

# Investigation of tensor-force effect on exotic nuclei by the HFB method

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# Outline

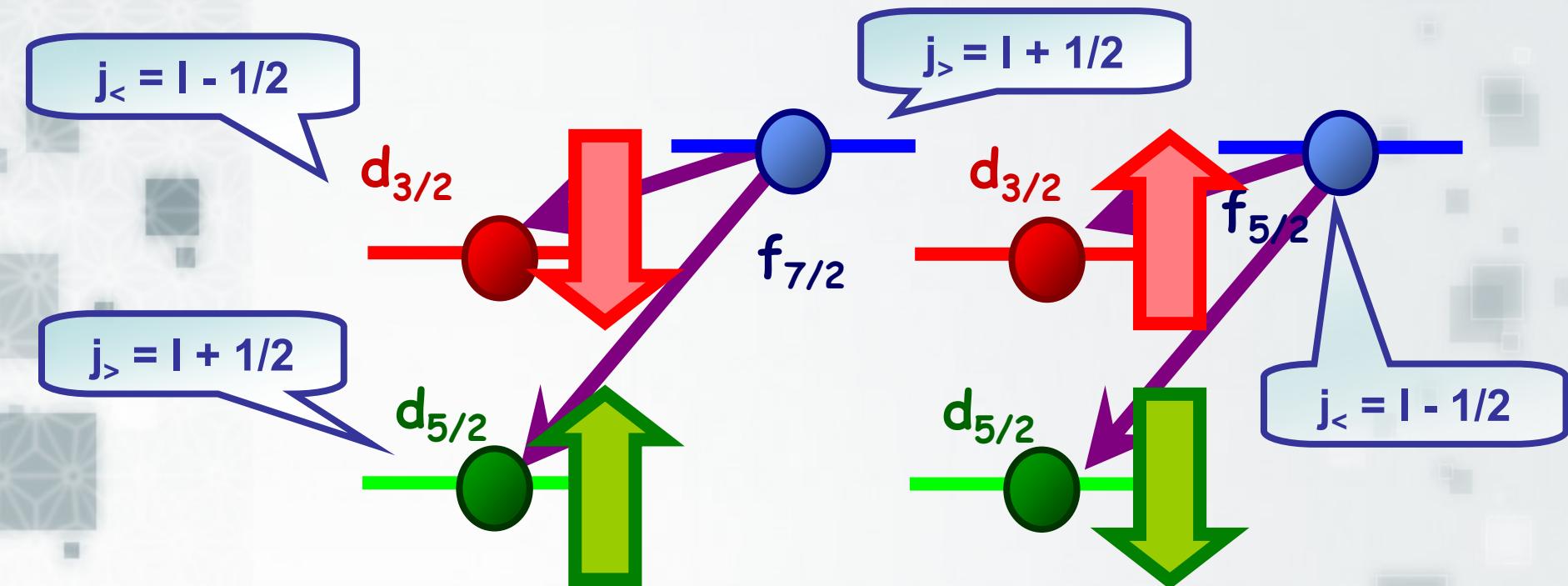
- Motivation
- Method
  - HFB + Gogny
  - How to implement the tensor force
  - How to adjust parameters
- Results and discussion
  - Ni isotopes
  - Sb isotopes
  - Bi isotopes
  - Kr isotopes
- Summary

# Motivation

- We investigate nuclear structures from viewpoint of the tensor force.
- The “tensor force” is a missing part in usual mean-field calculations such as Skyrme and Gogny.
- There are some experimental data which cannot be explained by those usual methods.
  - Sb isotopes: PRL 92, 16 (2004),
  - Ca and Ar: PRL 97, 092501 (2006)
  - $^{17}\text{F}$  and  $^{23}\text{F}$ : PLB 638, 146 (2006)
  - and so on...
- We can see interesting differences between the tensor force and spin-orbit force.
- And we expect that the tensor force plays a crucial role in exotic nuclear region as well as the spin-orbit force.

# Tensor-force effect

$$S_{12} = \frac{3}{r^2} (\sigma_1 \cdot r)(\sigma_2 \cdot r) - (\sigma_1 \cdot \sigma_2)$$



# HFB calculation with steepest gradient method

➤ Spherical symmetry

$$\beta_k^\dagger = \sum_l (c_l^\dagger U_{lk} + c_l V_{lk})$$

$$\beta_k = \sum_l (c_l^\dagger V_{lk}^* + c_l U_{lk}^*)$$

$$\hat{H} = \sum \varepsilon_{\alpha\beta} c_\alpha^\dagger c_\beta + \frac{1}{4} \sum \bar{v}_{\alpha\beta\gamma\delta} c_\alpha^\dagger c_\beta^\dagger c_\delta c_\gamma$$

➤ Basis

- spherical HO basis

➤ Hamiltonian

- Gogny force

➤ Iteration

- steepest gradient method

$$0 = \delta \langle \Phi | (\hat{H} - \lambda_N \hat{N} - \lambda_Z \hat{Z}) | \Phi \rangle$$

$$N = \langle \Phi | \hat{N} | \Phi \rangle, Z = \langle \Phi | \hat{Z} | \Phi \rangle$$

$$\delta E = \langle \Phi' | \hat{H} | \Phi' \rangle - \langle \Phi | \hat{H} | \Phi \rangle$$

$$\begin{aligned} &= \delta Z^* \frac{\partial E}{\partial Z^*} + \delta Z \frac{\partial E}{\partial Z} \\ &= \delta Z^* H^{20} + \delta Z H^{20*} \end{aligned}$$

# Gogny force

J. Decharge and D. Gogny. PRC 21, 4 (1980)

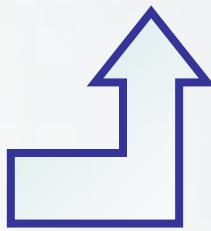
$$\begin{aligned}V_{12} = & \sum_{i=1,2} (W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau) \exp(-r_{12}^2/\mu_i^2) \\& + i W_0 (\sigma_1 + \sigma_2) \cdot \overleftarrow{\mathbf{k}} \times \delta(\mathbf{r}_{12}) \overrightarrow{\mathbf{k}} \\& + t_3 (1 + x_3 P_\sigma) \delta(\mathbf{r}_{12}) \rho^\alpha\end{aligned}$$

- Coulomb force and CM correction.
  - Two Gaussians as central force.
  - Good binding energies and other properties of nuclei
  - Good pairing property (Sn isotopes)
- But...
- No tensor force

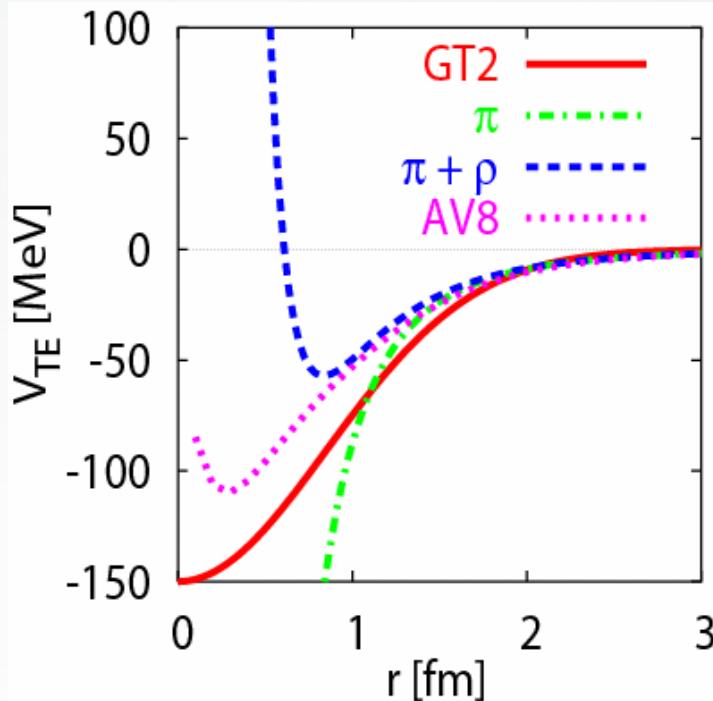
# Gogny + Tensor

T. Otsuka et al., PRL 97, 162501 (2006)

$$\begin{aligned} V_{12} = & \sum_{i=1,2} (W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau) \exp(-r_{12}^2/\mu_i^2) \\ & + i W_0 (\sigma_1 + \sigma_2) \cdot \overleftarrow{\mathbf{k}} \times \delta(\mathbf{r}_{12}) \overrightarrow{\mathbf{k}} \\ & + t_3 (1 + x_3 P_\sigma) \rho^\alpha \delta(\mathbf{r}_{12}) \\ & + (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) V_T^0 \exp(-r_{12}^2/\mu_T^2) S_{12} \end{aligned}$$

- Tensor force. 
- Parameters are readjusted.
- Single Gaussian.

# Radial dependences of the tensor force



Triplet-Even

$$\mu_T = 1.2[\text{fm}]$$

$$V_T^0 = 50.8[\text{MeV}]$$

- Similar dependence between GT2 and AV8'.
- Short-range dependence of GT2 is somehow different from others.
- This difference is irrelevant.
- Because the tensor force cannot affect on s-wave states.

