Green's function approach to nuclear structure
⇒ Correlations at large asymmetry

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- Correlations for stable closed-shell nuclei
- Exciting physics for N >> Z or Z >> N nuclei?
  - What to look for? Motivation
  - Solid framework for extrapolation
  - Some recent results and developments
  - Outlook

Theoretical tools:
nonrelativistic many-body theory

Green's function ingredients

- Spectral functions: $$S_h(\alpha;E) = \sum_n \left| \left\langle \Psi_n^{A-1} \middle| a_\alpha \middle| \Psi_0^A \right\rangle \right|^2 \delta\left(E - \left(E_0^A - E_n^{A-1}\right)\right)$$
- Spectroscopic factor: $$S_{\ell j}^n = \int dr\, r^2 \left| \left\langle \Psi_n^{A-1} \middle| a_{r\ell j} \middle| \Psi_0^A \right\rangle \right|^2$$
- Relation to single-particle propagator (Green's function):

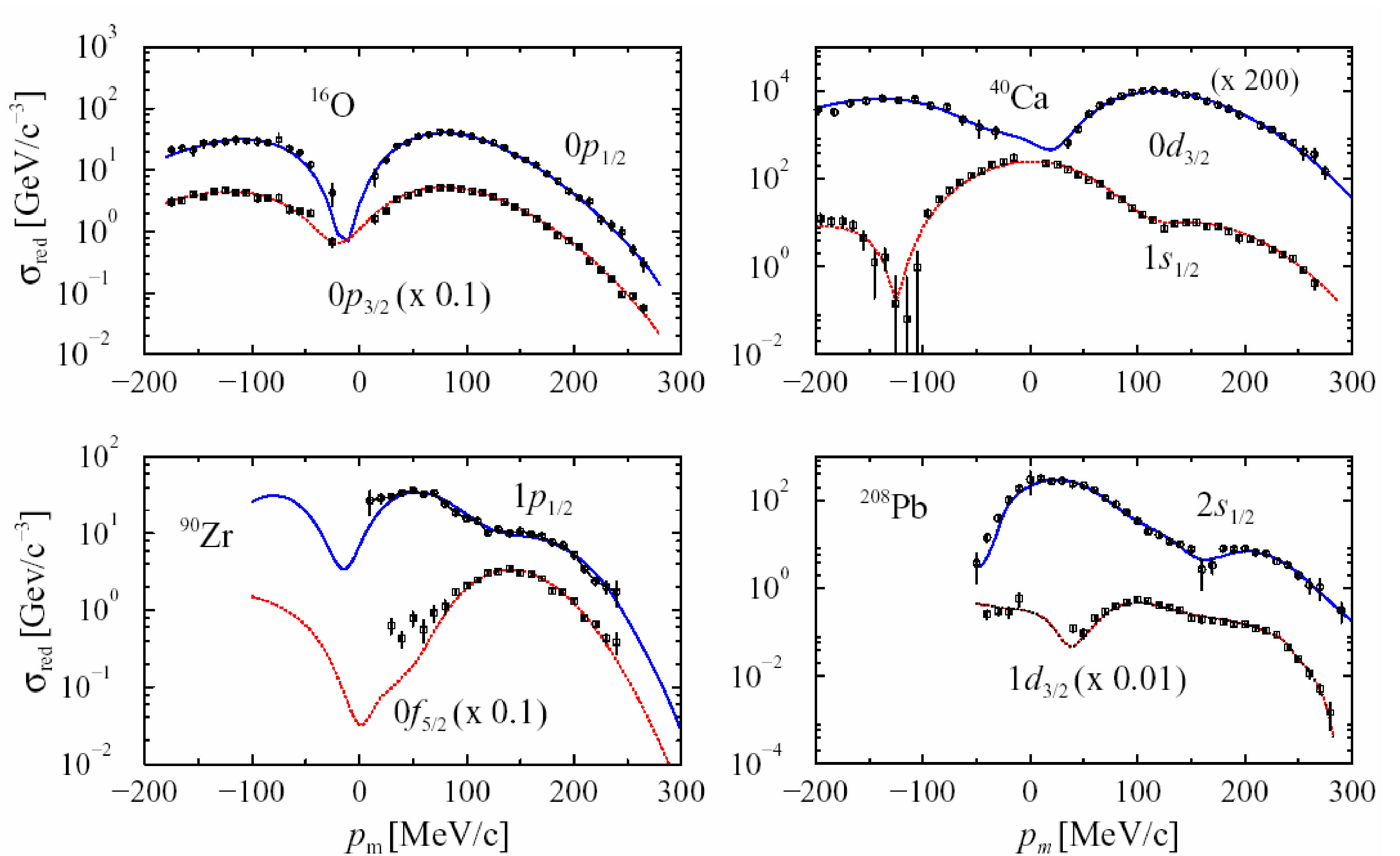
$$G(\alpha,\beta;E) = \sum_m \frac{\left\langle \Psi_0^A \middle| a_\alpha \middle| \Psi_m^{A+1} \right\rangle \left\langle \Psi_m^{A+1} \middle| a_\beta^\dagger \middle| \Psi_0^A \right\rangle}{E - \left(E_m^{A+1} - E_0^A\right) + i\eta} \quad \Leftarrow \text{Particle part}$$

$$+ \sum_n \frac{\left\langle \Psi_0^A \middle| a_\beta^\dagger \middle| \Psi_n^{A-1} \right\rangle \left\langle \Psi_n^{A-1} \middle| a_\alpha \middle| \Psi_0^A \right\rangle}{E - \left(E_0^A - E_n^{A-1}\right) - i\eta} \quad \Leftarrow \text{Hole part}$$

- Occupation numbers: $$n(\alpha) = \int_{-\infty}^{\varepsilon_F^-} S_h(\alpha;E)\, dE = \left\langle \Psi_0^A \middle| a_\alpha^\dagger a_\alpha \middle| \Psi_0^A \right\rangle$$
- Below $\varepsilon_F^- = E_0^A - E_0^{A-1}$ $\Rightarrow$ $$S_h(\alpha;E) = \frac{1}{\pi} \mathrm{Im}\, G(\alpha,\alpha;E)$$
- Density matrix; natural orbits; Galitskii-Migdal sum rule …

Nuclei (e,e'p) reaction

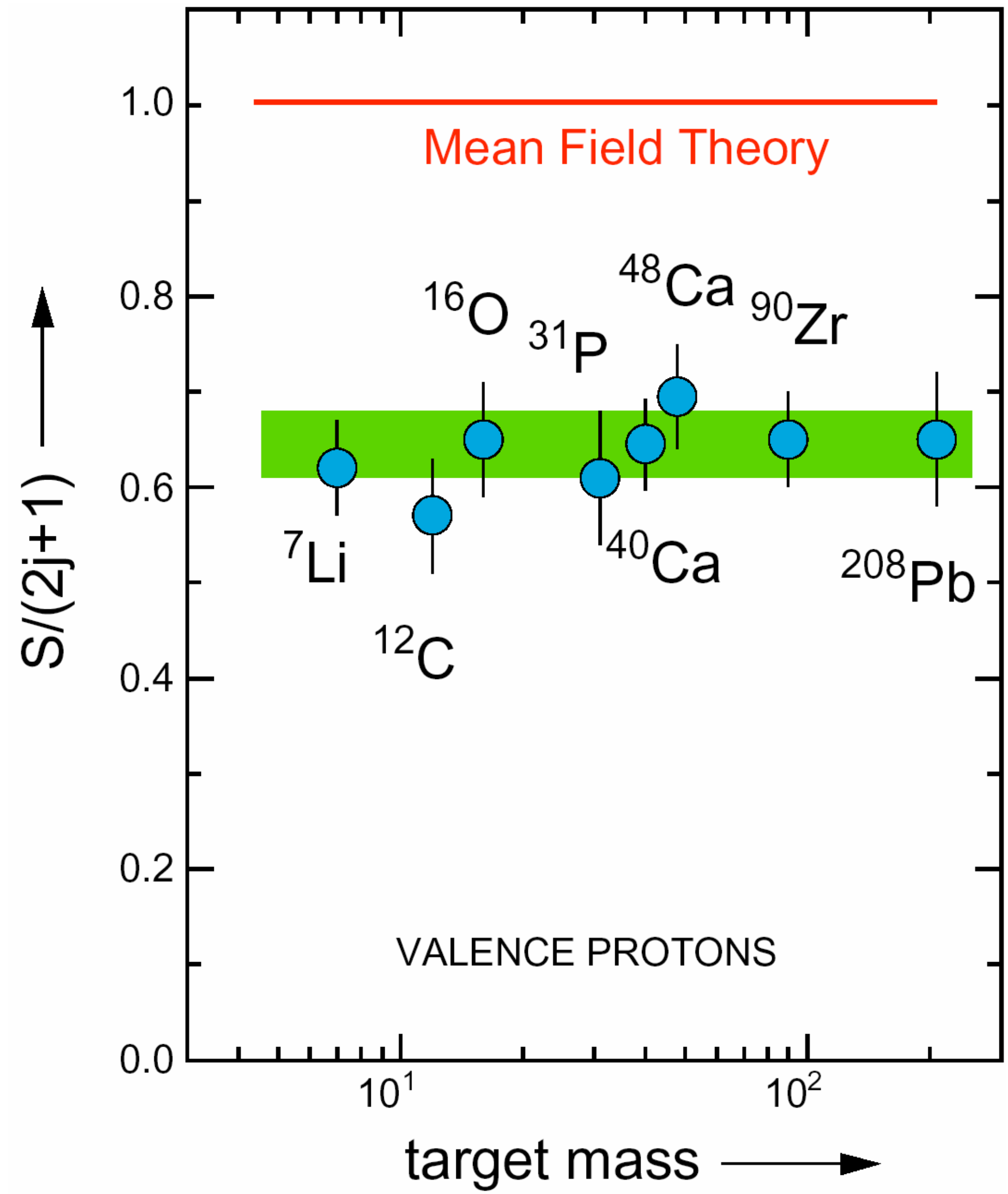
NIKHEF data, L. Lapikás, Nucl. Phys. A553, 297c (1993)



Wave functions as expected, except ....

Removal probability for valence protons from NIKHEF data

L. Lapikás, Nucl. Phys. A553,297c (1993)



S ≈ 0.65 for valence protons

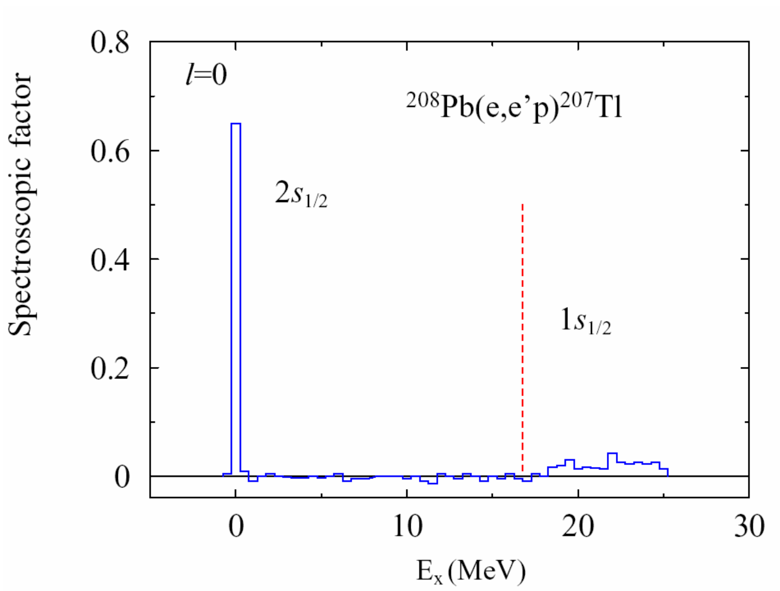
Reduction ⇒ both SRC and LRC

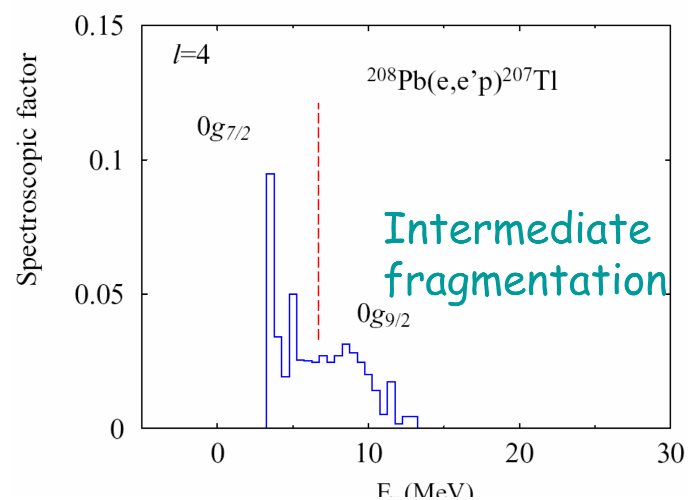
Note:

We have seen mostly data for removal of valence protons

Fragmentation of strength

E. Quint, Ph.D.thesis NIKHEF, 1988





Strong fragmentation of deeply-bound strength
$\Rightarrow$ self-energy has significant imaginary part!

