

Mass
parametry

A. Baran

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Tunnelling

Dynamics

Hamiltonian

GCM

Results

Sensitivity on the
pairing type

Skyrme vs Gogny
force

SkM*

Summary

Graphical
explanation

Collective inertia and fission barriers within the Skyrme-Hartree-Fock theory

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Outline

- Spontaneous fission
- GCM
- Energy corrections: vib & rot
- Results
- Summary

How do we describe fission?

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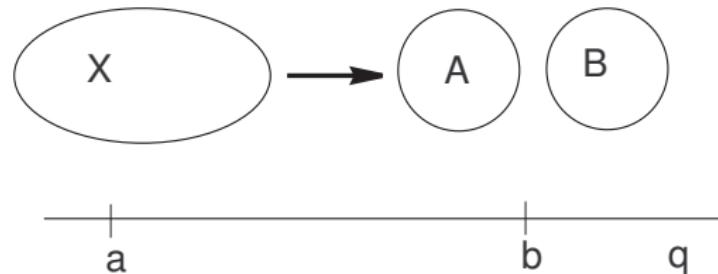
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Tunnelling of a potential barrier

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Conservative system:

$$H(p, q) = E = \text{const.}$$

Reduced action

$$S = \int_{q_1}^{q_2} p dq = \int_{t_1}^{t_2} p \dot{q} dt.$$

Maupertuis principle → Fission path.

$$\delta S = \delta \int_{\tau_1}^{\tau_2} d\tau \sqrt{2[E - V(q)]B_{ij}(q) \frac{dq^i}{d\tau} \frac{dq^j}{d\tau}}.$$

Tunnelling of a potential barrier

Quasiclassical approximation (WKB, JWKB,
 F^2 JWKB)

Spontaneous fission probability:

$$P \sim \exp(-2|S|), \quad \sim [1 + \exp(2|S|)]^{-1}.$$

Spontaneous fission half life:

$$T_{sf} = \frac{\ln 2}{nP}.$$

$$\min T = \max P = \min |S|$$

Elements: $V(\vec{q})$ (HFBCS), $B_{ij}(\vec{q})$ (GCM, ATDHF, CRA)

Dynamical programming

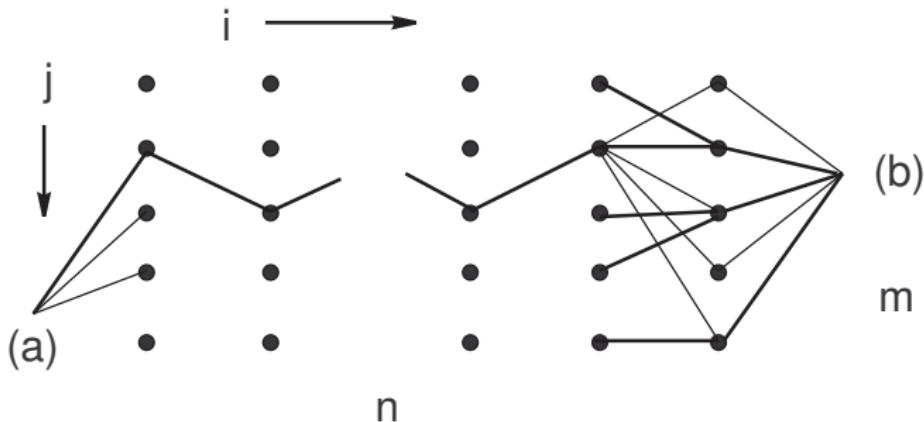


Figure: The *shortest path* determined by dynamic programming method. The grid points represent the mesh in a 2-dimensional space $\{x\}$ in which the functional $S[x(s)]$ is defined. Thick zig-zag line represents the shortest path ($\delta S = 0$) starting from the point (a) and ending at the final point (b). [Explanation](#)

$$S(a, b) = \min_{k_1} [S(a; (1, k_1)) + S((1, k_1); b)]$$

Hamiltonian

- One body mean field: \hat{H}_{SkHF}
- Pairing

$$\hat{H}_{pair} = - \sum_{k,l>0} G_{kl} a_k^\dagger a_k^\dagger a_l a_l$$

$$G_{kl} = \langle k\bar{k} | \hat{v}_{pair} | l\bar{l} \rangle$$

- monopole pairing: $G_{kl} = const$

$$\begin{aligned} G_n &= [24.70 - 0.108(N-Z)]/A, \\ G_p &= [14.76 + 0.241(N-Z)]/A. \end{aligned} \quad (1)$$

- Density dependent delta interaction (DDDI)

$$\hat{v}_{pair}(\vec{r}_1 - \vec{r}_2) = v_{0t} \delta(\vec{r}_1 - \vec{r}_2) \left[1 - \frac{\rho(\vec{r}_1)}{\rho_0} \right],$$

$$\rho_0 = 0.16 \text{ fm}^{-3}, v_{0n} = 842 \text{ MeV Fm}^3, v_{0p} = 1020 \text{ MeV fm}^3.$$

$$^{252}\text{Fm: } \Delta_n = 0.696, \Delta_p = 0.803 \text{ MeV.}$$

Generator coordinate method

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$$\Psi = \int dq f(q) \Phi(q).$$

$$\delta \frac{\langle \Psi | \hat{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle} = 0$$

$$h(q, q') = \langle \Phi(q) | \hat{H} | \Phi(q') \rangle$$

$$n(q, q') = \langle \Phi(q) | \Phi(q') \rangle$$

- Symmetric Moment Expansion (SME)
- Local Approximation (LA)
- Gaussian Overlap Approximation (GOA)

Generator coordinate method

GOA

$$q \rightarrow \frac{q + q'}{2}, \quad s = q' - q$$

Gaussian overlap approximation

$$n(q, q') \rightarrow n(q + s/2, q - s/2) = e^{-\gamma(q)s^2/2}.$$

$$\langle \Psi | \hat{H} | \Psi \rangle = \int \sqrt{\gamma} dq f^*(q) \mathcal{H}_{\text{coll}}(q, \frac{\partial}{\partial q}) f(q).$$

Collective Hamiltonian

$$\mathcal{H}_{\text{coll}}(q, \frac{\partial}{\partial q}) = -\frac{\hbar^2}{2\sqrt{\gamma}} \frac{\partial}{\partial q^\mu} \sqrt{\gamma} (\mathcal{M}^{-1})^{\mu\nu}(q) \frac{\partial}{\partial q^\nu} + V(q).$$

Generator coordinate method

GOA. Mass parameter and zero point energy

Mass parameter (1-dim; q)

$$M^{-1}(q) = \frac{1}{2\gamma^2(q)} \left[\left(\frac{\partial^2}{\partial q \partial q'} - \frac{\partial^2}{\partial q^2} + \frac{\partial \ln \gamma}{\partial q} \right) h(q, q') \right]_{q'=q} .$$

Potential:

$$V(q) = \langle \Phi | \hat{H} | \Phi \rangle - \varepsilon_0 ,$$

Zero point energy correction:

$$\varepsilon_0 = \frac{1}{2\gamma(q)} \left[\frac{\partial^2 h(q, q')}{\partial q \partial q'} + \langle q' | \frac{\partial \hat{H}}{\partial q} \frac{\partial}{\partial q} | q \rangle \right]_{q'=q} .$$