# Parameters

- **D1S**
  - J. F. Berger, et al., Nucl. Phys. A428, 23c (1984)
- **GT1,2 (our earlier study)**
  - T. Matsuo, Ph.D. Thesis, University of Tokyo (2004)
  - T. Otsuka et al., PRL 97, 162501 (2006)
- **GT3**
  - New parameter set

$W_1, B_1, H_1, M_1$   
 $W_2, B_2, H_2, M_2$   
 $t_3, x_3, \alpha$   
 $W_0$

- Nuclear Matter properties
- Binding energies of some nuclei
- Pairing property

# Nuclear matter properties

$$k_F \simeq 1.34 \text{ [fm}^{-1}]$$

$$-E/A \simeq 16.0 \text{ [MeV]}$$

$$\kappa = k_F^2 \frac{\partial^2(E/A)}{\partial k_f^2}$$

$$\frac{k_F}{M^*} = \frac{\partial \varepsilon}{\partial k_F}$$

$$a_s = \frac{1}{2} \frac{\partial^2(E/A)}{\partial^2 \eta_s}$$

$$a_t = \frac{1}{2} \frac{\partial^2(E/A)}{\partial^2 \eta_t}$$

$$a_{st} = \frac{1}{2} \frac{\partial^2(E/A)}{\partial^2 \eta_{st}}$$

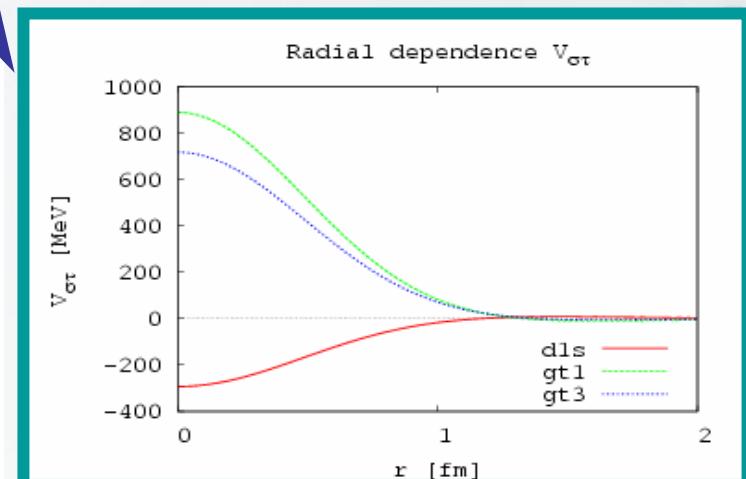
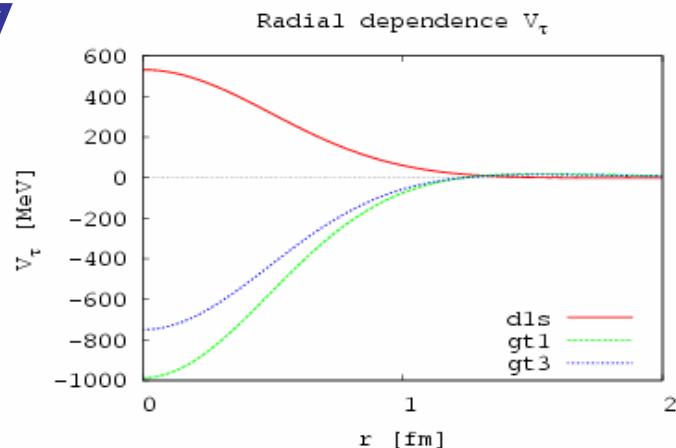
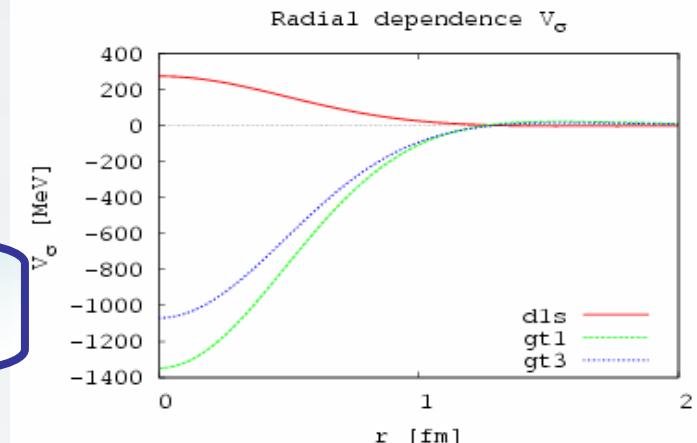
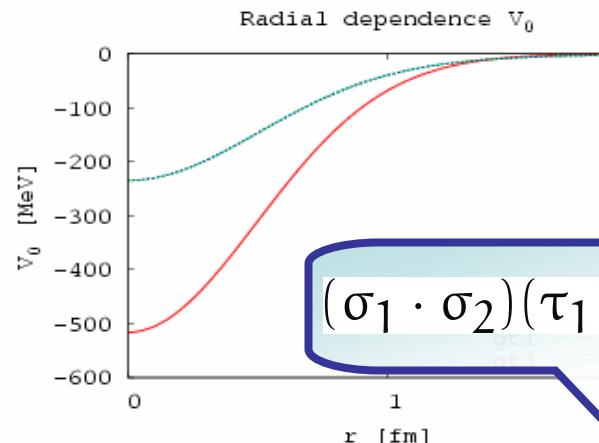
	D1S	GT1	GT2	GT3
$k_F$	1.342		1.336	
$-E/A$	16.01		16.02	
$\kappa$	202.8		228.1	
$M^*/M$	0.697		0.672	
$a_s$	26.18	35.19	38.40	24.31
$a_t$	31.13	31.29	33.94	28.60
$a_{st}$	29.12	26.64	24.69	31.17

# Radial dependences of central force

D1S

GT1

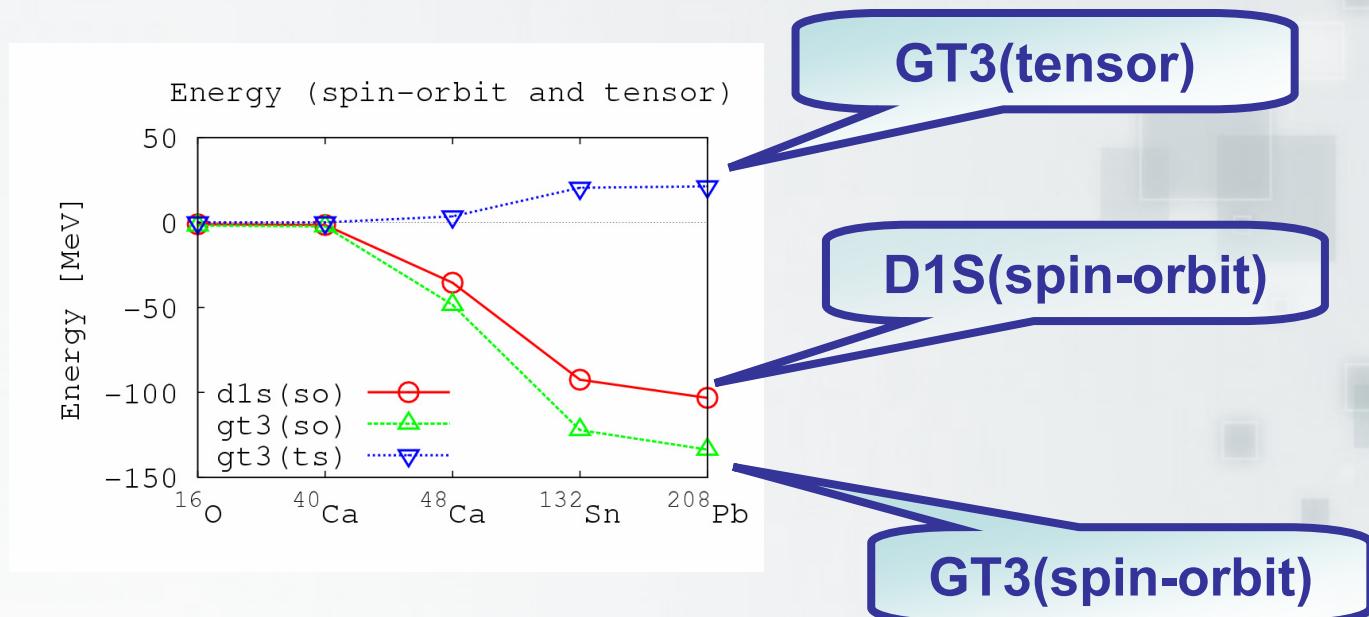
GT3



# Neutron $p_{1/2} - p_{3/2}$ splitting of $^{16}\text{O}$

- Spin-orbit force mainly contributes to this splitting
- No contribution of the tensor force
- D1S       $W_0 \simeq 130 \rightarrow 6 \text{ [MeV]}$
- GT3       $W_0 \simeq 160 \rightarrow 8 \text{ [MeV]}$ 
  - Larger gap
  - $^{28}\text{O}$  can be bound using  $W = 130$ , but cannot be bound using  $W = 160$

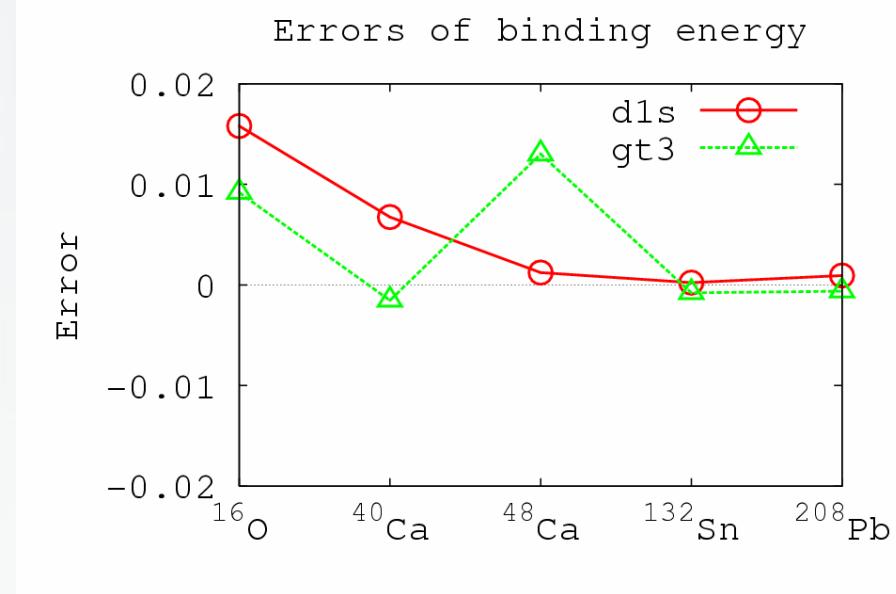
# Contributions to binding energies



- $D1S(\text{spin-orbit}) =$   
 $GT3(\text{spin-orbit}) + GT3(\text{tensor})$
- Contribution
  - spin-orbit force: more bound
  - tensor force: less bound