Quasihole strength or spectroscopic factor $S(2s_{1/2})$ =0.65

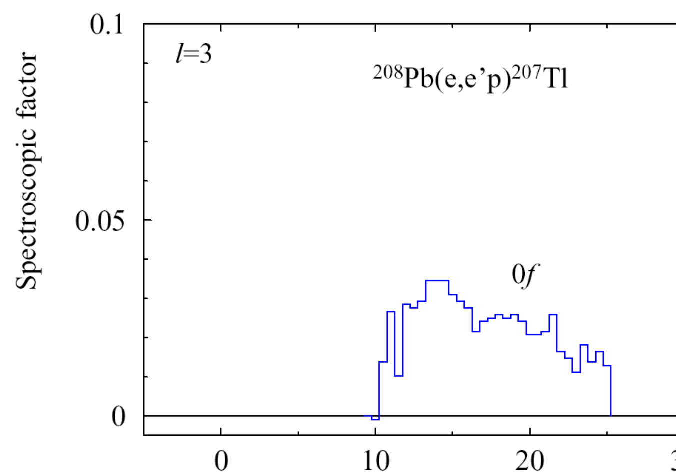
I. Sick & P de Witt Huberts, Comm. Nucl. Part. Phys.,**20**, 177 (1991)

$n(2s_{1/2})$ = 0.75

from elastic electron scattering

CERES analysis

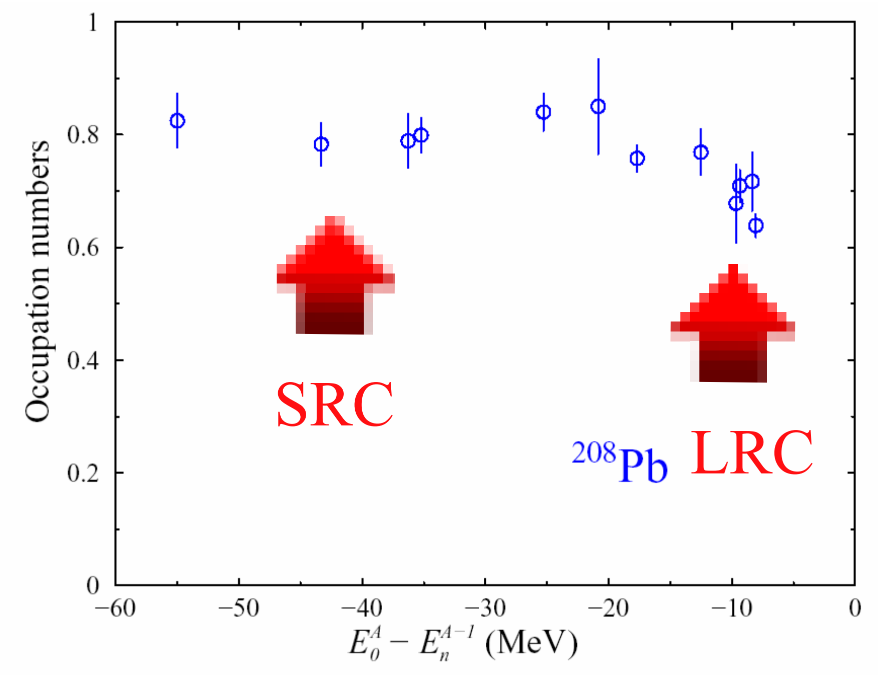
P. Grabmayr Prog. Part. Nucl. Phys. **29**,251(1992)



M. van Batenburg & L. Lapikás from ^{208}Pb (e,e´p) ^{207}Tl

NIKHEF in preparation

Occupation of deeply-bound proton levels from **EXPERIMENT**

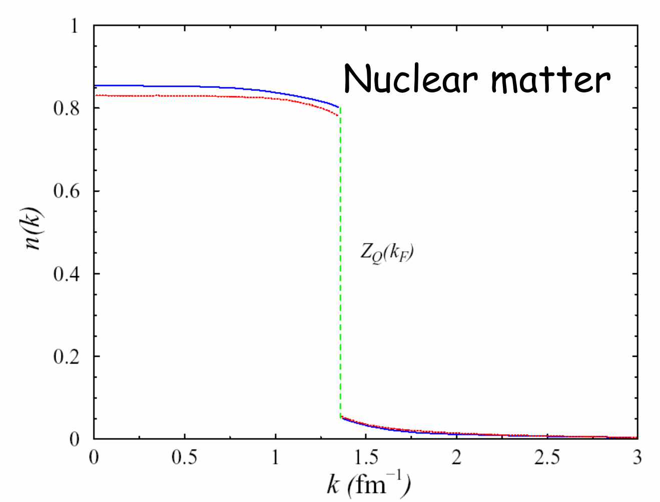


Up to 100 MeV missing energy and 270 MeV/c missing momentum

Covers the whole mean-field domain for the FIRST time!!

Confirms predictions for depletion

n(0) ⇒ 0.85 Reid
0.87 Argonne V18
0.89 CDBonn



Correlations for nuclei with N very different from Z?
$\Rightarrow$ Radioactive beam facilities

Nuclei are TWO-component Fermi liquids

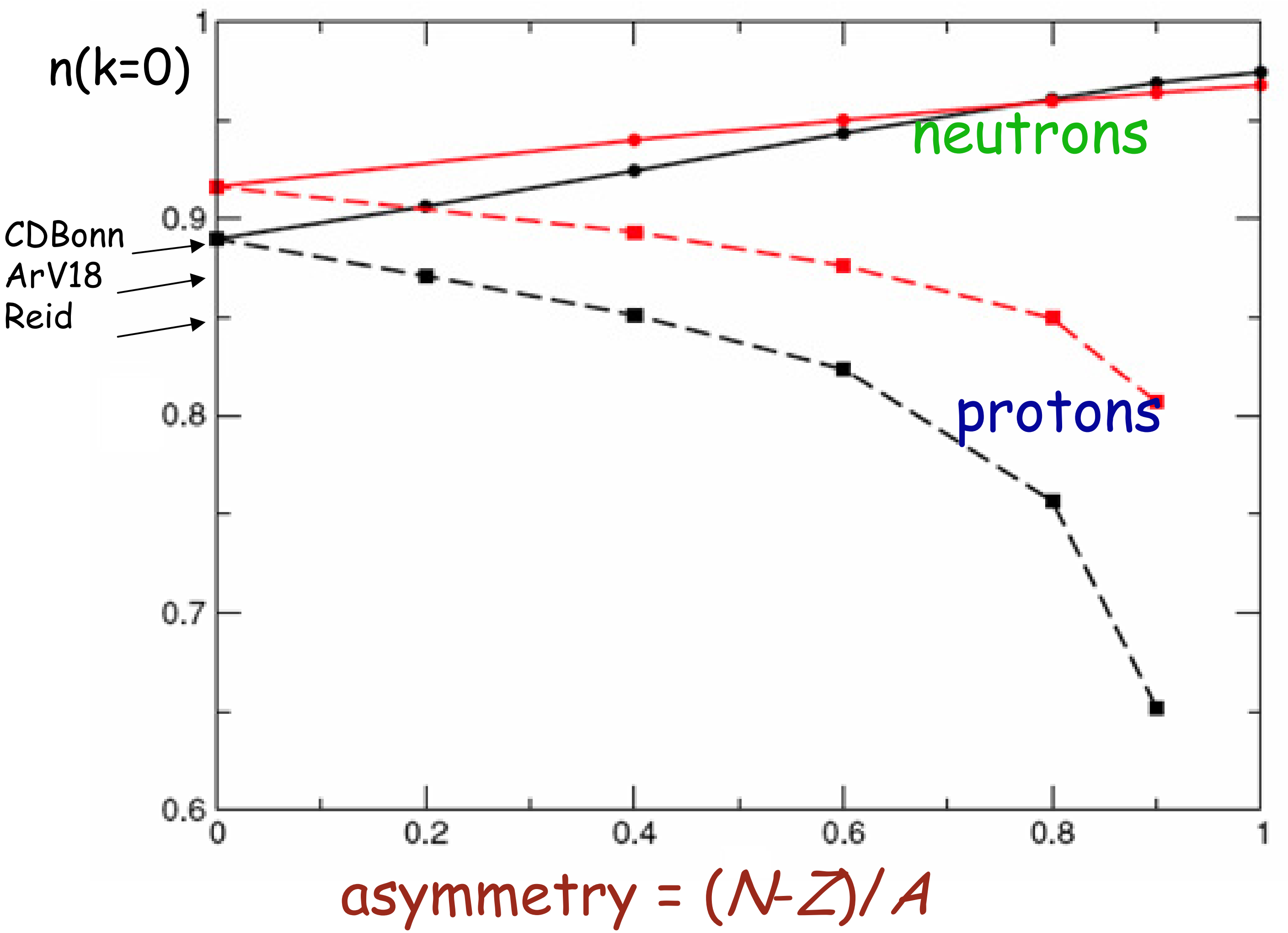
- SRC about the same between pp, np, and nn
- Tensor force disappears for n when N >> Z but …
- Empirically p more bound for N>Z systems than n