Generator coordinate method

GOA, n-dim, mass parameters

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Coordinates (n-dim):

$$q_i = \langle \hat{Q}_i \rangle.$$

$$\hat{H} = \hat{H}_0 - \sum_j \lambda_j \hat{Q}_j. \quad (2)$$

$$\delta \hat{H} = - \sum \delta \lambda_i \hat{Q}_i.$$

$$\frac{\partial \hat{H}}{\partial q_i} = \sum_j \left(\frac{\delta \lambda_j}{\delta q_i} \right) \hat{Q}_j = \frac{1}{2} \left(M^{(-1)} \hat{Q} \right)_i. \quad (3)$$

Generator coordinate method

GOA

$$B^{\text{GOA}} = 2\hbar^2 \Sigma^{(2)} [\Sigma^{(1)}]^{-1} \Sigma^{(2)} \quad (4)$$

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$$B^{\text{CRA}} = 2\hbar^2 \Sigma^{(3)} \quad (5)$$

$$\varepsilon_0 = \frac{1}{4} \text{Tr} \left\{ [\Sigma^{(2)}]^{-1} \Sigma^{(1)} \right\} \quad (6)$$

$$\Sigma^{(k)} = \frac{1}{4} M^{(1)-1} M^{(k)} M^{(1)-1} \quad (7)$$

$$M_{ij}^{(k)} = \sum_{\mu\nu} \frac{\langle \phi | \partial \hat{H} / \partial q_i | \mu v \rangle \langle \mu v | \partial \hat{H} / \partial q_j | \phi \rangle}{(E_\mu + E_v)^k}, \quad (8)$$

Rotational correction

Rotational symmetry breaking

In HFBCS ground state:

$$\langle \hat{j}^2 \rangle \neq 0 \quad \rightarrow \quad \text{rotation}$$

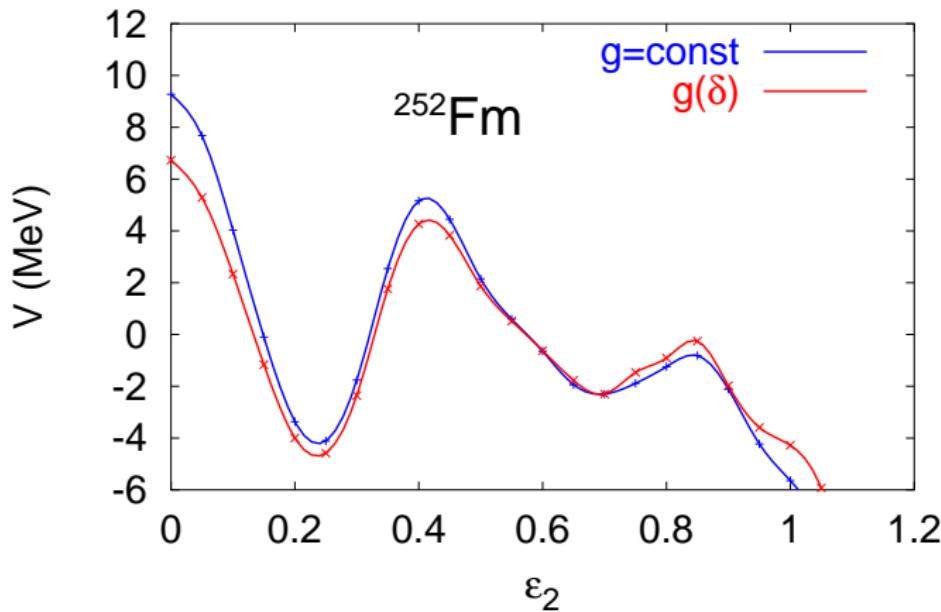
Energy correction:

$$E_{\text{rot}} = - \sum_{k=x,y,z} \frac{\langle \hat{j}_k^2 \rangle}{2 \mathcal{J}_k}$$

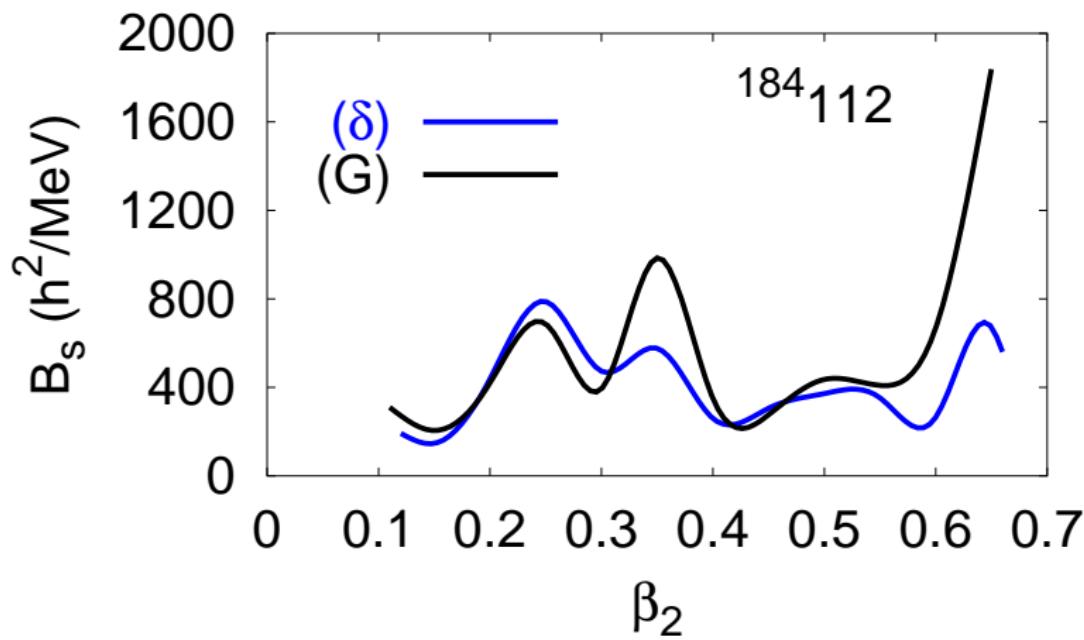
Quentin+
Baran, Höhenberger;