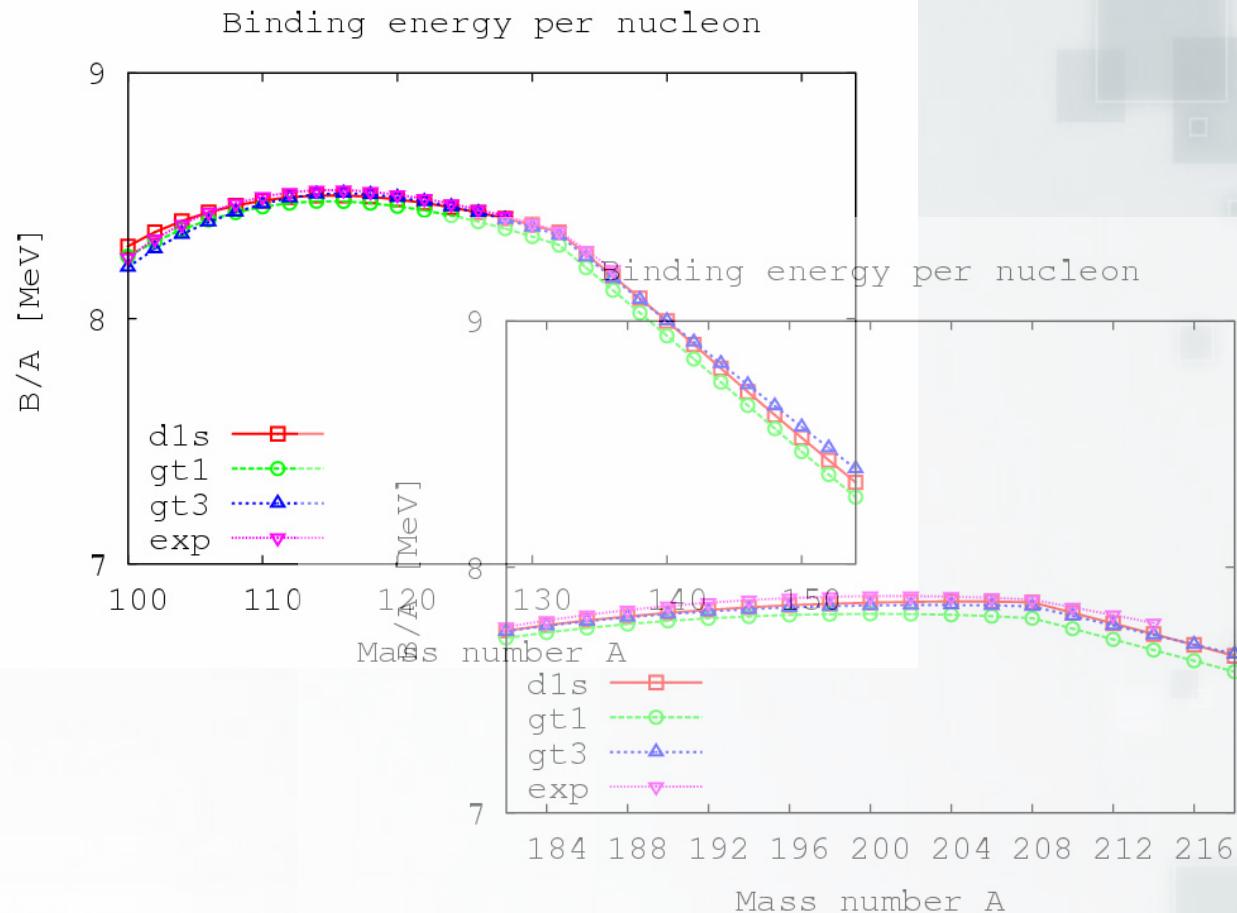
# Binding energy

$$\text{Error} = \frac{\text{BE}_{\text{calc}} - \text{BE}_{\text{exp}}}{\text{BE}_{\text{exp}}}$$



- B.E. errors
  - $^{16}\text{O}, ^{40}\text{Ca}, ^{48}\text{Ca}$  (bad result),  $^{132}\text{Sn}, ^{208}\text{Pb}$
- Fine tuning will be necessary