- Any surprises?
- Ideally: quantitative predictions based on solid foundation

Some pointers: one from theory and one from experiment

SCGF for isospin-polarized nuclear matter including SRC ⇒ momentum distribution

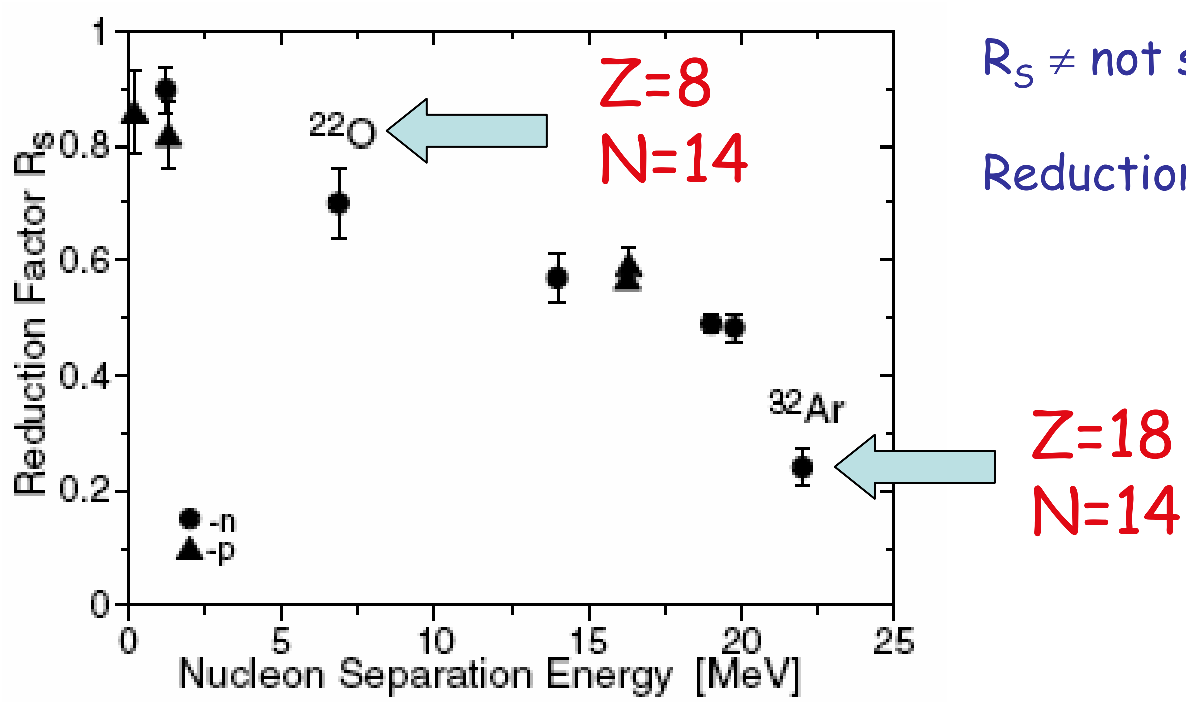
Frick *et al.* PRC71,014313(2005)



SRC
can be handled

A. Gade et al., Phys. Rev. Lett. 93, 042501 (2004)

Program at MSU initiated by Gregers Hansen
P. G. Hansen and J. A. Tostevin, Annu. Rev. Nucl. Part. Sci. **53**, 219 (2003)

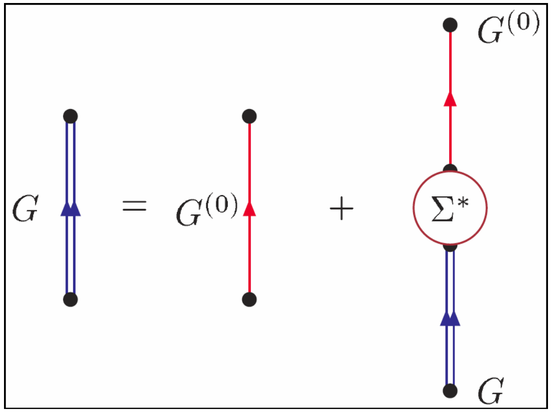


$R_s \neq$ not spectroscopic factor

Reduction w.r.t. shell model

neutrons more correlated with increasing proton number and accompanying increasing separation energy.

Dyson Equation and "experiment"



Equivalent to …

Schrödinger-like equation with: $E_n^- = E_0^N - E_n^{N-1}$

Self-energy: non-local, energy-dependent potential
With energy dependence: spectroscopic factors < 1
$\Rightarrow$ as observed in (e,e'p)

$$-\frac{\hbar^2\nabla^2}{2m}\left\langle \Psi_n^{N-1}\right| a_{\vec{r}m}\left|\Psi_0^N\right\rangle + \sum_{m'}\int d\vec{r}\,'\,\Sigma'^*(\vec{r}m,\vec{r}\,'m';E_n^-)\left\langle \Psi_n^{N-1}\right| a_{\vec{r}\,'m'}\left|\Psi_0^N\right\rangle = E_n^-\left\langle \Psi_n^{N-1}\right| a_{\vec{r}m}\left|\Psi_0^N\right\rangle$$

$$S = \left|\left\langle \Psi_n^{N-1}\right| a_{\alpha_{qh}}\left|\Psi_0^N\right\rangle\right|^2 = \frac{1}{1 - \left.\frac{\partial\Sigma'^*(\alpha_{qh},\alpha_{qh};E)}{\partial E}\right|_{E_n^-}}$$

α_{qh} solution of DE at E_n^-

DE yields

$$\left\langle \Psi_n^{N-1}\right| a_{\vec{r}m}\left|\Psi_0^N\right\rangle = \psi_n^{N-1}(\vec{r}m)$$ *Bound states in N-1*

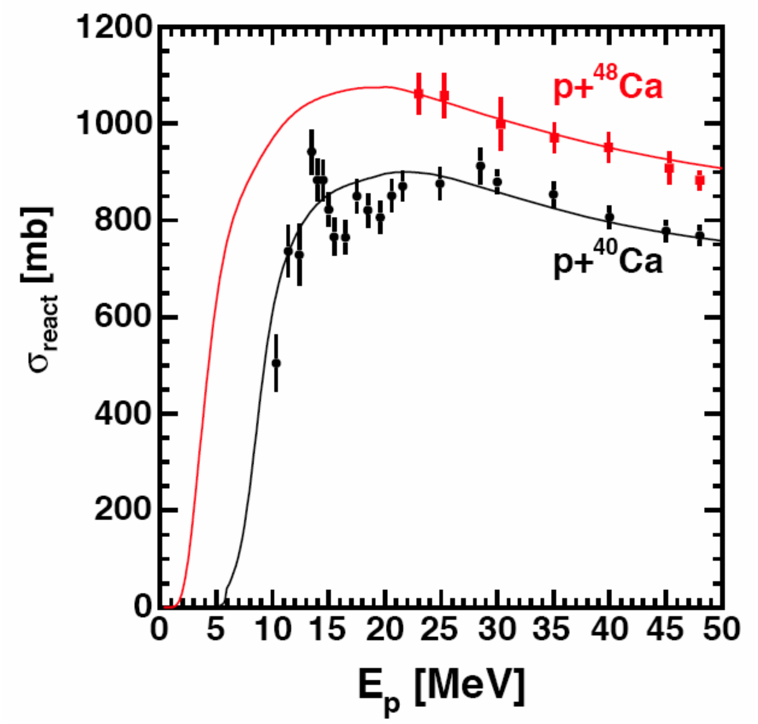
$$\left\langle \Psi_0^{N}\right| a_{\vec{r}m}\left|\Psi_k^{N+1}\right\rangle = \psi_k^{N+1}(\vec{r}m)$$ *Bound states in N+1*

$$\left\langle \Psi_E^{c,N-1}\right| a_{\vec{r}m}\left|\Psi_0^N\right\rangle = \chi_c^{N-1}(\vec{r}m;E)$$ *Scattering states in N-1*

$$\left\langle \Psi_0^{N}\right| a_{\vec{r}m}\left|\Psi_E^{c,N+1}\right\rangle = \chi_c^{N+1}(\vec{r}m;E)$$ *Elastic scattering in N+1*

Elastic scattering wave function for (*p*,*p*) or (*n*,*n*)

Does the nucleon self-energy also have an imaginary part above the Fermi energy?



Loss of flux in the elastic channel

Answer: YES!

FRAMEWORK FOR EXTRAPOLATIONS BASED ON EXPERIMENTAL DATA

"Mahaux analysis" ⇒ Dispersive Optical Model (DOM)

C. Mahaux and R. Sartor, Adv. Nucl. Phys. **20**, 1 (1991)

There is empirical information about the nucleon self-energy!!
⇒ Optical potential to analyze elastic nucleon scattering data
⇒ Extend analysis from A+1 to include structure information in A-1 ⇒ (e,e′p) data
⇒ Employ dispersion relation between real and imaginary part of self-energy

Recent extension

Combined analysis of protons in ^{40}Ca and ^{48}Ca
Charity, Sobotka, & WD nucl-ex/0605026, Phys. Rev. Lett. **97**, 162503 (2006)

Large energy window (> 200 MeV)

Goal: Extract asymmetry dependence ⇒ $\delta = (N - Z)/A$
⇒ **Predict** proton properties at large asymmetry ⇒ ^{60}Ca
⇒ **Predict** neutron properties … the dripline
based on data!

General dispersion relation

$$\operatorname{Re}\Sigma(\gamma,\delta;E)=\Sigma^{"HF"}(\gamma,\delta)-\frac{1}{\pi}P\int_{E_T^+}^{\infty}dE'\frac{\operatorname{Im}\Sigma(\gamma,\delta;E')}{E-E'}+\frac{1}{\pi}P\int_{-\infty}^{E_T^-}dE'\frac{\operatorname{Im}\Sigma(\gamma,\delta;E')}{E-E'}$$

At E_0 for example the Fermi energy

$$\operatorname{Re}\Sigma(\gamma,\delta;E_0)=\Sigma^{"HF"}(\gamma,\delta)-\frac{1}{\pi}P\int_{E_T^+}^{\infty}dE'\frac{\operatorname{Im}\Sigma(\gamma,\delta;E')}{E_0-E'}+\frac{1}{\pi}P\int_{-\infty}^{E_T^-}dE'\frac{\operatorname{Im}\Sigma(\gamma,\delta;E')}{E_0-E'}$$

Subtract

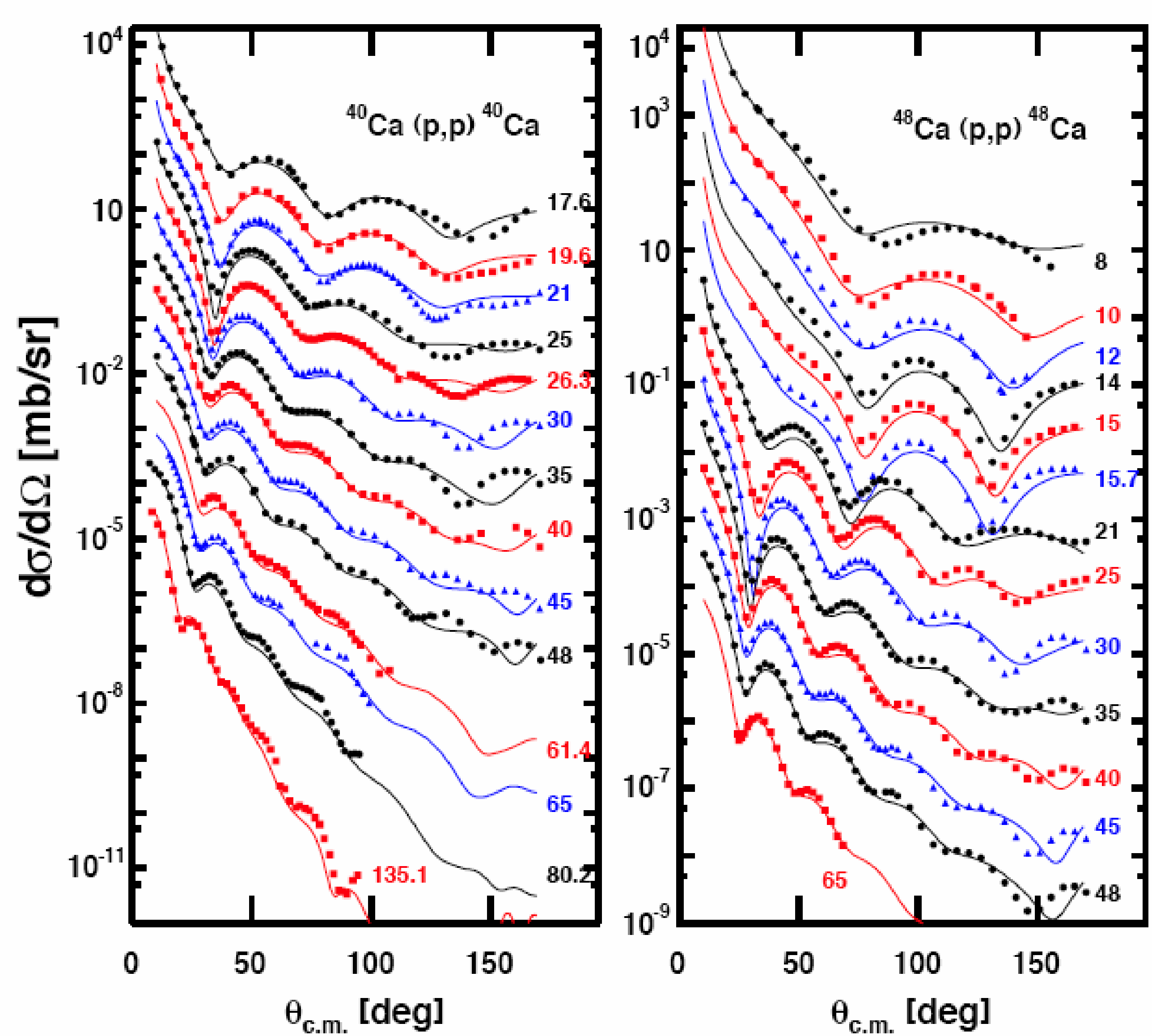
"HF" from Mahaux

$$\operatorname{Re}\Sigma(\gamma,\delta;E)=\operatorname{Re}\Sigma(\gamma,\delta;E_0)$$

