A partial paper as an example for ISOL01 proceedings

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Using random two-body interactions between valence space nucleons, we calculate various statistical measures of the many-body wave functions obtained from diagonalization. These measures include the entropy, spectral rigidity, inverse participation ratio, and related quantities. We also classify the interactions according to their mean-field characteristics by calculating the Hartree-Fock-Bogoloubov solution for selected random interactions and comparing to the spectra generated by shell model diagonalization.

I. MAIN BODY OF THE PAPER SHOULD BEGIN HERE

Many researchers have noticed that small changes in particular matrix elements improve upon the results generated from effective interaction theory and alleviate failures. For example, in *sd-pf* shell model calculations changing the difference between only two matrix elements completely changes the character of the monopole single-particle energy difference between the $d_{5/2}$ and $f_{7/2}$ [1].

This paper is organized as follows. We discuss in Section II the general form of the two-body random interactions employed in these calculations. We characterize the four random interactions using statistical measures in Section III. A correlation is shown between the various statistical pairing strengths in these interactions and the statistical measures. In Section IV we describe the intrinsic HFB structures that appear to be generic for $\mathcal{J} = 0$ and maximally aligned ground state configureations.

II. STATISTICAL STUDIES

In order to study the statistical nature of the random interactions we employ four basic ensembles that may be specified by the $c_{J,T}$ coefficients and the single-particle Hamiltonian, if present. The first of these we describe as the Random Quasiparticle Ensemble (RQE). In this case $c_{J,T} = [(2T+1)(2J+1)]^{-1}$. This relation between the $c_{J,T}$, which was discussed in [2], came from imposing on the ensemble the constraint that it should remain the same for the particle-particle interaction as for the particle-hole interaction. Our second ensemble is the two-body random ensemble (TBRE) for which c_{JT} = constant. Historically, this was the first two-particle random ensemble to be employed in studying statistical properties of many-particle spectra [3]. These two ensembles assume degenerate single particle energies. Realistic interactions do have nonzero single-particle energies and these will in principle affect various spectral properties.

Our interactions preserve rotational and isospin invariance as well as particle number conservation. We use the typical shell-model basis for calculations. The single particle states of the shell model are oscillator states classified by the quantum numbers $\{nljmt_z\}$ for the principle quantum number, the orbital angular momentum, the total momentum and its projection and the isospin projection. The hamiltonian is given by

$$H = \sum_{\alpha} \varepsilon_\alpha a^\dagger_\alpha a_\alpha + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} \sum_{JT} V^A_{JT}(\alpha\beta,\gamma\delta) \sum_{MT_z} A^\dagger_{JT;MT_z}(ab) A_{JT;MT_z}(cd) \; , \tag{1}$$

where the pair operator is

$$A^\dagger_{JT;MT_z}(ab) = \sum_{m_a m_b, t_a t_b} (j_a m_a j_b m_b \mid JM) \left(\frac{1}{2} t_a \frac{1}{2} t_b \mid TT_z \right) a^\dagger_{j_b m_b t_b} a^\dagger_{j_a m_a t_a} \; . \tag{2}$$

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*new address: the outer limits

TABLE I. Percentage of ground states with a given spin in the ^{24}Mg. The maximum spin obtainable by ^{24}Mg in this model space is $J_{\max} = 12$. We consider the two of the random ensembles discussed in this paper.

Spin	RQE	TBRE
$J = 0$	703	548
$J = 1$	39	30
$J = 2$	164	135
$J = 3$	12	20
$J = 4$	38	85
$J = 5$	4	3
$J = 6$	8	21

A. Subsection headers look like this

We diagonalized 1000 random interactions generated from the ensembles described in the previous section. In this paper we concentrate on the system ^{24}Mg which is comprised of 4 neutrons and 4 protons in the $1s$-$0d$ shell model space. Thus, the maximum spin state that we study in these systems is $\mathcal{J} = 12$.

In Table I, we indicate the relative abundances of J spin of ground states that are found in the four ensembles for the ^{24}Mg and ^{20}Ne systems. In each case $\mathcal{J} = 0$ dominates.