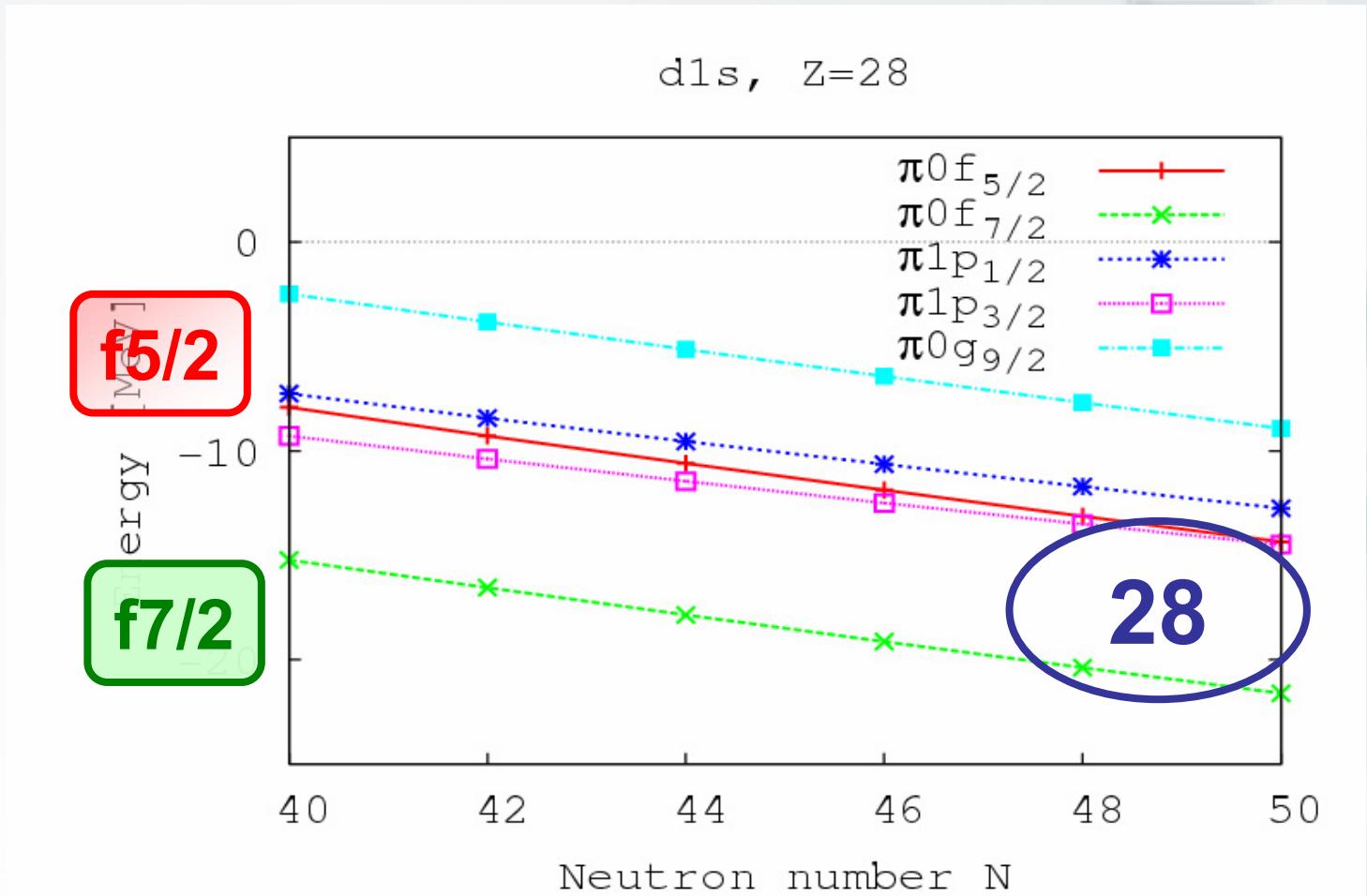
# B.E. of Sn and Pb isotopes



# Results

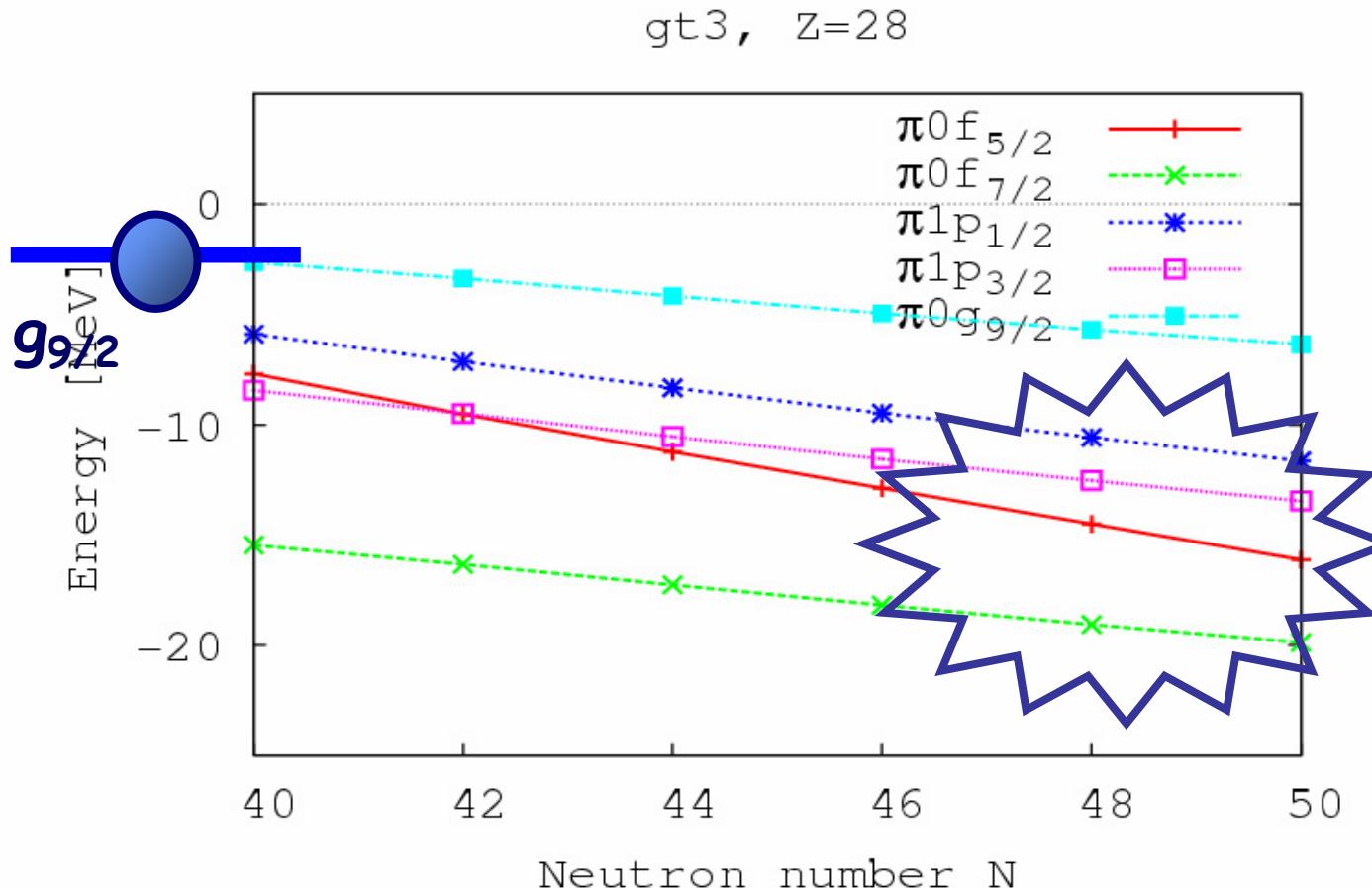
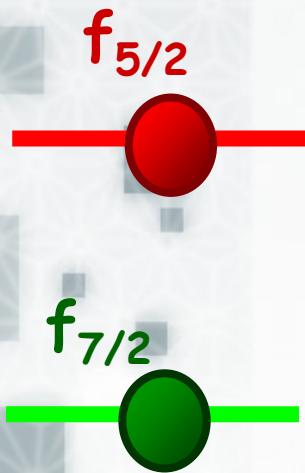
- Ni isotopes
  - Magic number Z=28
- Sb isotopes
  - h11/2 and g7/2 gap
- Bi isotopes
  - i13/2 and h9/2 gap
- Kr isotopes
  - two-neutron separation energy

# Proton single-particle energies of Ni isotopes, D1S



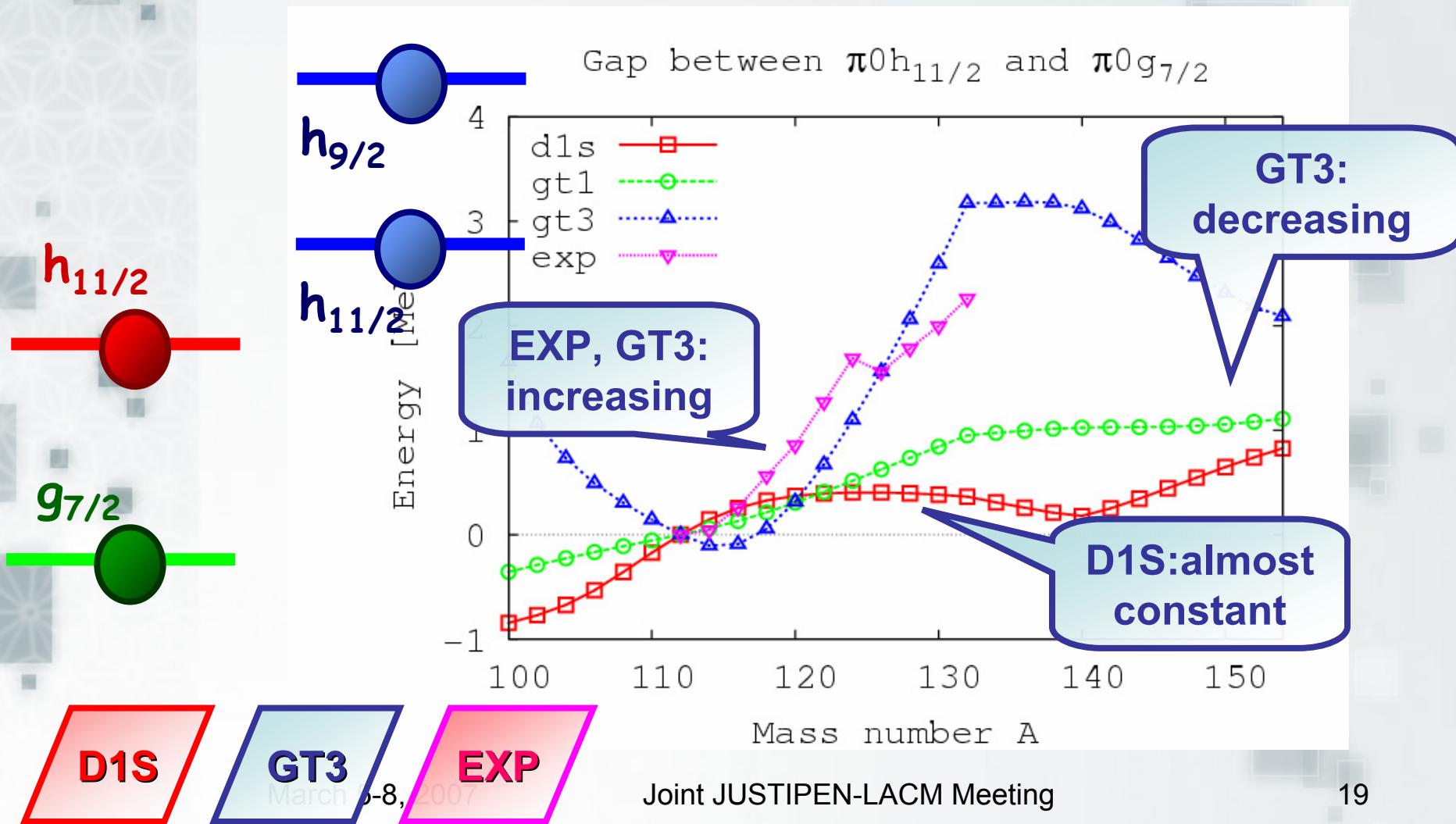
# Proton single-particle energies of Ni isotopes, GT3

HF calc :T. Otsuka et al., PRL 97, 162501 (2006)