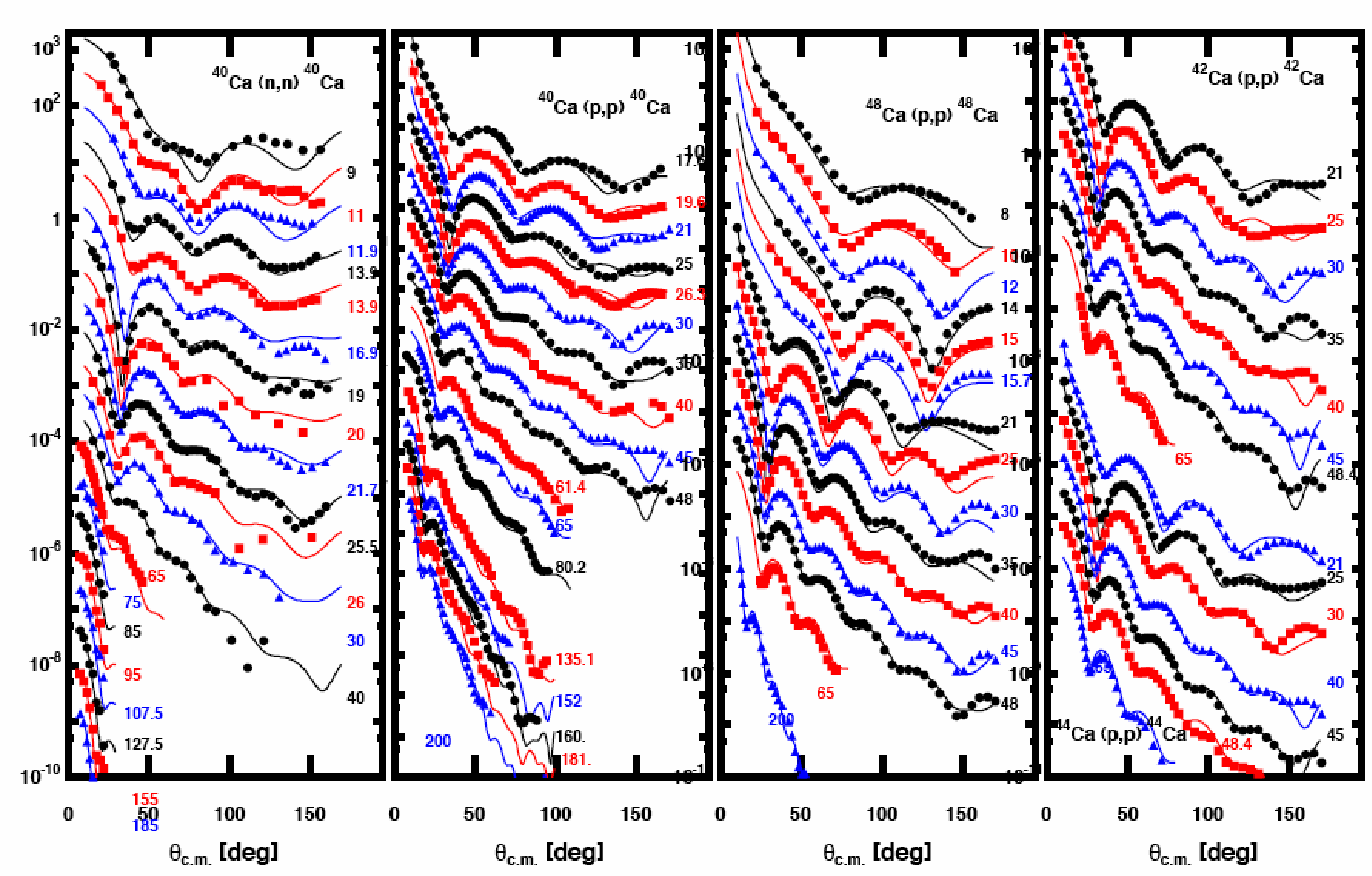
$$-\frac{1}{\pi}(E_0-E)P\int_{E_T^+}^{\infty}dE'\frac{\operatorname{Im}\Sigma(\gamma,\delta;E')}{(E-E')(E_0-E')}+\frac{1}{\pi}(E_0-E)P\int_{-\infty}^{E_T^-}dE'\frac{\operatorname{Im}\Sigma(\gamma,\delta;E')}{(E-E')(E_0-E')}$$

Note here: ImΣ < 0 for "2p1h" energies but >0 for "2h1p" energies

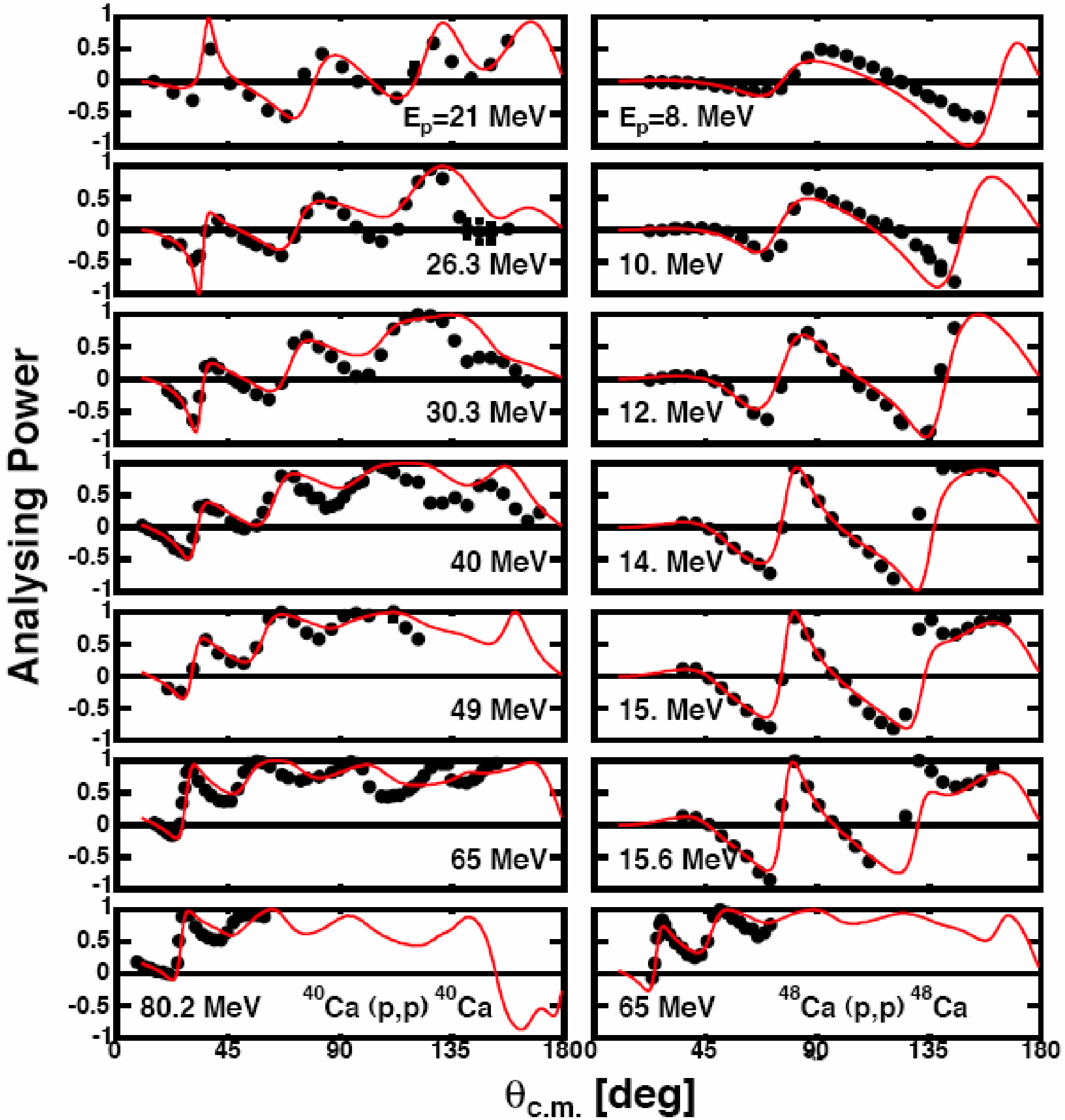
Fit of proton elastic scattering angular distributions



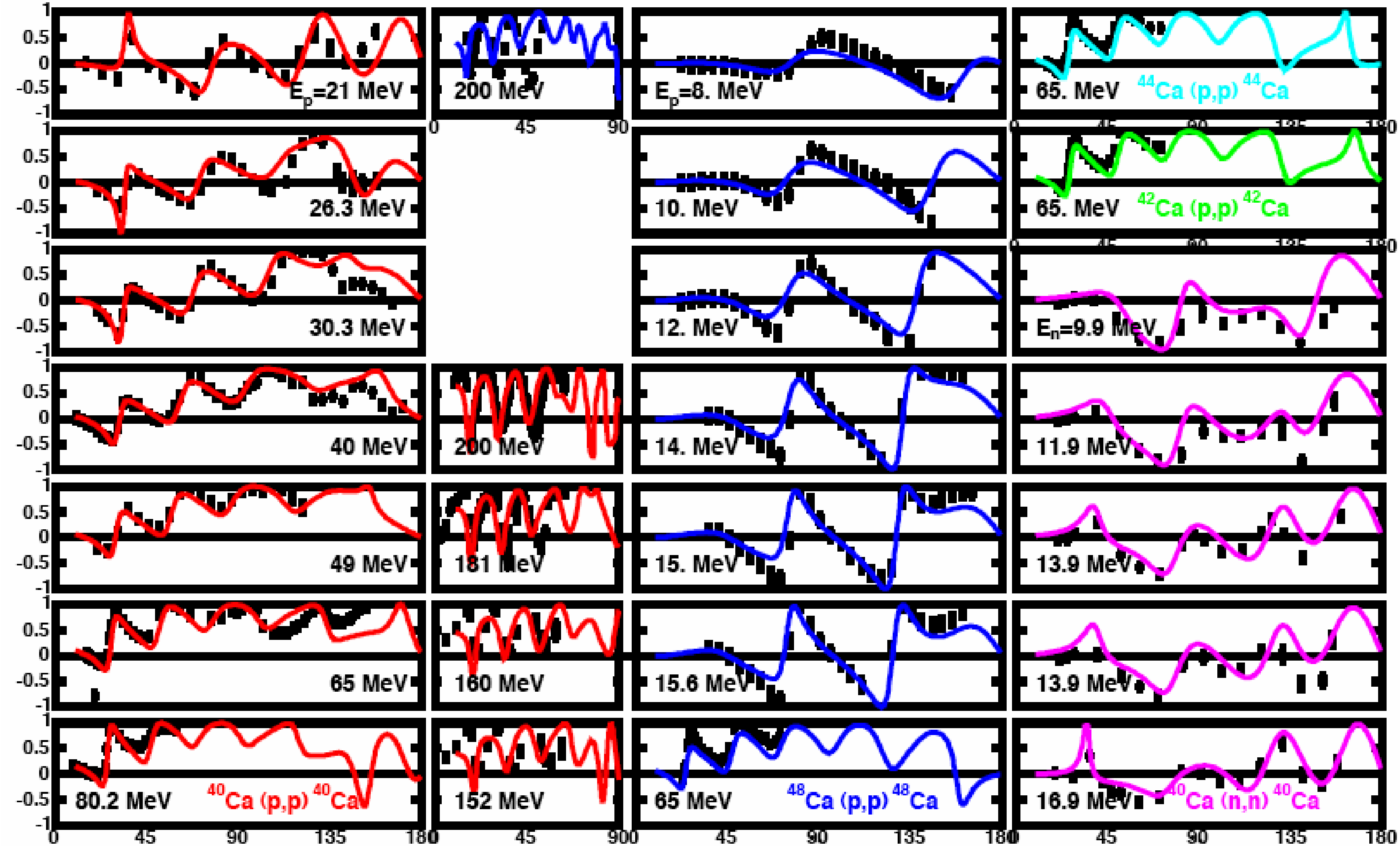
Current fit of n & p elastic scattering angular distributions