Sensitivity on the pairing type

^{252}Fm barrier



Sensitivity on pairing type

 $^{282}\text{112}$ mass parameter

Zero point energy correction

SLy4+GOA vs Gogny force

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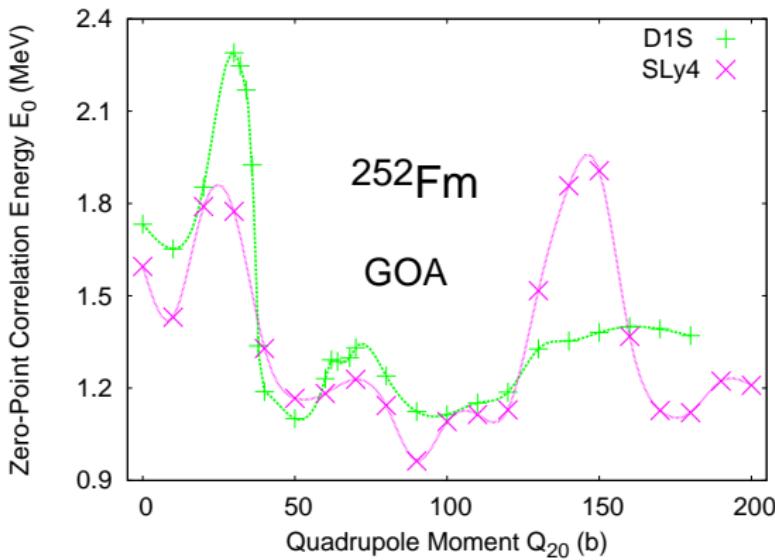
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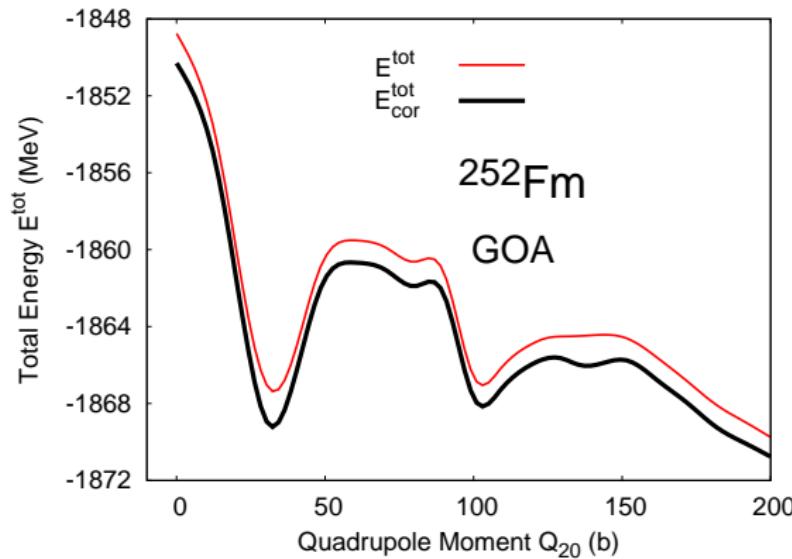
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Barriers

SLy4+GOA vs Gogny force



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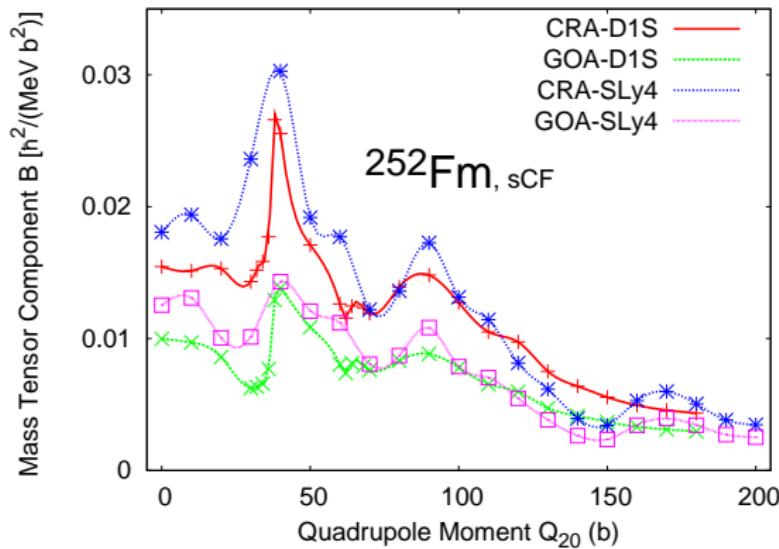
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Barriers & mass parameters

HF, SKM*, ^{256}Fm

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Definicje:

- sEF - symmetric(s), elongated(C), Fragments(F)
- aEF - asymmetric(s), elongated(E), Fragments(F)
- sCF - symmetric(s), compact(C), Fragments(F)

Barriers & mass parameters

HF, SkM*, ^{256}Fm

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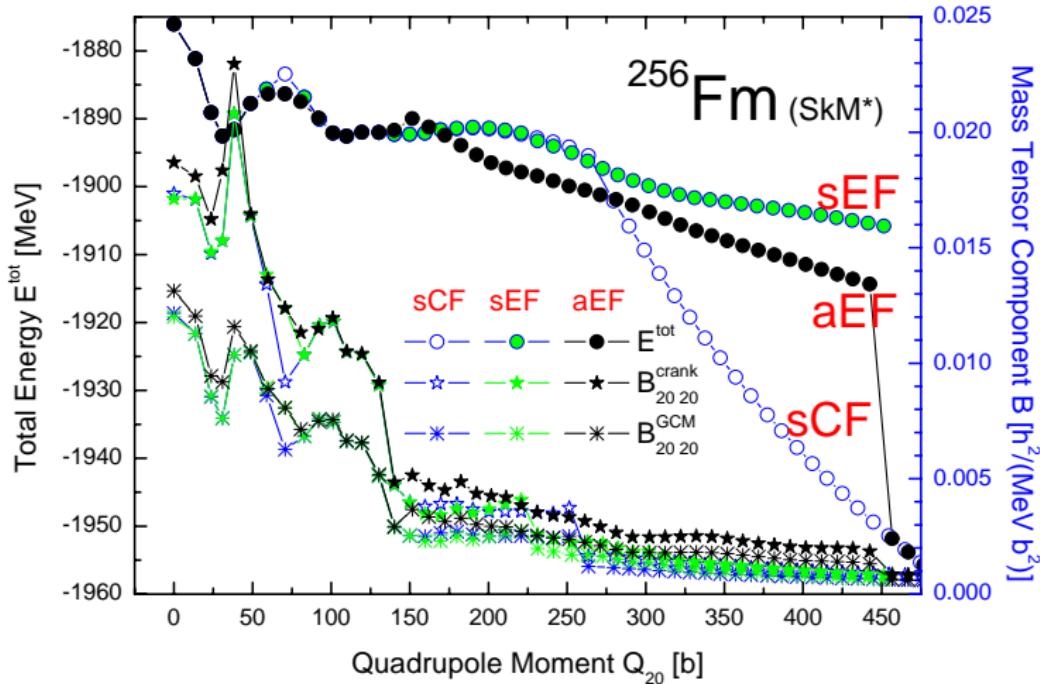
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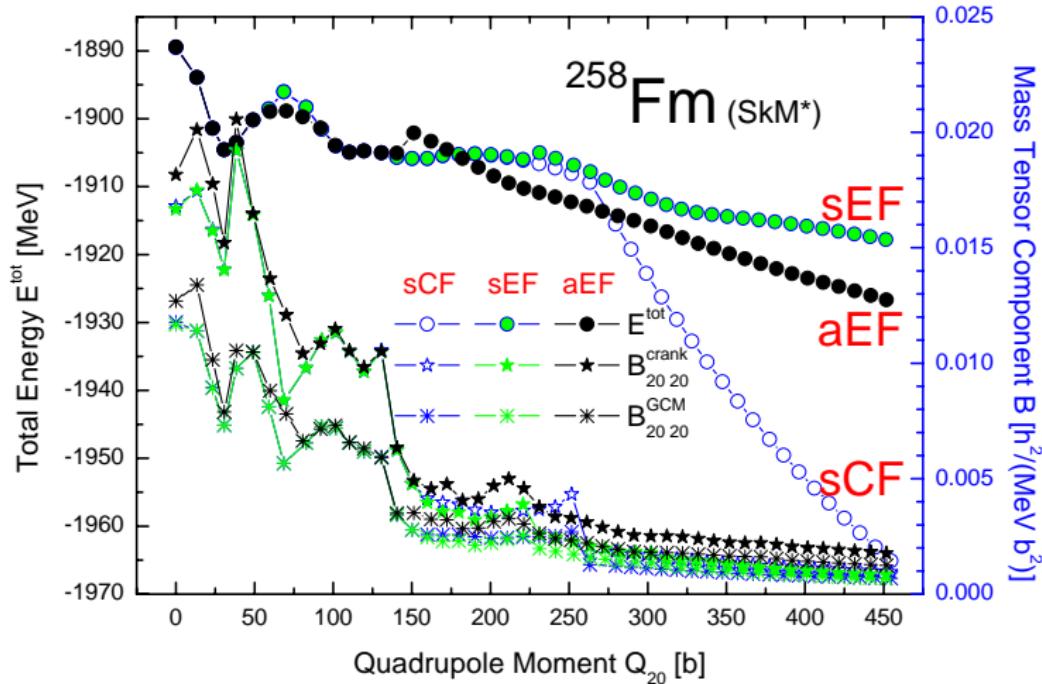
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Caculations