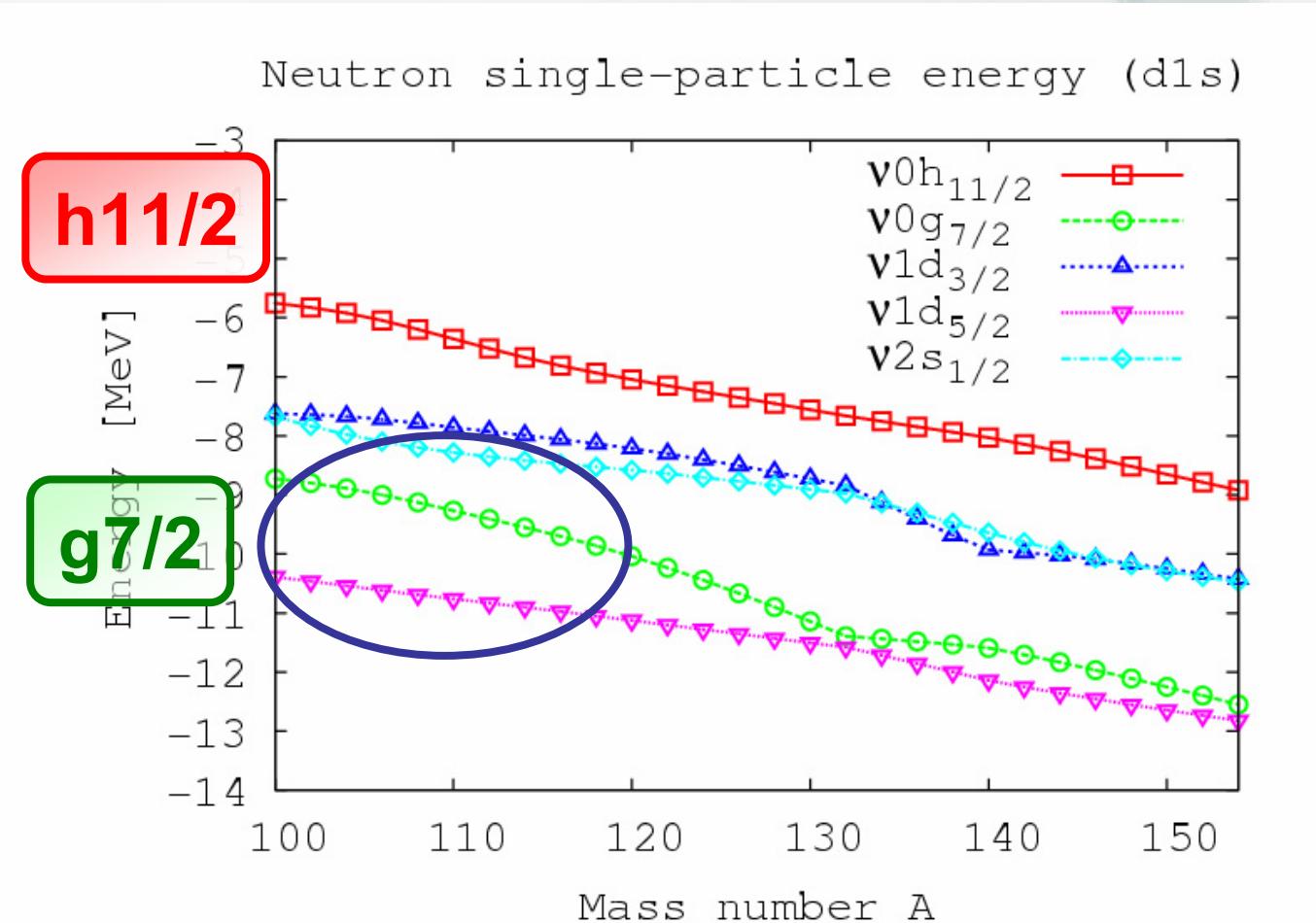


# Sb isotopes

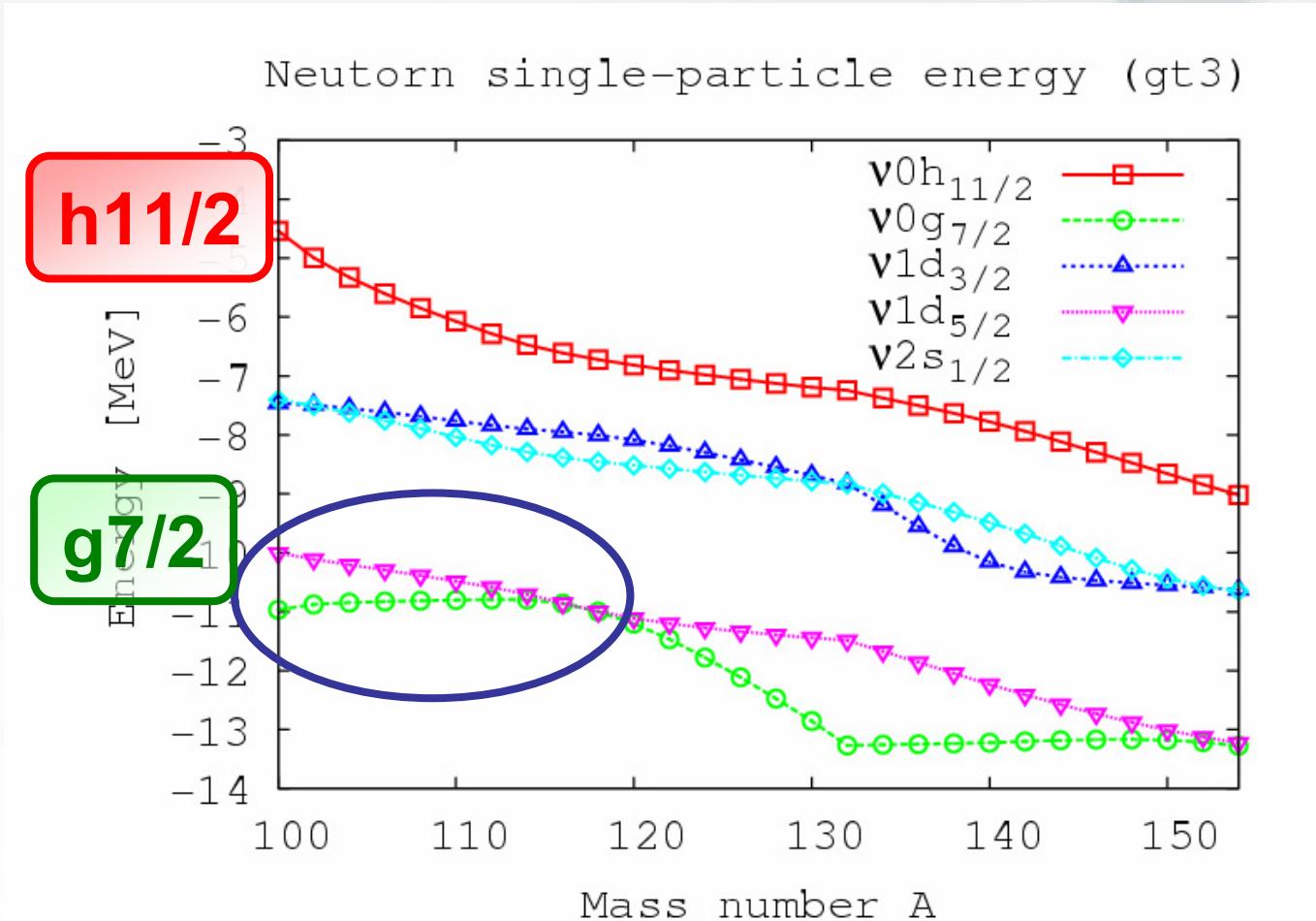
Exp: Schiffer et al., PRL 92, 16 (2004)