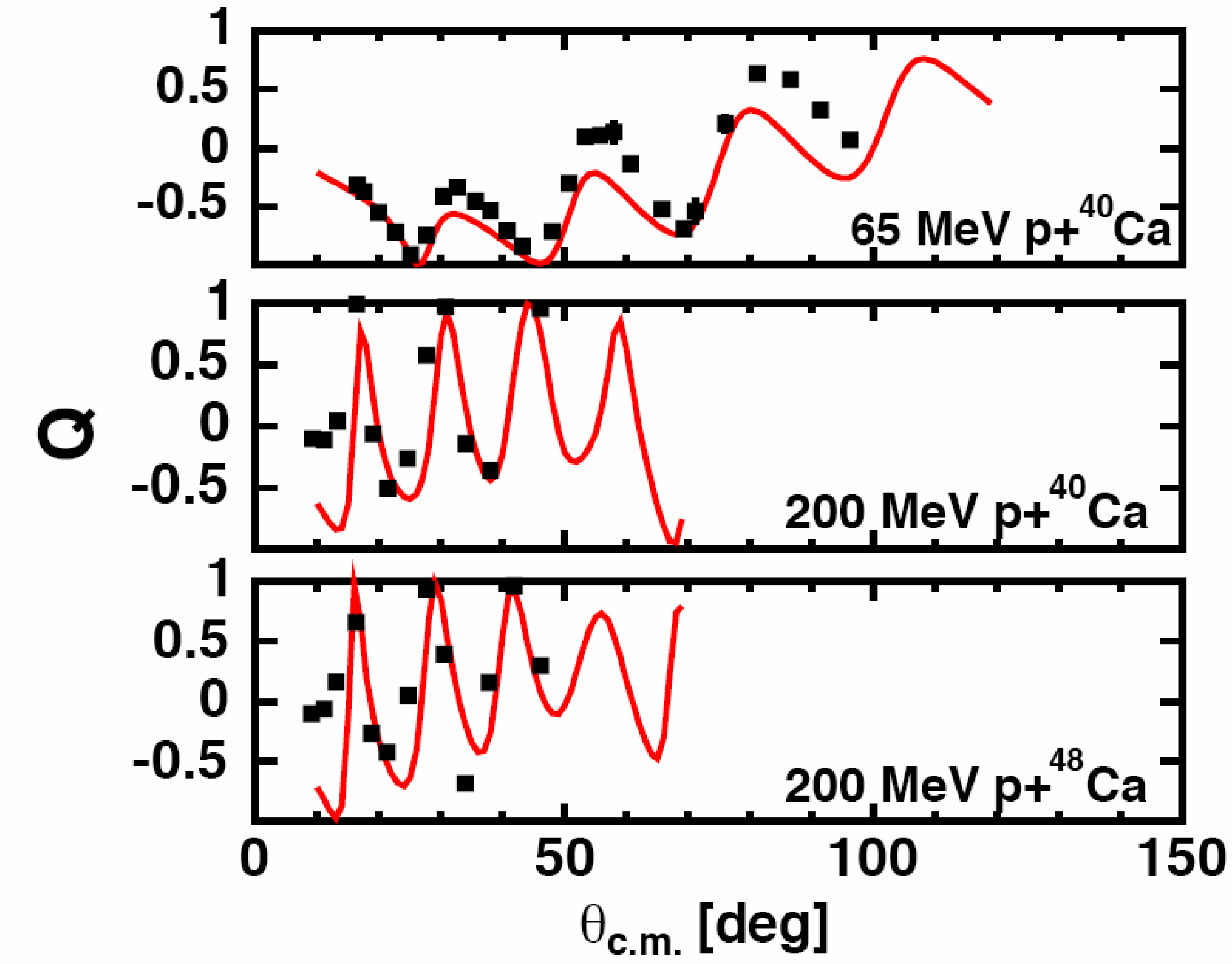
Fit of analyzing power data



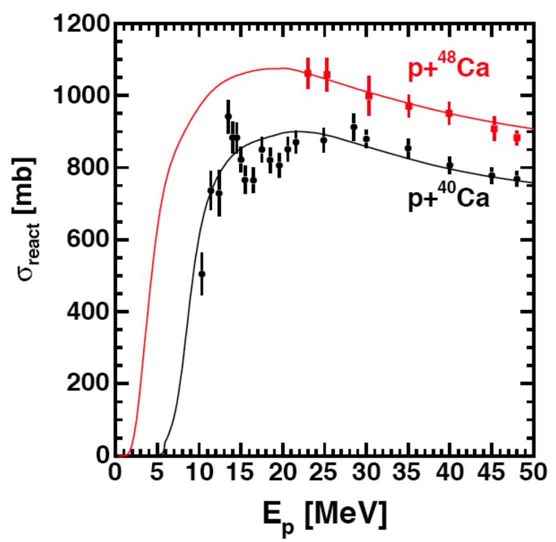
Present fit of polarization data

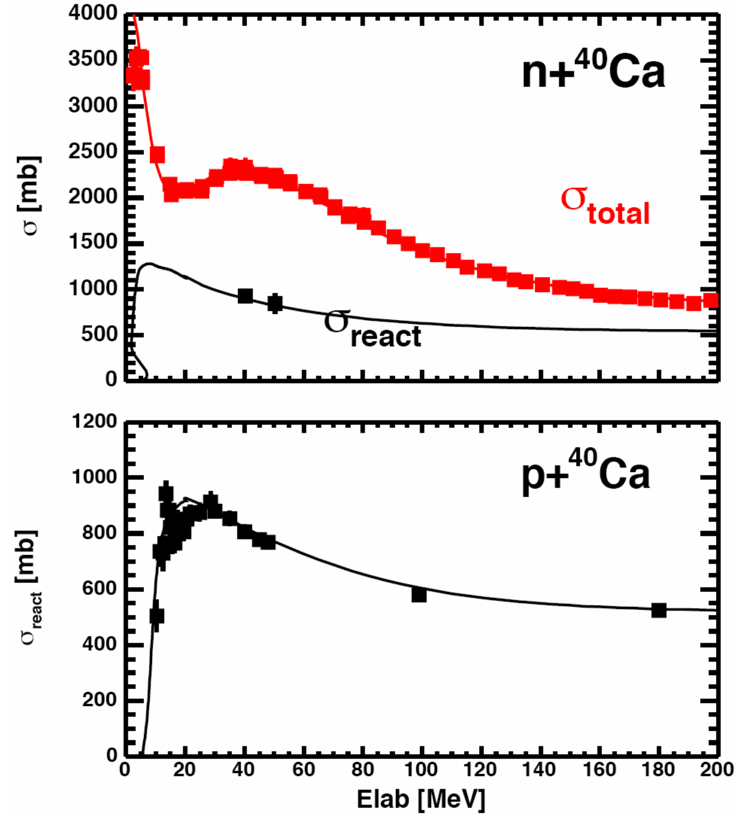


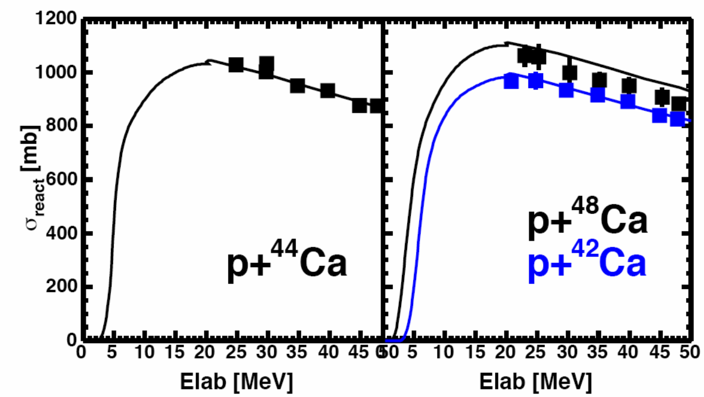
Spin rotation parameter



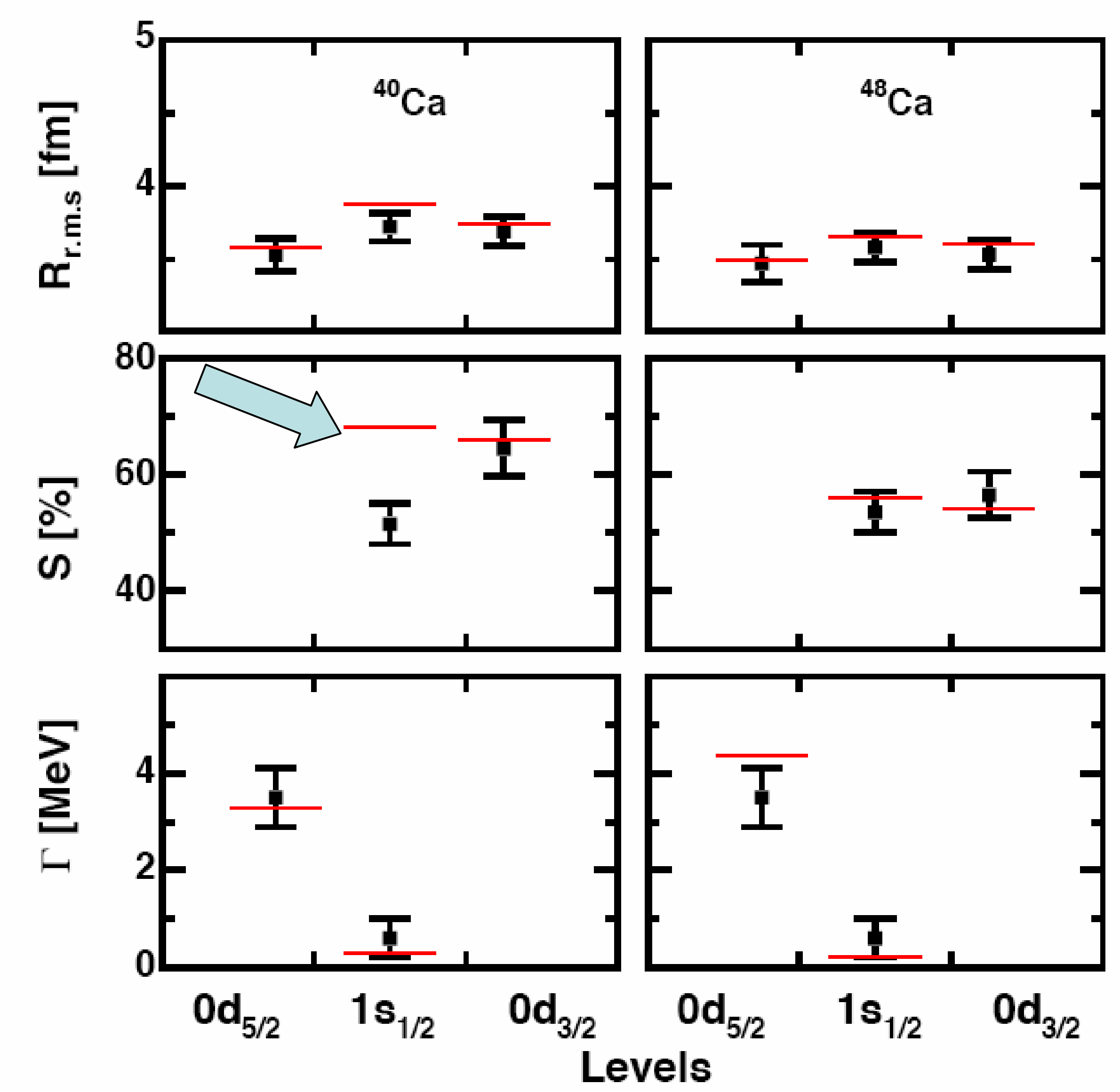
Fit of reaction cross sections







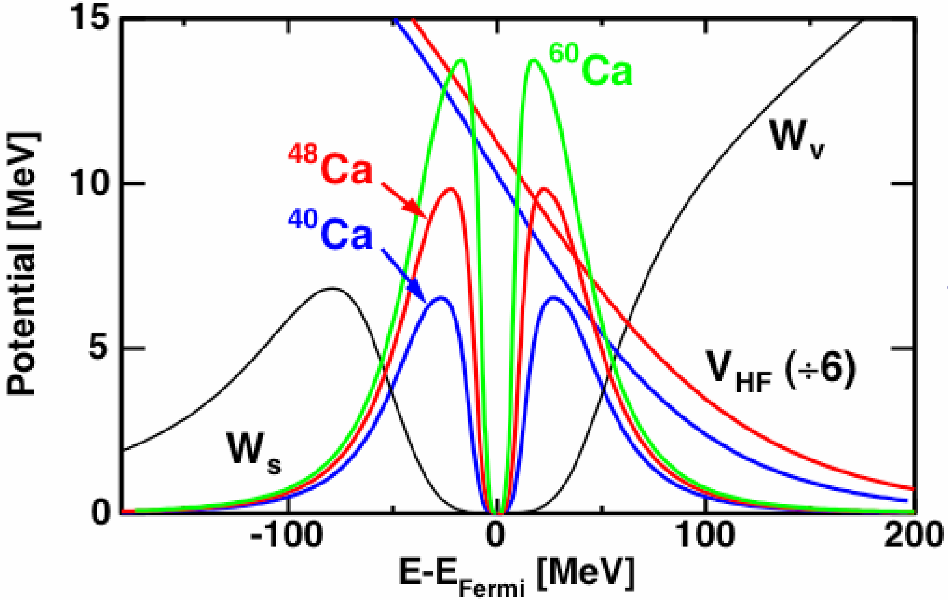
Present fit to (e,e'p) data



radii of
bound state