- Barriers (incl. **rot** and **zpe** corrections)
- GOA and cranking mass parameters
- → half lives, analysis of bimodal fission

Features and consequences of mass behaviour

- B in the barrier region —→ both V and B decides on T_{sf}
- asymptotic behaviour —→ the post fission motion of fragments depends mainly on V (bimodal fission)

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Graphical explanation of dynamic programing method

► Dynamics

(a)



1

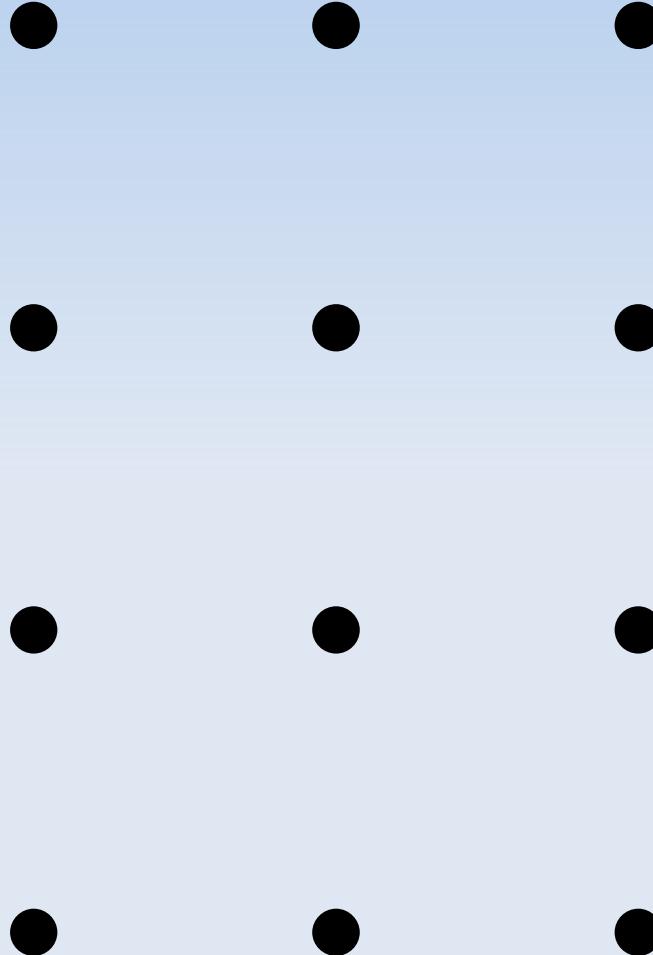


(b)



Shortest path...

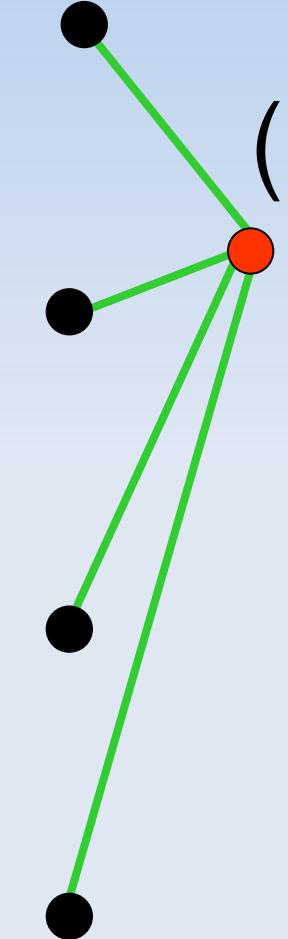
(a)



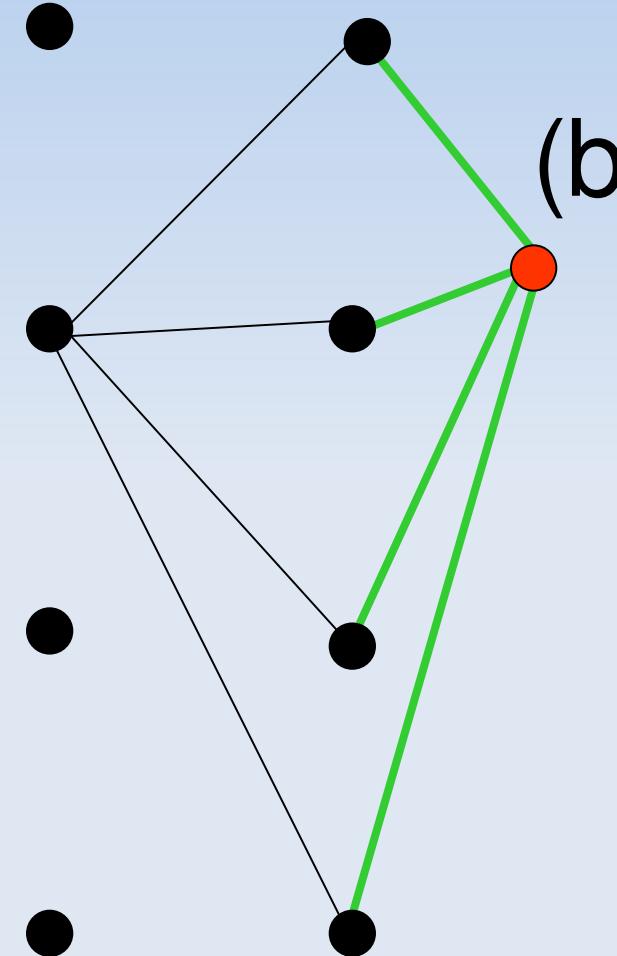
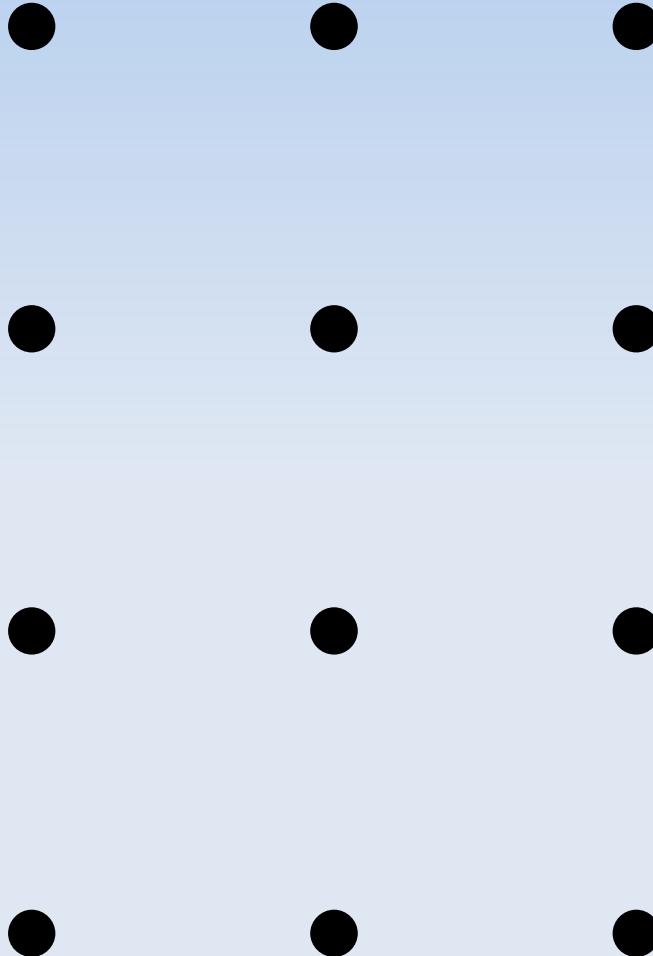
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Shortest path...