# Neutron single-particle energies, D1S, Z=50

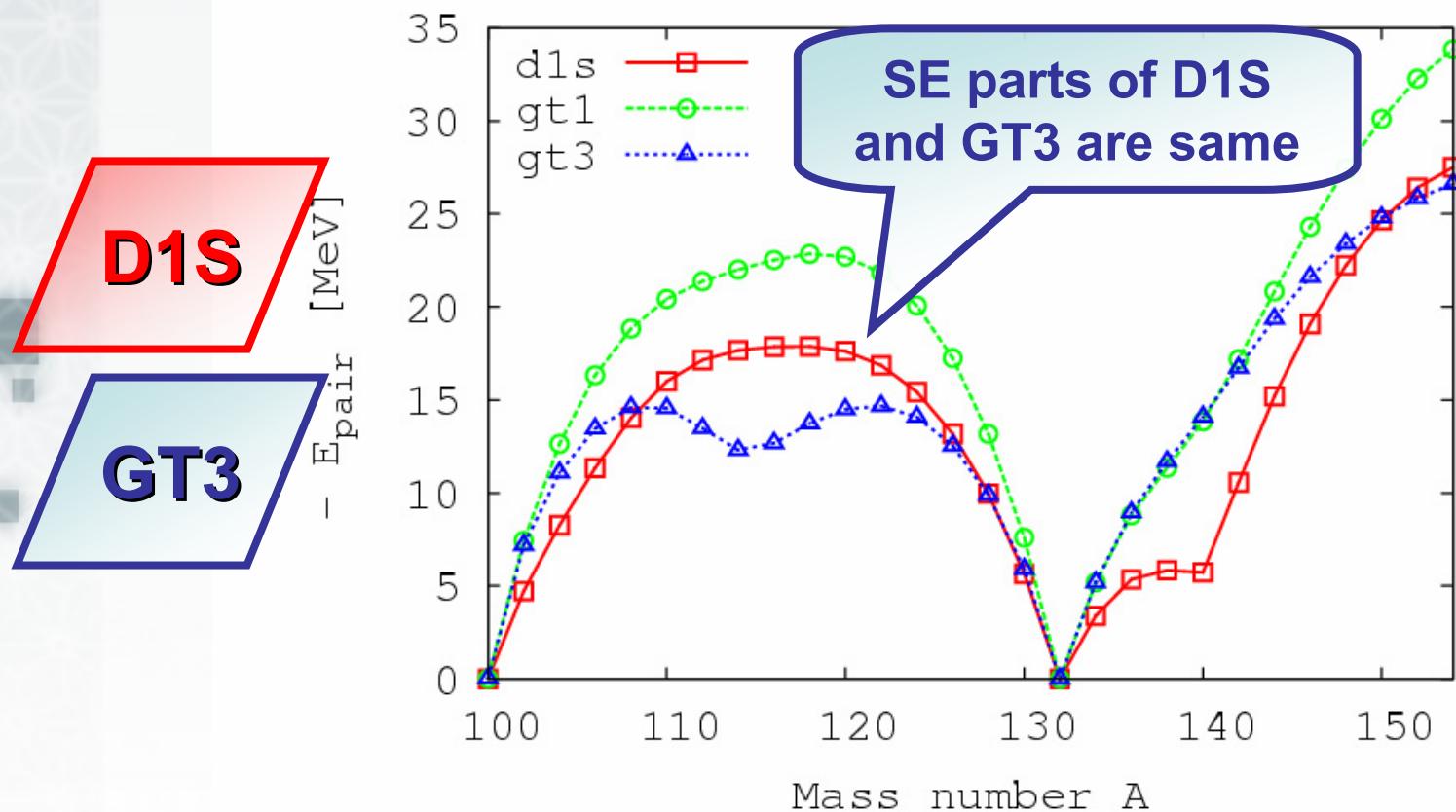


# Neutron single-particle energies, GT3, Z=50



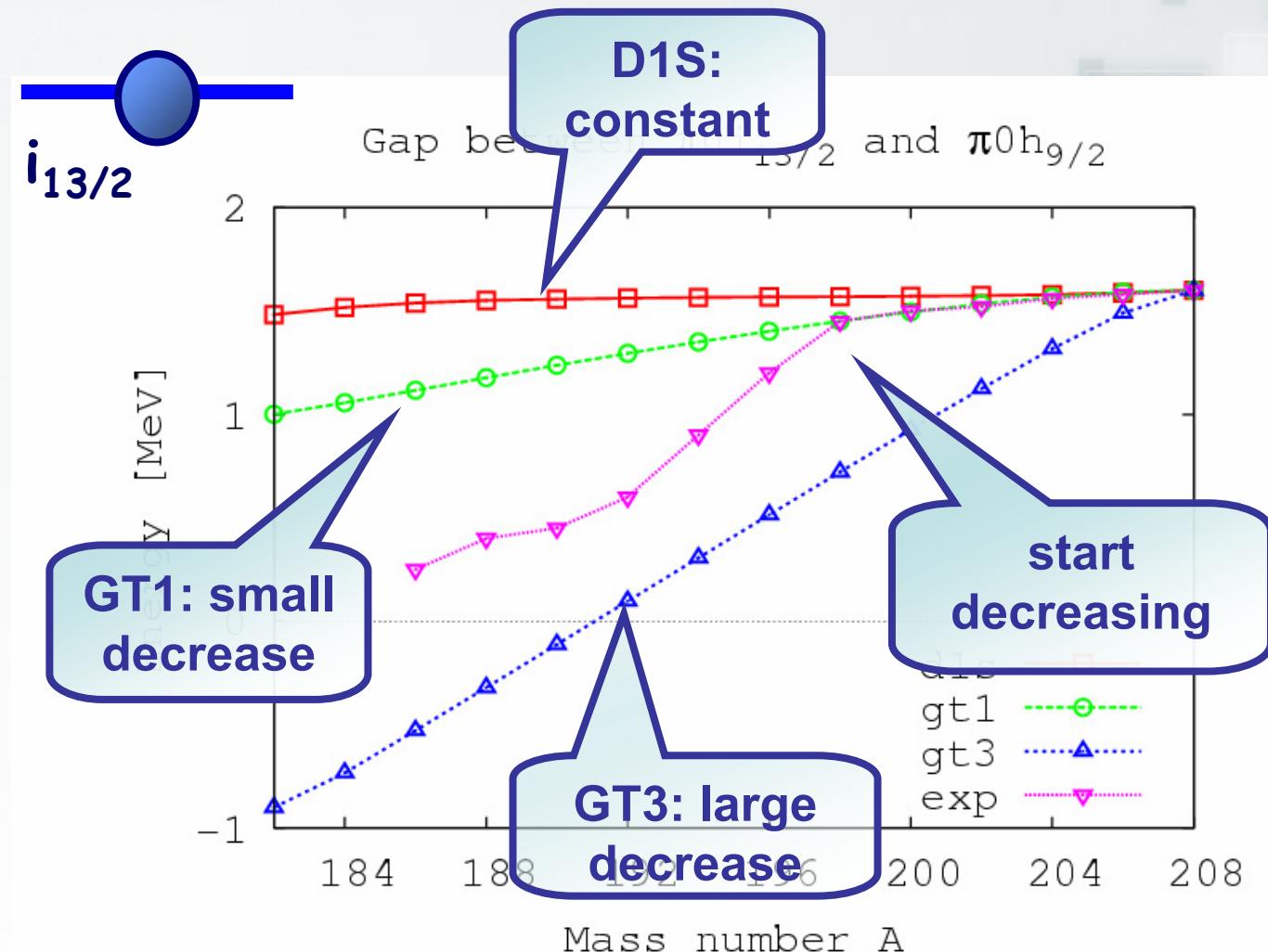
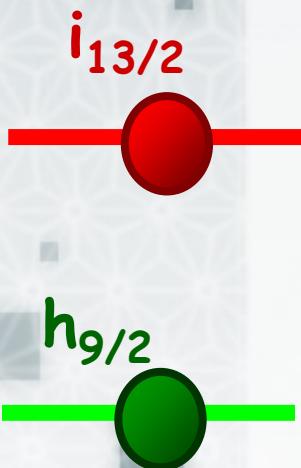
# Pairing property ( $\text{Sn}$ , $Z=50$ )

$$E_{\text{pair}} = -\frac{1}{2} \text{Tr} (\Delta \kappa^*)$$

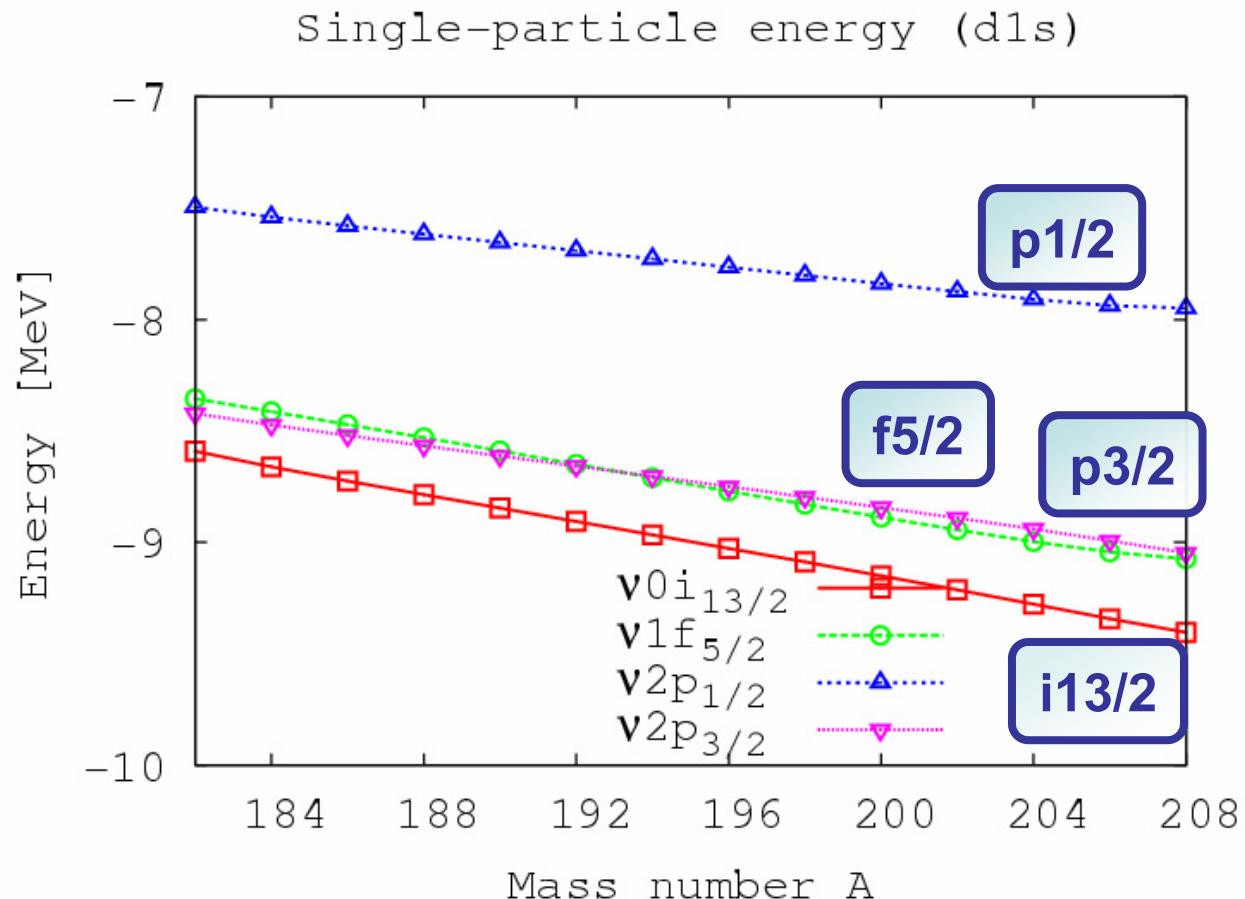


# Bi isotopes

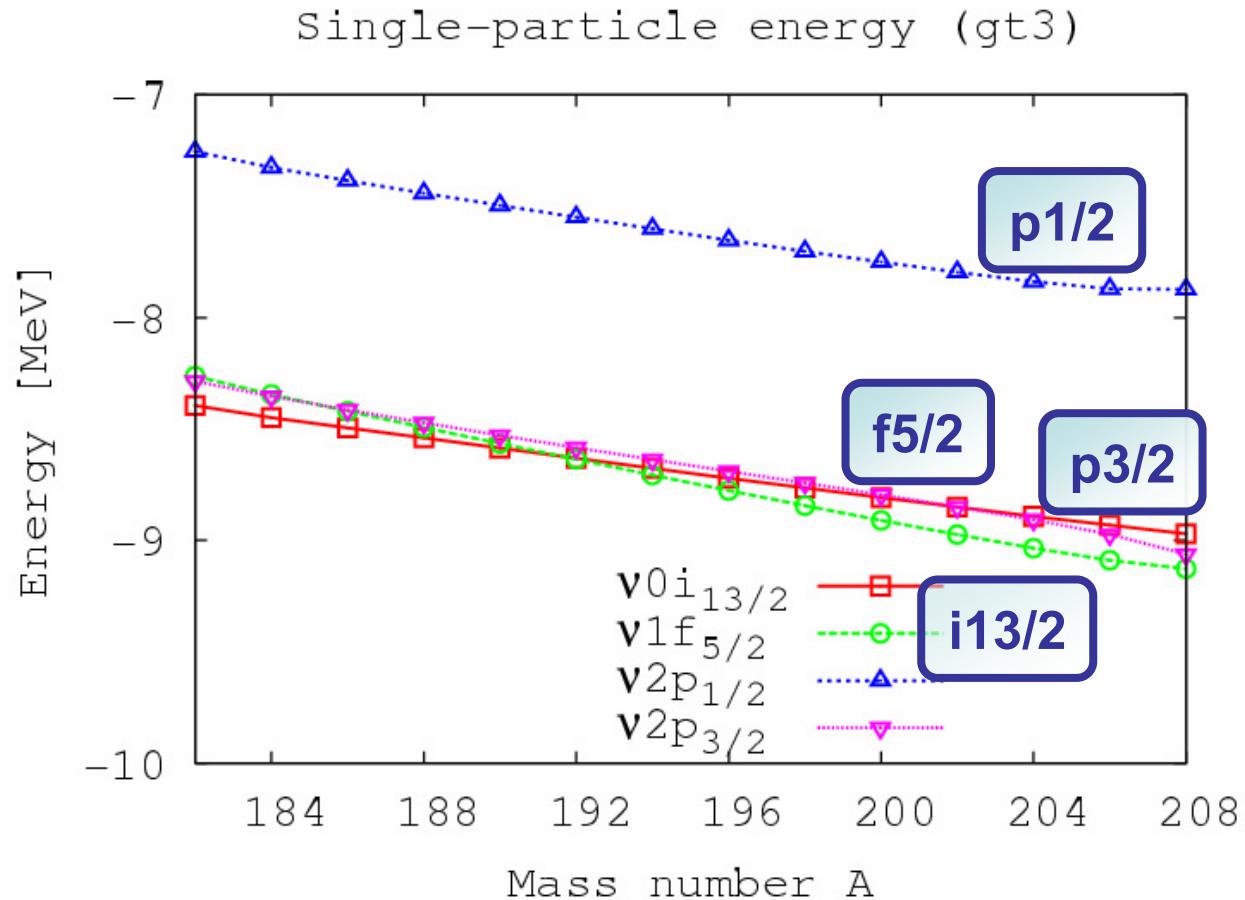
Exp: Eur. Phys. J 15, 329 (2002)