wave functions

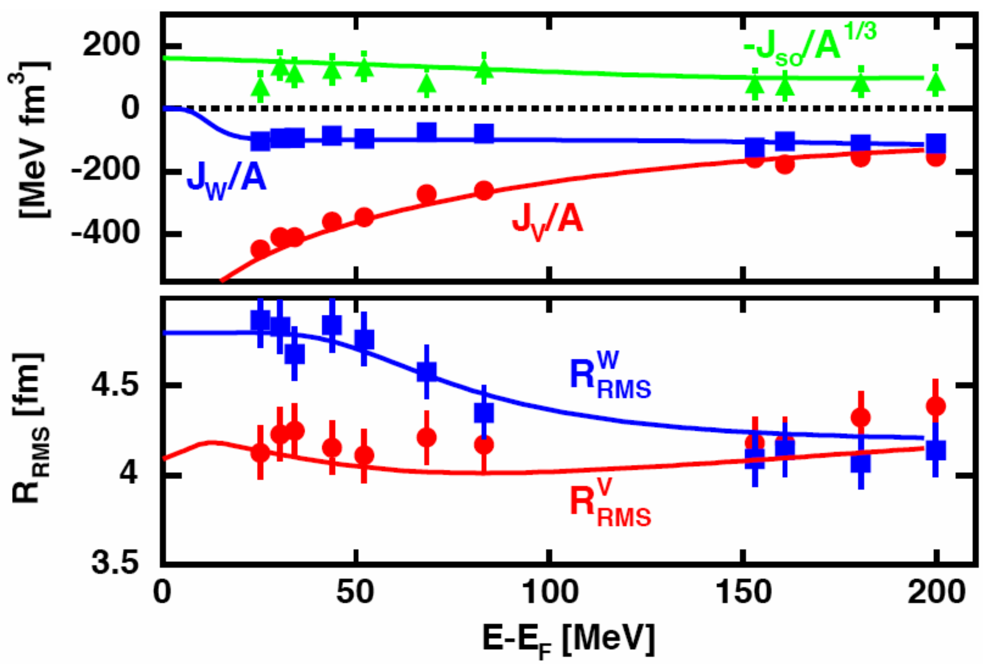
spectroscopic
factors

widths of strength
distribution



Potentials

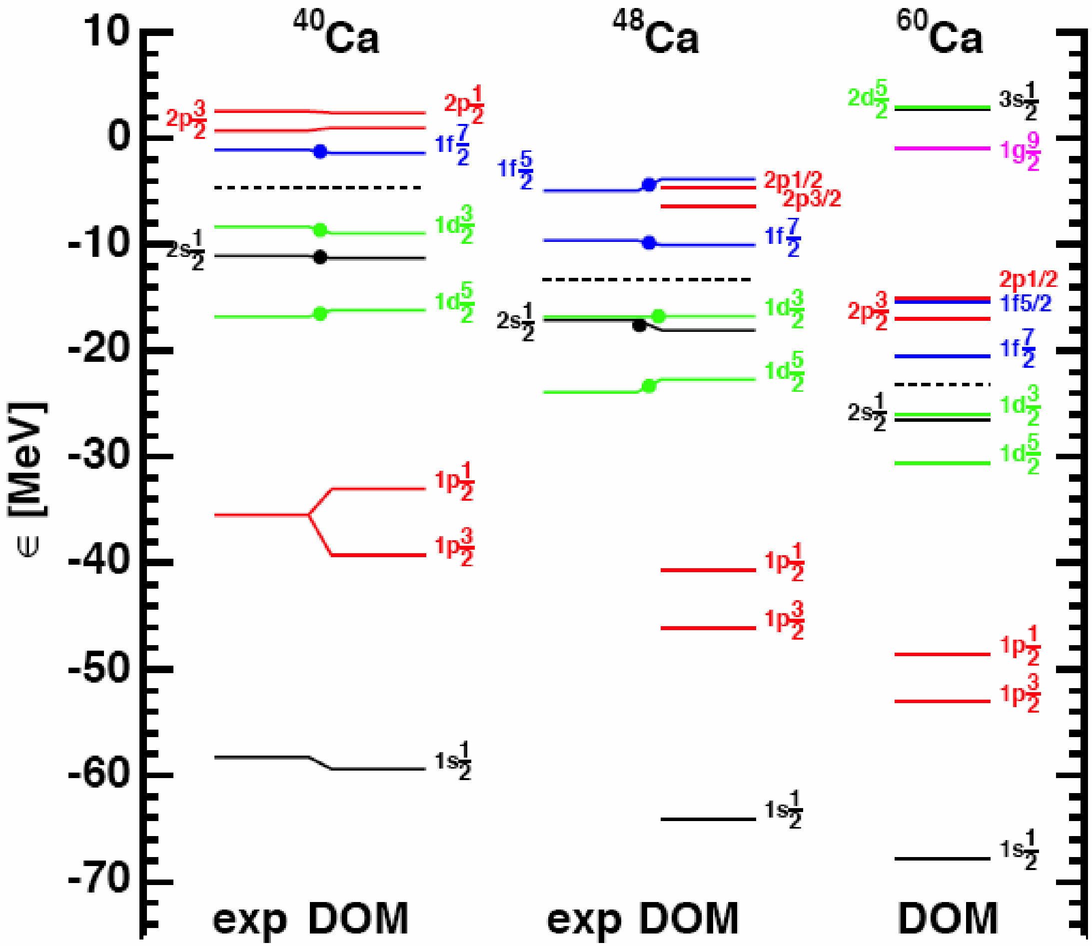
Surface potential strengthens
with increasing asymmetry



Volume integrals

RMS radii

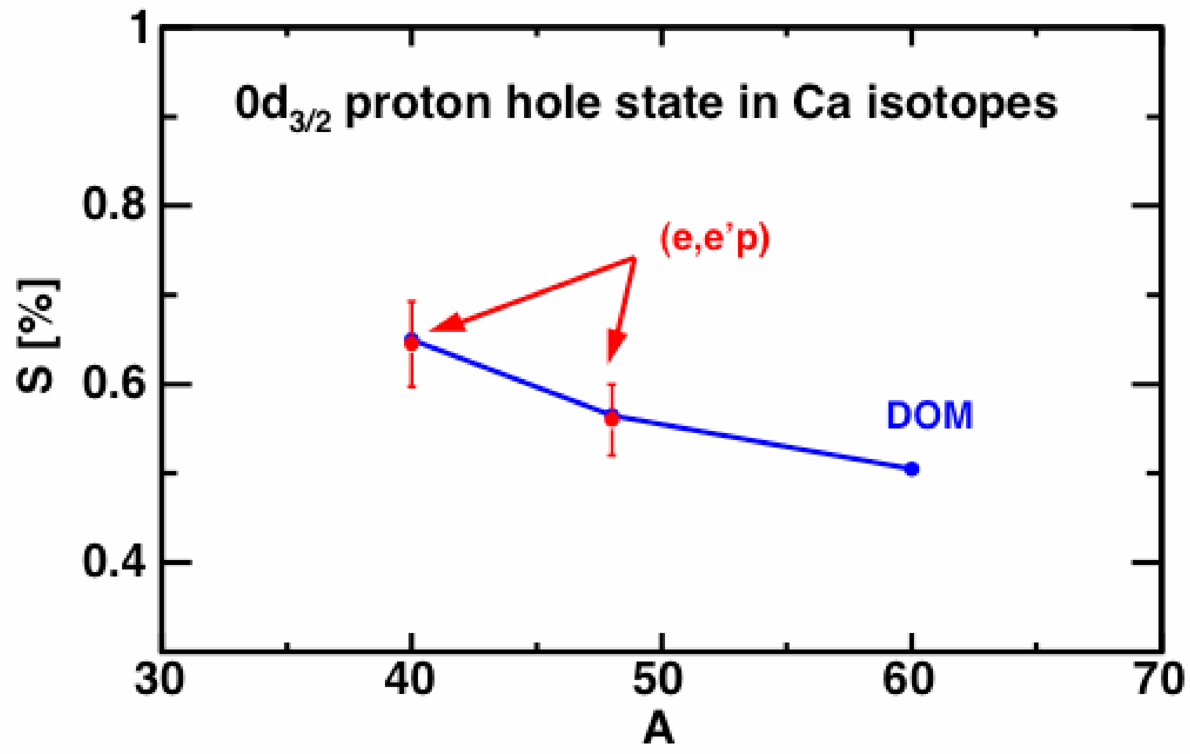
Proton single-particle structure and asymmetry



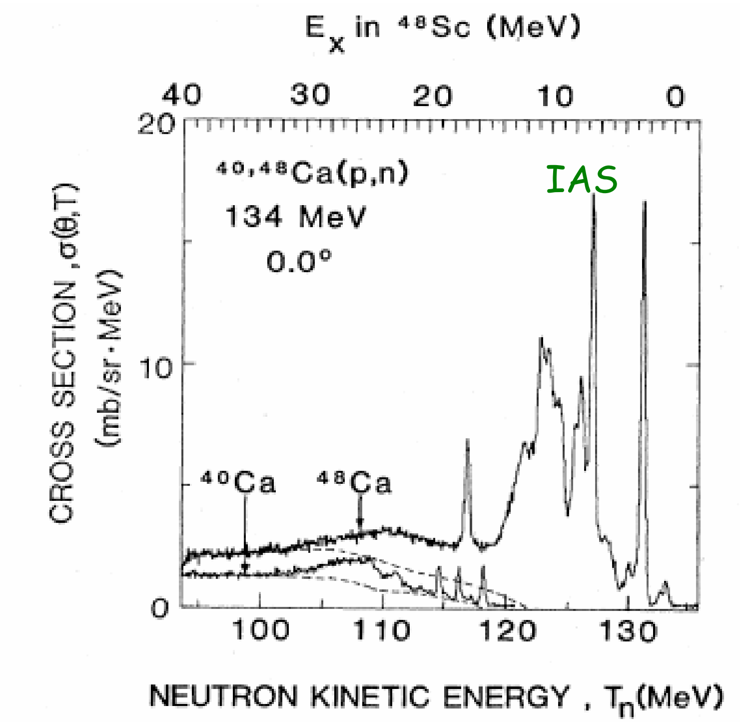
Pairing of protons due to *pn* correlations?!