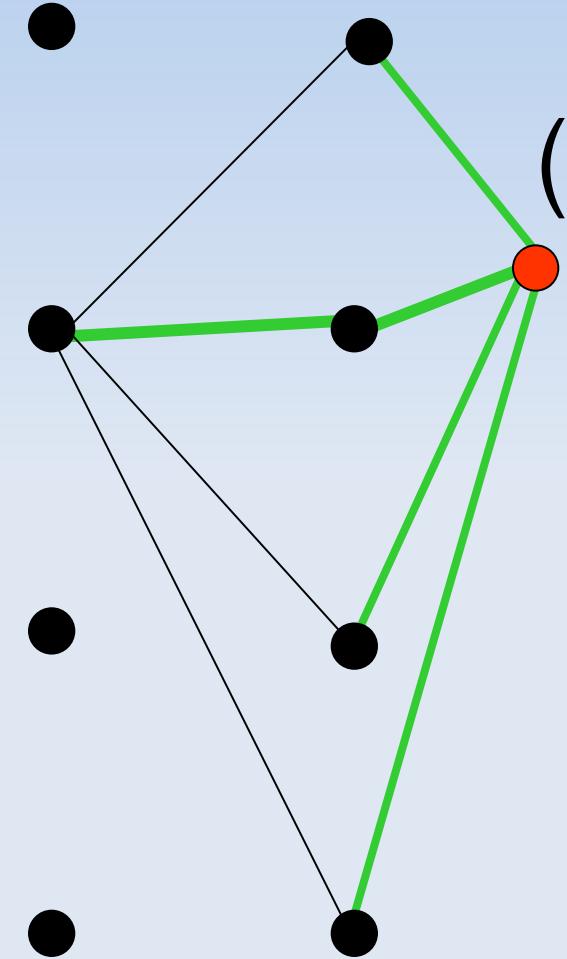
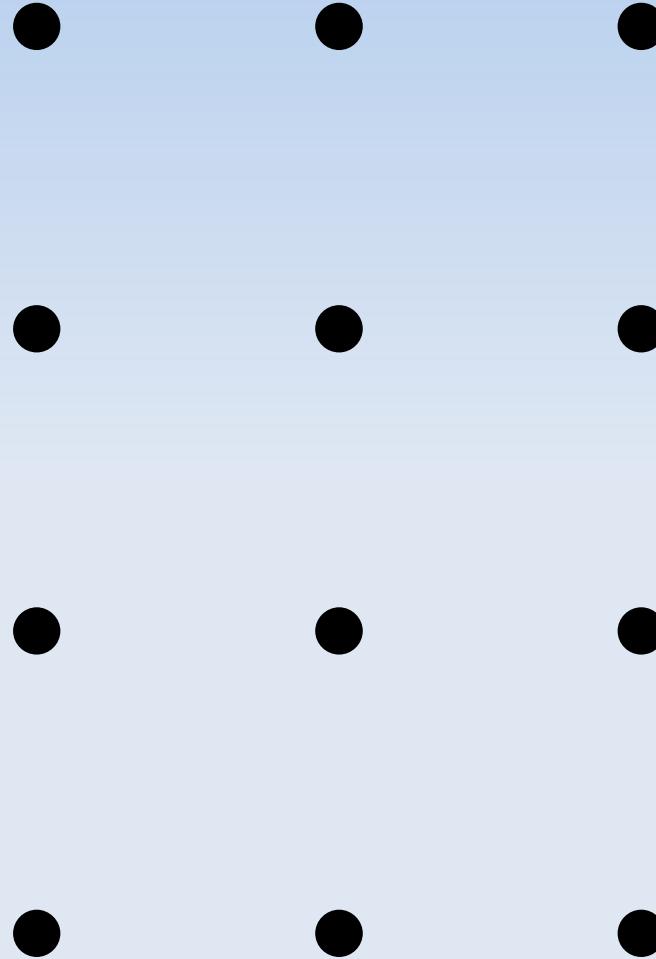
(b)