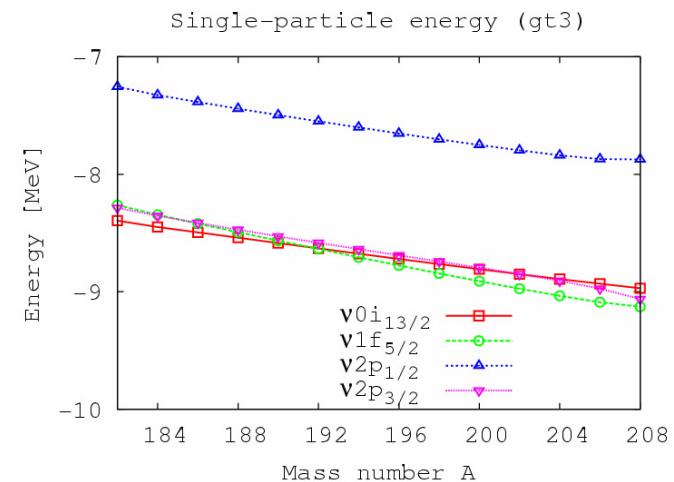
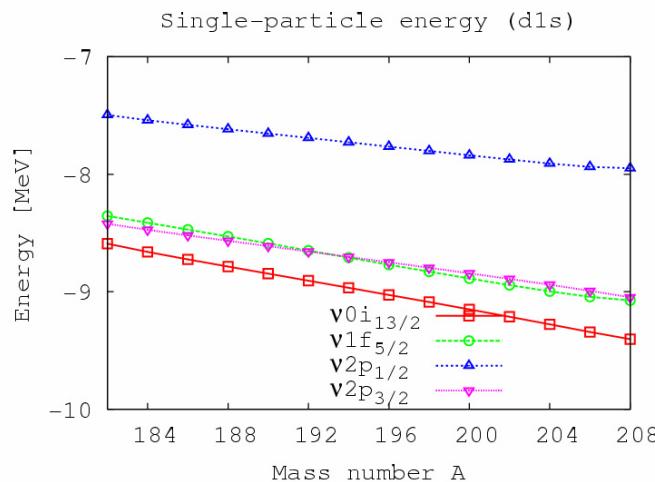
# Neutron single-particle energy, D1S, Z=82



# Neutron single-particle energy, GT3, Z=82



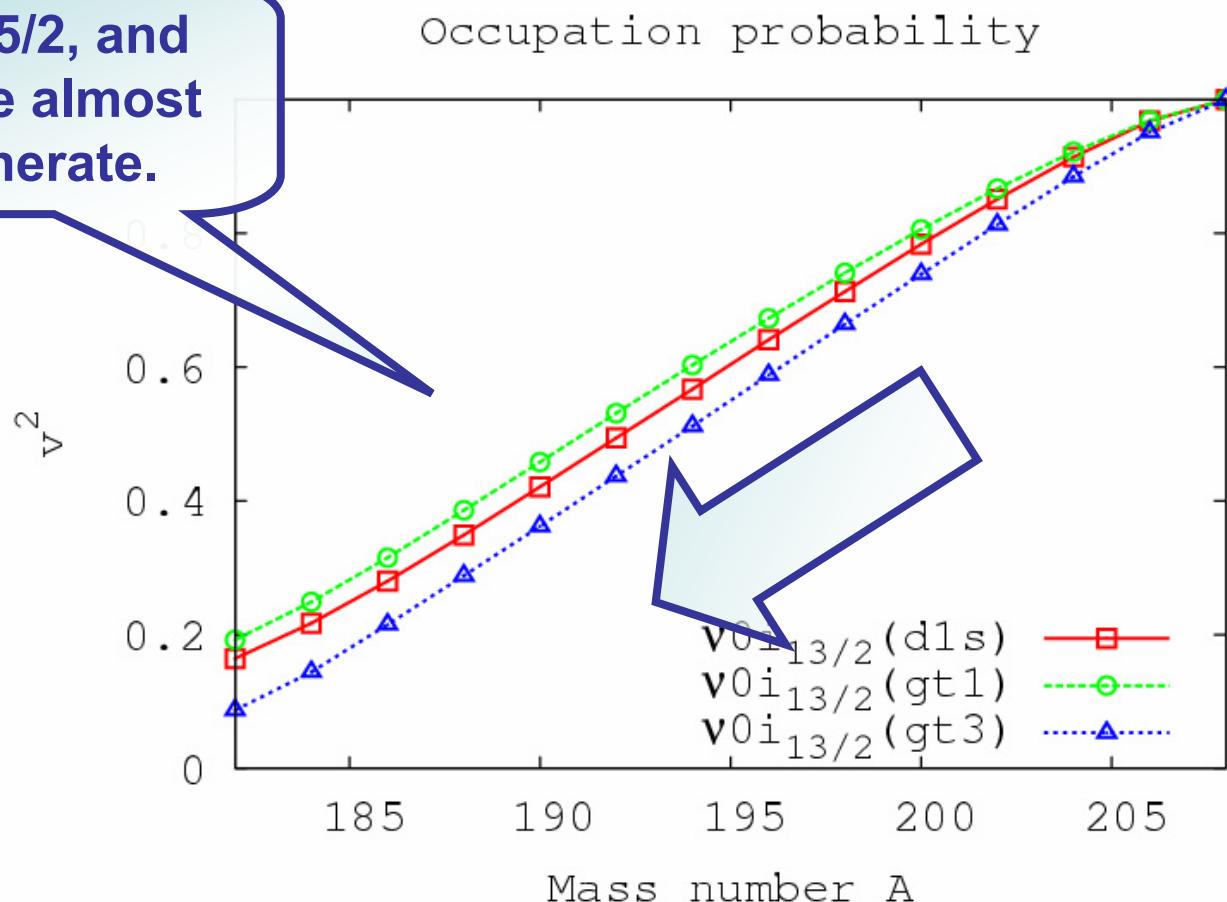
# Single-particle energy, Z=82



- i13/2, 1f5/2, and 2p3/2 are almost degenerate.
- This degeneracy produces inappropriate results of pairing property

# Occupation probability, Z=82

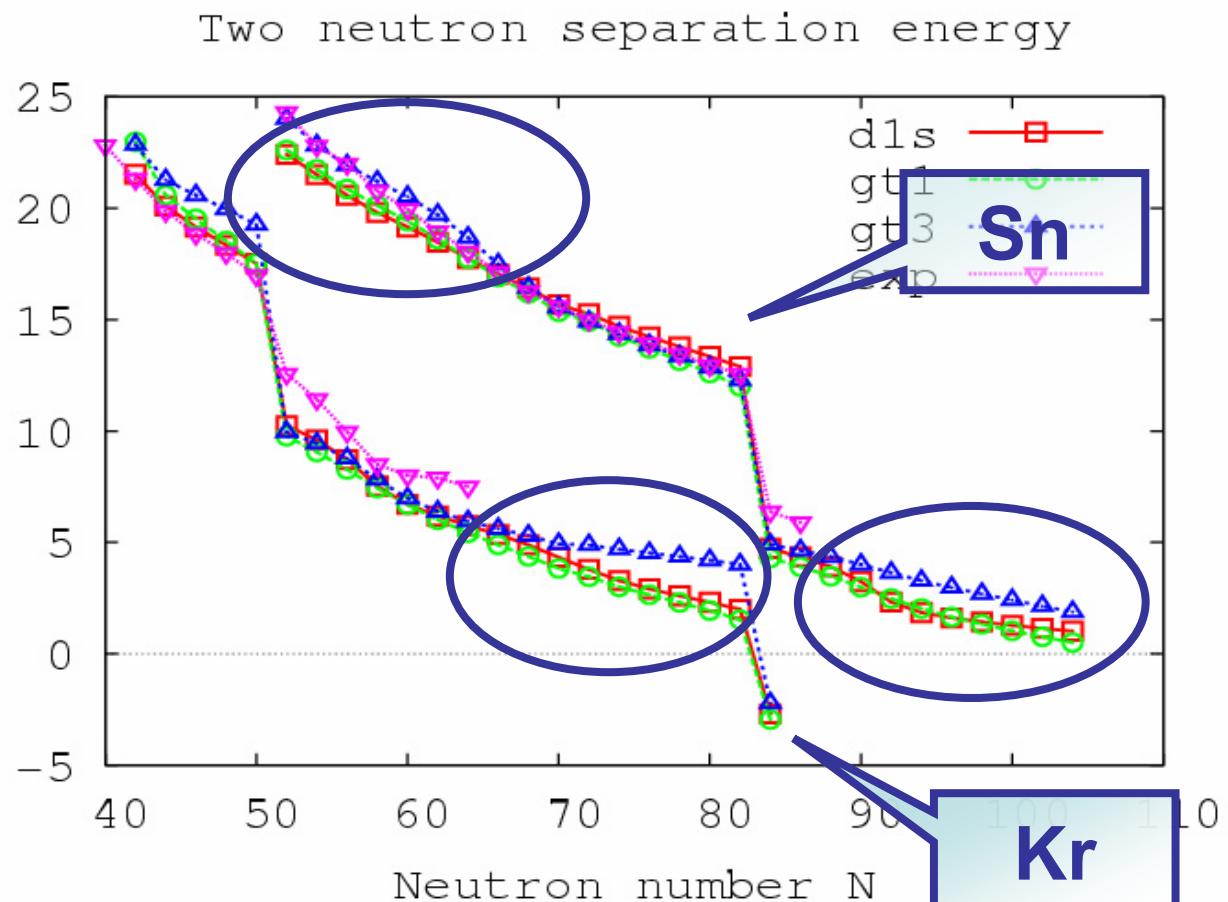
i<sub>13/2</sub>, f<sub>5/2</sub>, and p<sub>3/2</sub> are almost degenerate.