Increased correlations with increasing asymmetry!

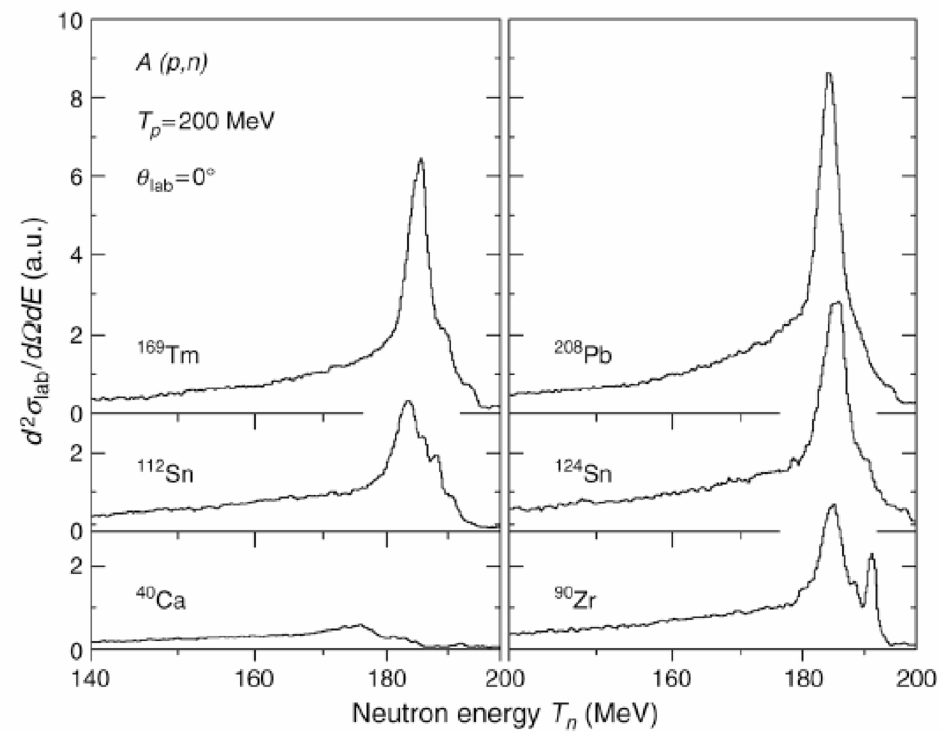
$d_{3/2}$ spectroscopic factor decreases with δ



What's the physics? GT resonance?



PRC31,1161(1985)

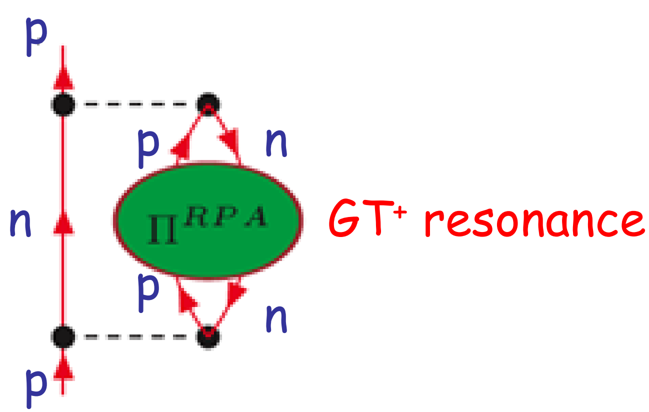


NPA369,258(1981)

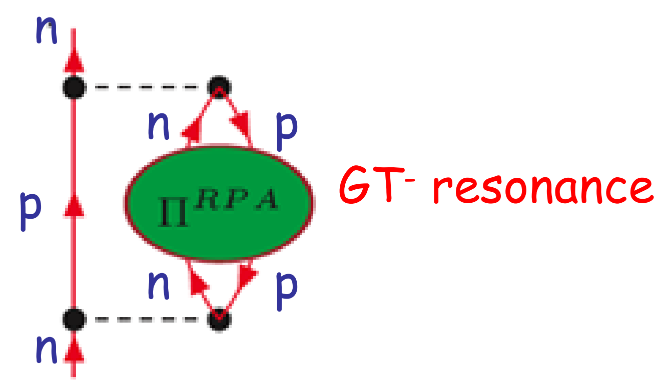
Influence of Gamow-Teller Giant Resonance or $\sigma_1.\sigma_2\ \tau_1.\tau_2$ (& tensor force) ph interaction

Sum rule for strength: $S(\beta^{+}) - S(\beta^{-}) = 3(N-Z)$

For N>Z only p affected



For Z>N only n affected



Related issue:

Change in magic numbers with increasing asymmetry

e.g. Otsuka et al., Phys. Rev. Lett. 95, 232502 (2005)

New framework to do self-consistent sp theory

Quasiparticle density functional theory ⇒ QP-DFT

D. Van Neck et al., Phys. Rev. A**74**, 042501 (2006)

Ground-state energy and one-body density matrix from self-consistent sp equations that extend the Kohn-Sham scheme.

Based on separating the propagator into a quasiparticle part and a background, expressing only the latter as a functional of the density matrix.
⇒ in addition yields qp energies and overlap functions

Reminder: DFT does not yield removal energies of atoms

Relative deviation [%]

		DFT	HF
He atom	1s	37.4	1.5
Ne atom	2p	38.7	6.8
Ar atom	3p	36.1	2.0

While ground-state energies are closer to exp in DFT than in HF

Can be developed for nuclei from DOM input!

Summary

- Proton sp properties in stable closed-shell nuclei understood (mostly)

Study of N≠Z nuclei based on DOM framework and experimental data

- Description of huge amounts of data
- More data necessary

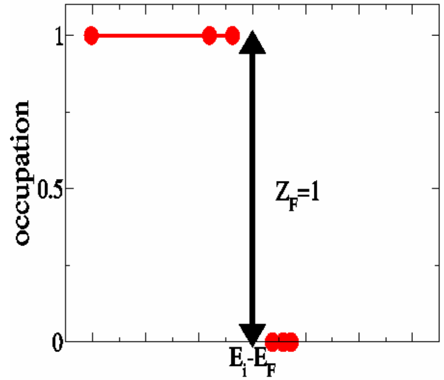
Predictions

- N≠Z protons more correlated while neutrons less (for N>Z) and vice versa
- Proton closed-shells with N>>Z ⇒ may ultimately favor pp pairing

Correlations in ...

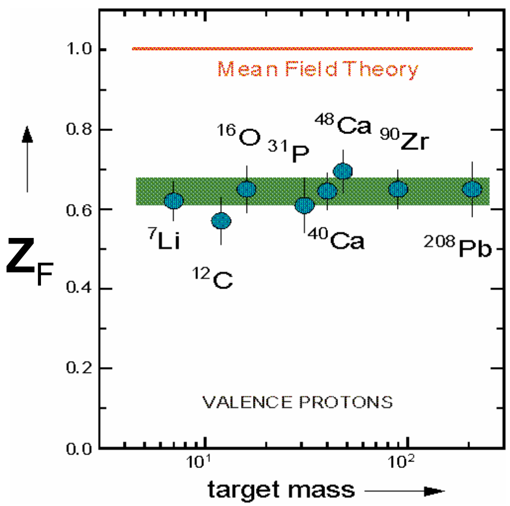
Atoms

weak correlations

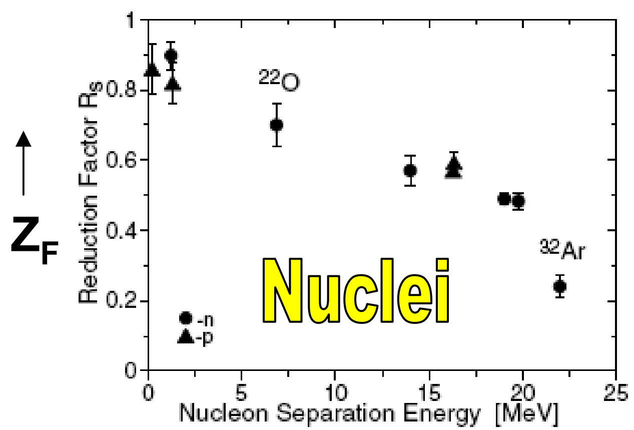


electrons in Ne
Data from (e,2e)

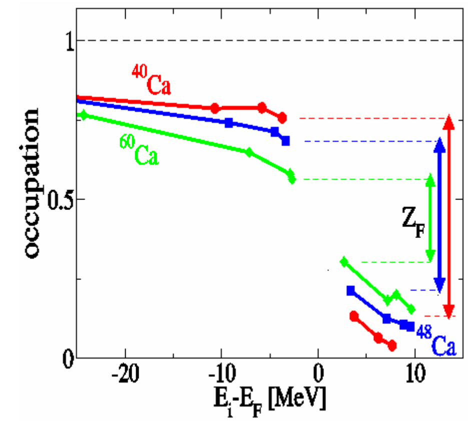
(e,e'p)



protons in stable
closed-shell nuclei

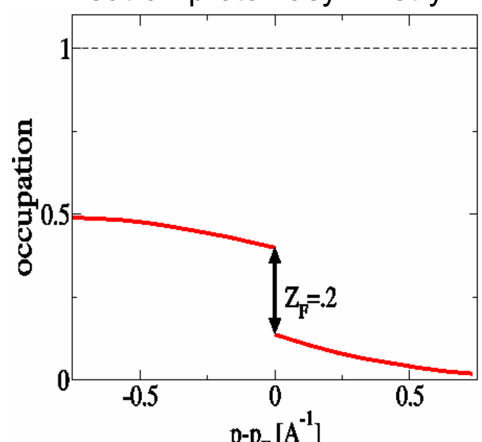


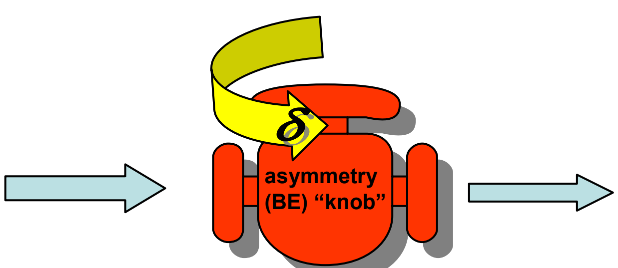
DOM



protons in Ca

Neutron-proton asymmetry





Liquid 3He

very strong correlations
Data from (n,n')

Outlook

- Explore the Gamow-Teller connection
  - link with excited states
- More experimental information from elastic nucleon scattering is important!
  - lots of informative experiments to be done with radioactive beams
- Neutron experiments ...