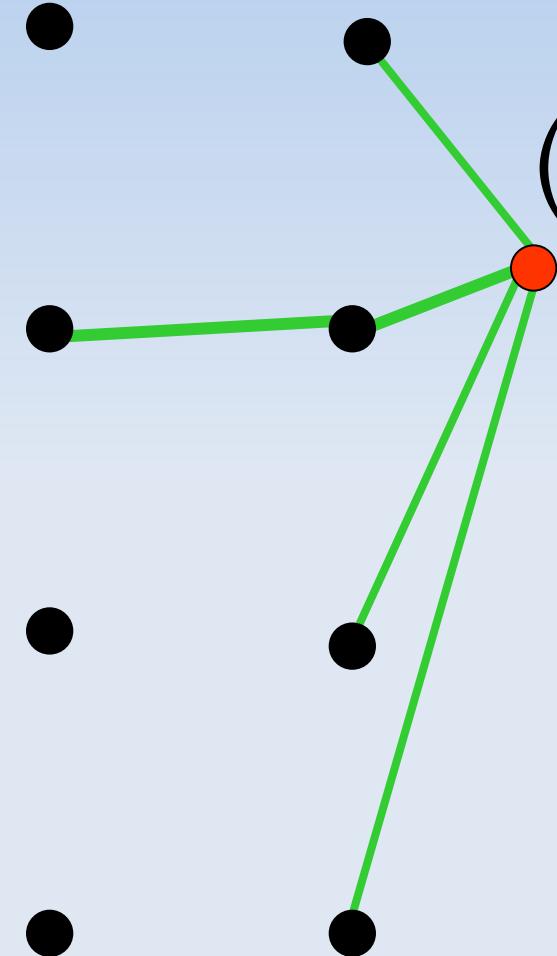
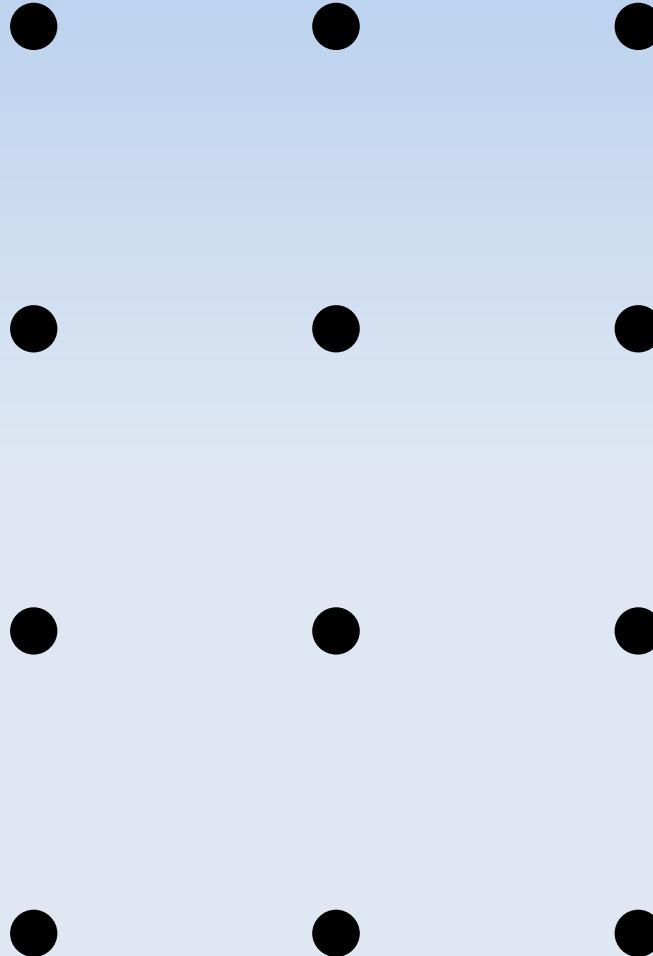
(a)



(a)



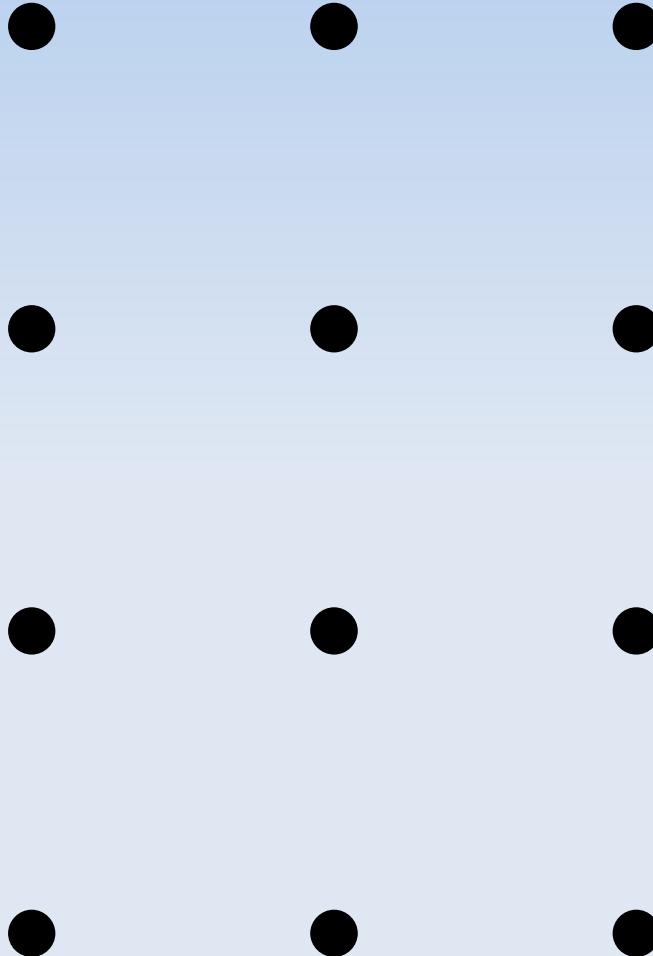
(a)



5

Shortest path...

(a)