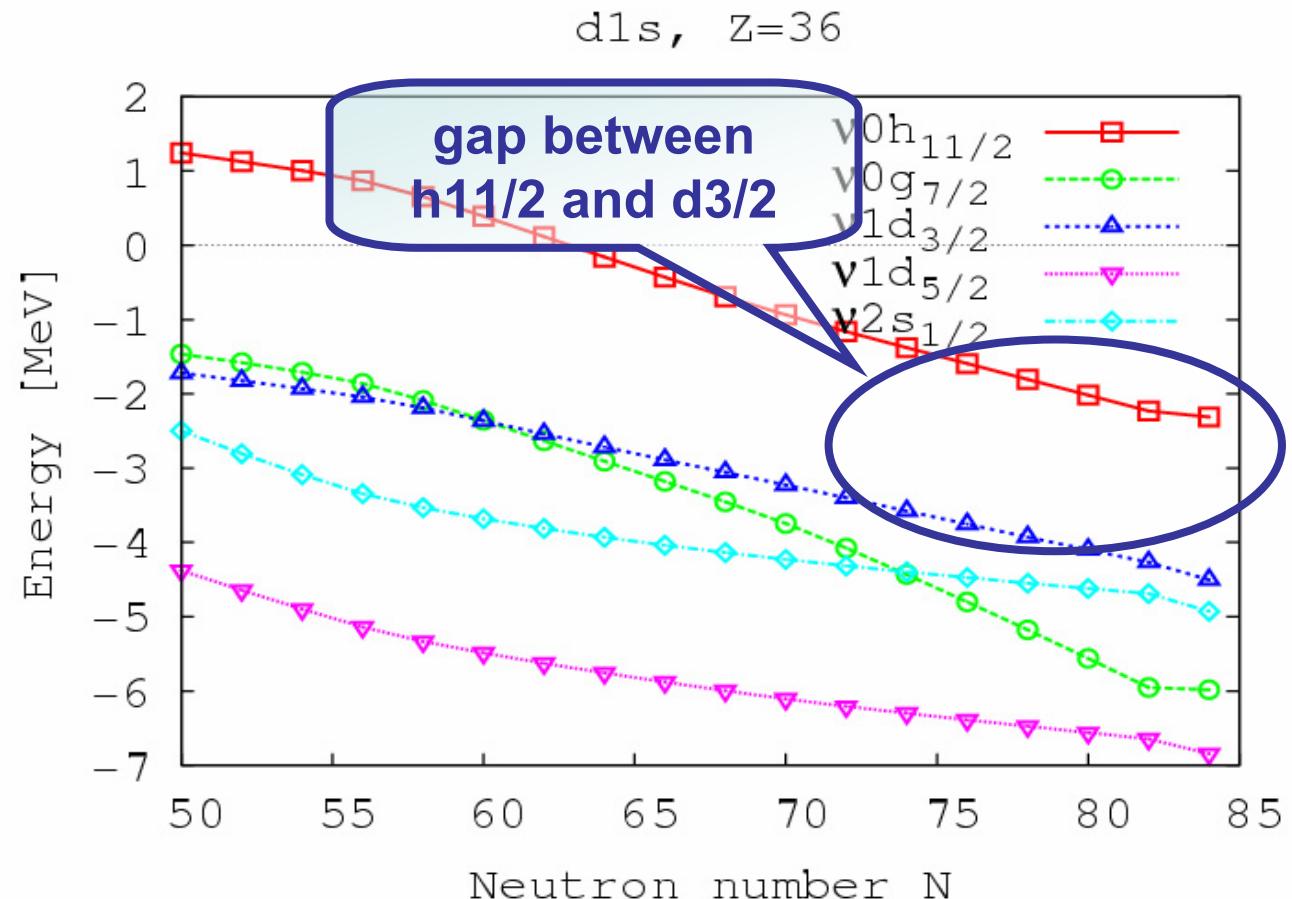
# Two neutron separation energy

Exp: Audi et al., NPA 595, 409 (1995)

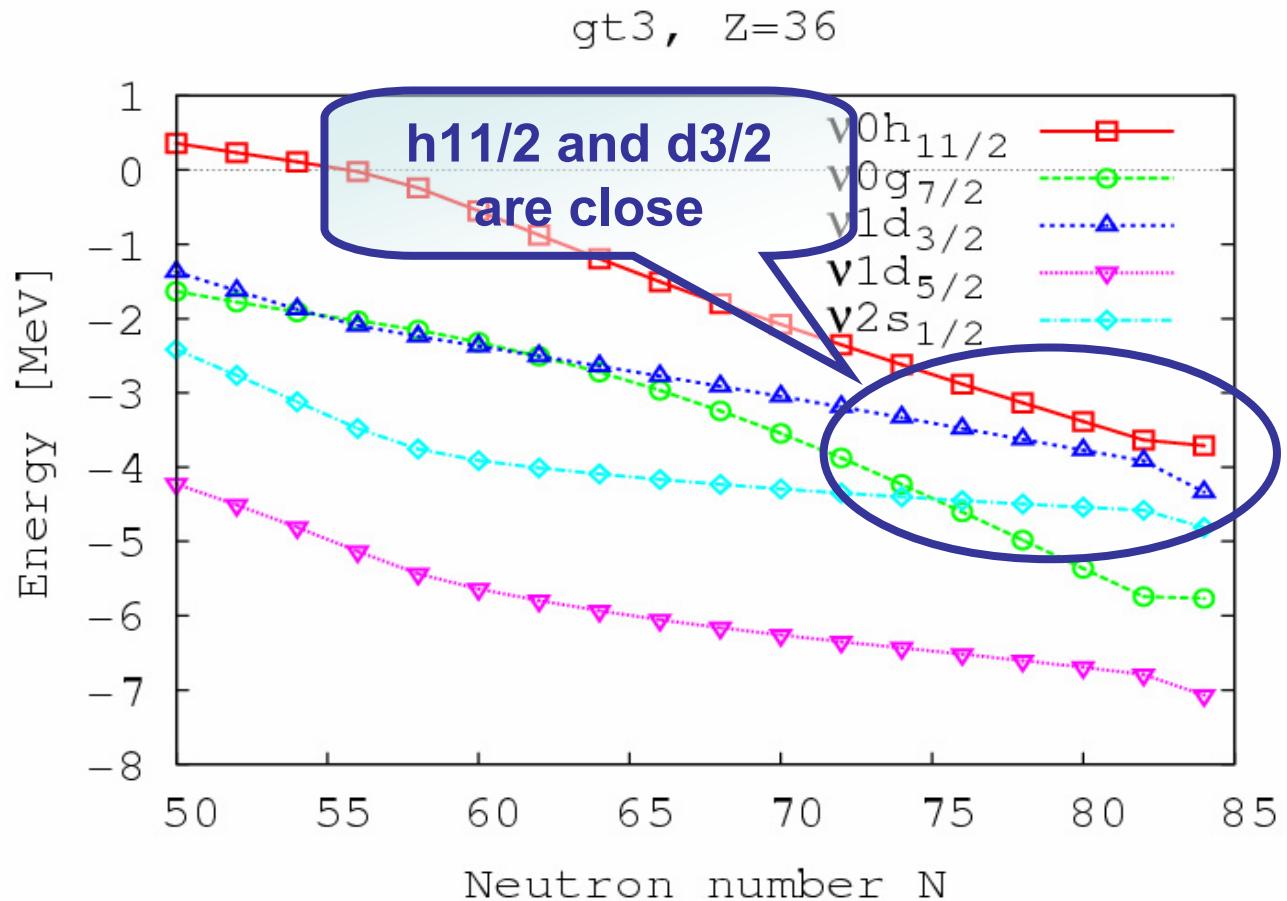
D1S  
GT1  
GT3  
EXP



# D1S, Kr isotopes



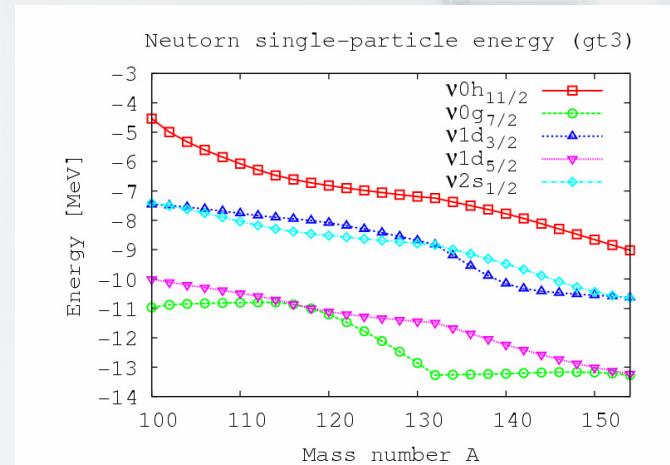
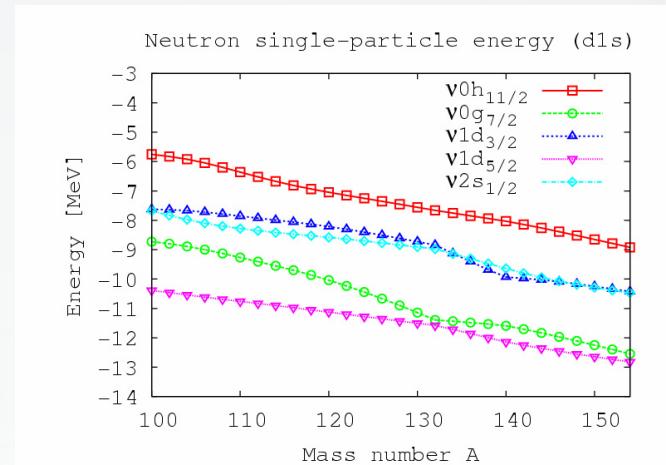
# GT3, Kr isotopes



# Summary

- We modified our new parameters including tensor force, considering appropriate pairing property of Sn isotopes.
- We can see influence of the tensor force on the pairing property of Sn isotopes.
- We re-examined the gap behavior of Ni and Sb isotopes using new parameters, GT3.
- In Bi isotopes, we can obtain similar trend as the experimental data, but we found some problems on single-particle levels.

# Single-particle energies, Z=50



- Changing shell gaps.
- GT3 produces larger gap than D1S.
- Tensor-force effect can be seen.
  - Gap between h<sub>11/2</sub> and g<sub>7/2</sub>