- More DOM analysis
- Exact solution of the Dyson equation with nonlocal potentials
- Employ information of nucleon self-energy to generate functionals for

  QP-DFT = Quasi-Particle Density Functional Theory (Van Neck et al. $\Rightarrow$ PRA)

  DFT that includes a correct description of QP properties!!

- More (funded) effort in nuclear many-body theory to support the above

Approximations to solving Dyson equation

- No l *j* dependence of self-energy apart from standard spin-orbit
- Assumed form of "*HF*" potential fixed geometry
- Imaginary part of self-energy at low-energy is spiky (poles)
  $\Rightarrow$ extra fragmentation at low energy (open-shell nuclei!)
- Expressions for occupation numbers "heuristic"
- *Z* factors not useful except near E_F *(exact there)*
- Division volume & surface "physical" but …
- Volume terms from nuclear matter should also include asymmetry
- Local-equivalent energy-dependent potential must be replaced by nonlocal potential

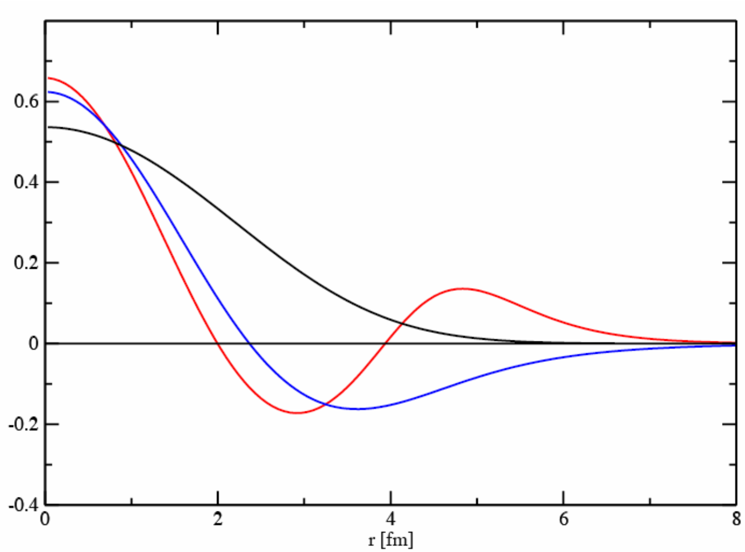
Improvements in progress

Replace treatment of nonlocality in terms of local equivalent but energy-dependent potential by explicitly nonlocal potential

$\Rightarrow$ Necessary for exact solution of Dyson equation

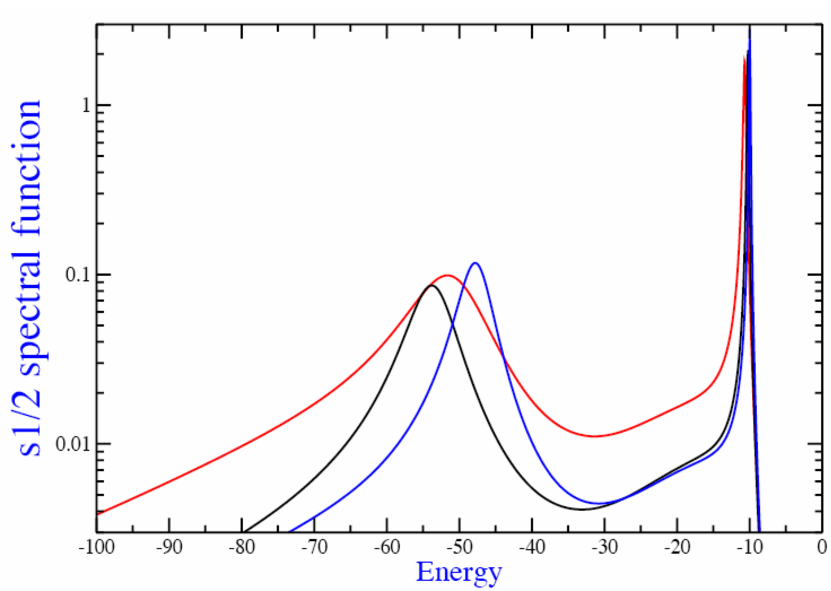
- Yields complete spectral density as a function of energy OK
- Yields one-body density OK
- Yields natural orbits OK
- Yields charge density OK
- Yields neutron density OK
- Data for charge density can be included in fit
- Data for (e,e′p) cross sections near E_F can be included in fit
- High-momentum components can be included (Jlab data)
- E/A can be calculated/ used as constraint $\Rightarrow$ TNI
- NN Tensor force can be included explicitly
- Generate functionals for QP-DFT

Examples



Natural orbits for $s_{1/2}$ in ^{40}Ca

$n_1 = 0.925$
$n_2 = 0.906$
$n_3 = 0.019$



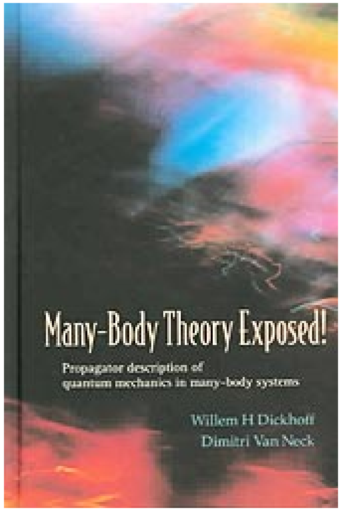
$s_{1/2}$ spectral function

Integrated:
all $s_{1/2}$ in ground state

Red curve: local-eq V_{HF}
Blue/Black: nonlocal V_{HF}

Description of the nuclear many-body problem

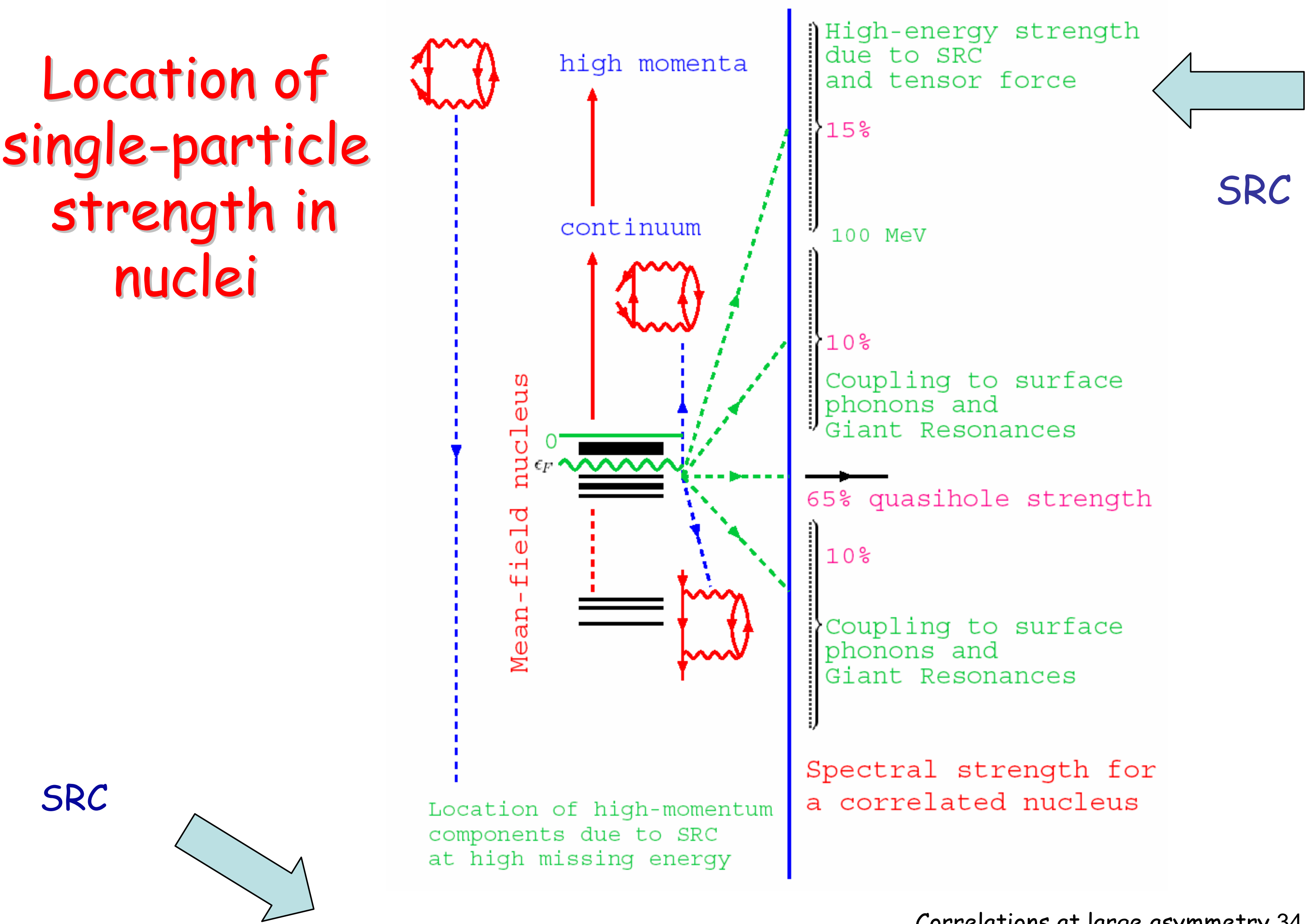
Ingredients: Nucleons interacting by "realistic interactions"
Nonrelativistic many-body problem
Method: Green's functions (Propagators)
☑ amplitudes instead of wave functions
keep track of all nucleons, including the high-momentum ones
Book:



Dimitri Van Neck & W.D.

Review: W.D. & C. Barbieri, Prog. Part. Nucl. Phys. **52**, 377 (2004)
Lecture notes: http://www.nscl.msu.edu/~brown/theory-group/lecture-notes.html

Location of single-particle strength in nuclei



Features of simultaneous fit to ^{40}Ca and ^{48}Ca data

- Surface contribution assumed symmetric around E_F
  Represents coupling to low-lying collective states (GR)
- Volume term asymmetric w.r.t. E_F taken from nuclear matter
- Geometric parameters r_i and a_i fit but the same for both nuclei
- Decay (in energy) of surface term identical also
- Possible to keep volume term the same (consistent with exp) and independent of asymmetry
- "*HF*" and surface parameters different and can be extrapolated to larger asymmetry

- Surface potential stronger and narrower around E_F for ^{48}Ca
- Both elastic scattering and (e,e′p) data included in fit

Employed equations

$$\Sigma(\boldsymbol{r}\mathrm{m},\boldsymbol{r}'\mathrm{m}';E) \Rightarrow \mathrm{U}(r,E) = -\mathrm{V}(r,E) + V_{so}(r) + V_C(r)$$
$$- iW_v(E)\, f(r,r_v,a_v) + 4ia_s W_s(E)\, f'(r,r_s,a_s)$$

$$f(r,r_i,a_i) = \left(1 + e^{\frac{r - r_i A^{1/3}}{a_i}}\right)^{-1}$$

Woods-Saxon form factor

$$\mathrm{V}(r,E) = V_{HF}(E)\, f(r,r_{HF},a_{HF}) + \Delta\mathrm{V}(r,E)$$

"*HF*" includes main effect of nonlocality $\Rightarrow$ *k*-mass

$$\Delta\mathrm{V}(r,E) = \Delta V_v(E)\, f(r,r_v,a_v) - 4a_s\, \Delta V_s(E)\, f'(r,r_s,a_s)$$

"Time" nonlocality $\Rightarrow$ *E*-mass

$$\Delta V_i(E) = \frac{P}{\pi} \int_{-\infty}^{\infty} W_i(E') \left(\frac{1}{E' - E} - \frac{1}{E' - E_F} \right) dE'$$

Subtracted dispersion relation