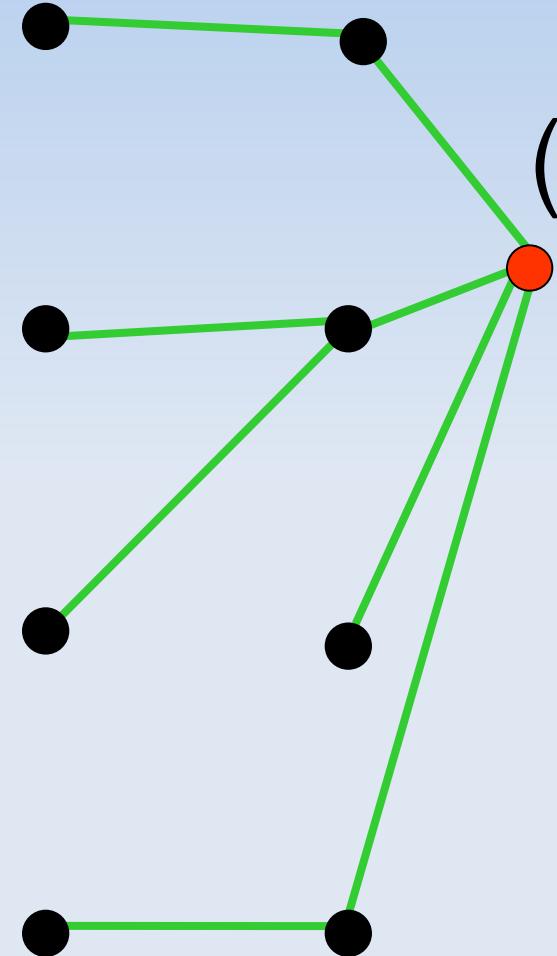


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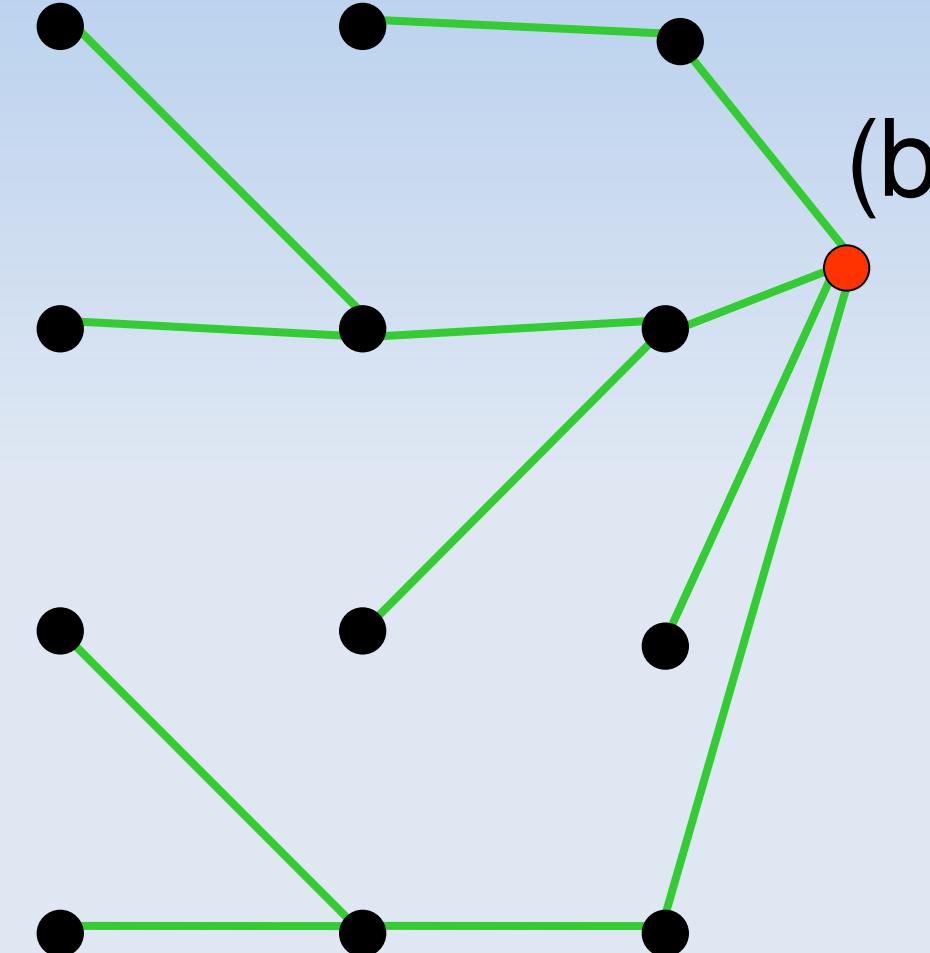
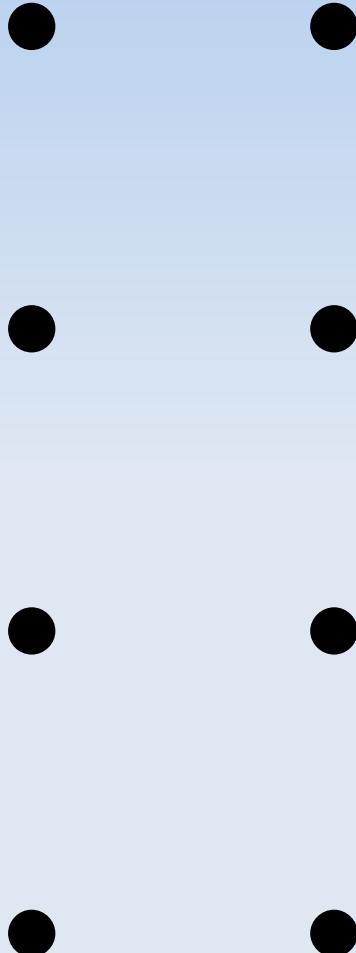
6

Shortest path...

(b)



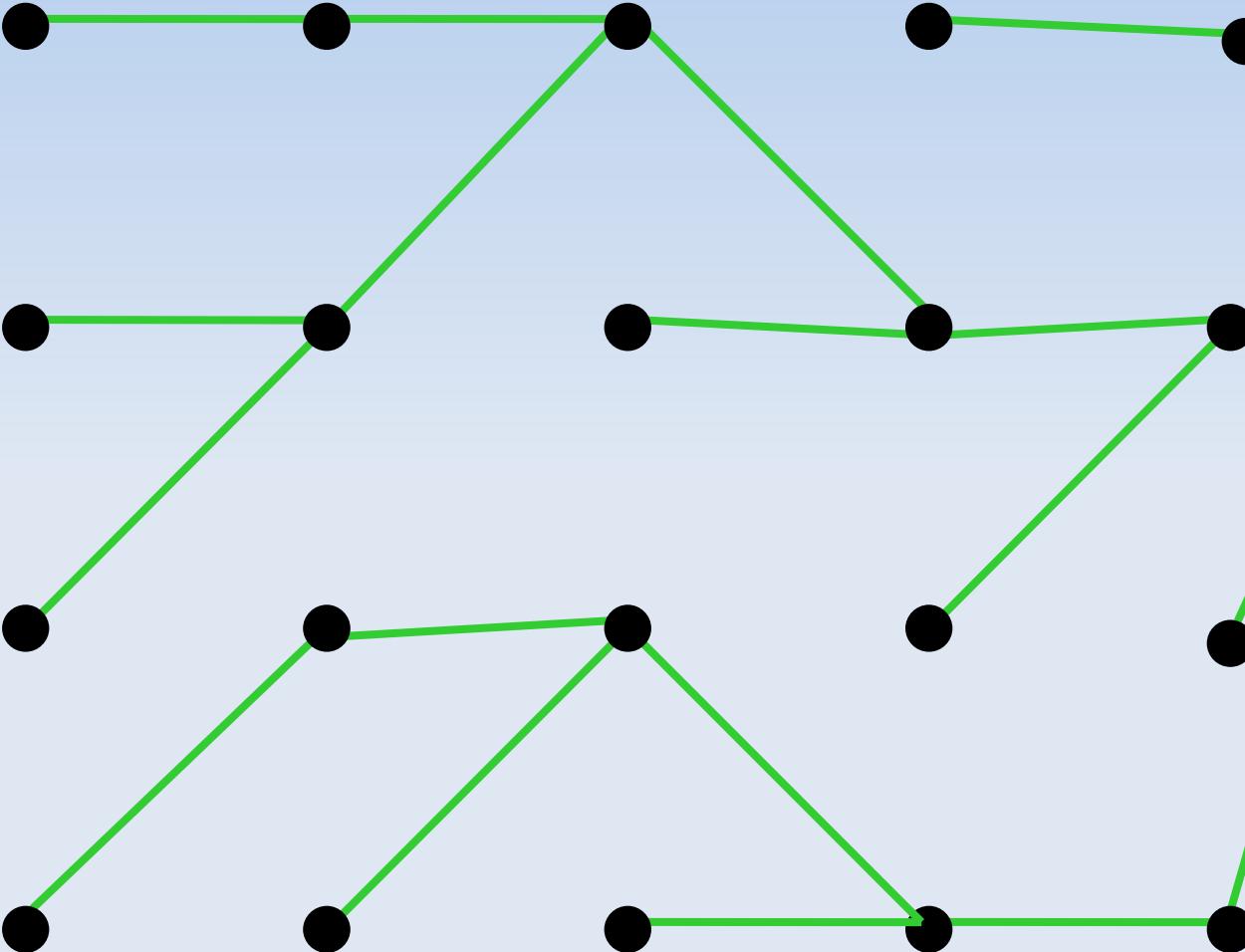
(a)



7

Shortest path...

