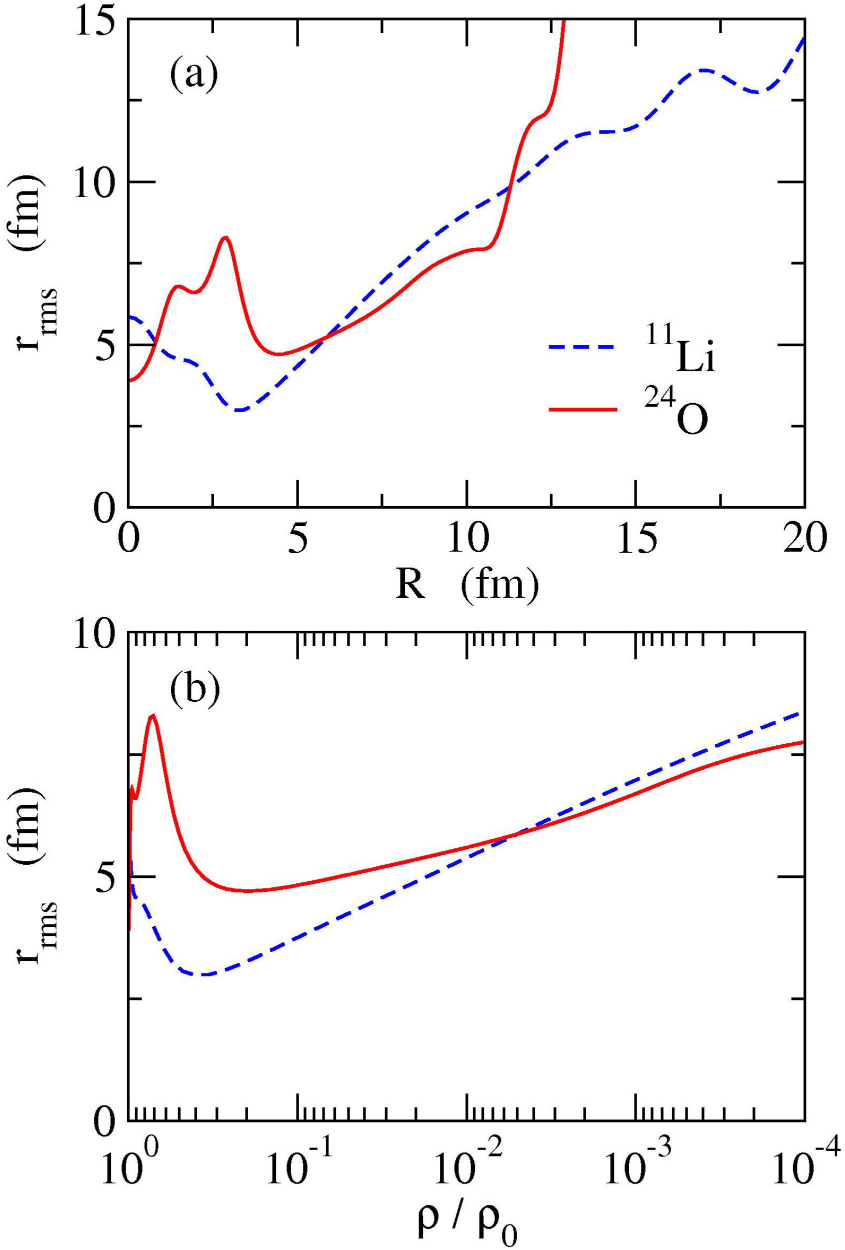
180
160
140
120
100
80
60
40
20
0
θ12 (deg.)
0 1 2 3 4 5 6 7 8
r (fm)
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0



8
7
6
5
4
3
2
1
0
R (fm)
0 2 4 6 8 10
r (fm)
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0



R = 0.5 fm
R = 1.0 fm
R = 1.5 fm
R = 2.0 fm
R = 2.5 fm
R = 3.0 fm
R = 4.0 fm
R = 6.0 fm
R = 8.0 fm
$r^2R^2\,|f_{L=0}(r,R)|^2$ (fm^{-2})
r (fm)



(a)
r_{rms} (fm)
R (fm)
^{11}Li
^{24}O
(b)
r_{rms} (fm)
ρ / ρ_0