Once the Hamiltonian is diagonalized, a given state is a superposition of the (normalized) many-body basis states

$$\phi^{\alpha} = \sum_k A_k^{\alpha} \mid k\rangle = \sum_k \langle k \mid \alpha\rangle \mid k\rangle \ , \tag{3}$$

where the coefficients of expansion obey the relation $\sum_k \mid A_k^{\alpha} \mid^2 = 1$ and the sum includes all many-body basis states D. We define the entropy within this basis as

$$S = -\sum_k W_k^{\alpha} \mathrm{ln} \mathrm{W}_{\mathrm{k}}^{\alpha} \ , \tag{4}$$

where $W_k^{\alpha} = \mid A_k^{\alpha} \mid^2$ are the overlap intensities. Thus, $S = 0$ if only one many-body basis state contributes to the sum (i.e. for a particular k we have $W_k^{\alpha} = 1$), whereas if all states equally contribute we obtain $S = \mathrm{lnD}$. Typically instead of plotting S directly, we discuss for a given eigenstate of the Hamiltonian $P(\alpha) = \exp(S^{\alpha})$. For the two extreme cases $P(\alpha) = \exp(S^{\alpha})$. For the two extreme cases $P(\alpha) = 1$ when the wave function is equivalent to a single basis state, and $P(\alpha) = D$ when all basis states equally contribute to the sum.

We now limit our discussion to those interactions that give $\mathcal{J} = 0$ ground states. We show in Fig. 1a the entropy for the first 150 $\mathcal{J} = 0^+, \mathcal{T} = 0$ wavefunctions for the four ensembles. No statistical difference exists among the four ensembles. We also show the entropy of the first 150 0^+ states in the USD interaction. Since there is no ensemble average, the line is much less smooth, but the general agreement between the USD and the random interactions appears obvious. None of these interactions reaches the GOE limit which is given by $0.48D = 8415$ in this case.

We also use the inverse participation ratio (IPR) to quantify the random ensembles. The IPR is given by

$$IPR_{\alpha} = D \sum_{k=1}^{D} (W_k^{\alpha})^2 \ , \tag{5}$$

and measures the inverse fraction of Fock states that participate in forming the full wavefunction $\mid \alpha\rangle$. This measure emphasises the contribution of the large components of the wave function. The extreme cases correspond to $IPR_{\alpha} = 1$ when all Fock states equally contribute to the wave function $\mid \alpha\langle$, while $IPR_{\alpha} = D$ when only one Fock state contributes. As point of reference, we calculated the IPR for the first $\mathcal{J} = 0^+$ state using the USD interaction and find $\mathrm{IPR}_1 = 64.18$ and $\mathrm{IPR}_1 = 13.48$ with and without single particle splitting, respectively. As one expects, the single-particle splitting enhances the $d_{5/2}$ occupation and hence acts as a filter for choosing many-body basis states. We show in Fig. 1b the IPR for the first 150 0^+ states. The inset shows the same quantitiy for the first 5 states. We see no statistically significant difference in the character of the random ensembles beyond the first ($\alpha = 1$) state. The inset shows in more detail the first state. We note that both the RQE-SPE and TBRE-SPE show enhanced IPR's compared to the RQE and TBRE. This property is somewhat different than we anticipated from our findings using the realistic two-body interaction.

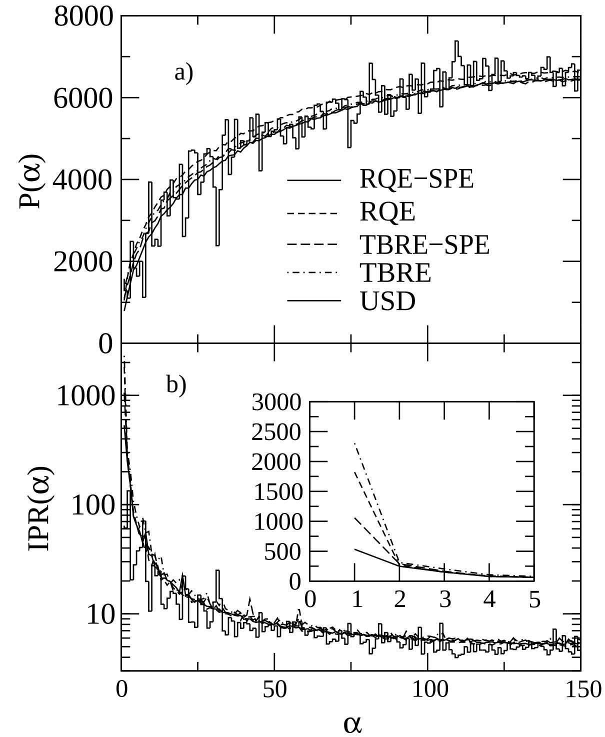

FIG. 1. Shown are two statistical quantities that describe the ensemble structure for the various random interactions in the $N = Z = 4$ system. A comparison to the USD interaction is also plotted. Shown are a) the entropy as a function of state number; and b) the inverse participation ratio (IPR). The inset to b) shows the IPR for the first five states.

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