(a)

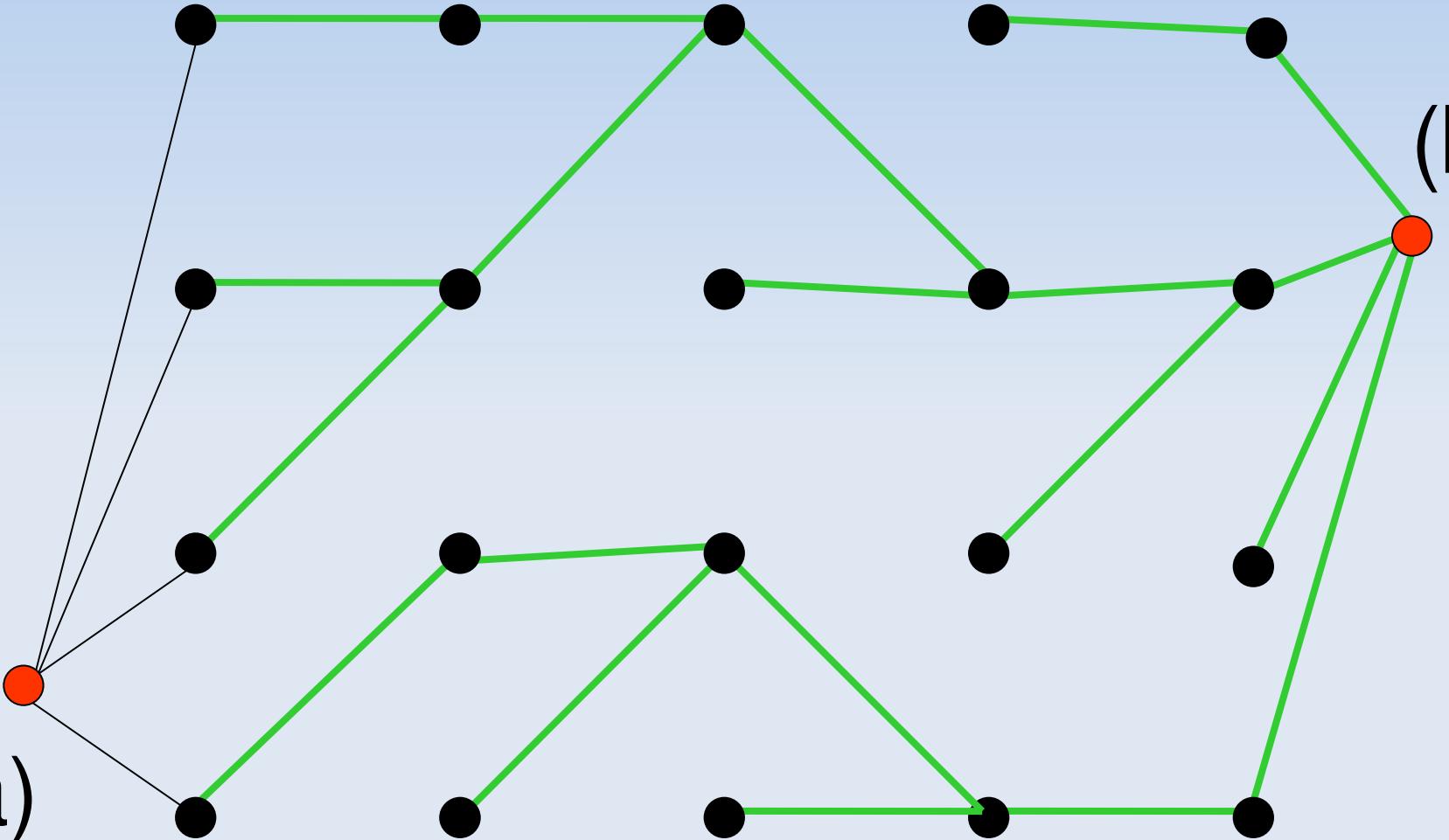


(b)

8

Shortest path...

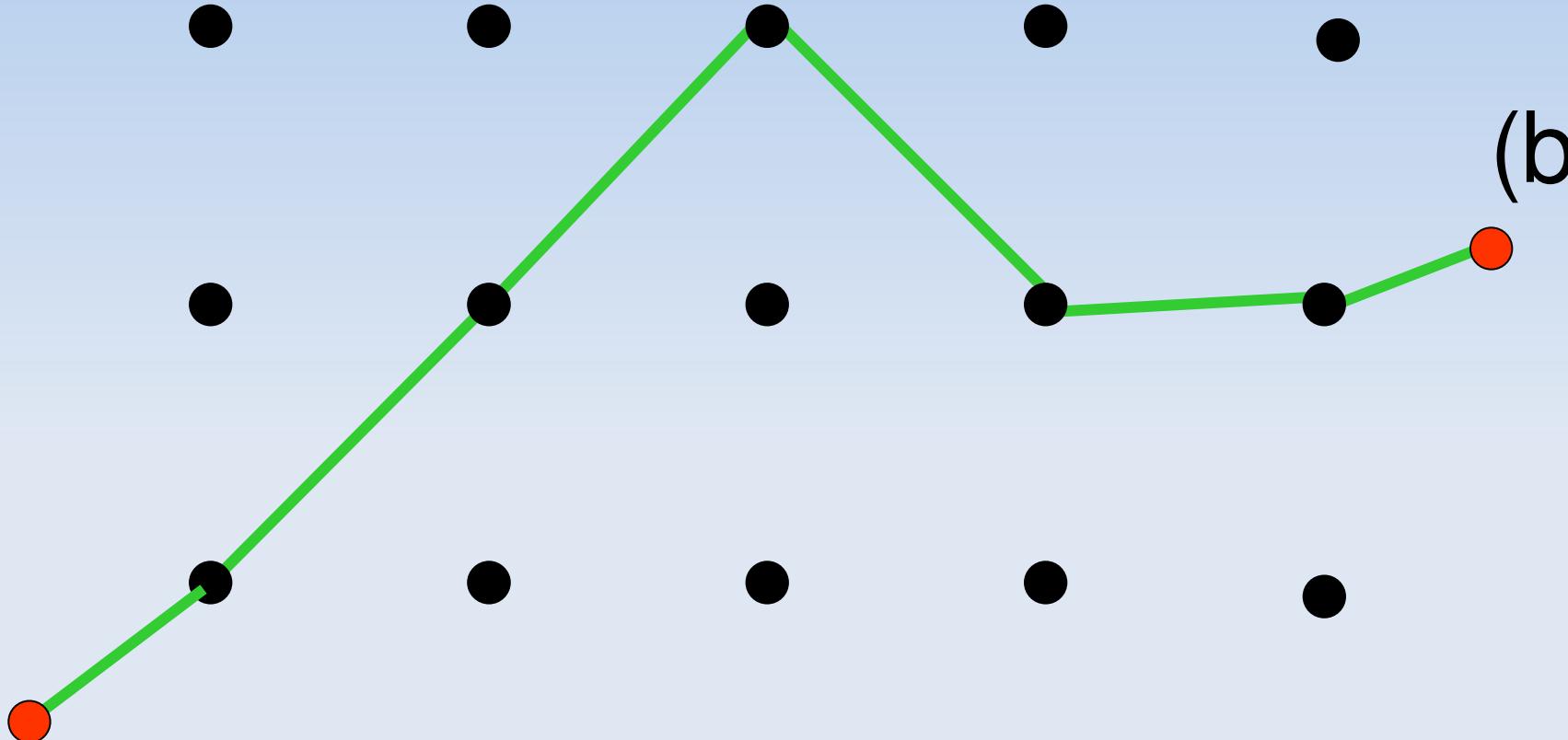
(a)



9

Shortest path...

(a)



10

Shortest path...